Grade 8
End of Year Teacher Pack

Includes:

- Lesson Plans
- Blackline Masters
- Assessment & Practice Book Answers
- Quizzes and Unit Tests
- Rubrics for Scoring Quizzes and Tests
- Common Core State Standards Correlations
Unit 4 Geometry: Pythagorean Theorem

This Unit in Context
In 8.1 Unit 3, students learned that congruent triangles have the same shape and size. Corresponding sides of congruent triangles are equal, as are the measurements of the corresponding angles. Students learned rules (SSS, SAS, and ASA) to help determine whether two triangles are congruent.

In 8.2 Unit 2, students learned about transformations, including translations, reflections, rotations, and dilations. They learned how transformations can be used to show that triangles are congruent (8.G.A.2), how dilations lead to similar triangles (8.G.A.4), and how similar triangles can be used to show that the slope of a linear function remains constant.

In this unit, students will learn a proof called the Pythagorean Theorem (8.G.B.6). This theorem shows how to calculate the length of the longest side in a right triangle, given the measurements of the other two sides. They will apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real world and mathematical problems of two and three dimensions (8.G.B.7). They will use the Pythagorean Theorem to calculate the lengths of line segments in coordinate grids, when given the coordinates of the endpoints of the line segments (8.G.B.8).

Mathematical Practices in This Unit
In this unit, you will have the opportunity to assess MP.1 to MP.8. The MP labels in this unit flag both opportunities to develop the mathematical practice standards and opportunities to assess them. Below is a list of places where we recommend assessing each standard, as well as some examples of how students can show that they have met a standard.

**MP.1:** G8-42 Extension 4, G8-46 Extensions 3–4

**MP.2:** G8-46 Extension 3

**MP.3:** G8-40 Extension 2, G8-42 Extensions 2–4, G8-44 Extension 4, G8-45 Extension 1, G8-48 Extension 4
In G8-42 Extension 2, students construct an argument to explain why the three sides of a right triangle cannot all be odd integers.

**MP.4:** G8-40 Extension 1, G8-46 Extension 1, G8-47 Extensions 1 and 3 and Question 7 on AP Book 8.2 p. 132, Performance Task: Fire Department Ladder Problems Question 5 on p. O-92

**MP.5:** G8-43 Extension 3
In G8-43 Extension 3, students use appropriate tools strategically when they choose a tool (such as tracing paper, a ruler and protractor, or a transparency) to check if two triangles are congruent.
**MP.6:** G8-40 Extension 1
In G8-40 Extension 1, students attend to precision when they explain the symbols they used—both what they mean and why they used them.

**MP.7:** G8-41 Extension 4, G8-42 Extensions 2–4, G8-43 Extension 2, G8-44 Extension 3, G8-46 Extension 4
In G8-46 Extension 4, students look for and make use of structure when they recognize a 3-D path as the hypotenuse of a triangle on a flat surface.

**MP.8:** G8-44 Extension 4, G8-47 Extension 2
Unit 4  Geometry: Pythagorean Theorem

Introduction
In this unit, students will develop the formulas for finding the area of a triangle, the area of a parallelogram, the circumference of a circle, and the area of a circle. Students will also estimate the value of \( \pi \) and use the rational approximation of that irrational number.

Students will use the area of a triangle and a square to develop the Pythagorean Theorem, and they will explain proofs of the Pythagorean Theorem. They will also develop the proof for the converse of the Pythagorean Theorem. Students will use the Pythagorean Theorem to solve for missing sides in a triangle, and they will solve word problems that involve right triangles in two and three dimensions. Finally, they will use the Pythagorean Theorem to help find the distance between two points on a coordinate grid.

Materials. Students will often need grid paper. Additionally, for some of the lessons of this unit, you will need a pre-drawn grid on the board. If you do not have such a grid, you can photocopy BLM 1 cm Grid Paper (p. S-1) onto a transparency and project it onto the board. Similarly, when a coordinate grid is required, use a transparency of BLM Large Coordinate Grid (p. S-2). Students should be working in grid paper notebooks. If they are not working in such notebooks, provide them with grid paper or BLM 1 cm Grid Paper. To save time, you can occasionally provide students with BLM Coordinate Grids (p. S-3).

Additionally, we recommend that students each have a scientific calculator, a pair of scissors, and a protractor for this unit.

Technology: dynamic geometry software. Students are encouraged to use dynamic geometry software. Some of the activities and extensions in this unit use a program called The Geometer’s Sketchpad®. If you are not familiar with The Geometer’s Sketchpad®, the built-in Help Center provides explicit instructions for many constructions. Use phrases such as “How to construct a line segment of given length” to search the Index. NOTE: If you use a different dynamic geometry program to complete these activities, the instructions provided may need to be adjusted.

Fraction notation. We show fractions in two ways in our lesson plans:

Stacked: \( \frac{1}{2} \)  
Not stacked: 1/2

If you show your students the non-stacked form, remember to introduce it as new notation.
Areas of Triangles and Parallelograms

Pages 110–113


Goals:
Students will develop and use formulas to find the area of a triangle and the area of a parallelogram by comparing with a rectangle with the same base and height.

Prior Knowledge Required:
- Can find the area of a rectangle
- Can write repeated addition as multiplication
- Can determine if triangles are congruent using the congruence rules SAS, SSS, or ASA

Vocabulary: acute triangle, alternate angles, base, congruent, height, obtuse triangle, opposite angles, parallelogram, perpendicular, rectangle, right triangle, triangle, vertex

Materials:
BLM Finding the Area of Triangles and Parallelograms (pp. O-64–65)
plain white paper
scissors
transparency of BLM Finding the Area of Triangles and Parallelograms (pp. O-64–65)
set of cutout shapes from BLM Finding the Area of Triangles and Parallelograms (pp. O-64–65)
overhead projector

(MP.2, MP.5) Finding the area of a rectangle. Distribute BLM Finding the Area of Triangles and Parallelograms and a sheet of plain white paper to each student—students will use the paper to trace and cut out required shapes.

Project the transparency of BLM Finding the Area of Triangles and Parallelograms (1) on the board. Point to rectangle 1.a) and ask students to count the number of square units in the rectangle. (28) ASK: What numbers can we add to find the total number of squares faster? (7 + 7 + 7 + 7 or 4 + 4 + 4 + 4 + 4 + 4 + 4) What multiplication can we use to represent 7 + 7 + 7 + 7? (4 × 7) What multiplication can we use to represent 4 + 4 + 4 + 4 + 4 + 4 + 4? (7 × 4) SAY: When we find area, we are counting the number of square units in the rectangle. Here the length is 7 units and the width is 4 units. To find the area, we can multiply the length and width. Draw on the board:
ASK: What is the length of the rectangle? (3) What is the width of the rectangle? (2) What is 3 \times 2? (6) Write on the board:

\[
\text{Area of rectangle} = \text{length} \times \text{width} = 3 \times 2 = 6 \text{ square units}
\]

SAY: Notice we would have gotten the same answer if we had multiplied 2 \times 3 instead of 3 \times 2.

Ask students to find the area of rectangle 1.b) on the BLM. (4 \times 6 or 6 \times 4 = 24) Remind students that area is represented in square units. Examples: cm\(^2\), in\(^2\), m\(^2\).

(MP.2) Exercises:
1. Find the area of the rectangle. Include the units.
   a) length = 7 cm, width = 4 cm  
   b) length = 6 in, width = 5 in
   \text{Answers:} a) 28 \text{ cm}^2, b) 30 \text{ in}^2

2. For the given rectangle measurements, find the missing width or length. Include the units.
   a) Area = 35 \text{ cm}^2, width = 7 cm  
   b) Area = 63 \text{ in}^2, length = 9 in
   \text{Answers:} a) length = 5 \text{ cm}, b) width = 7 \text{ in}

(MP.2, MP.5) Finding the area of a right triangle. Return to BLM Finding the Area of Triangles and Parallelograms (1). Have students trace the two shaded triangles in 2.a) and 2.b) onto plain white paper and then cut out the traced shapes. Then ask students to place the cutouts in their original positions on the BLM and rotate each to cover the non-shaded triangle, as shown below:

\[
\text{Area of a right triangle} = (\text{area of a rectangle with same length and width}) \div 2 = \text{length} \times \text{width} \div 2
\]
SAY: For a triangle, instead of saying “length” and “width,” we usually call the dimensions “base” and “height.” So we can write the formula a different way. Write on the board:

\[
\text{Area of a triangle} = \text{base} \times \text{height} \div 2
\]

ASK: For triangle 2.a), what is the base? (7) What is the height? (4) What is \(7 \times 4 \div 2\)? (14)

SAY: So the area is 14 square units. Write on the board:

\[
\begin{align*}
\text{Area} &= \text{base} \times \text{height} \div 2 \\
&= 7 \times 4 \div 2 \\
&= 14
\end{align*}
\]

Ask students to find the area of triangle 2.b) on the BLM. (6 \times 4 \div 2 = 12)

**MP.2 Exercises:** Find the area of the right triangle.

\[\text{a) } \begin{array}{c}
\text{3 cm} \\
\hline
\text{8 cm}
\end{array} \quad \text{b) } \begin{array}{c}
\text{4 in} \\
\hline
\text{5 in}
\end{array} \quad \text{c) } \begin{array}{c}
\text{6 m} \\
\hline
\text{10 m}
\end{array}\]

**Answers:** a) 12 cm\(^2\), b) 10 in\(^2\), c) 30 m\(^2\)

**MP.6 Labeling the height in a triangle.** SAY: In a right angle triangle, the length of one of the sides that forms the right angle can be considered the height of the triangle. The other side that forms the right angle can be considered the base of the triangle. Draw on the board:

SAY: Remember that an acute triangle is a triangle with all angles less than 90 degrees. An obtuse triangle is a triangle with one angle greater than 90 degrees. In an acute or obtuse triangle, we can pick any side as the base. The height of the triangle is the length of a line segment that runs perpendicular to the base and toward the opposite vertex. You can also think of it as the height of a rectangle around the triangle. Draw on the board:

SAY: Notice that for an obtuse triangle, the height can be drawn outside the triangle.
Draw several acute and obtuse triangles on the board. Label one side as the base, and ask students to come to the board to mark and label the height. Remind them that the height must be perpendicular to the base, and sometimes the height can be drawn outside the triangle.

**(MP.2, MP.5) Finding the area of an acute triangle.** Return to BLM Finding the Area of Triangles and Parallelograms (1). Have students trace the shaded triangles in 3.a) and 3.b) onto plain white paper and then cut out the traced shapes. Then ask students to place the cutouts in their original positions on the BLM and rotate each to cover the non-shaded triangles, as shown below:

![Diagram of acute triangle](image)

Rotate the cutouts on the projected BLM, as well. SAY: Notice that in 3.a) and 3.b), we split the large acute triangle into two smaller, right angle triangles and these match exactly the shapes of the non-shaded area in each rectangle. The acute triangle’s area times two equals the rectangle’s area. In other words, the area of the acute triangle is one-half the area of the rectangle. Notice the base of the triangle is the same as the length of the rectangle. The height of the triangle is the same as the width of the rectangle. Write on the board:

\[
\text{Area of an acute triangle} = \left(\text{area of a rectangle with same base and height}\right) \div 2 \\
= \text{base} \times \text{height} \div 2
\]

ASK: For triangle 3.a), what is the base of the triangle? (9) What is the height? (4) If students have trouble picturing the height, ask them to think of the width of the rectangle. ASK: What is \(9 \times 4 \div 2\)? (18) SAY: So the area of the triangle is 18 square units. Write on the board:

\[
\text{Area} = \text{base} \times \text{height} \div 2 \\
= 9 \times 4 \div 2 \\
= 18
\]

Have students repeat the process with triangle 3.b). (base = 4, height = 6, area = \(4 \times 6 \div 2 = 12\))

**MP.2 Exercises:** Find the area of the acute triangle.

a) \[\text{3 cm} \quad \text{7 cm}\]

b) \[\text{6 in} \quad \text{8 in}\]

c) \[\text{4 cm} \quad \text{2 cm}\]

**Answers:** a) 10.5 cm\(^2\), b) 24 in\(^2\), c) 4 cm\(^2\)
Finding the area of an obtuse triangle. Project the transparency of BLM Finding the Area of Triangles and Parallelograms (2) on the board. Have students trace the gray triangle in diagram 4.a) onto plain white paper and then cut out the traced shape.

SAY: The line $DB$ in diagram 4.b) passes through $E$, the midpoint of $AC$, and $DE = EB$. Ask students to use the cutout triangle to rotate $\triangle BCE$ 180 degrees about the point $E$ so that $\triangle BCE$ covers the area of $\triangle ADE$ in diagram 4.b) of the BLM. Rotate the cutout on the projected BLM, as well. SAY: Since we rotated $\triangle BCE$ so that it landed on $\triangle ADE$, $\triangle ABD$ and $\triangle ABC$ have the same area.

SAY: We want to show that $\triangle ABC$ and $\triangle ABD$ have the same height. In $\triangle DCE$ and $\triangle BAE$, we have constructed $AE = CE$ and $DE = BE$. ASK: Why is $\angle DEC = \angle BEA$? (they are opposite angles) What can you say about $\triangle DCE$ and $\triangle BAE$? (they are congruent) Why are $\triangle DCE$ and $\triangle BAE$ congruent? (SAS; $DE = EB$, $AE = EC$, and $\angle DEC = \angle AEB$) SAY: Since $\triangle DCE$ is congruent to $\triangle BAE$, we have $\angle CDE = \angle ABE$. ASK: How do we know $DC \parallel AB$? (they are alternate angles) SAY: So $\triangle ABD$ and $\triangle ABC$ have the same height.

ASK: What type of triangle is $\triangle ABD$? (acute) How do we find the area of an acute triangle? (base $\times$ height ÷ 2) SAY: But the base and height of the acute triangle are the same as the base and height of the obtuse triangle. So the area of an obtuse triangle can also be found using the formula $\text{Area} = \text{base} \times \text{height} ÷ 2$.

ASK: In the diagram, how long is the base? (5 units) How long is the height? (4) Write on the board:

$$\text{Area of } \triangle ABC = \text{base} \times \text{height} ÷ 2$$
$$= 5 \times 4 ÷ 2$$
$$= 10 \text{ square units}$$

(MP.2) Exercises: Find the area of the obtuse triangle using the formula $\text{Area} = \text{base} \times \text{height} ÷ 2$.

Answers: a) 7.5 cm², b) 3 m², c) 24 in²

(MP.2, MP.5) Finding the area of a triangle using different bases and heights. Return to BLM Finding the Area of Triangles and Parallelograms (2). SAY: Triangles 5.a) and 5.b) are congruent. Let’s find the areas of the triangles using different bases and heights. For triangle 5.a), ask students to measure the base and height to the nearest tenth of a centimeter. (6 cm, 4 cm) ASK: What is the formula for finding the area of this triangle? (Area = base $\times$ height ÷ 2) Write on the board:

$$\text{Area} = \text{base} \times \text{height} ÷ 2$$
$$= 6 \times 4 ÷ 2$$
ASK: What is the area of triangle 5.a)? (12 cm\(^2\)) Ask students to measure the base and height of triangle 5.b) to the nearest tenth of a centimeter (5 cm, 4.8 cm) Write on the board:

\[
\text{Area} = \text{base} \times \text{height} \div 2
= 5 \times 4.8 \div 2
\]

ASK: What is the area of the triangle 5.b)? (12 cm\(^2\)) SAY: Notice that the areas are the same. ASK: How could we have known the areas of the two triangles are the same before calculating? (the triangles are congruent) SAY: For every triangle, we can use three different bases and heights. The area should be the same no matter which base and height we measure.

(MP.2, MP.5) **Finding the area of a parallelogram.** Return to BLM Finding the Area of Triangles and Parallelograms (2). Have students trace the shaded triangle in 6.a) onto plain white paper and then cut out the traced shape. Then have them place the cutout in its original position in diagram 6.a) and move the cutout as shown in diagram 6.b). Move the cutout on the projected BLM, as well. ASK: What new shape is formed if we cut out this triangle and move it to the other end of the shape? (a rectangle) SAY: Notice the base and height for the rectangle are the same as the base and height for the parallelogram. The two shapes have the same area. ASK: What is the formula for finding the area of a rectangle? (Area = length \times width or Area = \text{base} \times \text{height}) What is the area of the rectangle shown? (24 square units) SAY: So the area of the parallelogram is also 24 square units. Write on the board:

\[
\text{Area of a parallelogram} = \text{base} \times \text{height}
= 6 \times 4
= 24
\]

**Exercises:** Find the area of the parallelogram.

a) ![Parallelogram](image-a) b) ![Parallelogram](image-b) c) ![Parallelogram](image-c)

**Answers:** a) 24 cm\(^2\), b) 48 in\(^2\), c) 1.5 m\(^2\)

**Extensions**

(MP.1) 1. Each side of the regular hexagon is 2 cm. The height of the hexagon is 3.464 cm. a) Find the area of the hexagon in three different ways. i) Divide the hexagon into six equilateral triangles. Find the area of each triangle. Then find the total area.

![Hexagon](image)
ii) Divide the hexagon into three parallelograms. Find the area of each parallelogram. Then find the total area.

![Hexagon diagram]

iii) Find the area of the rectangle around the hexagon. Then subtract the areas of the four triangles.

![Rectangle diagram]

b) Explain why the answers to parts a) should be the same.

**Answers:**

a) i) Area of each equilateral triangle = \(2 \times 1.732 \div 2 = 1.732\), total area = \(6 \times 1.732 = 10.392\) cm\(^2\)

ii) Area of each parallelogram = \(2 \times 1.732 = 3.464\), total area = \(3 \times 3.464 = 10.392\) cm\(^2\)

iii) Area of rectangle = \(4 \times 3.464 = 13.856\), area of each shaded triangle = \(1 \times 1.732 \div 2 = 0.866\), total area of shaded triangles = \(4 \times 0.866 = 3.464\), area of hexagon = \(13.856 - 3.464 = 10.392\) cm\(^2\)

b) The answers should be the same because the three hexagons are congruent.

(MP.1, MP.3) 2. A triangle and parallelogram have the same area. They also have the same length base. Explain why the triangle must have twice the height of the parallelogram by …

a) using an example with numbers

b) using formulas

**Answers:**

a) Suppose the triangle and parallelogram each have an area of 12 cm\(^2\) and a base of 3 cm.

![Triangle diagram]

For the triangle, \(12 = 3 \times H + 2\)

\[12 + 3 \times 2 = H\]

\[8 = H\]

![Parallelogram diagram]

For the parallelogram, \(12 = 3 \times h\)

\[12 + 3 = h\]

\[4 = h\]

So the height of the triangle is twice that of the parallelogram.

b) For the triangle, \(A = b \times H + 2\). For the parallelogram, \(A = b \times h\). If the areas are the same, then:

\[b \times H + 2 = b \times h\]

\[b \times H + 2 + b \times 2 = b \times h + b \times 2\]

\[H = h \times 2\]

So the height of the triangle is twice that of the parallelogram.
(MP.1, MP.3) 3. Explain why $\triangle ABC$, $\triangle ABD$, $\triangle ABE$, and $\triangle ABF$ all have the same area.

Answer:

All the triangles have the same length base ($AB$). They also have the same height because they fit in the same rectangle. So when we calculate the formula $\text{Area} = \text{base} \times \text{height} ÷ 2$, the answers will all be the same.
Goals:
Students will learn the definitions of a circle and its circumference.
Students will develop the formula for finding the circumference of a circle and learn that pi is the ratio of a circle’s circumference to its diameter.
Students will use an approximate value for pi and the formula to find the circumference of a circle.

Prior Knowledge Required:
Can use a compass to draw a circle with a given center and radius
Can substitute numbers into a formula to evaluate an expression

Vocabulary: center, circle, circumference, compass, diameter, perimeter, pi (π), polygon, radii, radius, regular polygon

Materials:
board ruler
board compass
class set of compasses
an assortment of about 10 cans of different sizes
masking tape

(MP.2) Defining a circle. Draw on the board:

Point to the dot labeled as the center and ask a student to come to the board and, using the board ruler, mark another dot on the board that is 5 cm from this point. Ask another student to come and mark a different dot that is 5 cm from the center. Continue asking students until everyone has had a chance to draw a different dot that is 5 cm from the center. (see sample answers below)
ASK: If we kept on drawing more dots that are 5 cm from the center, how many dots would there be? (an infinite number) SAY: Imagine that we could collect all the dots that are 5 cm from the center. ASK: What shape would they form together? (a circle) SAY: We call the distance from the center the *radius* of the circle. All points on a circle are the same distance, 5 cm, from its center. Draw on the board:

![Circle with radius](image)

SAY: The plural of radius is *radii*. Write “radii” on the board.

**(MP.5, MP.6) Drawing a circle with a known radius.** SAY: You can use a compass to draw a circle with a given center and radius. Draw a dot on the board. SAY: Suppose we want this dot to be the center of a new circle with radius 20 cm. Use the board compass and a ruler to open the compass so that the distance between the arms is 20 cm, as shown below:

![Compass set to 20 cm](image)

SAY: We place the sharp end of the compass on the center of the circle. While holding that arm on the center of the circle, we rotate the compass all the way around to draw the circle. Draw on the board:

![Circle with 20 cm radius](image)

**(MP.5, MP.6) Drawing a circle with radius equal to the length of a given line segment.** SAY: Suppose we want the radius of our circle to be the same length as a certain line segment. Draw on the board:
Place your board compass on the board so that the sharp end is at one end of the line segment. SAY: We can adjust our compass so that the other end will be at the far end of the line segment. Demonstrate on the board, as shown below:

![Compass demonstration](image1)

SAY: Now our compass is set to a radius that is the same as the line segment. We can pick any point as our center and draw the circle. Draw on the board:

![Circle drawing](image2)

**(MP.2)** Draw several line segments of different lengths on the board. For each line segment, draw a center point. Ask students to take turns adjusting the compass width so that it is the same as the length of one line segment and then draw a circle with that radius and with the center point.

**(MP.5)** **Diameter of a circle.** Draw on the board:

![Circle center point](image3)

Ask students to come to the board and use a ruler to draw a line segment so that each endpoint of the line segment touches the outside of the circle. (see sample line segments below)

![Sample line segments](image4)

Continue until at least a few line segments have passed through the center of the circle. ASK: Which line segments are the longest? After students have pointed out several of them,
ASK: What do all these long line segments have in common? (they pass through the center of the circle) SAY: The longest line segment that can be drawn inside the circle so that its endpoints touch the outside of the circle is called the **diameter**. ASK: How many different diameters can be drawn for a circle? (infinite) To convince students of this, you may have to draw several diameters, as shown below:

![Diameters](image)

Draw on the board:

![Diameter and Radius](image)

SAY: But before we said the distance from the center to the outside of the circle was called the **radius**. Label the radius on the same line as the diameter, as shown below:

![Diameter and Radius](image)

ASK: How many times longer is the diameter than the radius? (2) SAY: The diameter is twice the radius. Another way of saying this is that the radius is one-half the diameter. ASK: If the radius is 7 cm, what is the diameter? (14 cm) If the diameter is 20 in, what is the radius? (10 in)

**(MP.2) Exercises:** Find the diameter and radius.

a)

![Diameter 10 cm](image)

b)

![Diameter 3 in](image)

c)

![Diameter 14 m](image)

**Answers:** a) $r = 5$ cm, $d = 10$ cm; b) $r = 3$ in, $d = 6$ in; c) $r = 7$ m, $d = 14$ m
(MP.3, MP.6) Developing the formula for the circumference. Draw on the board:

ASK: What name do we give to the distance around the outside of a shape? (perimeter) What is the perimeter of the shape? (14 cm) How did you calculate the perimeter? (add the distances around the outside of the shape) SAY: Some polygons have all sides the same length. They are called regular polygons. Draw on the board:

ASK: How can we use addition to find the perimeter? (2 + 2 + 2 + 2) How can we use multiplication? (4 × 2)

With students, work through Question 3.a) on AP Book 8.2, p. 115, in which students calculate the perimeter of each polygon inside a circle. The measurement of each side of the polygon is shown to the nearest hundredth of a centimeter. Have volunteers write their answers on the board. (see answers below—note that students will fill in the last row later)

<table>
<thead>
<tr>
<th>Polygon</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of One Polygon Side</td>
<td>2.12</td>
<td>1.76</td>
<td>1.50</td>
<td>1.15</td>
<td>1.03</td>
<td>0.93</td>
</tr>
<tr>
<td>Number of Sides</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Perimeter</td>
<td>8.48</td>
<td>8.80</td>
<td>9.00</td>
<td>9.20</td>
<td>9.27</td>
<td>9.30</td>
</tr>
<tr>
<td>$P \div d$</td>
<td>2.83</td>
<td>2.93</td>
<td>3.00</td>
<td>3.07</td>
<td>3.09</td>
<td>3.10</td>
</tr>
</tbody>
</table>

SAY: Notice that, as the number of sides of the polygon inside the circle increases, the polygon looks more and more like the circle. ASK: How could we get the polygon to look even more like the circle? (increase the number of sides) How many sides would we need to make the polygon a perfect circle? (an infinite number)

SAY: For each circle, the diameter is 3 cm. Have students complete Question 3.c) on AP Book 8.2, p. 115, then ask volunteers to write the answers in the last row of the table on the board. The answers are shown in the final row of the table above, rounded to two decimal places.

ASK: As the number of sides increases, what happens to the decimal that is perimeter divided by diameter? (it gets bigger) SAY: Mathematicians have found that, for the perimeter of a circle,
or circumference, the value of the circumference divided by the diameter is exactly the same for all circles. This value is a ratio and called pi. Here is the symbol for pi. Write on the board:

\[ \pi \]

SAY: Pi is a very cool number. It has an infinite number of decimal places with no pattern in the digits. Ask students to find the value of \( \pi \) on their calculators. This may involve different buttons depending on the calculator. Write on the board:

\[ \pi \approx 3.14159265358979 \ldots \]

SAY: You can find the first 10,000 digits in the decimal on the Internet. For fun, see how many you can memorize! We don’t need all these decimal places for most of our questions, so we will just round pi to the first two decimal places. Write on the board:

\[ \pi \approx 3.14 \]

**Activity**

**Find the approximate ratio between the diameter and circumference of a circle.** Divide students into groups of three or four and give each student in the group a different-sized can. Have students each trace the base of a can onto plain white paper and then cut out the paper circle, fold it in half, and measure the diameter of the circle to the nearest millimeter. To find the circumference of the circle, ask students each to wrap masking tape around their can (without overlapping at the end), remove the tape, lay it straight, and measure its length to the nearest millimeter. ASK: About how many times larger is the circumference than the diameter? Have students compare their answers in small groups. Students will find that the circumference of the circle is always about 3.14 times the diameter.

(end of activity)

SAY: The formula we found for the ratio between the diameter and circumference of a circle is circumference \( \div \) diameter = \( \pi \). We can write this formula in several ways. Write on the board:

\[ \frac{C}{d} = \pi \], where \( C \) is circumference and \( d \) is diameter

SAY: We can write it as a fraction where \( C \) is circumference and \( d \) is diameter. ASK: If we multiply both sides of the formula by \( d \), what will the formula be?  \( (C = \pi \times d) \) Write on the board:

\[ C = \pi d \]

Remind students that there is a multiplication sign between the \( \pi \) and \( d \). ASK: Since the diameter is twice the radius, what can we replace \( d \) with in the formula? \( (2r) \) Continue writing on the board:

\[ = \pi (2r) \]
\[ = 2\pi r \]
SAY: We can calculate the approximate circumference of a circle using either the formula $C = \pi d$ or $C = 2\pi r$, depending on whether we know the diameter or the radius of the circle. Remember to use 3.14 for pi except when told otherwise.

**Exercise:** Use the appropriate formula to approximate the circumference.

a) diameter = 10 cm  
b) radius = 8 in  
c) radius = 20 km  
d) diameter = 2 mi

**Answers:** a) 31.4 cm, b) 50.24 in, c) 125.6 km, d) 6.28 mi

**Extensions**

**Exercise:** 1. The radius of the Earth is approximately 6,371 km.

a) How fast does a person standing at the equator travel as Earth spins on its axis, in km/h?  
b) Explain what the symbols you used mean and why you used them.

**Answers:** a) Let $d$ be the distance someone standing on the equator will travel when the Earth spins one full rotation on its axis. Then $d = 2\pi r \approx 2(3.14)(6,371) \text{ km} \approx 40,009.88 \text{ km}$. The person travels that distance when the Earth spins one full rotation, which takes 24 hours. So the person’s speed is approximately $40,009.88 \text{ km}/24 \text{ h} \approx 1,667 \text{ km/h}$; b) I used “=” for exactly equal, “≈” for approximately equal, $d$ for the distance the person travels, $\pi$ for pi, and $r$ for the radius of the Earth. I used brackets around the numbers being multiplied. I had to use $= \approx$ in the last two steps, not $= \approx$, since I used the rational value 3.14, which is just a rational approximation of the irrational number pi, and since 6,371 km is only an approximation of the radius of the Earth.

Redirecting students: ASK: For part a), what information do you need to answer the question?  
**PROMPT:** What do you need to know if you want to know someone’s speed? (how far they go and how long it takes them) Can you figure out how far they go from the information in the question? (yes, it is the circumference of the Earth, so I can use the radius) Can you figure out how long it takes? If students say “no, because there is no other information given,” you may need to point out that as it rotates, the side closer to the sun experiences daytime and the side farther from the sun experiences nighttime.

**NOTE:** Students might also note that they are using symbols such as “km” for kilometer, “h” for hour, a dot for the decimal point, and a comma for the large number, but they are not required to explain these symbols because they are from much earlier grades.

2. Don says that the number $\pi$, to two decimal places, is 3.14, so you can find $\pi^2$, accurate to two decimal places, by finding $3.14 \times 3.14$ and then rounding the answer to two decimal places. Do you agree with Don? Why or why not?

**Answer:** I disagree with Don. $3.14 \times 3.14 = 9.8596$. Rounded to two decimal places, this is 9.86. But when I did $\pi^2$ on my calculator, I got 9.869604401, which is 9.87 to two decimal places. This is not exactly the same as rounding first. It is more accurate to wait to the end of the calculations to round the numbers, because you keep all the information until the end.
G8-41  Area of Circles

Pages 117–118

Standards: 8.NS.A.2

Goals:
Students will develop the formula for finding the area of a circle.
Students will use the formula and the approximate value of pi to find the area of a circle.

Prior Knowledge Required:
Can find the radius of a circle given its diameter
Can find the diameter of a circle given its radius
Knows that an approximate value for \( \pi \) is 3.14
Can find the circumference of a circle
Can find the area of a parallelogram given its base and height

Vocabulary: area, base, circle, circumference, diameter, height, parallelogram, pi (\( \pi \)), radii, radius

Materials:
BLM Finding the Area of a Circle (p. O-66)
scissors
transparency of BLM Finding the Area of a Circle (p. O-66)
set of cutout wedges from BLM Finding the Area of a Circle (p. O-66)
overhead projector

(MP.2, MP.4.) Developing the formula to find the area of a circle. Distribute BLM Finding the Area of a Circle. Project the transparency of the BLM on the board and ask students to each cut out the 12 pie-shaped wedges from the circle at the top of their BLM. Have students place the cutout wedges from the circle on the parallelogram at the bottom of their BLM as you do the same on the projected BLM.

ASK: Do the wedges fit perfectly in the parallelogram? (no) Why not? (the curved part of the wedge leaves spaces uncovered) Ask students to shade in the spaces. ASK: If we use hundreds of wedges instead of 12, will the uncovered spaces become bigger or smaller? (smaller) SAY: If we use millions of wedges, the wedges will start to completely fill the parallelogram.

ASK: Where did the curved part of the wedges come from? (the circumference of the circle) Since half the wedges are at the bottom of the parallelogram, what can we write for the approximate length of the base of the parallelogram? (base = half of the circumference)

SAY: If we use more and more wedges, the wedges will become thinner. Each will look more vertical and less wide. ASK: Where did the straight sides of the wedges come from? (the radius of the circle) What can we write for the approximate height of the parallelogram? (height = the radius)
Draw on the board:

\[ \text{base} \times \text{height} \]

ASK: What is the formula for the area of a parallelogram? (base \times height) Write on the board:

\[ A = \frac{1}{2} \times \text{base} \times \text{height} \]

ASK: Which formula for circumference uses the radius? (C = 2\pi r) Continue writing on the board:

\[ = \frac{1}{2} (2\pi r) \times r \]

ASK: What is \( 1/2 \times 2 \)? (1) Continue writing on the board:

\[ \pi r \times r = \pi r^2 \]

SAY: So to find the area of a circle, multiply pi times the square of the radius.

**Exercises:**

1. Find the approximate area of the circle. Use \( \pi \approx 3.14 \).
   a) \( r = 5 \text{ cm} \)  
   b) \( r = 10 \text{ in} \)  
   c) \( r = 6 \text{ km} \)
   **Answers:** a) 78.5 cm\(^2\), b) 314 in\(^2\), c) 113.04 km\(^2\)

2. Find the radius, then find the approximate area of the circle.
   a) \( d = 20 \text{ mm} \)  
   b) \( d = 40 \text{ cm} \)  
   c) \( d = 2.4 \text{ in} \)
   **Answers:** a) 314 mm\(^2\); b) 1,256 cm\(^2\); c) 4.5216 in\(^2\)

3. Find the area of the semi-circle.
   a)
   \[ A = \frac{1}{2} \times \pi \times (5)^2 \approx 39.25 \text{ m}^2 \]
   b)
   \[ A = \frac{1}{2} \times \pi \times (6)^2 \approx 56.52 \text{ cm}^2 \]
   **Solutions:** a) \( A = \frac{1}{2} \times \pi \times (5)^2 \approx 39.25 \text{ m}^2 \), b) \( A = \frac{1}{2} \times \pi \times (6)^2 \approx 56.52 \text{ cm}^2 \)

4. Find the area of the picture shaped like an ice-cream cone.

\[ A = (1/2 \times 6 \times 4) + (1/2 \times 3.14 \times 2^2) \approx 18.28 \text{ cm}^2 \]
(MP.1) Solving word problems involving areas of circles. SAY: When solving word problems involving circumference and area, it is important to determine whether the question is looking for the distance around a circle (the circumference) or the amount of space inside the circle (area). It is also important to note what information is given—for example, the radius or the diameter.

Write on the board:

A circular clock has an hour hand that is 30 cm long. To the nearest tenth of a centimeter, how far does the tip of the hour hand travel in 12 hours? 24 hours?

ASK: Are we looking for a distance traveled around the circle or the area of the circle? (distance traveled) Ask a volunteer to draw the clock and hour hand on the board. ASK: Does the question tell us the radius or the diameter? (the radius) Label the radius on the board, as shown below:

\[ r = 30 \text{ cm} \]

ASK: What is the formula for the circumference using the radius? (\( C = 2\pi r \)) What decimal can we use to approximate pi? (3.14) Write on the board:

\[
C = 2\pi r \\
= 2 \times 3.14 \times 30 \\
= 188.4 \text{ cm}
\]

ASK: Does this represent the distance traveled in 12 hours or 24 hours? (12 hours) Why? (the hour hand travels, or rotates, clockwise from 12 until it reaches 12 again, which takes 12 hours) How far does the hour hand travel in 24 hours? (188.4 \times 2 = 376.8 \text{ cm})

Write on the board:

A bakery charges $8 for an 8-inch pie and $12 for a 12-inch pie. Which pie is the better purchase?

ASK: Is the question looking for the area or circumference? (area) Why? (we eat the area of the pie) What is the formula for the area of a circle? (\( A = \pi r^2 \)) Are we given the radius or diameter? (diameter, 8 inch and 12 inch) What are the radii for the two pies? (4 inch and 6 inch) Ask volunteers to calculate the areas to the nearest tenth of a square inch on the board. (see answers below)

<table>
<thead>
<tr>
<th>8 inch diameter pie</th>
<th>12 inch diameter pie</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A = \pi r^2 )</td>
<td>( A = \pi r^2 )</td>
</tr>
<tr>
<td>\approx 3.14 \times 4^2</td>
<td>\approx 3.14 \times 6^2</td>
</tr>
<tr>
<td>= 50.24 \text{ in}^2</td>
<td>= 113.04 \text{ in}^2</td>
</tr>
</tbody>
</table>
ASK: How do we know which is the better purchase? (find which pie gives you the greater amount, or area, of pie for each dollar) What can we divide to find the better purchase? (divide the area by the amount of money) Write on the board:

\[
\begin{align*}
50.24 \text{ in}^2 / \$8 & \quad \approx 6.28 \text{ in}^2 \text{ per dollar} \\
113.04 \text{ in}^2 / \$12 & \quad \approx 9.42 \text{ in}^2 \text{ per dollar}
\end{align*}
\]

Extensions

1. Find the approximate area of the shaded region.

   a) \[16^2 - \pi(8)^2 \approx 55.04 \text{ cm}^2\]
   
   b) \[\pi(3)^2 - \frac{1}{2}(6)(3) \approx 19.26 \text{ in}^2\]

   c) \[\pi(10)^2 - \pi(5)^2 - \pi(5)^2 \approx 157 \text{ cm}^2\]

2. Find the area of the shaded region.

   Solution: (Area of semi-circle with diameter 7 in) + (Area of semi-circle with diameter 3 in) - (Area of semi-circle with diameter 4 in)
   
   \[
   = (\pi \times 3.5^2 + 2) + (\pi \times 1.5^2 + 2) - (\pi \times 2^2 + 2)
   \]
   
   \[
   \approx 3.14 \times 12.25 + 2 + 3.14 \times 2.25 + 2 - 3.14 \times 4 + 2
   \]
   
   \[
   = 16.485 \text{ in}^2
   \]

3. a) Use \(A = \pi r^2\) and \(r = \frac{1}{2}d\) to develop a formula for the circumference that uses only the diameter, and not the radius.

   b) Use the formula in part a) to find the area of a circle with diameter 14 cm.

   Solutions: a) \(A = \pi r^2 = \pi \left(\frac{1}{2}d\right)^2 = \frac{1}{4}\pi d^2\),
   b) \(\frac{1}{4}\pi d^2 = \frac{1}{4} \times 3.14 \times 14^2 \approx 153.86 \text{ cm}^2\)

(MP.7) 4. In Extension 1.b), how long are the two short sides of the triangle? Explain.

   Answer: The angle at the top is a right angle, because when you draw the height, you make two isosceles triangles that have 45° base angles. The area of the triangle is 9 in\(^2\), so if \(x\) is the length of the two short sides of the triangle, then \(\frac{1}{2}x^2 = 9\), so \(x^2 = 18\), so \(x = \sqrt{18} \approx 4.24\ \text{ in.}\)
G8-42 Introduction to the Pythagorean Theorem

Pages 119–121

Standards: 8.G.B.7

Goals:
Students will find the width of a square that has been rotated on a grid by breaking up the square into four right-angled triangles and a smaller inner square.
Students will observe that for a right-angled triangle with squares formed on all sides, the square on the side opposite the right angle has an area equal to the sum of the areas of the squares on the other two sides. Students will use this observation to find a missing length of right-angled triangle.

Prior Knowledge Required:
Can find the area of a square
Can find the area of a right angle triangle
Can find the square root of perfect squares mentally
Can find the square root of a number using a calculator
Is familiar with $a^2$ and $\sqrt{a}$ notations

Vocabulary: right triangle, side length, slanted square, square, square root

Materials:
BLM Finding the Area of a Slanted Square (p. O-67)
BLM Finding the Side Length of a Slanted Square (p. O-68)

NOTE: In this lesson, students come to an understanding of the relationship among side measurements of right triangles, but do not yet name the Pythagorean Theorem. They will name and address the Pythagorean Theorem formally in lesson G8-43.

(MP.5) Review finding the area of a triangle. Display a grid on the board. Draw on the board:

ASK: How can we find the area of the rectangle? (length × width or base × height) What is the area of the triangle compared to that of the rectangle? (1/2) What is the area of the triangle? (9 square units) How did you calculate it? (6 × 3 ÷ 2)
**Exercises:** Find the area of the right triangle.

- **a)**
  ![Diagram](image)
  
  **Answer:** $3 \times 4 \div 2 = 6 \text{ units}^2$

- **b)**
  ![Diagram](image)
  
  **Answer:** $4 \times 5 \div 2 = 10 \text{ units}^2$

- **c)**
  ![Diagram](image)
  
  **Answer:** $1 \times 6 \div 2 = 3 \text{ units}^2$

**MP.7 Finding the area of a slanted square.** Display a grid on the board. Draw on the board:

![Diagram](image)

**SAY:** It looks like this square has been rotated on the grid. We are going to call it a “slanted square.” It is difficult to find the area of the slanted square by counting the small squares on the grid paper because the square is not parallel to the grid lines. It’s easier to calculate the area if we draw four right-angled triangles and a smaller central square. Draw on the board:

![Diagram](image)

**ASK:** What are the dimensions of each of the right triangles? (6 x 8) How can we calculate the area of each right triangle? (6 x 8 ÷ 2 = 24) What are the dimensions of the smaller square? (2 x 2) How can we calculate the area of the smaller square? (2 x 2 = 4) How can we calculate the total area of the slanted square? (add the areas of the right triangles and the smaller, inner square) Write on the board:

\[
\text{Area} = 4 \times 24 + 4 \\
= 96 + 4 \\
= 100
\]
Distribute BLM Finding the Area of a Slanted Square. Ask students to find the area of each slanted square using the technique above. **NOTE:** For parts d) and e), students will need to complete the lines to draw the right angled triangles and small squares. (Area of A = 4(7.5) + 4 = 34 units², Area of B = 4 × 2.5 + 16 = 26 units², Area of C = 4 × 8 + 36 = 68 units², Area of D = 4 × 12 + 4 = 52 units², Area of E = 4 × 24 + 64 = 160 units²)

**(MP.5) Review square roots.** Write on the board:

Since $5 \times 5 = 25$, we can write $\sqrt{25} = 5$.

SAY: Remember that we say “The square root of 25 is 5 since $5 \times 5 = 25$.”

**Exercise:** Evaluate without a calculator.

a) $\sqrt{36}$  
   b) $\sqrt{81}$  
   c) $\sqrt{4}$  
   d) $\sqrt{100}$  
   Bonus: $\sqrt{40,000}$

**Answers:** a) 6, b) 9, c) 2, d) 10, Bonus: 200

**(MP.7) Finding the width of a square given its area.** Draw on the board:

SAY: The shapes are all squares. Remember that a square has equal length and width.

ASK: What is the length and width of the first square? (5 cm) Why? (5 × 5 = 25) Repeat with the other squares. (4 in, 8 km)

**(MP.7) Finding the side length of a slanted square.** Display a grid on the board. Draw on the board:

SAY: We want to find the length or width of this square. First we are going to find the total area of the square by breaking the square into four right triangles and a smaller, inner square. For each vertex we are going to draw either a vertical or horizontal line along a grid line and toward
the inside of the square. Extend each line until it touches the outside of the square, as shown below:

Use the dotted lines to help create four right triangles. Draw one of the right triangles as shown, and ask volunteers to draw the other three right triangles. Erase any extra lines so that the drawing looks like this:

ASK: What is the base and height of each triangle? (base = 5, height = 12) What is the area of each triangle? (5 × 12 ÷ 2 = 30) What are the dimensions of the smaller square? (7 × 7) What is the area of the smaller square? (49)

ASK: What is the total area of the slanted square? (4 × 30 + 49 = 169) How can we find the width of the square? (find the square root of the area) What is the width? (√169 = 13) Write on the board as you SAY: We can write this in fewer steps like this:

\[
\text{Side length} = \sqrt{4 \times 30 + 49} \\
= \sqrt{169} \\
= 13
\]

(MP.3) Review the order of operations with square roots. SAY: You must be careful when evaluating expressions like \(\sqrt{4 \times 30 + 49}\). The order in which you perform the operations is important. Write on the board:

\[
\sqrt{9 + 16} \quad \quad \sqrt{9 + \sqrt{16}}
\]
SAY: These two expressions look similar. ASK: But are the answers the same? Encourage a discussion to determine whether the answers are the same. ASK: What is $9 + 16$? (25) What is the square root of $25$? (5) What is the square root of $9$? (3) What is the square of $16$? (4) What is $3 + 4^2$? (7) Are the answers the same for both expressions? (no, $\sqrt{9 + 16} = 5$, but $\sqrt{9} + \sqrt{16} = 7$)

Investigate with the following expressions:

\[
\begin{align*}
\text{a) } & \sqrt{36 + 64} \text{ and } \sqrt{36} + \sqrt{64} \\
\text{b) } & \sqrt{25 + 144} \text{ and } \sqrt{25} + \sqrt{144} \\
\text{c) } & \sqrt{64 + 225} \text{ and } \sqrt{64} + \sqrt{225} \\
\text{d) } & \sqrt{100 + 576} \text{ and } \sqrt{100} + \sqrt{576}
\end{align*}
\]

(selected sample answer: a) $\sqrt{100} = 10$, $\sqrt{36} + \sqrt{64} = 6 + 8 = 14$, $10 \neq 14$)

SAY: When using your calculator, it is a good idea to enter brackets around the operations being performed underneath the square root symbol. So when calculating $\sqrt{36 + 64}$, think of it as $(36 + 64)$.

**Exercises:** Calculate.

a) $\sqrt{5\times3+1}$ 

b) $\sqrt{8\times4 - 7}$ 

c) $\sqrt{5^2 + 11}$

**Answers:** a) $\sqrt{16} = 4$, b) $\sqrt{25} = 5$, c) $\sqrt{36} = 6$

Distribute BLM Finding the Side Length of a Slanted Square. Ask students to find the side length of each square using the techniques shown on p. O-23—in other words, break the slanted square into four right triangles and a smaller, central square; find the areas of each part; add them to find the area of the slanted square; and find the side measure of the slanted square by finding the square root of the slanted square’s area. Where the area is not a perfect square, ask students to find the side length accurate to one decimal place. (Side length of A = $\sqrt{4\times6+1} = 5$, Side length of B = $\sqrt{4\times24+4} = 10$, Side length of C = $\sqrt{4\times2+9} = \sqrt{17} \approx 4.1$, Side length of D = $\sqrt{4\times5+9} = \sqrt{29} \approx 5.4$, Side length of E = $\sqrt{4\times20+9} = \sqrt{89} \approx 9.4$)

(MP.2, MP.7) Informally observing the Pythagorean Theorem. Display a grid on the board. Draw a right triangle and label the sides with $a$, $b$, and $c$, as shown below. Ensure the longest, slant side is $c$.

ASK: What type of triangle is this? (a right triangle) SAY: I’m going to draw a square using the vertical side of the right triangle as the square’s width. Draw the square with side $a$, below, and draw the others in turn as you SAY: I’m going to draw another square using the horizontal side of the right triangle as its width. These are easy to draw because I can use the grid to count and help me draw vertical and horizontal lines. Next, I’ll draw a third square with a side that is $c$. 

Teacher Resource for Grade 8 — Unit 4 Geometry O-25
ASK: What is the area of a square with each side \(a\)? \((a^2)\)

SAY: In the same way, the other squares have area \(b^2\) and \(c^2\). Label the area of the squares on the diagram, as shown below:

![Diagram of squares with areas labeled]

SAY: In this diagram, count the squares or multiply to find the area of the square on side \(a\). \((3^2 = 9)\) Count the squares or multiply to find the area of the square on side \(b\). \((8^2 = 64)\)

Point to the large slanted square with area \(c^2\) and SAY: for the last side and square, side \(c\) and the square on side \(c\), we need to find the area and side length in the same way we did earlier with slanted squares. Continuing with the diagram from above, ask for a volunteer to draw four triangles and a smaller, inner square in the square on side \(c\), as shown below. SAY: This will help us find the area of the slanted square on side \(c\). The completed picture is shown below.

![Completed picture with additional elements]

ASK: What are the dimensions of the right triangles? \((8 \times 3)\) What is the width of the small, inner square? \((5)\) What is the total area of the slanted square with side \(c\)? \((4 \times (8 \times 3 + 2) + 25 = 73)\)

Write the areas of the squares on the board, as shown below:

\[
c^2 = 73 \quad a^2 = 9 \quad b^2 = 64
\]

ASK: How can we calculate 73 using 9 and 64? \((73 = 9 + 64)\) SAY: For this triangle, it seems that \(c^2 = a^2 + b^2\). Let’s investigate whether this relationship is true for other right triangles. Have students complete Question 5 on AP Book 8.2, p. 120. After students have had enough time to
complete the question, ASK: Did the relationship seem to hold for other right triangles? (yes)
Have students do Question 6 on AP Book 8.2, p. 121 to test the relationship further.

SAY: It seems that for right triangles, \( c^2 = a^2 + b^2 \). In the next lesson, we will prove that the relationship is true for all right triangles. For now, let's use the rule to help us find the lengths of missing sides in a diagram.

(MP.5) Finding the missing side in a right triangle using \( c^2 = a^2 + b^2 \). Ask students to draw a right triangle to scale in their notebooks using the measurements below:

\[
\begin{align*}
\text{SAY: We would like to find the length of the longest side. We labeled it } c. \text{ Write on the board:} \\
\quad c^2 &= a^2 + b^2 \\
\text{ASK: What can we substitute for } a \text{ and } b? \ (3, 4) \text{ Write on the board:} \\
\quad c^2 &= 3^2 + 4^2 \\
\text{ASK: What is } 3^2? \ (9) \text{ What is } 4^2? \ (16) \text{ What is } 9 + 16? \ (25) \text{ Write on the board:} \\
\quad c^2 &= 9 + 16 \\
\quad c^2 &= 25 \\
\text{SAY: This is the value of } c^2. \text{ ASK: How can we find the value of } c? \ (\text{find the square root}) \text{ What is the square root of } 25? \ (5) \text{ Write on the board:} \\
\quad c &= \sqrt{25} = 5 \\
\end{align*}
\]

Have students measure the length of \( c \) in the diagram in their notebook to see if this is correct.

Exercises:
1. Find the length of \( c \) rounded to one decimal place.
   a) \( a = 6, \ b = 8, \ c = ? \) \quad b) \( a = 15, \ b = 8, \ c = ? \) \quad c) \( a = 6, \ b = 11, \ c = ? \)
   Answers: a) 10, b) 17, c) 12.5

(MP.3) 2. Terry thinks that in the diagram, \( c^2 = 5^2 + 8^2 \). Is he correct? Explain.

\[
\begin{align*}
\text{Answer: No, the triangle is not a right-angled triangle. So } c^2 &\neq a^2 + b^2. \\
\end{align*}
\]
Extensions
1. A Pythagorean triple is a set of three positive integers that can form the sides of a right triangle. For example, \{3, 4, 5\} is a Pythagorean triple because \(3^2 + 4^2 = 5^2\). Is the set of numbers a Pythagorean triple?
   a) \{8, 15, 17\}   b) \{5, 12, 13\}   c) \{6, 8, 9\}
   d) \{4, 11, 12\}   e) \{4, 5, 6\}   f) \{7, 24, 25\}

Sample solutions:
   a) \(8^2 + 15^2 = 64 + 225 = 289 = 17^2\), so \{8, 15, 17\} is a Pythagorean triple.
   c) \(6^2 + 8^2 = 36 + 64 = 100 = 9^2\), so \{6, 8, 9\} is not a Pythagorean triple.

Answers: b) and f) are Pythagorean triples; d), and f) are not Pythagorean triples.

(MP.3, MP.7) 2. Can the numbers in a Pythagorean triple all be odd? Explain.
Answer: No. Suppose \(a, b,\) and \(c\) are a Pythagorean triple. If \(a\) and \(b\) are odd, then \(a^2\) and \(b^2\) are odd, because the product of two odd numbers is odd, so \(a^2 + b^2\) is even, because the sum of two odd numbers is even. But then \(c^2\) is even, because \(c^2\) is equal to \(a^2 + b^2\). So, \(c\) is even.

(MP.3, MP.7) 3. a) Show that 3, 4, 5 and 6, 8, 10 are both Pythagorean triples.
   b) Make and prove a conjecture to find many Pythagorean triples by using the 3, 4, 5 triple.
   c) Show that 5, 12, 13 is a Pythagorean triple.
   d) Can you use your conjecture from part b) and your answer to part c) to make new Pythagorean triples? If not, improve your conjecture so that you can.

Selected answers:
   a) \(3^2 + 4^2 = 9 + 16 = 25 = 5^2\) and \(6^2 + 8^2 = 36 + 64 = 100 = 10^2\)
   b) Let \(x\) be a positive integer, and \(a, b, c\) a Pythagorean triple, then \(ax, bx, cx\) is also a Pythagorean triple:
      \[(ax)^2 + (bx)^2 = a^2x^2 + b^2x^2\] because \((ax)^2 = a^2x^2\) for any numbers \(a\) and \(x\)
      \[= (a^2 + b^2)x^2\] from the distributive property
      \[= c^2x^2\] because \(c^2 = a^2 + b^2\)
      \[= (cx)^2\] because of the same rule above: \((cx)^2 = c^2x^2\)

Alternatively, the triangle with side lengths \(ax, bx, cx\) is similar to the triangle with side lengths \(a, b, c\), and so is a right triangle.

(MP.1, MP.3, MP.7) 4. Use \(\sqrt[3]{4,913} = 17\) to evaluate the number. Do not use a calculator. Then explain to a partner how you know your answer is correct. Do you agree with each other? Discuss why or why not.
   a) \(\sqrt[3]{4,913,000}\)   b) \(\sqrt[3]{0.004913}\)   c) \(\sqrt[3]{(4,913)^2}\)

Answers: a) 4,913,000 = \(17 \times 10^3 = (17 \times 10)^3\), so 4,913,000 = 170^3, so \(\sqrt[3]{4,913,000} = 170\);
   b) 0.004913 = 4,913 \times 10^{-3} = 17^3 \times 10^{-3} = (17 + 10)^3 = 1.7^3\), so \(\sqrt[3]{0.004913} = 1.7\);
   c) \(\sqrt[3]{(4,913)^2} = \sqrt[3]{4,913 \times 4,913} = \sqrt[3]{4,913^2} \times \sqrt[3]{4,913} = 17 \times 17 = 289\)
Goals:
Students will identify the hypotenuse in a right triangle and write the Pythagorean Theorem so that the square of the hypotenuse is the sum of the squares of the other sides.
Students will use the Pythagorean Theorem to find the length of the missing side in a right triangle.

Prior Knowledge Required:
Can identify the longest side in a right triangle
Can find the square of a number
Can find the square root of perfect squares mentally
Can find the square root of a number using a calculator

Vocabulary: hypotenuse, Pythagorean Theorem, right angle

(MP.6) Finding the hypotenuse in a right triangle. Draw on the board:

Have volunteers draw a thick line on top of the longest side in each triangle. (in each case, it is the side opposite the right triangle) SAY: The longest side in a right triangle is always opposite the right angle. We call this side the hypotenuse. Label the diagrams as shown below:

SAY: In the next lesson, we will prove the theorem that, in a right triangle, the square of the hypotenuse length is the sum of the squares of the lengths of the other two sides. The theorem was discovered by a Greek mathematician named Pythagoras. It is called the Pythagorean Theorem. Draw on the board another right triangle, and label it as you SAY: If we label the hypotenuse as $c$ and the other two sides $a$ and $b$, we can write the Pythagorean Theorem as:

$$c^2 = a^2 + b^2$$
SAY: The variables used to label the sides will not always be $a$, $b$, and $c$ as shown here. It is important, however, that in any equation using the Pythagorean Theorem that the variable for the hypotenuse is alone on one side of the equation.

**Exercises:** Write the Pythagorean Theorem for the triangle using the variables shown.

a) ![Triangle](image1.png)  
\[ a^2 = b^2 + c^2 \]

b) ![Triangle](image2.png)  
\[ m^2 = r^2 + p^2 \]

c) ![Triangle](image3.png)  
\[ q^2 = w^2 + x^2 \]

**Answers:**  
a) $a^2 = b^2 + c^2$, b) $m^2 = r^2 + p^2$, c) $q^2 = w^2 + x^2$

(MP.6) **Finding the hypotenuse in a right triangle given the other sides.** Draw on the board:

![Triangle](image4.png)

ASK: Which side is the hypotenuse? ($m$) How do we write the Pythagorean Theorem for this diagram? ($m^2 = 4^2 + 6^2$) Write the equation on the board and solve for $m$. If needed, 
PROMPT: What is $4^2$? (16) What is $6^2$? (36) How do we find $m^2$? (find the square root) 
The solution should look like this:

\[
\begin{align*}
m^2 &= 4^2 + 6^2 \\
     &= 16 + 36 \\
     &= 52 \\
m &= \sqrt{52} \\
     &\approx 7.2
\end{align*}
\]

**Exercises:** Find the length of the hypotenuse to the nearest tenth.

a) ![Triangle](image5.png)  
\[ 5^2 + 4^2 = w^2 \]

b) ![Triangle](image6.png)  
\[ 23^2 + 6^2 = s^2 \]

c) ![Triangle](image7.png)  
\[ 8^2 + 11^2 = f^2 \]

**Answers:**  
a) $\sqrt{41} \approx 6.4$, b) $\sqrt{4,373} \approx 66.1$, c) $\sqrt{185} \approx 13.6$

(MP.6) **Finding the missing side in a right triangle given the hypotenuse and another side.** Draw on the board:

![Triangle](image8.png)
SAY: Notice this time the hypotenuse is given. Remember that in the Pythagorean Theorem, the hypotenuse squared is the sum of the other two sides squared. ASK: What is the equation for this triangle? \( 8^2 = 4^2 + r^2 \)

Write on the board:

\[
8^2 = 4^2 + r^2
\]

ASK: What is \(8^2\)? (64) What is \(4^2\)? (16) Continue writing on the board:

\[
64 = 16 + r^2
\]

ASK: How do we isolate for \(r^2\)? (move 16 to the left side) Continue writing on the board:

\[
64 - 16 = r^2
\]

\[
48 = r^2
\]

ASK: How do we find \(r\)? (find the square root) Write on the board:

\[
r \approx 6.9
\]

SAY: Remember that when you write the Pythagorean Theorem, you must write the equation so that the square of the hypotenuse is the sum of the other sides squared.

**Exercises:** Find the missing side to the nearest tenth.

a) \( t \) 

b) \( s \) 

c) \( n \) 

**Answers:** a) 6, b) 5.2, c) 6.9

**(MP.6) Finding missing sides when one side is a radical.** Write on the board:

\[
\sqrt{16}
\]

ASK: What is the square root of 16? (4) If we square 4, what do we get? (16) Write on the board:

\[
(\sqrt{16})^2 = (4)^2 = 16
\]

SAY: Notice that if we start with 16, find the square root of 16, then square the answer, we get back to 16. ASK: Why does that work? (because finding the square root and squaring are opposites)
Exercises: Without a calculator, evaluate the expression.

a) \((\sqrt{25})^2\)  
b) \((\sqrt{36})^2\)  
c) \((\sqrt{81})^2\)  

Bonus: \((\sqrt{11})^2\)

Answers: a) 25, b) 36, c) 81, Bonus: 11

SAY: Notice that for \((\sqrt{11})^2\), we could have tried to find \(\sqrt{11}\) as a decimal, but \(\sqrt{11}\) as a decimal goes on forever. We can approximate it, but if we round off the decimal and square it, we might get 10.999999. If we were able to keep all the decimals, and then square it, we would get back to exactly 11.

Draw on the board:

![Diagram]

SAY: In this triangle, one of the sides has the length expressed as the root of 12, so we can use what we have learned about squaring roots here. ASK: Which side is the hypotenuse? (c) How do we write the Pythagorean Theorem for this triangle? \(c^2 = (\sqrt{12})^2 + 5^2\) 
What is \((\sqrt{12})^2\)? (12) What is \(5^2\)? (25) Write on the board:

\[c^2 = 12 + 25\]
\[= 37\]

ASK: How do we find \(c\)? (find the square root) Write on the board:

\[c = \sqrt{37} \approx 6.1\]

Exercises: Find the length of the missing side to the nearest tenth.

a) 

![Diagram]

Answers: a) 3, b) 6.2, c) 22
Extensions

1. Find the value of $y$.

   a) \[
   \sqrt{y + 4} \]
   b) \[
   8 \sqrt{y - 3}
   \]
   c) \[
   \sqrt{y + 2} \]

   Answers: a) 21, b) 228, c) 49

(MP.7) 2. Find the area of an equilateral triangle with 10 cm sides to the nearest tenth of a square centimeter.

   Answer:

   \[
   h^2 + 5^2 = 10^2
   \]
   \[
   = 75
   \]
   \[
   h \approx 8.7
   \]
   \[
   A = 10 \times 8.7 \div 2 = 43.5 \text{ cm}^2
   \]

(MP.5) 3. a) Draw a triangle with two 4 cm sides and a right angle. Use any tools you think will help.

     b) Compare your triangle with a partner’s triangle. Are your triangles congruent? Use any tool you think will help you decide.

   Sample answers:

   a) • I used a compass, ruler, and a set square to draw the triangle. I drew the right angle between the two 4 cm sides. I tried drawing only one of the 4 cm sides adjacent to the right angle, but the side opposite the right angle was always longer than 4 cm, no matter how short I made the third side. That makes sense because the side opposite the right angle in a right triangle is always the longest side.

     • I drew the triangle on centimeter grid paper, because grid paper already has right angles on it. I know that the right angle has to be between the two 4 cm sides, because the side opposite the right angle has to be longer than either of the other two sides.

   b) • We used tracing paper, a pencil, and scissors. We traced my copy, cut it out, and rotated and flipped it to see that it fit exactly over my partner’s triangle. They are congruent.

     • We measured all the side lengths and angles using a ruler and protractor, and found that all the corresponding sides and angles are equal, so the triangles are congruent.

     • We used a transparency and a marker. We put the transparency on top of my triangle, traced it with the marker, and then moved the transparency over to my partner’s triangle. We rotated and flipped the transparency and it fit exactly, so the triangles are congruent.
G8-44 Proving the Pythagorean Theorem

Pages 124–125

Standards: 8.G.B.6

Goals:
Students will develop a proof for the Pythagorean Theorem using algebra.
Students will use a hands-on proof using dissection and prove the theorem with dynamic geometry software.

Prior Knowledge Required:
Can isolate for a variable in an equation
Can find the area of a triangle and rectangle

Vocabulary: hypotenuse, proof, Pythagorean Theorem

Materials:
transparency of BLM Proving the Pythagorean Theorem (pp. O-69–70)
overhead projector
erasable marker
BLM Proving the Pythagorean Theorem by Dissection (p. O-71)
plain white paper
scissors
BLM The Geometer’s Sketchpad® Pythagorean Theorem (pp. O-72–73)
The Geometer’s Sketchpad®

NOTE: To prove the Pythagorean Theorem in this lesson, you and students will work through the proofs that are on the BLMs and AP Book pages together. First, students will prove the theorem using unit measures—that is, they will use a right angle with sides of 3, 4, and 5 to prove that \(5^2 = 3^2 + 4^2\). Then they will work through a proof using algebra, thus proving that \(c^2 = a^2 + b^2\). Later in the lesson, students also prove the theorem by dissection (specifically, by cutting out and re-arranging parts of a BLM) and using dynamic geometry software. The extensions provide an additional option for proving the theorem.

(MP.3) Proving that for a 3-4-5 right triangle, \(5^2 = 3^2 + 4^2\). Project the transparency of BLM Proving the Pythagorean Theorem (1) onto the board. SAY: Let’s work through the proof together. Follow the model on the BLM and write your answers on AP Book pp. 124–125.

Point to Figure 1 on the projected BLM, which duplicates Figure 1 on AP Book 8.2 p. 124.
ASK: What is the width of square P? (3) Count the squares to show how you found the answer. Mark the width on the diagram. ASK: What is the width of square Q? (4) Count the squares and mark the width on the diagram. Have students complete Questions 1.a–b) on AP Book 8.2 p. 124.
ASK: What are the length and width of rectangle R? (4, 3) Count the squares on the BLM. Then trace the outline of rectangle S so that you clearly include the entire rectangle bisected by line $c$ and with the dashed outline. ASK: What are the length and width of rectangle S? (3, 4) Count the squares on the BLM for rectangle S. ASK: Do rectangles R and S have the same area? (yes) How do you know? (count the squares, multiply $4 \times 3$ and $3 \times 4$, or they have the same shape) Have students complete Question 1.c) on AP Book 8.2 p. 124.

ASK: How do you find the area of a square? (width $\times$ width—in other words, the width squared) What is the width of square P? (3) How do we calculate the area of square P? ($3^2$) What is the width of square Q? (4) How do we calculate its area? ($4^2$) How do we calculate the area of a rectangle? (multiply the length and width) How do we calculate the areas of rectangles R and S? ($3 \times 4$ or $4 \times 3$ in each case) Have students fill in the first three lines of Question 1.d) on AP Book 8.2 p. 124.

Point to the shape with the dashed outline in Figure 1—in other words, the combined areas of P, Q, R, and S. ASK: How can we calculate the total area of the square with the dashed outline using P, Q, R, and S? (add them up) Write on the board:

\[
\text{Area of square with dashed outline} = 3^2 + 4^2 + 3 \times 4 + 4 \times 3
\]

Have students fill in the last line of Question 1.d) on AP Book 8.2 p. 124.

SAY: Now we are going to calculate the area of the slanted square that has one side equal to the hypotenuse of right triangle Z. Point to Figure 2 on the projected BLM, which duplicates Figure 2 on AP Book 8.2 p. 124.

Point to triangle Z in Figure 2 and trace the three sides. ASK: What are the base and height of triangle Z? (triangle Z’s base and height are the same as rectangle S’s length and width in Figure 1; they are 4, 3) SAY: Look at triangle X in Figure 2. ASK: What do you notice about the triangle X’s base and height? (triangle X’s base and height are the same as rectangle S’s length and width in Figure 1; they are 4, 3) If we put triangles Z and X together, what shape does it make? (a rectangle) What is the length and width of the rectangle formed by Z and X? (4, 3) How do we calculate the area of this rectangle? ($4 \times 3$) SAY: Look at triangles W and Y. If we put them together, what shape do they make? (a rectangle) How do we calculate the area of this rectangle? ($3 \times 4$)

Point to the slanted square T in Figure 2. ASK: What variable can we use for the length and width of the slanted square? (c) How do we calculate the area of this slanted square? ($c \times c$ or $c^2$)

Point to the shape with the dashed outline—in other words, the combined area of Z, X, W, Y, and T—and SAY: Look at the square with the dashed outline. Point to the square and trace its
outline. ASK: How can we find the total area of this square using the triangles Z, X, W, Y, and the square T? (add them up) Write on the board:

\[
\text{Area of dashed square} = Z + X + W + Y + T
\]

SAY: Earlier we noted that \(Z + X\) made a 4 by 3 rectangle, and \(W + Y\) made a 3 by 4 rectangle. We also calculated the area of \(T\) as \(c^2\). So we can write the total area of the dashed square like this. Write on the board:

\[
\text{Area of dashed square} = 4 \times 3 + 3 \times 4 + c^2
\]

Have students complete **Question 2** on AP Book 8.2 p. 124.

Return to Figure 1 on the projected BLM and point to the square with the dashed outline. Ask students to count the squares to find the length and width of the square. \((7, 7)\) Then return to Figure 2 on the BLM and point to the square with the dashed outline. Ask students to count the squares to find the length and width of the square with the dashed outline in Figure 2. \((7, 7)\) Have students complete **Questions 3.a–b** on AP Book 8.2 p. 124. ASK: What can you say about the areas of the two dashed squares? (they are the same; they are equal) Write on the board:

\[
\frac{4 \times 3 + 3 \times 4 + c^2}{\text{Area of 2nd square}} \quad \frac{3^2 + 4^2 + 3 \times 4 + 4 \times 3}{\text{Area of 1st square}}
\]

SAY: From both sides of the equation, we can subtract \(3 \times 4\) and \(4 \times 3\). Cross out “\(3 \times 4\)” and “\(4 \times 3\)” on each side of the equation, as shown below:

\[
4 \times 3 + 3 \times 4 + c^2 = 3^2 + 4^2 + 3 \times 4 + 4 \times 3
\]

SAY: So \(c^2 = 3^2 + 4^2\)

Have students complete **Questions 3.c–d** on AP Book 8.2 p. 124.

**MP.3** Proving the Pythagorean Theorem with algebra. Project the transparency of BLM Proving the Pythagorean Theorem (2) onto the board. Point to Figure 3, which duplicates Figure 3 on AP Book 8.2 p. 125. SAY: In Figure 1, we counted the squares to find the length and width of squares P and Q. In Figure 3, each square does not represent one unit so we must label the sides and work out our answers with algebra. In Figure 3, we labeled the width of square P with the variable \(a\) and the width of square Q, with the variable \(b\). Point to the horizontal measure of P and ASK: What is the length of square P? \((a)\) Mark the length of square P on the display with another variable \(a\). Point to the vertical measure of square Q and
ASK: What is the length of square Q? (b) Mark the length of square Q on the display with another variable \( b \), as shown below:

![Diagram]

Point to the rectangle R and ASK: What variables can we use for the length and width of rectangle R? (\( b, a \)) Trace the outline of the rectangle marked as S and ASK: What variables can we use for the length and width of rectangle S? (\( a, b \)) Do rectangles R and S have the same area? (yes) How do you know? (they have the same length and width—\( a, b \)) What is the area of each S and R? (\( a \times b \)) What is the area of square P? (\( a \times a = a^2 \)) What is the area of square Q? (\( b \times b = b^2 \)) Point to the larger square with a dashed outline—in other words, the area combining P, Q, R, and S. ASK: How can we find the area of this square using the squares P, Q, R, and S? (add the area of those squares) What is an expression for the total area of the square with the dashed outline? (\( a^2 + b^2 + a \times b + b \times a \)) Have students complete Question 4 on AP Book 8.2 p. 125.

Point to Figure 4 on the projected BLM, which duplicates Figure 4 on AP Book 8.2 p. 125.

Point to the square with the dashed outline. SAY: This square is made of four triangles (W, X, Y, and Z) and a slanted smaller square labeled T. ASK: What variables can we use for the base and height of triangle Z? (\( b, a \)) If we place triangles X and Z together, what shape does it make? (a rectangle) What expression can we use for the area of the rectangle formed from triangles X and Z? (\( a \times b \)) SAY: Triangles W and Y are rotations of triangle Z. ASK: What variables can we use for the base and height of each triangle W and Y? (\( b, a \)) If we place triangles W and Y together, what shape does it make? (a rectangle) What expression can we use for the area of the rectangle formed from W and Y? (\( a \times b \)) What variable represents the width of the slanted square T? (\( c \)) What expression can we use for the area of the slanted square T? (\( c \times c \), or \( c^2 \)) Point to the area with the dashed outline again and ASK: How can we find the total area of the square with the dashed outline from the triangles W, X, Y, Z, and the smaller square T? (add them up) What expression can we use for the total area of the square with the dashed outline? (\( c^2 + a \times b + a \times b \)) Have students complete Question 5 on AP Book 8.2 p. 125.

Return to Figure 3 on the projected BLM and point to the square with the dashed outline. Point to the variable \( a \) that marks the width of the square P, then point to the variable \( b \) that marks the width of the square Q. ASK: What expression represents the length and width of the square with the dashed outline in Figure 3? (\( a + b \)) Return to Figure 4 on the projected BLM and point to the variable \( b \) that shows the base of triangle Z. SAY: Triangle Y has base \( a \). ASK: What expression can we use for the total length and width of the square with the dashed outline in Figure 4?
(a + b) SAY: So the two squares with dashed outlines in Figures 3 and 4 have the same width and length measurements: a + b. ASK: What can we say about their areas? (they are the same) SAY: But we used the expressions \(a^2 + b^2 + a \times b + a \times b\) for the area of the outlined square in Figure 3 and \(c^2 + a \times b + a \times b\) for the area of the outlined square in Figure 4. Write on the board:

\[a^2 + b^2 + a \times b + a \times b = c^2 + a \times b + a \times b\]

SAY: If we subtract \(a \times b + a \times b\) from both sides, what is left? \((c^2 = a^2 + b^2)\) Write on the board:

\[c^2 = a^2 + b^2\]

Have students complete Question 6 on AP Book 8.2 p. 125.

**Activities 1–2**

1. Give each student BLM Proving the Pythagorean Theorem by Dissection and a sheet of plain white paper for tracing and cutting out shapes. Have students follow the instructions on the BLM. When students have solved the puzzle, point out that what they have done is shown that the area of the square aligned with the hypotenuse (the square with width \(c\)) is equal to the sum of the areas of the squares on the other two sides (the squares with widths \(a\) and \(b\)).

**Answer:** Rotate quadrilaterals 1, 2, 3, and 4 to fill the square with side \(b\). Move square 5 to fill the square with side \(a\).

2. Distribute BLM The Geometer’s Sketchpad® Pythagorean Theorem. Ask students to follow the instructions using The Geometer’s Sketchpad® to verify that the Pythagorean Theorem holds true.

*(end of activities)*
Extensions

1. Before he became the twentieth president of the United States, James A. Garfield discovered a proof of the Pythagorean Theorem that uses the area of a trapezoid. Remember, a trapezoid is a quadrilateral with one pair of opposite sides parallel. The formula for finding the area of a trapezoid is shown below:

\[ A = \frac{1}{2} h(b_1 + b_2) \]

In the formula, \( h \) is the distance between the parallel lines, and \( b_1 \) and \( b_2 \) are the lengths of the parallel sides, or bases, as shown below:

![Trapezoid Diagram](image)

a) Find the area of the trapezoid.
   i) \[ \text{2 cm} \]
   ii) \[ \text{8 in} \]

b) Follow the steps to show for a right triangle with sides 3 and 4, and hypotenuse \( c \), that \( c^2 = 3^2 + 4^2 \).

i) What are the lengths of \( b_1 \) and \( b_2 \) for the trapezoid?
ii) What numbers can be added to find the height of the trapezoid?
iii) Fill in the blanks for the first line of finding the area of the trapezoid using the formula.

\[
\frac{1}{2} h(b_1 + b_2) = \frac{1}{2} (\_ + \_)(\_ + \_)
\]

iv) Fill in the blanks to find the area of each of the three triangles that make up the trapezoid.

\[
\frac{1}{2} (\_)(\_) + \frac{1}{2} (\_)(\_) + \frac{1}{2} (\_)(\_)
\]

v) The area of the trapezoid must be the same as the sum of the areas of the triangles. Use your answers to iii) and iv) to write an equation.

vi) Multiply all the terms on both sides of the equation by 2.
vii) The distributive property allows us to write the right side of the equation in answer vi) as:

\[(3 + 4)(3 + 4) = (3 + 4)(3) + (3 + 4)(4)\]

Use this and your answer to vi) to show that \(c^2 = 3^2 + 4^2\).

**Answers:**

a) i) \(A = \frac{1}{2}(3)(2 + 4) = 9 \text{ cm}^2\), ii) \(A = \frac{1}{2}(5)(10 + 8) = 45 \text{ in}^2\)

b) i) 3, 4; ii) 3, 4; iii) \(\frac{1}{2}(3 + 4)(3 + 4)\); iv) \(\frac{1}{2}(3)(4) + \frac{1}{2}(3)(4) + \frac{1}{2}(c)(c)\)

v) \(\frac{1}{2}c^2 + \frac{1}{2}(3)(4) + \frac{1}{2}(3)(4) = \frac{1}{2}(3 + 4)(3 + 4)\); vi) \(c^2 + 3(4) + 3(4) = (3 + 4)(3 + 4)\)

vii) \(c^2 + 3(4) + 3(4) = (3 + 4)(3 + 4) = (3)(3) + (4)(3) + (3)(4) + (4)(4)\). Subtract 3(4) + 3(4) from both sides: \(c^2 = (3)(3) + (4)(4) = 3^2 + 4^2\).

2. The Pythagorean Theorem states that, for a right triangle, the area of the square on the hypotenuse is the sum of the areas of the squares on the other two sides. Is the same statement true if you replace squares with equilateral triangles? Check using The Geometer’s Sketchpad®.

(MP.7) 3. Solve the equation \(x^2 = 30 \frac{1}{4}\). Do not use a calculator.

**Sample answers:**

• I wrote 30 1/4 as 30.25, and then I tried finding the square root. I know it’s between 5 and 6 because \(5^2 = 25\) and \(6^2 = 36\). I tried 5.4 because 30.25 is a little closer to 25 than to 36, but \(5.4 \times 5.4 = 29.26\), then I tried 5.5 \(\times 5.5 = 30.25\), so \(x = 5.5\) or \(-5.5\).

• I wrote \(30 \frac{1}{4} = \frac{121}{4}\), and I noticed that \(\frac{121}{4} = \left(\frac{11}{2}\right)^2\), so \(x = \pm \frac{11}{2}\).

Whole-class follow-up: Have different volunteers present their solutions. When both solutions above have been presented, ASK: Which way is faster? (changing to an improper fraction) Would it be faster for all numbers or was there something special that made it fast for these numbers? (121 and 4 are both perfect squares; it wouldn’t have worked out as well for 30 \(\frac{1}{2} = 61/2\), because neither 61 nor 2 are perfect squares)

(MP.3, MP.8) 4. a) To find \(\frac{7}{15}\), Kathy divides \(\frac{7.000}{15}\) \[\begin{array}{ccc}
-6 & 0 \\
1 & 0 & 0 \\
-9 & 0 \\
1 & 0 & 0 \\
-9 & 0 \\
1 & 0 & 0
\end{array}\]

She stops dividing and writes \(\frac{7}{15} = 0.4\bar{6}\). How does she know the 6 will keep repeating?

**Answer:** The 6 will always repeat because 15 \(\times 6\) will always be 90, when you subtract 90 from 100 you always get 10, and when you bring down the 0 you always get 100.
The Converse of the Pythagorean Theorem

Pages 126–128

Standards: 8.G.B.6

Goals:
Students will develop a proof that shows that if \( c^2 = a^2 + b^2 \), then the triangle is a right triangle.

Prior Knowledge Required:
Knows the differences between an acute, an obtuse, and a right angle
Can correctly use the symbols <, >, and =
Can use the Pythagorean Theorem to find missing sides in a right triangle

Vocabulary: acute, converse, obtuse, proof by contradiction, Pythagorean Theorem, right

(MP.3) Determining whether \( c^2 \) is less than, greater than, or equal to \( a^2 + b^2 \). Draw the following three triangles to scale on the board—label the sides (\( a, b, c \)) but omit the measurements of the sides from your drawing:

Point to each of the triangles, and ask whether angle \( C \) is acute, right, or obtuse. PROMPT: Is the angle greater than, equal to, or less than 90°? (acute, right, obtuse)

Ask students to measure the lengths of the sides of the triangles to the nearest tenth of a centimeter. Draw the following chart on the board, and ask students to fill in the blanks for the first six columns, as shown below, in italics:

<table>
<thead>
<tr>
<th></th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
<th>( a^2 )</th>
<th>( b^2 )</th>
<th>( c^2 )</th>
<th>( c^2 ) &gt; ( a^2 + b^2 )</th>
<th>( \text{Is } \angle C \text{ acute, right, or obtuse?} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle 1</td>
<td>20</td>
<td>15</td>
<td>18</td>
<td>400</td>
<td>225</td>
<td>324</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangle 2</td>
<td>20</td>
<td>15</td>
<td>25</td>
<td>400</td>
<td>225</td>
<td>625</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangle 3</td>
<td>20</td>
<td>15</td>
<td>30.4</td>
<td>400</td>
<td>225</td>
<td>924.16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ASK: What is the value of \( a^2 + b^2 \) in each case? (625) Fill in the "\( a^2 + b^2 \)" column. For each row, ask whether \( c^2 \) is less than, equal to, or greater than \( a^2 + b^2 \). (<, =, >) Fill in the "\( c^2 \) > \( a^2 + b^2 \)" column. Point to each triangle and ask whether \( \angle C \) is acute, right, or obtuse. (acute, right, obtuse) Fill in the last column.
ASK: Can you guess how we can predict whether \(\angle C\) is acute, right, or obtuse, just by looking at \(c^2\) and \(a^2 + b^2\)? (if \(c^2 < a^2 + b^2\), \(\angle C\) is acute; if \(c^2 = a^2 + b^2\), \(\angle C\) is right; if \(c^2 > a^2 + b^2\), \(\angle C\) is obtuse)

Have students test the conjecture by completing Question 1 on AP Book 8.2 pp. 126–127. Then ask them to fill in the answers to Question 2 on p. 127.

(MP.3) **Explain the method of labeling sides and vertices in a triangle.** Draw on the board:

![Diagram of a triangle labeled A, B, and C with sides a, b, and c]

SAY: Mathematicians label the vertices of triangles with capital letters and the sides of the triangles with the non-capitalized, small letters. Here we used capitals \(A\), \(B\), and \(C\) for the vertices, so we will use small letters \(a\), \(b\), and \(c\) to label the sides. However, we must label the sides in a particular way: we must label any side with the small letter that corresponds to the capital letter for the vertex opposite. Draw the arrow below on the board and label the side with the letter \(a\), as shown below:

![Diagram showing vertex A labeled with side a]

SAY: Notice the small letter \(a\) is opposite vertex \(A\), but between vertices \(B\) and \(C\). Ask for volunteers to label the other sides in the diagram with the corresponding small letters. (see diagram below)

![Diagram showing vertices A, B, and C labeled with sides a, b, and c]

(MP.2) **Comparing \(c^2\) with \(a^2 + b^2\) to find whether \(\angle C\) is acute, right, or obtuse.**

SAY: Earlier, we compared \(c^2\) with \(a^2 + b^2\) to find out whether \(\angle C\) is acute, right, or obtuse. Summarize the results using your arms and chest to represent the sides and vertices of a triangle. SAY: My arms represent sides \(a\) and \(b\) of a triangle. Point to your upper chest and SAY: Here is vertex \(C\), where sides \(a\) and \(b\) meet. Place your arms so that \(\angle C\) is acute, as shown in the diagram below. ASK: How can I imagine a line to represent side \(c\)? (join a line between my hands) SAY: Here we have \(c^2 < a^2 + b^2\). (see diagram below)

![Diagram showing vertex C with sides a and c]

O-42 Teacher Resource for Grade 8 — Unit 4 Geometry
Now arrange your arms so that one arm is vertical and the other is horizontal, as shown in the
diagram below. SAY: Here we have $c^2 = a^2 + b^2$.

![Diagram](image1)

Finally, arrange your arms so that one arm points out to the left and the other to the right, as
shown in the diagram below. SAY: Here we have $c^2 > a^2 + b^2$.

![Diagram](image2)

Draw on the board:

![Diagram](image3)

Ask a student to label the sides of the triangles with the appropriate small letters. ASK: What is
the value of $a$? (6) of $b$? (10) of $c$? (5.2) What is the value of $a^2$? (36) of $b^2$? (100) of $c^2$? (27.04)
What is the value of $a^2 + b^2$? (136) SAY: Now compare $c^2$ with $a^2 + b^2$. ASK: Is $c^2$ less than,
equal to, or greater than $a^2 + b^2$? (less than) Since $c^2 < a^2 + b^2$, what do we know about $\angle C$?
(it is acute)

SAY: It is important to remember that the vertices of the triangles will not always be labeled $A$,
$B$, and $C$. However, we can still determine whether an angle is acute, right, or obtuse just by
changing the letters of the sides and angles to $a$, $b$, $c$, and $A$, $B$, $C$. Write on the board:

If $c^2 < a^2 + b^2$, we know that $\angle C$ is acute.

SAY: Or we can change the corresponding letters to come up with a comparison for different
triangles but still using the Pythagorean Theorem to compare the angles. Suppose the vertices
of the triangles were labeled $D$, $E$, and $F$ instead of $A$, $B$, and $C$. Write on the board:

If $f^2 < d^2 + e^2$, we know that $\angle ____ is acute.
If $m^2 > n^2 + p^2$, we know that $\angle ____ is obtuse.
If $w^2 = x^2 + y^2$, we know that $\angle ____ is right.
Ask for a student to fill in the blanks. \((F, M, W)\) PROMPT: In the first inequality above, \(f^2 < d^2 + e^2\), which variable was by itself in the inequality? \(f\) So which angle can we gather information about? \(\angle F\)

**Exercise:** For the triangle, calculate whether the given angle is acute, right, or obtuse.

a) \(\angle D\)

![Diagram](image1)

b) \(\angle N\)

![Diagram](image2)

c) \(\angle Q\)

![Diagram](image3)

**Answers:**

a) \(d^2 = 14.6^2 = 213.16\)

\[e^2 + f^2 = 10^2 + 8^2 = 164\]

Since \(d^2 > e^2 + f^2\),

\(\angle D\) is obtuse

b) \(n^2 = 1.7^2 = 2.89\)

\[m^2 + p^2 = 0.8^2 + 1.5^2 = 2.89\]

Since \(n^2 = m^2 + p^2\),

\(\angle N\) is a right angle

c) \(q^2 = 6.5^2 = 42.25\)

\[x^2 + j^2 = 13^2 + 15^2 = 394\]

Since \(q^2 < x^2 + j^2\),

\(\angle Q\) is acute

**Converse of the Isosceles Triangle Theorem**

If a triangle has two equal sides then the angles opposite the equal sides are equal.

Point to the triangle on the left and SAY: If we are given that two sides in a triangle are equal, then the Isosceles Triangle Theorem tells us that the angles opposite the equal sides are equal.

SAY: For the Converse of the Isosceles Triangle Theorem, we must switch what we are given and what we are proving. Tell students that you are switching the diagram, and draw on the board:

If a triangle has two equal angles then the sides opposite the equal angles are equal.
Read the theorem aloud, then SAY: We have spent time proving the Pythagorean Theorem. Write on the board:

**Pythagorean Theorem**

\[
\text{If a triangle is right-angled then } \quad c^2 = a^2 + b^2
\]

Read the theorem aloud, then SAY: For the converse of the theorem, we have to switch what we are given with what we are proving. ASK: What do you think the Converse of the Pythagorean Theorem says? (if \(c^2 = a^2 + b^2\), then the triangle is right-angled) Write on the board:

**Converse of the Pythagorean Theorem**

\[
\text{If } \quad c^2 = a^2 + b^2 \quad \text{then} \quad \text{the triangle is right-angled.}
\]

(MP.3) Proof by contradiction. SAY: Let’s work through the proof of the Converse of the Pythagorean Theorem together. Follow along with Question 4 on AP Book p. 128. We are going to prove this theorem by contradiction. In a proof by contradiction, we start by assuming the opposite of the theorem. For example, there is a theorem that a triangle can have at most one right angle. So we will assume that a triangle can have more than one right angle. Suppose there is a triangle with two right angles. Draw on the board:

Say: We know that the sum of the angles in a triangle is always 180°. Write on the board:

\[
\angle A + \angle B + \angle C = 90° + 90° + \angle C
\]

\[
180° = 180° + \angle C
\]

ASK: What value of \(\angle C\) would make this equation true? (0°) But if \(\angle C\) is 0°, how many sides would the triangle have? (two) SAY: But a triangle has to have three sides, so our original idea that a triangle could have two right angles must be wrong! So, it is not possible for a triangle to
have more than one right angle. In our proof by contradiction, we have proven that a triangle can have at most one right angle.

(MP.3) Proving the Converse of the Theorem of Pythagoras by contradiction. SAY: Let’s assume that it is possible for a triangle that is not right-angled to have $c^2 = a^2 + b^2$. Draw on the board:

![Diagram of a triangle with sides labeled $a$, $b$, and $c$.]

Ask a volunteer to draw a line segment through the point $C$ that is perpendicular to $AC$ and the same length as $CB$. Label the endpoint of the new line segment $D$. Have another volunteer join $D$ to $A$ to make the new triangle $DCA$. SAY: Since $CD$ is the same length as $CB$, we can label $CD$ as $a$. Let’s label $AD$ as $y$. The drawing should look like this:

![Diagram with points labeled $A$, $B$, $C$, and $D$.]

ASK: Since we drew $DC$ perpendicular to $AC$, what is the measurement of $\angle ACD$? (90°) What type of triangle is $\triangle ACD$? (a right triangle) What does the Pythagorean Theorem tell us about $y^2$? ($y^2 = a^2 + b^2$) Write on the board:

$$c^2 = a^2 + b^2$$

and

$$y^2 = a^2 + b^2$$

SAY: Look at the two equations. ASK: What do we now know about $c^2$ and $y^2$? (they are equal because both are equal to $a^2 + b^2$) Write on the board:

$$c^2 = y^2$$

ASK: If $c^2 = y^2$, what does that tell us about $c$ and $y$? (they are equal) Write on the board:

$$c = y$$

ASK: If we look at $\triangle ACD$ and $\triangle ACB$, what do we know about the lengths of the corresponding sides? (they are equal; $a = a$, $b = b$, $c = y$) SAY: Think back to congruence rules about triangles. ASK: Which congruence rule allows us to say that the triangles are congruent? (side, side,
side—SSS) SAY: If two triangles are congruent, they have exactly the same shape and size. Their corresponding sides are equal in length and their corresponding angles are equal. ASK: What angle in \( \triangle ACB \) is equal to \( \angle ACD \)? (\( \angle ACB \)) SAY: But we drew \( \angle ACD = 90^\circ \). So \( \angle ACB \) is 90°.

SAY: When we started, we said that \( \angle ACB \) was not 90°. This is a contradiction. So our assumption that a triangle that is not right-angled can have \( c^2 = a^2 + b^2 \) was false! If \( c^2 = a^2 + b^2 \), then the triangle is right-angled.

Have students complete Question 4 on AP Book 8.2 p. 128.

**Exercises:** Check if \( c^2 = a^2 + b^2 \) to determine whether the triangle is a right triangle.

   a) \[ \begin{array}{c}
   C \\
   9.9 \\
   B
   \end{array} \quad \begin{array}{c}
   A \\
   8.0 \\
   \end{array} \]

   b) \[ \begin{array}{c}
   A \\
   1.3 \\
   B
   \end{array} \quad \begin{array}{c}
   C \\
   0.5 \\
   \end{array} \]

   c) \( a = 4, b = 7.5, c = 8.5 \)

**Answers:** a) \( 9.9^2 \neq 5.8^2 + 8.0^2 \), so the triangle is not a right triangle; b) \( 0.5^2 + 1.2^2 = 1.3^2 \), so the triangle is a right triangle; c) \( 4^2 + 7.5^2 = 8.5^2 \), so the triangle is a right triangle

**Extensions**

(MP.3) 1. Prove by contradiction that if \( y = \frac{x + 3}{x + 2} \), where \( x \neq -2 \), then \( y \neq 1 \).

**Answer:** Assume that \( y = 1 \). Then \( 1 = \frac{x + 3}{x + 2} \). Multiplying both sides by \( x + 2 \) gives \( x + 2 = x + 3 \). Subtracting \( x \) from both sides gives \( 2 = 3 \). This is a contradiction. So our original assumption that \( y = 1 \) must have been false. So, \( y \neq 1 \).

2. On the Island of Math, the inhabitants either always tell the truth, or they always lie. A man there says, “I am a liar.” Use a proof by contradiction to show that man is not an inhabitant of the Island of Math.

**Answer:** Assume the speaker is an inhabitant of the Island of Math. Since he is an inhabitant of the island, he either always tells the truth or he always lies. If he always tells the truth, when he says “I am a liar,” he would be lying. But this would be a contradiction, since he cannot be both a liar and a truth teller when saying “I am a liar.” If he always lies and he says “I am a liar,” he would be telling the truth. Again, this would be a contradiction, since he cannot be both a liar and a truth teller when saying “I am a liar.” So our original assumption that the particular man is an inhabitant of the island must be false.
Goals:
Students will use the Pythagorean Theorem to solve word problems involving one right triangle.

Prior Knowledge Required:
Can identify the hypotenuse in a right triangle
Can apply the Pythagorean Theorem to find a missing side in a right triangle

Vocabulary: hypotenuse, Pythagorean Theorem

(MP.3) Review finding the hypotenuse in a right triangle. SAY: It is important when using the Pythagorean Theorem to first identify the hypotenuse. ASK: What are two ways to help identify the hypotenuse? (the longest side, the side opposite the right angle) Draw on the board:

Ask for volunteers to label or mark the hypotenuse. For each triangle, point out that the hypotenuse is the longest side and it is opposite the right angle. (see answers below)

(MP.3) Review correctly writing the formula for the Pythagorean Theorem. SAY: Remember, the formula for the Pythagorean Theorem is often written as $c^2 = a^2 + b^2$, but different letters can be used. To use the formula correctly, the variable for the hypotenuse should be on the left side of the equation, all alone. On the right side of the formula is the sum of the squares on the other two sides of the triangle. Draw on the board:
Remind students that we use the same letter for the side and the vertex opposite that side, but the small letter for the side and the capital for the vertex. Ask for a volunteer to come to the board and label the sides of the triangle. (see answer below)

![Diagram of a triangle with sides labeled](image)

**ASK:** Which variable is used for the hypotenuse? *(g)* **SAY:** So the formula for the Pythagorean Theorem must start with “*g*² = .” **ASK:** What should we put on the other side of the equation? *(f² + h²)* Write on the board:

\[ g^2 = f^2 + h^2 \]

**MP.3** **Solving word problems involving the Pythagorean Theorem.** **SAY:** We can find the missing side in a right triangle using the Pythagorean Theorem. We have to know that it is a right triangle, and we have to be given the lengths of two of the sides. Write and draw on the board:

A carpenter needs to build the framework for a wall that is 6 feet by 8 feet. Carpenters use a trick to measure the diagonals of the wall to check the angles. If the diagonals are the same length, carpenters know all the angles in the rectangle are exactly 90°.

The carpenter builds the frame and checks that the diagonals are indeed the same length. How long are the diagonals?

**ASK:** Is there a right triangle in this word problem? *(yes)* Ask a volunteer to outline one right triangle—either of the two larger triangles will do. Ask a second volunteer to label the vertices of the triangle with capital letters of their own choosing. Ask a third volunteer to label the sides of the triangle with corresponding letters. (see sample answer below)

![Diagram of a triangle with sides labeled](image)

**ASK:** In this case, what is the value of *g* and *b*? *(6 and 8)* Which variable is unknown? *(f)* What is the equation for the Pythagorean Theorem in this diagram? *(f² = b² + g²)* Write on the board:

\[ f^2 = b^2 + g^2 \]

\[ = 8^2 + 6^2 \]
ASK: What is $8^2$? (64) What is $6^2$? (36) What is $64 + 36$? (100) Continue writing on the board:

$$= 64 + 36$$
$$= 100$$

ASK: How do we find $f$? (find the square root) What is the square root of 100? (10) Write on the board:

$$f = 10$$

SAY: So the length of the diagonal is 10 ft.

SAY: Here is another example, but this time we know the length of the hypotenuse. Write and draw on the board:

A 15-foot ladder leans against a wall so that the base of the ladder is 8 feet from the base of the wall. To the nearest tenth of a foot, how high above the ground is the top of the ladder?

![Diagram of a right triangle with a ladder leaning against a wall. The ladder is 15 feet long, and one end is 8 feet from the base of the wall.]

Ask a volunteer to complete the right triangle, and label the vertices and sides with the appropriate letters. (see sample answer below)

ASK: Which sides are we given this time? What are their values? ($q = 15$, $r = 8$) Ask a volunteer to label the lengths of the known sides in the diagram. Which variable represents the hypotenuse? ($q$) Which variable is unknown? ($b$) How do we write the equation for the Pythagorean Theorem this time? ($q^2 = r^2 + b^2$) Write on the board:

$$q^2 = r^2 + b^2$$
$$15^2 = 8^2 + b^2$$

ASK: What is $15^2$? (225) What is $8^2$? (64) Continue writing on the board:

$$225 = 64 + b^2$$

ASK: How do we get $b^2$? (subtract 64 from 225) What is $225 - 64$? (161) Write on the board:

$$b^2 = 161$$
ASK: How do we find \( b \)? (find the square root) Write on the board:

\[
b = \sqrt{161} \approx 12.7
\]

SAY: So the top of the ladder is approximately 12.7 feet from the ground.

**Extensions**

(MP.4) 1. A dog is 64 feet away from a squirrel that is at the base of a tree. When the dog sees the squirrel, the dog starts running toward the tree at a speed of 8 ft/s. The squirrel immediately starts running up the tree at a speed of 4 ft/s. Three seconds later, how far apart are the dog and the squirrel to the nearest tenth of a foot?

**Solution:** The dog starts at \( D \). The squirrel starts at \( B \). In three seconds, the dog runs \( 3 \times 8 = 24 \) ft, so from 64 ft (\( D \)) to 40 ft from the base (\( E \)); the squirrel runs \( 3 \times 4 = 12 \) ft in the same time to \( S \).

Start of chase: \begin{align*}
\text{Three seconds later:} \\
&\begin{array}{c}
D & 64 \text{ ft} & B \\
E & 40 \text{ ft} & B \\
S & 12 \text{ ft} & \\
\end{array}
\end{align*}

\[
b^2 = 40^2 + 12^2 \\
= 1600 + 144 \\
= 1744 \\
b = \sqrt{1744} \approx 41.8 \text{ feet}
\]

2. Use The Geometer’s Sketchpad® to verify the following:

If a triangle is drawn inside a circle so that its longest side is the diameter of the circle and its third vertex is any point on the outside of the circle, the triangle will be a right triangle.

**Answer:** Students should produce a diagram similar the one shown below, then check that \( a^2 + b^2 = c^2 \), in which case the triangle they have drawn is a right triangle.

Sample solution:

\[
\begin{align*}
a &= 4.93400 \text{ cm} & a^2 &= 24.34438 \text{ cm}^2 \\
b &= 2.77957 \text{ cm} & b^2 &= 7.72601 \text{ cm}^2 \\
c &= 5.636307 \text{ cm} & c^2 &= 32.07039 \text{ cm}^2 \\
\end{align*}
\]

\[
a^2 + b^2 = 32.07039 \text{ cm}^2
\]
3. The diagram below shows the cross-section of a pipe with a 12-inch diameter that is partially filled with water. Find the depth of the water to the nearest hundredth of an inch.

**Solution:**

\[ d = 12 \text{ in, so } r = 6 \text{ in} \]
\[ x^2 + 2^2 = 6^2 \]
\[ x^2 + 4 = 36 \]
\[ x^2 = 36 - 4 \]
\[ x = \sqrt{32} \approx 5.66 \text{ in} \]

So, the depth of the water is \( 6 - 5.66 = 0.34 \) inches.

4. An ant is crawling along the walls inside a box with the dimensions shown in the diagram below. To the nearest tenth of a centimeter, what is the shortest distance the ant can crawl to get from point \( A \) to point \( B \)? How does drawing the net help you solve the problem?

**Solution:**

The shortest distance is the length of the hypotenuse in the dotted triangle.
\[ x^2 = 9^2 + 10^2 \]
\[ = 81 + 100 \]
\[ = 181 \]
\[ x = \sqrt{181} \approx 13.5 \text{ cm} \]

Drawing the net helped me see the distance on a flat surface.
Goals:
Students will use the Pythagorean Theorem to solve word problems involving two right triangles. Students will find the missing side in one right triangle, and then use the answer to help find a missing side in the other triangle.

Prior Knowledge Required:
Can identify the hypotenuse in a right triangle
Can apply the Pythagorean Theorem to find a missing side in a right triangle
Can solve word problems that require them to use the Pythagorean Theorem to find a missing side in a right triangle

Vocabulary: hypotenuse, Pythagorean Theorem

(MP.3) Solving problems involving two right triangles. SAY: Sometimes a problem will involve not just one but two right triangles. You will have to find the missing side in one triangle, and use that answer to help find a missing side in another right triangle.

SAY: Special wires called guy wires can be used to brace or anchor a building or a tower. Write on the board:

Mary and Tom attached guy wires to the top of a tower. Mary’s wire is 50 ft long. Tom’s wire is 78 ft long. If the tower is 30 ft tall, how far apart are the ends of Mary’s and Tom’s wires?

Draw the following diagram on the board, but without the measurements or labels for the vertices. Ask a volunteer to label the height of the tower, the lengths of the guy wires, and where each wire ends (M for Mary and T for Tom). Label the top and bottom of the tower as A and B. The final diagram should look like this:
ASK: Which triangles are right triangles? (\(\triangle ABM, \triangle ABT\)) In \(\triangle ABM\), which side is the hypotenuse? (AM) SAY: Let’s label BM as x. ASK: How can we use the Pythagorean Theorem to find x? (\(50^2 = 30^2 + x^2\)) Write on the board:

\[50^2 = 30^2 + x^2\]

ASK: What is 50\(^2\)? (2500) What is 30\(^2\)? (900) How can we find \(x^2\)? (subtract 900 from 2,500) Continue writing on the board:

\[
\begin{align*}
2,500 &= 900 + x^2 \\
2,500 - 900 &= x^2 \\
1,600 &= x^2 
\end{align*}
\]

ASK: How can we find x? (find the square root) What is the square root of 1,600? (40) Write on the board:

\[x = 40\]

Ask the class to use \(\triangle ABT\) to find the length of BT in the same way. Ask them to label BT as y. (y = 72) Label the distances x and y on the board, as shown below:

![Diagram of triangles and distances](image)

ASK: What line segment represents the distance between the ends of Mary’s and Tom’s wires? (MT) How can we find this distance? (y − x) What is 72 − 40? (32) SAY: So the ends of Mary’s and Tom’s wires are 32 feet apart.

**Extensions**

(MP.4) 1. The distance to Earth’s horizon is a measure of how far you can see along the surface of the planet. The radius of Earth is approximately 6,400 km. If your eyes are 1.8 m above the ground surface of Earth, then the distance to the horizon, D, is one side of a right triangle where the other two sides are the radius r and the radius + 1.8 m.

![Diagram of horizon distance](image)
a) Find the distance to the horizon (D) in the diagram to the nearest tenth of a kilometer.
b) How much farther can you see to the horizon if you are on top of a mountain so that your
eyes are 200 m above the ground?

**Solutions:**
a) \( D^2 + 6400^2 = 6400.0018^2 \)
\( D^2 = 40960023.04 - 40960000 \)
\( D = 4.8 \) km

b) \( D_1^2 + 6400^2 = 6400.200 \)
\( D_1^2 = 40962560.04 - 40960000 \)
\( D_1 = 50.6 \) km

So, when standing on the mountain, you can see 45.8 km farther (50.6 minus 4.8 km).

(MP.8) 2. Right triangles are drawn in a series in a counterclockwise direction. The first right
triangle has two sides of 1 cm. The hypotenuse of the first triangle becomes one of the sides of
the next triangle. Each of the following triangles has one side with length 1 cm.

Find the length of the hypotenuse in the twentieth triangle.

**Solution:**

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Calculation</th>
<th>Hypotenuse</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( x^2 = 1^2 + 1^2 )</td>
<td>( \sqrt{2} )</td>
</tr>
<tr>
<td></td>
<td>( x = \sqrt{2} )</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>( x^2 = 1^2 + (\sqrt{2})^2 )</td>
<td>( \sqrt{3} )</td>
</tr>
<tr>
<td></td>
<td>( x = 3 )</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( x^2 = 1^2 + (\sqrt{3})^2 )</td>
<td>( \sqrt{4} )</td>
</tr>
<tr>
<td></td>
<td>( x = \sqrt{4} )</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>( \sqrt{21} )</td>
</tr>
</tbody>
</table>
3. An air traffic controller detects an airplane at a height of 10 km, 4 km north and 5 km east of the control tower. How far is the plane from the tower’s base to the nearest tenth of a kilometer?

Solution:

\[ AC^2 = 4^2 + 5^2 \]
\[ AC^2 = 41 \]

\[ AD^2 = AC^2 + 10^2 \]
\[ AD^2 = 41 + 100 \]
\[ AD^2 = 141 \]
\[ AD = \sqrt{141} \]
\[ AD \approx 11.9 \text{ km} \]

Whole-class follow-up: Have volunteers show their solutions. Point out that \( AC = \sqrt{41} \), but they don’t need to calculate that because it is \( AC^2 \), not \( AC \), that they need to use in the calculation of \( AD \).

4. Find the length of \( AB \).

Solution:

\[ AD^2 = 4^2 + 1^2 \]
\[ AD^2 = 17 \]
\[ BD^2 = 2^2 + 3^2 \]
\[ BD^2 = 13 \]
\[ AB^2 = AD^2 + BD^2 \]
\[ AB^2 = 17 + 13 \]
\[ AB^2 = 30 \]
\[ AB = \sqrt{30} \]
\[ AB \approx 5.5 \]

MP.4 Assessment Opportunity: Question 7 on AP Book 8.2 p. 132
The Pythagorean Theorem in a Coordinate Grid

Goals:
Students will use the Pythagorean Theorem to find the distance between two points on a coordinate grid.

Prior Knowledge Required:
- Can find the distance between two points on a coordinate grid that are on the same horizontal line
- Can find the distance between two points on a coordinate grid that are on the same vertical line
- Can subtract integers
- Can find the absolute value of a number or expression
- Can use the Pythagorean Theorem to find the length of the missing side in a right triangle

Vocabulary: absolute value, change in $x$, change in $y$, coordinate grid, horizontal, hypotenuse, Pythagorean Theorem, vertical

(MP.3) Review finding the absolute value of a number. SAY: Remember: the absolute value of an integer is the number’s distance from zero on a number line. Draw on the board:

ASK: What is the distance between 0 and 3? (3) SAY: So the absolute value of 3 is 3.
ASK: What is the distance between 0 and −4? (4) SAY: So the absolute value of −4 is 4.
Remember that we use the notation for absolute value as follows. Write on the board:

$$|3| = 3 \quad |-4| = 4$$

Exercises: Evaluate the expression.
- a) $|5|$
- b) $|-7|$
- c) $|3 - 5|$
- d) $|4 - (-2)|$

Bonus: $|2 - 5| - |3 - 7|$

Answers: a) 5, b) 7, c) $|-2| = 2$, d) $|6| = 6$, Bonus: $|-3| - |-4| = |3 - 4| = |-1| = 1$
(MP.3) Review order of operations and absolute values. Write on the board:

\[ |3 - 5| \quad |3 - 5| \]

Encourage the class to discuss whether these two expressions are the same. After some time, SAY: Let’s look at the expression on the left. ASK: What is \(3 - 5\)? (-2) What is \(|-2|\)? (2) Write on the board underneath the first expression:

\[
= |-2|
\]
\[
= 2
\]

ASK: For the second expression, what is the absolute value of 3? (3) What is the absolute value of 5? (5) What is \(3 - 5\)? (-2) Write on the board underneath the second expression:

\[
= 3 - 5
\]
\[
= -2
\]

ASK: Are the expressions \(|3 - 5|\) and \(|3 - 5|\) equivalent? (no) SAY: It is a good idea to think of there being brackets around the expression with absolute value notation. You must completely evaluate the expression to which that notation is applied first, before doing any more operations. When you are done, then you can take the absolute value of the result.

Exercise: Evaluate.

a) \(|3 - (-5)| - |3 - 5|\)

b) \(|2 - |-5 - 2|\|

Answers: a) \(8 - 2 = 6\), b) \(|2 - 7| = |2 - 7| = |-5| = 5\)

(MP.3) Review finding the length of a horizontal or vertical line segment. Draw on the board:

ASk: What are the coordinates of points A and B? ((-3, 2) and (1, 2)) What do these coordinates have in common? (same y-coordinate) What is the change in x from A to B? (4)
ASK: How can we get this answer just by looking at the x-coordinates of A and B? (1 - (-3) = 4)
SAY: The distance is the absolute value of the change in x, so the distance from A to B is 4.
Write on the board:

Distance from A to B = \(|1 - (-3)| = |4| = 4\)
ASK: What are the coordinates of C and D? ((4, 2) and (4, −4)) What do these coordinates have in common? (same x-coordinate) What is the change in y from C to D? (−6) How can we get this answer just by looking at the y-coordinates of C and D? (2 − (−4) = −6) SAY: The distance from C to D is the absolute value of the change in y, so the distance from C to D is 6. Write on the board:

\[ \text{Distance from } C \text{ to } D = |−4 − 2| = |−6| = 6 \]

Project a grid on the board and draw several vertical line segments and several horizontal line segments on the grid on the board. Ask students to find the lengths of the line segments using the above method.

Exercises: Find the distance between the points.

a) A (3, −1) B (3, −5)
b) C (4, −2) D (−6, −2)
c) E (0, −8) F (0, −3)

Answers: a) \(|−5 − (−1)| = |−4| = 4 \), b) \(|−6 − 4| = |−10| = 10 \), c) \(|−3 − (−8)| = |5| = 5 \)

(MP.3) Finding the lengths of line segments that are neither vertical nor horizontal by drawing right triangles. Display another grid on the board. Draw on the grid:

ASK: Why is it more difficult to find the length of AB by counting on the grid? (not counting horizontally or vertically) What are the coordinates of A and B? ((1, 1) and (5, 4)) Draw a right triangle that uses AB as the hypotenuse and label the sides c, a, and b, as shown below:
ASK: What type of triangle did I draw? (a right triangle) What are the coordinates of point C? (5, 1) SAY: Notice that AB is the hypotenuse of the triangle. ASK: How can we use the Pythagorean Theorem to find the length of c? \(c^2 = a^2 + b^2\) How do we calculate the length of b? (|5 – 1| = 4) How do we calculate the length of a? (|4 – 1| = 3) Write on the board:

\[
\begin{align*}
    a &= |4 - 1| = 3 \\
    b &= |5 - 1| = 4 \\
    c^2 &= a^2 + b^2 \\
    &= 3^2 + 4^2
\end{align*}
\]

ASK: What is \(3^2 + 4^2\)? (25) How can we find c? (find the square root) Write on the board:

\[
\begin{align*}
    c^2 &= 25 \\
    c &= \sqrt{25} \\
    &= 5
\end{align*}
\]

Repeat with coordinates A (3, 4) and B (−5, −2). \((a = |−5 − 3| = 8, b = |−2 − 4| = 6, c^2 = 8^2 + 6^2 = 100, c =10)\)

Draw several lines that are neither vertical nor horizontal and have students follow the above steps to find the lengths of the lines. You can either leave answers as square roots, or approximate to the nearest tenth.

**MP.3** Developing a formula for finding the length of a line segment or the distance between two points. Display another grid on the board and draw:

\[
\begin{align*}
    A &= (-3, 3) \\
    B &= (4, -1) \\
    C &= (-3, -1)
\end{align*}
\]

SAY: To find the distance between \(A\) and \(B\), we draw a right triangle and then calculate the length of the horizontal side of the triangle (in this case \(a\)) and the vertical side (in this case \(b\)). The length of the horizontal line is the absolute value of the change in \(x\). The length of the vertical line is the absolute value of the change in \(y\). Then we use the Pythagorean Theorem to find the length of the hypotenuse. ASK: In this example, how do we calculate the length of the
horizontal line or \(a\)? \(|4 - (-3)|\) How can we calculate the length of the vertical line or \(b\)? \(|-1 - 3|\) Write on the board:

\[
\text{Change in } x: \quad a = |4 - (-3)| = |7| = 7 \\
\text{Change in } y: \quad b = |-1 - 3| = |-4| = 4
\]

ASK: How can we use the Pythagorean Theorem to find \(c\)? \((c^2 = a^2 + b^2)\) Write on the board:

\[
c^2 = 7^2 + 4^2
\]

ASK: What is \(7^2 + 4^2\)? \((65)\) How can we find \(c\)? (find the square root) Write on the board:

\[
c^2 = 65 \\
c = \sqrt{65} \approx 8.1
\]

**Exercises:** Find the change in \(x\) and the change in \(y\). Then find the distance between the points.

\begin{align*}
a) & \quad A (2, -3) \quad B (-1, 4) \quad b) C (-4, 7) \quad D (-2, -1) \quad \text{Bonus: } E (-103, -98) \quad F (-101, -96) \\
\text{Answers:} & \\
a) & |\text{change in } x| = 3, \ |\text{change in } y| = 7, \ c^2 = 9 + 49 = 58, \ c = \sqrt{58} \approx 7.6 \\
b) & |\text{change in } x| = 2, \ |\text{change in } y| = 8, \ c^2 = 4 + 64 = 68, \ c = \sqrt{68} \approx 8.2 \\
\text{Bonus: } & |\text{change in } x| = 2, \ |\text{change in } y| = 2, \ c^2 = 4 + 4 = 8, \ c = \sqrt{8} \approx 2.8
\end{align*}

**MP.3** Using a formula to find the length of a line segment or the distance between two points. SAY: In each of the previous questions, we found the change in \(x\) and the change in \(y\), then we found \(c^2\), and took the square root to find \(c\). Instead of doing this in several steps, we can combine all the steps into one. Write on the board:

\[
d_{AB} = \sqrt{|\text{change in } x|^2 + |\text{change in } y|^2}
\]

Point out that in this formula we are still finding the absolute value of the change in \(x\) and the absolute value of the change in \(y\), and then finding \(c^2 = a^2 + b^2\) to find the square root or \(c\). This we call \(d_{AB}\) or distance between \(A\) and \(B\). Write on the board:

\[
A (-2, -3) \quad B (3, 9)
\]

ASK: For these two points, what is the absolute value of the change in \(x\)? \((5)\) What is the absolute value of the change in \(y\)? \((12)\) What is \(5^2 + 12^2\)? \((169)\) What is the square root of 169? \((13)\) Write on the board:

\[
d_{AB} = \sqrt{|3 - (-2)|^2 + |9 - (-3)|^2} \\
= \sqrt{5^2 + 12^2} \\
= \sqrt{169} \\
= 13
\]
Exercises: Use the formula to find the distance between the points.

a) \( A (-1, 6) \quad B (-4, 10) \)  

b) \( C (-3, -2) \quad D (-5, 7) \quad \text{Bonus:} \ E (-452, -242) \quad F (-455, -235) \)

Answers:

a) \( d_{AB} = \sqrt{(-4 - (-1))^2 + (10 - 6)^2} = \sqrt{25} = 5 \)

b) \( d_{CD} = \sqrt{(-5 - (-3))^2 + (7 - (-2))^2} = \sqrt{85} \approx 9.2 \)

Bonus: \( d_{EF} = \sqrt{(-455 - (-452))^2 + (-235 - (-242))^2} = \sqrt{58} \approx 7.6 \)

Extensions

(MP.1) 1. Find the perimeter of a triangle with vertices \( A (-1, 3) \), \( B (-4,-1) \), and \( C (9, -1) \).

Solution:

\[
\begin{align*}
    d_{AB} &= \sqrt{(-4 - (-1))^2 + (-1 - 3)^2} = \sqrt{25} = 5 \\
    d_{BC} &= |9 - (-4)| = |13| = 13 \\
    d_{AC} &= \sqrt{116} \approx 10.8 \\
\end{align*}
\]

Perimeter \( \approx 5 + 13 + 10.8 = 28.8 \)

(MP.1) 2. Prove that \( \triangle ABC \) is a right triangle by following the steps:

a) Find the lengths of \( BC, AB, \) and \( AC \).

b) Show that \( b^2 = a^2 + c^2 \).

\[
\begin{align*}
    &d_{AC} = \sqrt{(5 - (-3))^2 + (1 - 1)^2} = \sqrt{68} \\
    &d_{BC} = \sqrt{|0 - (-3)|^2 + (2 - (-1))^2} = \sqrt{18} \\
    &d_{AB} = \sqrt{|5 - 0|^2 + |3 - (-2)|^2} = \sqrt{50} \\
\end{align*}
\]

Answers:

a) \( d_{AC} = \sqrt{68} \quad d_{BC} = \sqrt{18} \quad d_{AB} = \sqrt{50} \)

b) \( a = \sqrt{18} \quad b = \sqrt{68} \quad c = \sqrt{50} \)

\[
a^2 + c^2 = (\sqrt{18})^2 + (\sqrt{50})^2 = 18 + 50 = 68 = b^2
\]

So, \( \triangle ABC \) is a right triangle.
3. In three dimensions, or 3-D, the coordinate system has three coordinates: $x$, $y$, and $z$. For example, the coordinates $(3, 4, 12)$ describe a point that is 3 units east of the origin, 4 units north, and 12 units up, above the “ground” of the $xy$ plane.

a) $\triangle OBC$ is a right triangle. Find the length of $OC$.
b) $\triangle OAC$ is a right triangle. Find the length of $OA$.
c) Show that the length of $OA$ could have been found using the formula:

$$d_{OA} = \sqrt{\text{change in } x^2 + \text{change in } y^2 + \text{change in } z^2}$$

Answers:
a) $OC^2 = OB^2 + BC^2$
   $= 3^2 + 4^2 = 25$
   $OC = \sqrt{25} = 5$
b) $OA^2 = OC^2 + AC^2$
   $= 5^2 + 12^2 = 169$
   $OA = \sqrt{169} = 13$
c) $d_{OA} = \sqrt{3^2 + 4^2 + 12^2} = \sqrt{169} = 13$

4. Tony says $\sqrt{\frac{36}{4}} = 6 \frac{1}{2}$ because $\sqrt{36} = 6$ and $\sqrt{\frac{1}{4}} = \frac{1}{2}$.

a) Do you agree with Tony? Why or why not?
b) In pairs, explain your answers to part a). Do you agree with each other? Discuss why or why not.

Selected answer: a) No. Sample explanations:

- $\left(6 \frac{1}{2}\right)^2 = \left(\frac{13}{2}\right)^2 = \frac{169}{4} = 42 \frac{1}{4}$, so $6 \frac{1}{2}$ is greater than the actual square root.
- $\left(6 \frac{1}{2}\right)^2 = \left(6 \frac{1}{2} + \frac{1}{2}\right)^2 = \left(6 + \frac{1}{2}\right)^2 = 36 + 3 + \frac{1}{4} = 42 \frac{1}{4}$, which is not $36 \frac{1}{4}$.
- Tony’s method doesn’t make sense because $\frac{36}{4} = 36 + \frac{1}{4}$, and it’s not true that the square root of the sum of two numbers is the same as the sum of the square roots.
Finding the Area of Triangles and Parallelograms (1)

1. Area of a Rectangle

   a) 
   b) 

2. Area of a Right Triangle

   a) 
   b) 

3. Area of an Acute Triangle

   a) 
   b)
Finding the Area of Triangles and Parallelograms (2)

4. Area of an Obtuse Triangle

5. Area of a Triangle Using Different Bases and Heights

6. Area of a Parallelogram
Finding the Area of a Circle
Finding the Area of a Slanted Square
Finding the Side Length of a Slanted Square
Proving the Pythagorean Theorem (1)

Figure 1

Figure 2
Proving the Pythagorean Theorem (2)

Figure 3

Figure 4
Proving the Pythagorean Theorem by Dissection

Trace the diagram below on another sheet of paper. Cut out all the pieces of the slanted square with width $c$ from the traced diagram. Rearrange the pieces so that they completely fill the squares with widths $a$ and $b$. 
The Geometer’s Sketchpad® Pythagorean Theorem (1)

Use The Geometer’s Sketchpad® to verify the Pythagorean Theorem by following these steps:

**Step 1:** Construct the right triangle ABC.

- a) Use the Straightedge tool to draw a horizontal line. Label it \( l \).
- b) Use the Point tool to create a point not on the line. Label it \( A \).
- c) Select point \( A \) and line \( l \). Use the Construct menu to draw a line through \( A \) perpendicular to \( l \).
- d) Use the Point tool to label the intersection of the two lines as point \( C \).
- e) Use the Point tool to create another point on \( l \) to the left of point \( C \). Label it \( B \).
- f) Use the Polygon tool to create \( \triangle ABC \). Select the option that creates sides but no interior. Note that \( \triangle ABC \) is a right triangle. Move point \( A \) around and notice that \( \triangle ABC \) stays a right triangle even as point \( A \) is moved.
- g) Label line segment \( AC \) as \( b \), line segment \( AB \) as \( c \), and line segment \( BC \) as \( a \).
- h) Use the Measure menu to find the lengths of \( a \), \( b \), and \( c \).

**Step 2:** Construct a square on the side \( AC = b \) and find its area.

- a) Select point \( C \) and the measurement of \( b \). Use the Construct menu to construct a circle by using Center & Radius, with \( C \) as the center and \( b \) as the radius.
- b) The circle intersects line \( l \) at 2 points. Select the point to the right of \( C \). Label it \( D \).
- c) Select line \( l \) and point \( D \). Use the Construct menu to draw a line through \( D \) perpendicular to \( l \). Label the line \( m \).
- d) Select line segment \( AC \) and point \( A \). Use the Construct menu to draw a line through \( A \) perpendicular to \( AC \). Label it \( n \).
- e) Use the Point tool to find the intersection of lines \( m \) and \( n \). Label the intersection point \( E \).
- f) Use the Polygon tool to create square \( AEDC \). Use the option that has both sides and interior. Notice that we have created a square on side \( b \) of the triangle.
- g) Hide the circle.
- h) Use the Measure menu to find the area of \( AEDC \) (\( b^2 \)).

**Step 3:** Repeat the process in Step 2 to construct a square on side \( a \) of the triangle. Label the square \( BCFG \). Measure its area (\( a^2 \)).

**Step 4:** Repeat the process in Step 2 to construct a square on side \( c \) of the triangle. Label the square \( CBHI \). Measure its area (\( c^2 \)).
The Geometer’s Sketchpad® Pythagorean Theorem (2)

Step 5: Hide all the lines and circles. The diagram should look something like this:

Step 6: Verify the Pythagorean Theorem.

a) Use the Number menu to calculate the sum of the areas $AEDC$ and $BCFG$ ($a^2 + b^2$).

b) What do you notice about $c^2$ and $a^2 + b^2$?

c) Select point $A$. Move point $A$ anywhere. Notice that $\triangle ABC$ remains a right triangle. Also notice that although the areas of the squares on $AC$, $BC$, and $AB$ change, $c^2$ is always equal to $a^2 + b^2$. 

Teach this lesson after: 8.2 Unit 4

Goals: Students will use pictures to help visualize a problem and will add information to an already given picture to solve a problem.

Prior Knowledge Required: Can apply the Pythagorean Theorem Can apply the congruence rules for triangles: SSS, SAS, ASA Can evaluate square roots to the nearest whole number, using a calculator

Vocabulary: alternate and corresponding angles (with transversals), auxiliary line, congruence rule, corresponding angles and sides (in congruent triangles), hypotenuse, parallel, parallelogram, perpendicular, rhombus

Materials: grid paper or BLM 1 cm Grid Paper (p. O-87) calculators (see Performance Task) BLM Fire Department Ladder Problems (pp. O-90–93, see Performance Task)

The importance of seeing given information visually. Tell students to think of a word that contains the letters b, l, and p (pebble, bullpen, bleep, pliable, public, blimp). ASK: What strategy did you use? Did anyone write the letters down so they didn't have to remember them? SAY: Seeing things on paper helps you not have to remember as much in your head, and that leaves more room for solving the problem.

Drawing a picture to solve a problem. Write on the board:

John decided to go for a walk in his neighborhood.
He started by going 1 block east.
Then, he turned left and went 2 more blocks.
Then, he turned left again and went 3 more blocks.
He kept turning left and going one more block than after the previous turn.
After he had walked a total of 45 blocks, he got to his school.
He took a shortcut and walked home in a straight line.
To the nearest whole number, how many blocks did he walk in total?

NOTE: If students live in the country, you might point out that city blocks are much shorter than country blocks!
Give students time to read the problem. ASK: What makes this problem hard? (there are a lot of words, it's hard to picture what is happening) SAY: Close your eyes and imagine picture what is happening as I read the problem aloud. Read the problem aloud, and then have a volunteer draw a map of the first few turns of John’s walk, as shown below:

![Map of John's walk](image)

SAY: When you draw a picture, you don’t have to keep everything in your head. That means you can focus on solving the problem. ASK: What is the next step to solving the problem? (we need to figure out where the school is) SAY: We know how many blocks John walked to get to school, and we know where his home is, so we have to use this information to figure out where the school is. ASK: How can you tell if he got to the school yet? (by counting the total number of blocks he's gone so far and seeing if that is equal to 45) How can you get the number of blocks he's gone so far? (add 1 + 2 + 3 + ... + 7 = 28) SAY: He's gone 28 blocks so far, so he needs to keep going. Have a volunteer draw the next turn. ASK: Now how far has he gone? (36 blocks) Repeat for the next turn (45 blocks) SAY: So, now we know where the school is. Label the school. The final picture should look like this:

![Final map of John's walk](image)

ASK: Is the school east or west of John’s home? (east) Is the school north or south of John's home? (south) Draw on the board:
ASK: How many blocks east of John's home is the school? (students might guess various answers, but the correct answer is 5) How do you know? (allow some students to explain their reasoning; students could come to the board and point to each corner of the triangle while explaining) Have students draw the picture of John's trip on grid paper, or **BLM 1 cm Grid Paper**, as shown below:

![Grid Paper Diagram](image)

ASK: Does the grid paper make it easier to tell how many blocks east of John's home the school is? (yes) How does it make it easier? (I can just count the blocks instead of figuring out a calculation) How many blocks south of his home is the school? (4 blocks) How do you know? (I counted the blocks on grid paper) SAY: If you are ever taking a test and you don’t have grid paper, you can draw a grid yourself. Show students a rough drawing of a grid on the board. SAY: Now that you know how far south and east of John’s home the school is, you can add those numbers to the right triangle diagram. Label the side lengths of the right triangle on the board, as shown below:

![Right Triangle](image)

ASK: How can we find the length of the shortcut? (use the Pythagorean theorem) Have students find the distance of the shortcut. (approximately 6.4 blocks) ASK: To the nearest whole number, what is the total distance that John walked? (45 + 6 = 51 blocks)
(MP.1) Exercises: Draw a picture to solve the problem.

a) Yu goes for a walk. She starts at home. She walks 6 blocks south, then 3 blocks east, then 3 blocks north, and then 1 more block east. Then, she walks home in a straight line. How many blocks did Yu walk in total?

b) Yu is showing a friend around her neighborhood. She starts at home. Then, she walks 2 blocks north, 3 blocks east, 4 blocks south, 9 blocks west, 8 blocks north, 3 blocks east, 1 block south, and 15 blocks east. They end up at Yu’s school. Yu knows a shortcut home that goes in a straight line. How far do Yu and her friend need to walk from the school to home, measured in blocks?

c) Yu walks 1 block east, then turns right and walks 2 blocks, then turns right and walks 3 blocks, and then turns right again and walks 4 blocks. She then turns left and walks 4 blocks, turns left again and walks 3 blocks, turns left again and walks 2 blocks, and then turns left again and walks 1 block. How far does she end up from home, and in which direction?

Bonus: Yu follows the same pattern as in part c), but changes the direction of her turns from right to left after 4n blocks instead of after 4 blocks. How far does she end up from home and in what direction?

Selected solution: b) Draw a picture on grid paper to make an easier problem:

From the picture, the horizontal distance is 12 and the vertical distance is 5. The total distance is $\sqrt{5^2 + 12^2} = \sqrt{169} = 13$ blocks.

Answers: a) 18 blocks, c) 4 blocks west, Bonus: 4n blocks west

Proofs by using auxiliary lines. SAY: Sometimes you are given a picture of the situation, but you need to add something to it to help you solve the problem. This type of situation comes up in angle problems with parallel lines. ASK: What types of angles are equal when you have parallel lines and a transversal? (corresponding angles and alternate angles) Have volunteers draw examples on the board, as shown below:
Draw on the board:

\[ \angle ABC = \_\_\_\_ \]

SAY: I want to figure out what \( \angle B \) is. ASK: What line can I add to the picture so that I will have alternate angles? Have a volunteer draw it as a dashed line on the board, as shown below:

ASK: Can you describe the dashed line you drew? (I drew it parallel to the other two lines and through \( B \)) To emphasize these features, draw lines that are obviously incorrect one at a time and then erase them. First, draw a dashed line that is parallel to the other lines but not through \( B \), and then draw a dashed line that goes through \( B \) but is not parallel to the other lines. Mark the correct dashed line as parallel. SAY: By adding the dashed line, you have changed the problem into an easier one. Have volunteers solve the problem on the board. As they work, ASK: How did the volunteer know that angle was 50°? (it is alternate to the top 50° angle) How did the volunteer know that the other angle was 20°? (it is alternate to the 20° angle) The final picture should look like this:

ASK: So what is \( \angle B \)? (70°) Tell students that when they add a line to a picture to help them solve a problem, they are drawing an \textit{auxiliary line}. Draw on the board:

Conjecture: \( \angle A = \angle B \)
SAY: I think that when both lines in two angles are parallel that the angles will be equal, but I want to prove it for sure. Sometimes, there are different ways to add lines to a picture, and you can still prove the result you want whichever way you use.

(MP.1) Exercises: Use three different ways of adding lines to the picture to show that $\angle A = \angle B$.

a) 

b) 

c) 

Answers: a) $\angle A$ and $\angle B$ are both corresponding to the same angle, so they are equal; b) $\angle A$ and $\angle B$ are both alternate to the same angle, so they are equal; c) The angles to the left of $\angle A$ and $\angle B$ are corresponding and the angles to the right of $\angle A$ and $\angle B$ are corresponding, so $\angle A$ and $\angle B$ are both $180^\circ$ minus the same two angles and so are equal.

Review congruence rules for triangles. SAY: Another situation where it is useful to draw a picture is when you are using the congruence rules. It would be hard to keep track in your head which sides and angles are equal without seeing a picture. Write on the board:

SSS (side-side-side) SAS (side-angle-side) ASA (angle-side-angle)

SAY: Remember that for side-angle-side to work, you need to know that two sides and the angle between them are equal in both triangles. For angle-side-angle to work, you need to know that two angles and the sides between them are equal.

Exercises: Can you prove the triangles are congruent? If so, which congruence rule can you use?

a) 

b) 

c) 

d) 

e) 

Bonus:
**Answers:** a) SAS; b) no; c) SSS; d) ASA; e) no; Bonus: yes, the third angle is equal if the other two are, and the equal sides are in between corresponding angles, so ASA

SAY: Sometimes, the triangles you are trying to prove congruent have a side in common. That side is always an equal side. Draw on the board:

![Diagram of two triangles](attachment:triangle.png)

SAY: There are two triangles in the picture. Have a volunteer name them. (\(\triangle ABC\) and \(\triangle ACD\))

ASK: Are they congruent? (yes) What congruence rule did you use? (SSS) Write on the board:

\[
AB = AC = BC
\]

Have volunteers tell you what the corresponding equal sides are in \(\triangle ACD\). Write them in as volunteers tell you how to complete the equations. (\(AD, AC, CD\))

**Exercises:** Can you prove the triangles are congruent with the given information? If so, which congruence rule can you use? If not, draw a counterexample.

a) ![Counterexample](attachment:counterexample.png)

b) ![Counterexample](attachment:counterexample.png)

c) ![Counterexample](attachment:counterexample.png)

**Answers:** a) yes, SSS; b) yes, SAS; c) no, a counterexample would be an isosceles trapezoid:

![Counterexample](attachment:counterexample.png)

**Solving congruence problems using auxiliary lines.** Draw on the board:

\[AB = BC\] and \[AD = DC\]. Prove that \(\angle A = \angle C\).
Have a volunteer mark the equal sides in the picture. ASK: What line could you draw so that you could use a congruence rule? \(BD\) Show this on the board:

ASK: What congruence rule can you use? (SSS) Do angles \(A\) and \(C\) correspond to each other? (yes) Remind students that the phrase “corresponding angles” has two different meanings, both of which you are using in this lesson. In this case, corresponding angles in congruent triangles are equal.

Exercises:
(MP.1, MP.3) 1. Show that \(\angle B = \angle D\).

a) \hspace{1cm} b) Bonus: \(AB = DE\) and \(AD = BE\)

Answers:

a) Draw line segment \(AC\). Then, \(\triangle ADC \cong \triangle ABC\) by SSS, so \(\angle B = \angle D\).

b) Join \(A\) and \(C\). Then, \(\triangle ABC \cong \triangle CDA\) by SSS, so \(\angle B = \angle D\).

Bonus: Join \(AE\). Then, \(\triangle ABE \cong \triangle EDA\) by SSS, so \(\angle B = \angle D\).

(MP.1, MP.3) 2. A parallelogram is a quadrilateral with opposite sides parallel. Show that the opposite sides of a parallelogram are equal by following the steps below.

a) Draw a parallelogram and a diagonal to split it into two triangles.

b) Show that the two triangles are congruent. Hint: What side do the triangles have in common?

c) What are the corresponding sides of the triangles?

Solutions:

a) Draw and label a parallelogram as shown, and join \(AC\).

b) \(\angle BAC = \angle ACD\) by alternate angles, \(\angle BCA = \angle CAD\) by alternate angles, and the side between the equal angles, \(AC\), is common to both \(\triangle ABC\) and \(\triangle CDA\), so the triangles are congruent by ASA.

c) \(AB = CD\) and \(BC = AD\) by corresponding sides of congruent triangles.
(MP.1, MP.3) 3. How can you show that the diagonals of a rectangle are equal without using a congruence rule?
Answer: Use the Pythagorean Theorem.

4. Remember, a parallelogram has opposite sides equal and parallel. Follow the steps to show that the diagonals of the parallelogram bisect each other; i.e.,

- Label the vertices of the parallelogram $ABCD$.
- Draw diagonals $AC$ and $BD$. Label the point of intersection $E$.
- Show that $\triangle ABE \cong \triangle CDE$

Solutions:

a–b) 

- $\angle EAB = \angle ECD$ because they are alternate angles,
- $\angle EBA = \angle EDC$ because they are alternate angles, and
- $AB = DC$ because they are opposite sides of the parallelogram. So, $\triangle ABE \cong \triangle CDE$, so $DE = BE$ and $AE = EC$, and so the diagonals bisect each other.

(MP.1, MP.3) 5. A rhombus is a quadrilateral with all sides equal. Show that a rhombus is a parallelogram using the steps below.

- Draw a rhombus and a diagonal to divide it into two triangles.
- Prove that the two triangles are congruent.
- Label all the equal angles.
- Are alternate angles equal? Are opposite sides parallel?

Selected solutions:

- $\triangle ABD \cong \triangle CDB$ by SSS, since $AB = CD$, $BD = DB$, and $DA = BC$
- $\angle ABD$ and $\angle CDB$ are corresponding angles in congruent triangles and hence equal. They are alternate angles, so $AB$ and $CD$ are parallel. Also, $\angle ADB = \angle CBD$, since they are corresponding angles in congruent triangles, and they are alternate angles, so $AD$ and $BC$ are parallel. So, the rhombus is a parallelogram.

(MP.1, MP.3) 6. Prove that the angles opposite the equal sides in an isosceles triangle are equal by following the steps below.

- Sketch an isosceles triangle and mark the equal sides.
- Join the midpoint of the unequal side to the vertex opposite it.
- Your picture now has two smaller triangles. Prove that those triangles are congruent.
- Do the angles opposite the equal sides correspond to each other in the congruent triangles?
Problem Bank

**Problem Bank**

(MP.1, MP.3) 1. Reproduce all the steps for these proofs on your own, without looking at your notes from the lesson.

a) A quadrilateral has opposite sides parallel. Show that opposite sides are equal.

b) In a parallelogram, the diagonals bisect each other.

c) A quadrilateral has all sides equal. Show that opposite sides are parallel.

(MP.1, MP.3) 2. Show that the diagonals of a rhombus are perpendicular.

**Solution:** Label the rhombus $ABCD$. Draw the diagonals $AC$ and $BD$.

\[ \triangle ABC \cong \triangle ADC \] by SSS. So, $\angle BAC = \angle DAC$ and $\angle BCA = \angle DCA$. Also, $\triangle DAB \cong \triangle DCB$ by SSS. So, $\angle ABD = \angle CBD$ and $\angle ADB = \angle CDB$. Now look at all four triangles in the diagram. They have two angles the same, so the third one is the same, too. That means all four angles around the intersection point of $AC$ and $BD$ are the same and they add to $360^\circ$, so they are all $90^\circ$.

(MP.1, MP.3) 3. Show that, in an isosceles triangle, the line joining the midpoint of the unequal side to the opposite vertex is perpendicular to that side.

**Answer:** The two triangles are congruent in the picture below, by SSS.

So, $\angle BDA = \angle BDC$. But together, they make a straight line, so $\angle BDA = \angle BDC = 90^\circ$. So, $BD$ is perpendicular to $AC$. 

---

**Answers:**

a–b)

\[ \triangle ABD \cong \triangle CDB \] by SSS; d) $\angle BAD = \angle BCD$
(MP.1, MP.3) 4. Does the congruence rule for triangles hold for parallelograms, too? Either prove that it works or show a counterexample.

a) SSS  

b) SAS  

c) ASA

Answers:

a) No.

b) Yes. Once you know two sides and the angle between them, you can determine all the angles, and you can determine all the sides in order around the parallelogram.

c) No.

(MP.1, MP.3, MP.7) 5. A square is drawn on the hypotenuse of an isosceles right triangle.

Determine the ratio of the area of the square to the area of the triangle in two ways. Make sure you get the same answer using both ways.

a) Cut the square into four parts by drawing two diagonals. How does each part compare to the triangle? Use congruence rules to justify your answer.

b) Use $x$ for the length of the equal sides of the isosceles triangle. Then, use the Pythagorean Theorem to find the side length of the square.

Answers:

a) $	riangle DAB \cong ADC$ by SAS, so $\angle ABD = \angle DCA$ and $\angle DCA = \angle BAC$ because they are alternate angles. In fact, all eight angles around the square are equal and they add to $360^\circ$, so they are each $45^\circ$ and so $\triangle ABF \cong \triangle ADE$ by ASA. This is true of all four triangles, so the area of each triangle is one fourth the area of the square.

b) Using $x$ for the length of the equal sides of the isosceles triangle, the area of the triangle is $x^2 + 2$, and the side length of the square is by the Pythagorean Theorem, $\sqrt{x^2 + x^2} = \sqrt{2x^2} = \sqrt{2}x$. So, the area of the square is $(\sqrt{2}x)^2 = 2x^2$, the ratio of the areas is $2x^2/(x^2 + 2) = 4$. 

(MP.1, MP.7) 6. Find the area and perimeter of the trapezoid. All measurements are in centimeters.

Solution: Draw a perpendicular from B to DC, and label the point on DC as E. Then, 

\[ EC = (x + 7) - (x + 1) = 6. \] 

Then, by the Pythagorean Theorem, the height of the trapezoid is \( \sqrt{10^2 - 6^2} = \sqrt{100 - 36} = \sqrt{64} = 8. \) Then, the area of the trapezoid is \( 8(x + 4) \) cm\(^2\) and the perimeter is \( 2x + 26 \) cm.

(MP.3, MP.7) 7. Here is another way to show that isosceles triangles have base angles equal. Make two copies of the same triangle, as shown below:

You can say the following are corresponding sides: \( AB = EF, BC = ED, \) and \( AC = DF. \)

a) What are the corresponding angles? How does this show that the base angles of an isosceles triangle are equal?

b) Use the same method to show that all angles in an equilateral triangle are equal.

Answers: a) The corresponding angles are \( \angle A = \angle F \) and \( \angle C = \angle D, \) but \( \angle D \) is a copy of \( \angle A, \) and \( \angle F \) is a copy of \( \angle C, \) so \( \angle A = \angle C; \) b) You can pick any sides in any order as corresponding, so all angles can be picked as corresponding to any other, so all angles are equal.
1 cm Grid Paper
Performance Task: Fire Department Ladder Problems

Materials:
BLM Fire Department Ladder Problems (pp. O-90–93)
calculators

Performance Task: Fire Department Ladder Problems. Give students BLM Fire Department Ladder Problems. Students will solve problems that require them to choose what length of ladder to use and how to place the ladder safely. They will need to apply the Pythagorean Theorem and find the slope of a line.

Answers:
1. a) \( \sqrt{24} \approx \sqrt{25} = 5 \), too high; b) \( \sqrt{50} \approx \sqrt{49} = 7 \), too low; c) \( \sqrt{38} \approx \sqrt{36} = 6 \), too low
2. a) 12 feet, b) 7 feet
3. a) about 79 feet high, b) about 105 feet high
4. a) the truck with the 75 foot ladder, b) the truck with the 100 foot ladder, c) the truck with the 100 foot ladder
5. a) Option A

b) Option A: -5, Option B: -2.4; c) B only; d) about 22 feet long; e) no; Bonus: about 201 feet
Fire Department Ladder Problems (1)

You may use a calculator to complete this task.

1. Mentally estimate the square root to the nearest whole number by using a perfect square. Say whether your estimate too high or too low.
   a) \(\sqrt{24}\)
   b) \(\sqrt{50}\)
   c) \(\sqrt{38}\)

2. a) The base of a 13 foot ladder is placed 5 feet from the building. How high can the ladder reach up the side of the building?

   b) The base of an 8 foot ladder is placed 4 feet from the building. How high, to the nearest foot, can the ladder reach up the side of the building?
Fire Department Ladder Problems (2)

3. A fire department has ladders of different lengths mounted on different trucks. The ladder needs to reach the top of the building. The base of the ladder is 8 feet above the ground and cannot go closer than 25 feet to the building.
   a) How high up the side of a building can a 75 foot ladder reach, to the nearest foot?
   b) How high up the side of a building can a 100 foot ladder reach, to the nearest foot?

4. The fire department wants to send the truck with the shortest ladder that will reach the top of the building, in case there is a taller building that needs emergency help while the truck is away.
   Which truck should the fire department send?
   a) The building is 75 feet tall.
   b) The building has 9 floors and each floor is 10 feet high.
   c) The building has 8 floors and each floor is 13 feet high.
5. Two firefighters need to reach the top of a building. The building is 20 feet high and has a stone wall around it. The stone wall is:

- 8 feet high
- 4 feet out from the building
- 1 foot thick

The firefighters need to bring a ladder (not one mounted on the truck). The firefighters have two options.

**Option A**

**Option B**

a) Label each picture with the distances given.

b) What is the slope of the ladder in each option?

<table>
<thead>
<tr>
<th>Option A</th>
<th>Option B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To be safe to climb, the absolute value of the slope of the ladder has to be less than 4. Circle the option(s) that are safe.

A only  B only  A and B  No option is safe
Fire Department Ladder Problems (4)

d) How long of a ladder do the firefighters need to use the safe option from part c)?

e) Two firefighters, with all their equipment, weigh a total of 500 pounds. They estimate that a ladder with slope having absolute value 3 or greater can hold the 500 pounds. Using the safe option from part c), can both firefighters climb the ladder at the same time?

Bonus ► There are square buildings, each as tall as they are wide, laid out as shown and a fireman places a ladder along all three of them. The ladder touches all three buildings as shown below, with its base on the ground.

The shortest building is 40 feet tall and 40 feet wide, and the middle building is 60 feet tall and 60 feet wide. How long does the ladder need to be to reach the top of the tallest building?
PS8-7 Using Logical Reasoning

Teach this lesson after: 8.2 Unit 4


Goals:
Students will understand and reproduce a proof of the Pythagorean Theorem different than those studied in Lesson G8-44.
Students will use logical reasoning to prove theorems.

Prior Knowledge Required:
Can state and apply the Pythagorean Theorem
Can apply the distributive property with more than one variable
Can identify corresponding angles and sides in similar triangles
Knows the commutative property of multiplication
Knows that the sum of the angles in a triangle is 180°
Can apply the congruence rules for triangles: SSS, SAS, ASA

Vocabulary: congruent, Pythagorean Theorem, scale factor, similar triangles, variable

Materials:
grid paper or BLM 1 cm Grid Paper (p. O-107)
BLM Proving the Pythagorean Theorem Using Area (pp. O-108–109, see Problem Bank 10)

Using reasoning to prove equations. Start by providing the following exercises.

Exercises: Find four positive integers that are in a true addition equation so that all four numbers add to 12.
Bonus: Find as many ways as you can that work.
Sample answers: $4 + 2 = 5 + 1$, $1 + 2 + 3 = 6$, $3 + 3 = 3 + 3$, $1 + 5 = 3 + 3$, $1 + 5 = 1 + 5$, $1 + 1 + 4 = 6$

Have volunteers show different answers on the board, so students see both addition equations where two pairs of numbers add to the same total and where three numbers add to a total. Write on the board:

Four whole numbers are in a true addition equation.
Together, all four numbers add to 7.

Either find four numbers that work, or prove that you cannot.

Read the problem aloud. SAY: Trying examples is a good idea, because you might find an example that works or you might see why an example cannot work.
Ask several volunteers to write true addition equations on the board. For example:

\[
\begin{align*}
1 + 2 + 3 &= 6 \\
3 + 4 &= 2 + 5
\end{align*}
\]

ASK: What do all four numbers add to in each case? (for the examples above, 12 and 14) What property do these totals have? (they are all even) Can you prove that the totals will always be even? (yes, the total is the sum of two equal whole numbers) SAY: Both sides of the equation are equal whole numbers, so their total is an even number and cannot be 7. You've just proven that you cannot find four numbers that will work.

**Exercises:** Either find the numbers, or prove that you cannot.

a) Three whole numbers are in a true addition equation. Together, all three numbers add to 7.  
b) Eight integers are in a true addition equation. Together, all eight numbers add to 7.  
c) Four whole numbers are in a true subtraction equation. Together, all four numbers add to 7.

**Answers:**

a–b) The total of the numbers on each side of the equation must be half of the total of all numbers, 7. So, the total on each side must be 3.5, but then all the numbers can't be whole numbers or integers, which is not possible for parts a) or b).  
c) Any subtraction equation can be rearranged to make an addition equation—simply change the subtraction to an addition and move the subtracted number to the other side. So, by the same reasoning as for parts a) and b), it is not possible to find four numbers that work.

SAY: We could have looked at many examples to satisfy ourselves that it is probably true that the sum of all four numbers has to be even, but when you have a reason why it’s always true, you have a proof. Today, we’re going to look at how to prove things so that you can be sure.

**The sum of two multiples of a number is still a multiple of that number.** SAY: I want to show that the sum of two multiples of a number is still a multiple of that number. Let’s start with multiples of 2. Have a volunteer to tell you any multiple of 2 less than 100 (for example, 56).  
SAY: Let's add 56 to various even numbers. Write on the board:

\[
\begin{align*}
56 + 12 &= \\
56 + 300 &=
\end{align*}
\]

Have volunteers do the additions. (68, 356) ASK: Is 12 even? (yes) Is 56 + 12 even? (yes) Is 300 even? (yes) Is 56 + 300 even? (yes) SAY: It looks like it’s true that adding two even numbers still gets an even number, but let’s prove it starting with any two even numbers, not just 56 and 12 or 56 and 300. Any even number can be written as 2 times something. ASK: 56 is 2 times what? (28) Have volunteers name other even numbers, and ASK: That number is 2 times what? (answers will depend on the numbers named) SAY: Let’s call the even number we are starting with 2x. Write on the board:

\[
\begin{align*}
\text{Start with } 2x & & \text{Example: Start with } 56 = 2 \times 28
\end{align*}
\]
SAY: Now, let’s say we add another even number. It will also be 2 times something. ASK: Can we write it as $2x$? (no) PROMPT: If we started with $56 = 2 \times 28$, could we write the even number we are adding as $2 \times 28$, or will it be $2 \times$ something else? (it’s 2 times something else) SAY: Let’s add 12 to the number we started with. Write on the board:

\[
56 + 12 = 2 \times 28 + 2 \times 6 \\
= 2 \times _____
\]

Have a volunteer fill in the blank. (34) ASK: How did you get 34? (28 + 6) What property of multiplication allows you to do that? (multiplication distributes over addition) SAY: In general, we know both the even numbers are 2 times something, but they are 2 times different things. Write on the board:

\[
2x + 2y = 2(x + y)
\]

ASK: Is the result of adding two even numbers still even? (yes) SAY: This is something you probably knew for even numbers from lower grades; but now you can prove it algebraically, and that is very powerful, because you can prove it for much more than even numbers.

(MP.3, MP.7) Exercises: a) Show that the sum of two multiples of 3 is also a multiple of 3. b) Show that the sum of two multiples of 7 is also a multiple of 7. c) Show that the difference of two multiples of 3 is also a multiple of 3.

Answers:

a) Let the two numbers be $3x$ and $3y$. Then, $3x + 3y = 3(x + y)$ is also a multiple of 3.

b) Let the two numbers be $7x$ and $7y$. Then, $7x + 7y = 7(x + y)$ is also a multiple of 7.

c) Let the two numbers be $3x$ and $3y$. Then, $3x - 3y = 3(x - y)$ is also a multiple of 3.

When you add a multiple of a number, you don’t change the remainder when dividing by that number. SAY: When you add a multiple of 3 to a whole number, you don’t change the remainder when dividing by 3. Draw on the board:

0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16

SAY: The dots show the multiples of 3, and if I start two more than a multiple of 3 and add three, I get two more than the next multiple of 3, and it doesn’t matter how many 3s I add. I will never change the remainder when dividing by 3.
I want to prove that algebraically. If the number I start with is also a multiple of 3, then adding a multiple of 3 still gets a multiple of 3. You proved that algebraically in the previous exercises. So, let’s say the number is not a multiple of 3. Ask: What can its remainder be when dividing by 3? (1 or 2) Say: Let’s say it has remainder 1. Write on the board:

\[
7 + 6 = \_
\]
\[
4 + 12 = \_
\]
\[
13 + 9 = \_
\]

Have volunteers fill in the blanks. (13, 16, 22) Say: I started with numbers with remainder 1. Ask: What is the remainder of the answers? (still 1) Write on the board:

\[
4 + 12 = 3 \times \_
\]
\[
1 + 3 \times \_
\]

Say: Because 4 has remainder 1, it is 3 \times something + 1. Ask: What is the something? (1) Write “1” in the first blank. Say: 12 is a multiple of 3, so it is 3 times something else. Ask: What is the something else? (4) Write “4” in the second blank. Continue writing on the board:

\[
= 3 \times 1 + 3 \times 4 + 1
\]
\[
= 3 \times 5 + 1
\]

Have volunteers tell you the property you are using to go from one line to the next, starting at the second line. (commutative property of addition, then distributive property of multiplication over addition) Say: You can combine the multiples of 3 into one larger multiple of 3 by the distributive property. Ask: How did I get 5 in the last expression? (added 1 and 4) Say: It looks like starting with any number with remainder 1 and adding a multiple of 3 gets another number with remainder 1. In the following exercises you will prove this by using \(x\) for the starting number. Then you will have proven it for every starting number at the same time.

(MP.7) Exercises: a) A number \(3x + 1\) has remainder 1 when divided by 3. Another number, \(3y\), is a multiple of 3. What is the remainder when you divide their sum by 3? b) A number \(3x + 2\) has remainder 2 when divided by 3. When you add a multiple of 3, what is the remainder when you divide by 3? c) A number has remainder 4 when divided by 7. When you add a multiple of 7, what is the remainder when you divide by 7? Bonus: A number has remainder \(m\) when divided by \(n\). When you add a multiple of \(n\), what is the remainder when you divide by \(n\)?

Selected solution: a) \(3x + 1 + 3y = 3x + 3y + 1 = 3(x + y) + 1\) is 1 more than a multiple of 3, so it has remainder 1 when you divide by 3.

Answers: b) 2, c) 4, d) \(m\)

Geometric proofs. Say: Remember how you proved the Pythagorean Theorem first by using whole numbers and then by using letters. Let’s try this approach to prove the Pythagorean Theorem another way. Let’s start with a right triangle with side lengths 3, 4, and 5 units. Draw on the board:

```
3
\_
\_
4
\_
5
\_
```

Problem-Solving Lessons for Grade 8
SAY: Now we will use three different scale factors to scale the original triangle. Draw on
the board:

![Triangles](image)

SAY: The first triangle is scaled by a factor of 3, the second one by a factor of 4, and the third
one by a factor of 5. I used the same numbers for scale factors as the side lengths of the
triangle, and that was on purpose. Have students draw the three triangles on grid paper or
**BLM 1 cm Grid Paper** and cut them out. Have students rearrange the three triangles to make a
rectangle. Ask them to think about why the triangles fit the way they did, in terms of both the
side lengths and the angles.

(MP.3, MP.7) **Exercises:**
a) Draw a diagram to show how the triangles fit together. Label all the side lengths.
b) All the triangles are similar to the original triangle with side lengths 3, 4, and 5. In your
picture, mark the angles in each triangle: use 1 mark for the angle opposite the shorter leg and
2 marks for the angle opposite the longer leg, as shown below:

![Angles](image)

**Answers:**

![Answer](image)

Have a volunteer draw the answer on the board. SAY: The three angles at the top look like they
make a straight line. ASK: How can you know this for sure? (they are the same as the three
angles in the original triangle with side lengths 3, 4, and 5) How does marking the angles in the
triangles help you know that? (I can see they are the three different angles in the triangle) Point
to the two bottom angles in the rectangle, and ASK: These look like right angles, but how do you
know for sure? (they are the two acute angles in the original right triangle, so they add to 90°)

SAY: Remember, the goal is to prove the Pythagorean Theorem for any right triangle with side
lengths \(a-b-c\), not just one with side lengths 3-4-5. ASK: Which numbers occur in two triangles?
(12, 15, and 20) Where do they occur in the rectangle? (the 12s are two opposite sides of the
rectangle and the 15 and 20 are common sides of two triangles) SAY: Having the 15 and the 20
sides as common sides ensures that the third triangle fits the gap left by the other two triangles.
If we keep track of how we got the numbers in the first place, it might help us see what is going on. Draw on the board:

Point to the side marked 5 × 4 and 4 × 5 and ASK: How do you know that these side lengths are the same? (because 5 × 4 = 4 × 5) Do you think this will be true with any numbers, or does it just work for 4 and 5? (it will be true for any numbers) How do you know? (multiplication is commutative; the order you multiply numbers in doesn’t matter) What is the length of the top side of the rectangle? (3 × 3 + 4 × 4 = 25) What is the length of the bottom side of the rectangle? (5 × 5 = 25) How does this prove the Pythagorean Theorem in the special case? (the top and bottom sides of the rectangle are equal)

(MP.3, MP.7, MP.8) Exercise: Draw three triangles similar to a right triangle with sides a, b, and c, where a ≤ b ≤ c:
• one triangle scaled by a factor of a
• one triangle scaled by a factor of b
• one triangle scaled by a factor of c
Then, prove that the three triangles can be rearranged to make a rectangle. Explain why:
• the three angles at the top make a straight line
• the four corners of the shape are right angles
• the sides of the triangles that touch in your picture are identical
Then, find equal opposite sides that prove the Pythagorean Theorem.
Solution:
The three angles at the top make a straight line because they are equal to the three angles in the original triangle. The bottom corners make right angles because they are made from the two acute angles in the original right triangle, which together add to 90°. The two top corners are 90° because they correspond to the right angles in the original triangle. The sides of the triangles fit exactly together because \(a \times c = c \times a\) and \(b \times c = c \times b\) (i.e., the commutative property). So, the shape is a rectangle. Then, the top side length equals the bottom side length, so \(b \times b + a \times a = c \times c\), which is exactly what the Pythagorean Theorem says.
1. a) Explain why the number of white squares in the pictures below is a multiple of 3.

```
234
```

```
172
```

b) How do the pictures in part a) show that you can subtract a multiple of three from a number to get the sum of its digits?

c) Cam says that a number has the same remainder when it is divided by 3 as when divided by the sum of its digits. Is he right? Explain how you know.

d) Use your answer to part b) to create an easy way to check if any number is divisible by 3.

ed) Try out your method from part c) on the numbers.

i) 251  ii) 252  iii) 804  iv) 2,345,678

Answers: 
a) Each hundreds block has 99 white squares and each tens block has 9 white squares and both 99 and 9 are multiples of 3; b) When you take away the white squares, which are in total a multiple of 3, you are left with the grey squares, which add to the sum of the digits of the number; c) yes, because adding a multiple of 3 does not change the remainder when dividing by 3; d) add the digits, if the sum is a multiple of 3, so is the number, and if not, neither is the number; e) i) 2 + 5 + 1 = 8 is not a multiple of 3; ii) 2 + 5 + 2 = 9 is a multiple of 3; iii) 8 + 0 + 4 = 12 is a multiple of 3; iv) 2 + 3 + 4 + 5 + 6 + 7 + 8 = 35 is not a multiple of 3

NOTE: To do Problem Banks 3 and 4, students need to use prime factorization. Be sure that students can do Problem Bank 2 before providing Problem Banks 3 and 4.

2. Any whole number greater than 1 can be written as a product of prime factors.

Examples: 15 = 3 × 5  36 = 2 × 2 × 3 × 3

a) Write the number as a product of prime factors.

i) 10  ii) 21  iii) 18  iv) 30  v) 72

b) You can write a prime factorization for a number by picking any two factors that multiply to the number and then breaking them down until you get prime factors.

Example: 60 = 4 × 15 = 2 × 2 × 3 × 5

Finish the prime factorization of 60.

i) 60 = 6 × 10  ii) 60 = 20 × 3  iii) 60 = 5 × 12

c) How are the prime factorizations of 60 in part b) the same? How are they different?
Answers:

a) i) 2 × 5, ii) 3 × 7, iii) 2 × 3 × 3, iv) 2 × 3 × 5, v) 2 × 2 × 2 × 3 × 3
b) i) 2 × 3 × 2 × 5, ii) 2 × 2 × 5 × 3, iii) 5 × 2 × 2 × 3
c) they multiply all the same prime factors, but in a different order

(MP.3, MP.7) 3. In this exercise, you will create an easy way to check if a number is divisible by 7.

a) You can write any number in the form 10a + b, where b is a single digit. For example, 413 = 10 × 41 + 3, so a = 41 and b = 3. Write the number in the form 10a + b, where b is a single digit.
   
   i) 45  ii) 126  iii) 900  iv) 1,852  v) 274,056
b) Simplify the expression: 2(10a + b) + a - 2b.

c) Is the expression in a) always divisible by 7? How do you know?

d) The whole numbers 2(10a + b) and a - 2b add to a multiple of 7.
   
   i) If a - 2b is a multiple of 7, what does that tell you about 2(10a + b)?
   ii) If a - 2b is not a multiple of 7, what does that tell you about 2(10a + b)?

e) How can you tell from the prime factorization of a number if the number is a multiple of 7?

f) How does the prime factorization of 10a + b and 2(10a + b) compare?

g) Show that 10a + b is a multiple of 7 when and only when 2(10a + b) is.

h) Show that 10a + b is a multiple of 7 when and only when a - 2b is.

i) Use your method from part i) to check if the number is a multiple of 7.
   
   i) 413  ii) 546  iii) 136  iv) 455  v) 287  vi) 368

Answers:

a) i) 10 × 4 + 5, ii) 10 × 12 + 6, iii) 10 × 90 + 0, iv) 10 × 185 + 2, v) 10 × 27,405 + 6,

b) 21a

c) yes, because it is a multiple of 21 which is a multiple of 7

d) i) 2(10a + b) would also be a multiple of 7, ii) 2(10a + b) would also not be a multiple of 7

e) if 7 is a prime factor

f) they are the same except for an extra 2 in the prime factorization of 2(10a + b)

g) 10a + b is a multiple of 7 when and only when 7 is in its prime factorization, but that is when and only when 7 is in the prime factorization of 2(10a + b)

h) 10a + b is a multiple of 7 when and only when 2(10a + b) is a multiple of 7 which is when and only when a - 2b is a multiple of 7

i) Write the number in the form 10a + b where b is a single digit and then subtract a - 2b. Do this repeatedly until it is clear if the result is a multiple of 7 or not. If the result is a multiple of 7, so is the number; if not, neither is the number.

j) i) 41 - 6 = 35, so yes; ii) 54 - 12 = 42, so yes; iii) 13 - 12 = 1, so no; iv) 45 - 10 = 35, so yes; v) 28 - 14 = 14, so yes; vi) 36 - 16 = 20, so no

4. a) Simplify the expression 4(10a + b) - (a + 4b).

b) Is the expression from part a) always divisible by 13? Explain how you know.

c) Show that 4(10a + b) is divisible by 13 when and only when a + 4b is divisible by 13.

d) Show that 4(10a + b) is divisible by 13 when and only when 10a + b is divisible by 13.

e) Create a divisibility rule for 13 and explain it to a partner.

f) Use your divisibility rule for 13 to check whether the number is divisible by 13.

   i) 104  ii) 105  iii) 376  iv) 3,277  Bonus: 319,553
Answers:
a) $40a + 4b - a - 4b = 39a$
b) yes, it is $13 \times 3 \times a$
c) If one of the two numbers is divisible by 13 and the other one isn’t, you can’t subtract to get a multiple of 13. So, either both are multiples of 13 or neither are.
d) $10a + b$ is divisible by 13 when and only when $4(10a + b)$ is, because $2 \times 2$ and 13 would be separate parts of the prime factorization of $4(10a + b)$, so the 13 would have to be from the $10a + b$ if it’s in $4(10a + b)$, so then $10a + b$ is divisible by 13 when and only when $4(10a + b)$ is divisible by 13.
e) $10a + b$ is divisible by 13 when and only when $4(10a + b)$ is, which is when and only when $a + 4b$ is. So, write the number as $10a + b$ where $b$ is a single digit, and then evaluate $a + 4b$. If the result is a multiple of 13, so is the number. You can repeat the process until the number is small enough to tell easily.
f) i) $10 + 16 = 26$, so yes; ii) $10 + 20 = 30$, so no; iii) $37 + 24 = 61$, and $6 + 4 = 10$, so no; iv) $327 + 28 = 355$ and $35 + 20 = 55$ and $5 + 20 = 25$, so no; Bonus: $31,955 + 12 = 31,967$ and $3,196 + 28 = 3,224$ and $322 + 16 = 338$ and $33 + 32 = 65$ and $6 + 20 = 26$, so yes.

**MP.7, MP.8**

5. a) Calculate the area of each shaded semicircle in terms of $\pi$.

\[
i) \quad \text{Area} = \frac{9\pi}{2}, \quad \text{Area} = \frac{16\pi}{2}, \quad \text{Area} = \frac{25\pi}{2}, \quad \text{Area} = \frac{25\pi}{2}, \quad \text{Area} = \frac{144\pi}{2}, \quad \text{Area} = \frac{169\pi}{2}, \quad \text{ii}) \quad \text{Area} = \frac{\pi}{2} \times (a/2)^2 = \frac{\pi a^2}{8}, \quad \text{Area} = \frac{\pi}{2} \times (b/2)^2 = \frac{\pi b^2}{8}, \quad \text{Area} = \frac{\pi}{2} \times (c/2)^2 = \frac{\pi c^2}{8}, \quad \text{iii}) \quad \text{Area} = \frac{\pi}{2} \times (a/2)^2 = \frac{\pi a^2}{8}, \quad \text{Area} = \frac{\pi}{2} \times (b/2)^2 = \frac{\pi b^2}{8}, \quad \text{Area} = \frac{\pi}{2} \times (c/2)^2 = \frac{\pi c^2}{8}
\]

b) Is the sum of the areas of the semicircles on the legs of a right triangle always equal to the area of the semicircle drawn on the hypotenuse? How do you know? Did leaving the areas in parts i) and ii) of part a) in terms of $\pi$ help you notice the pattern?

Answers:
a) i) $9\pi/2, 16\pi/2, 25\pi/2$, ii) $25\pi/2, 144\pi/2, 169\pi/2$, iii) $\pi/2 \times (a/2)^2 = \pi a^2/8, \pi/2 \times (b/2)^2 = \pi b^2/8, \pi/2 \times (c/2)^2 = \pi c^2/8$
b) yes; because $\pi a^2/8 + \pi b^2/8 = \pi c^2/8$; yes

**MP.3**

6. In the triangle, $AD = BD$ and $AE = EC$.

\[
\begin{aligned}
&\text{a)} \text{ Prove that } DE \text{ and } BC \text{ are parallel.} \\
&\text{b)} \angle ABC = 75^\circ \text{ and } \angle AED = 35^\circ. \text{ Find the } \angle BAC.
\end{aligned}
\]

Answers:
a) $\triangle DAE \sim \triangle BAC$ by the SAS rule for similar triangles. That means $\angle ADE$ equals $\angle ABC$, so lines $DE \parallel BC$.
b) $\angle ABC = 75^\circ$ and $\angle BCA = \angle DEA = 35^\circ$, so $\angle BAC = 180^\circ - 75^\circ - 35^\circ = 70^\circ$
(MP.1) 7. a) Construct a trapezoid from a right triangle by joining the midpoints of the two shorter sides of the right triangle. Label the sides as shown below:

Use the picture to show that \( 3^2 + 5^2 + 4^2 = \frac{1}{2}(10^2) \).

b) Construct a trapezoid from a right triangle as shown:

Use the picture to write \( 4^2 + 10^2 + 3^2 \) as a fraction of \( 15^2 \).

**Bonus:**

(c) Construct a trapezoid from a right triangle by joining the midpoints of the two shorter sides of the right triangle. Label the sides as shown:

Show that \( a^2 + b^2 + c^2 = \frac{1}{2}d^2 \).

d) Construct a trapezoid from a right triangle as shown:

Write \( a^2 + b^2 + c^2 \) as a fraction of \( d^2 \).
**Answers:**

a) \(3^2 + 5^2 + 4^2 = 3^2 + 3^2 + 4^2 + 4^2 = 2 \times 3^2 + 2 \times 4^2 = 2 \times 5^2\), but \(10^2 = (2 \times 5)^2 = 2^2 \times 5^2\), which is twice \(2 \times 5^2\).

b) \(4^2 + 10^2 + 3^2 = 4^2 + 8^2 + 6^2 + 3^2 = 4^2 + (2 \times 4)^2 + (2 \times 3)^2 + 3^2 = 4^2 + 4 \times 4^2 + 4 \times 3^2 + 3^2 = 5 \times 4^2 + 5 \times 3^2\) and \(15^2 = (3 \times 3)^2 + (3 \times 4)^2 = 9 \times 3^2 + 9 \times 4^2\), so the fraction is \(5/9\).

**Bonus:**

c) \(b^2 = a^2 + c^2\) by the Pythagorean Theorem, so \(a^2 + b^2 + c^2 = a^2 + a^2 + c^2 + c^2 = 2a^2 + 2c^2\) and \(d^2 = (2a)^2 + (2c)^2 = 4a^2 + 4c^2\), so \(a^2 + b^2 + c^2 = 1/2\ d^2\).

d) \(a^2 + b^2 + c^2 = a^2 + (2a)^2 + (2c)^2 + c^2 = 5a^2 + 5c^2\), and \(d^2 = (3a)^2 + (3c)^2 = 9a^2 + 9c^2\), so \(a^2 + b^2 + c^2 = 5/9\ d^2\).

(MP.1, MP.3, MP.7) 8. Two sides of a shape are called adjacent if they have a common vertex.

a) If you connect all the midpoints of adjacent sides of a rectangle, what shape do you get? Explain how you know.

b) If you connect all the midpoints of adjacent sides of the shape from part a) what shape do you get? Explain how you know.

**Answers:** a) a rhombus, because all four sides are the hypotenuse of congruent right triangles; b) a rectangle, because the line joining the midpoints are parallel to the diagonals by SAS for similar triangles, but the diagonals of a rhombus are perpendicular because all four angles around the center are equal and so must be \(90^\circ\).

(MP.3) 9. Another proof of the Pythagorean Theorem. In the picture below, Lily says that the shaded square has the same area as the shaded part of the big square and the white square has the same area as the white part of the big square.

![Diagram of Pythagorean Theorem proof](image_url)

Explain how it would prove the Pythagorean Theorem if she is right.

**Answer:** The area of the square on the hypotenuse is the sum of the areas of the shaded and white rectangles, so if the shaded rectangle has the same area as the shaded square and the white rectangle has the same area as the white square, then the area of the square on the hypotenuse is equal to the sum of the areas of the other two squares.
(MP.3, MP.7) 10. Prove the Pythagorean Theorem by using BLM Proving the Pythagorean Theorem Using Area.

Answers:
1. a) i) 35 cm², ii) 17.5 cm², iii) 17.5 cm²
b) Area of ΔPRS = Area of ΔPTS = 1/2 Area PQRS
2. a) ii) AB or FG, AB or FG; iii) JK or CL, JK or CL; iv) HK or BL, HK or BL
b) ii) AFGB, iii) LKJC, iv) BHKL
3. a) ii) square, iii) square, iv) rectangle, v) rectangle
b) 1/2, CD, CA
c) 1/2, CLKJ, CL
d) SAS, AC, CJ, ACB, ACJ
e) area of CDEA = twice of area of ΔCBD = twice of area of ΔCAJ (because they are congruent) = area of LKJC
f) 1/2, AFGB, BG, AB
g) 1/2, BLKG, BH, HK
h) ΔHBA, SAS, BA, HB, ABC, ABH
i) area of AFGB = twice of area of ΔCBG = twice of area of ΔABH (because they are congruent) = area of BHKL
j) \(AB^2 = \text{area of BHKL and } AC^2 = \text{area of LKJC, so } AB^2 + AC^2 = \text{area of CBJH} = BC^2\)
Proving the Pythagorean Theorem Using Area (1)

REMINDER: Area of triangle = base × height ÷ 2

1. PQRS is a rectangle.
   a) Find the area of the shape.
      i) PQRS
      ii) ΔPRS
      iii) ΔPTS
   b) Describe how the three areas in part a) are related.

2. CDEA, AFGB, and CBHJ are squares.
   a) The base of the two shapes is given. Find the heights of the shapes.
      i) ΔCBD
         base: DC
         height: AC
      ii) ΔCBG
         base: BG
         height: AC
      iii) ΔCAJ
         base: JC
         height: ______
      iv) ΔHBA
         base: BH
         height: ______
   b) Write the area of the triangle in terms of another shape.
      i) Area of ΔCBD = \( \frac{1}{2} \) of area of CDEA
      ii) Area of ΔCBG = \( \frac{1}{2} \) of area of________
      iii) Area of ΔCAJ = \( \frac{1}{2} \) of area of________
      iv) Area of ΔHBA = \( \frac{1}{2} \) of area of________
Proving the Pythagorean Theorem Using Area (2)

3. Fill in the blanks to finish proving the Pythagorean Theorem the same way Euclid did 2,300 years ago: In a triangle $ABC$ with $\angle A = 90^\circ$, $AB^2 + AC^2 = BC^2$.

a) The picture at right shows the Pythagorean Theorem.
   What special quadrilateral is the of these quadrilaterals?
   i) $CDEA$  
   ii) $AFGB$  
   iii) $CBHJ$  
   iv) $LKJC$  
   v) $BHKL$  

b) Area of $\triangle CBD = \underline{\text{of area of square } CDEA}$, because
   they both have base _____ and height ____.  

c) Area of $\triangle CAJ = \underline{\text{of area of rectangle } \underline{\text{}}}$, because
   they both have base $CJ$ and height ____.  

d) $\triangle CBD \cong \triangle CAJ$ by the ______ congruence rule, because
   $DC = \underline{\text{}}$ and $CB = \underline{\text{}}$
   $\angle DCB = 90^\circ + \angle = \angle$.  

e) Area of $CDEA = \underline{\text{area of } LKJC}$, because

f) Area of $\triangle CBG = \underline{\text{of area of square } \underline{\text{}}}$, because
   they both have base _____ and height ____.  

g) Area of $\triangle HBA = \underline{\text{of area of rectangle } \underline{\text{}}}$, because
   they both have base _____ and height ____.  

h) $\triangle CBG \cong \underline{\text{by the } \underline{\text{ congruence rule, because}}}$
   $BG = \underline{\text{}}$ and $CB = \underline{\text{}}$
   $\angle CBG = 90^\circ + \angle = \angle$.  

i) Area of $AFGB = \underline{\text{area of } BHKL}$, because

j) $AB^2 + AC^2 = BC^2$, because

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Blackline Master — Problem-Solving Lessons — Teacher’s Guide for Grade 8  
O-109
Unit 5  Expressions and Equations:
Systems of Linear Equations

This Unit in Context
From Kindergarten onward, students wrote numerical equations and simple equations involving one operation with a variable. In Grade 6, students learned to create and solve equations to find answers to real world problems. They extended their knowledge of solving equations to include equations with rational coefficients and investigated how the distributive property can be used to find equivalent expressions (6.EE.A.3). They analyzed the relationship between dependent and independent variables using graphs and tables, and related these to the equations that define the relationship between the variables (6.EE.C.9). They also solved inequalities and recognized that, while the equations they have solved thus far have only one solution, solving inequalities leads to infinitely many solutions (6.EE.B.8).

In Grade 7, students expanded their knowledge of solving equations and began to see whole numbers, integers, and positive and negative fractions as belonging to a single system of rational numbers. They solved multistep problems involving rational numbers presented in various forms. They used variables to represent quantities in real world and mathematical problems, and constructed simple equations and inequalities to solve problems by reasoning about the quantities (7.EE.B.4).

In 8.1 Unit 4, students gave examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. They showed which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form \( x = a, a = a, \) or \( a = b \) results, where \( a \) and \( b \) are different (8.EE.C.7a).

In this unit, students will learn to solve pairs of linear equations simultaneously. They will understand that solutions to a system of two linear equations in two variables correspond to points of intersection on the respective graphs because points of intersection satisfy both equations simultaneously (8.EE.C.8a). They will solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations (8.EE.C.8b). They will solve real world and mathematical problems involving two linear equations in two variables (8.EE.C.8c).

Solving equations and inequalities is fundamental in high school algebra, functions, and statistics and probability.
Mathematical Practices in This Unit
In this unit, you will have the opportunity to assess MP.1 to MP.4, MP.6, and MP.8. The MP labels in this unit flag both opportunities to develop the mathematical practice standards and opportunities to assess them. Below is a list of where we recommend assessing each standard, as well as some examples of how students can show that they have met a standard.

**MP.1:** EE8-49 Extension 6, EE8-50 Extension 5, EE8-53 Extension 1, EE8-54 Extensions 2–3, EE8-55 Extensions 1–4
In EE8-53 Extension 1, students make sense of, and persevere in solving, a non-routine problem when they find the area of a triangle given the equations of the lines that form the triangle by finding the base and height. Students need to recognize the horizontal side as the base and the vertical distance to the intersection of the other two lines as the height.

**MP.2:** EE8-49 Extension 6, EE8-51 Extension 2, EE8-55 Extensions 1–3
In EE8-49 Extension 6, students reason abstractly and quantitatively when they use equations to solve a real-world problem and say what each mathematical step means in the context of the problem.

**MP.3:** EE8-50 Extensions 1 and 5, EE8-56 Extension 2

**MP.4:** EE8-49 Extension 6, EE8-51 Extension 2, EE8-54 Extension 3, EE8-55 Extensions 1–4
In EE8-55 Extension 4, students model with mathematics when they create and solve a system of linear equations in order to determine the cost of adult and child tickets to a museum.

**MP.6:** EE8-50 Extension 5, EE8-51 Extension 2, EE8-54 Extension 3, EE8-56 Extension 2

**MP.8:** EE8-50 Extension 1
Unit 5  Expressions and Equations:  
Systems of Linear Equations

Introduction
In this unit, students will learn about systems of linear equations, analyze them, and solve them. They will learn how to find the intersection of a system of equations using three different methods: by looking at the tables of values, by graphing the lines, and by using algebra. Students will also solve word problems that involve a system of equations and look for solutions by inspection.

Grids. Students will often need to have grid paper in this unit. Additionally, for some of the lessons of this unit, you will need a pre-drawn grid on the board. If you do not have such a grid, you can photocopy BLM 1 cm Grid Paper (p. S-1) onto a transparency and project it onto the board. This will allow you to draw and erase shapes and lines on the grid without erasing the grid itself. Similarly, when a coordinate grid is required, you can project a transparency of BLM Large Coordinate Grid (p. S-2). Students should be working in grid paper notebooks. If they are not working in such notebooks, provide them with grid paper or BLM 1 cm Grid Paper. To save time, you can occasionally provide students with BLM Coordinate Grids (p. S-3).

Technology: dynamic geometry software. Students will use dynamic geometry software to investigate systems of equations. Some of the activities in this unit use a program called The Geometer’s Sketchpad®, and some are instructional—they help you teach students how to use the program. If you are not familiar with The Geometer’s Sketchpad®, the built-in Help Center provides explicit instructions for many constructions. Use phrases such as “How to construct a line segment of given length” to search the Index.

NOTE: If you use a different dynamic geometry program to complete activities, the instructions provided may need to be adjusted.

Fraction notation. We show fractions in two ways in our lesson plans:

Stacked: $\frac{1}{2}$
Not stacked: 1/2

If you show your students the non-stacked form, remember to introduce it as new notation.
**EE8-49 Introduction to Systems of Linear Equations**

Pages 136–137

**Standards:** 8.EE.C.8, 8.EE.C.8a

**Goals:**
Students will find the common ordered pair for a pair of function tables.

**Prior Knowledge Required:**
Knows the standard order of operations
Can evaluate an expression by substituting values for the variable
Can complete a table of values for an expression
Can write ordered pairs from a function table

**Vocabulary:** common ordered pair, compound inequality, constants, expression, function table, inequality, ordered pair, variable, x-coordinate, y-coordinate

(MP.2) **Review how to evaluate expressions.** SAY: Remember that a variable represents a number. An expression may include variables, constants, and arithmetic operations. Write on the board:

\[ 2x + 3 \]

ASK: What is the variable in this expression? (x) What arithmetic operation is understood to be between the 2 and the x? (multiplication) SAY: So, the expression tells us to multiply an unknown number by 2, and then add 3. This expression can represent many different numbers. ASK: How can we get different answers for the expression? (substitute different values for x, then perform the arithmetic) SAY: Suppose we substitute \( x = 4 \) in the expression. Write on the board:

\[ 2(4) + 3 \]

ASK: What operation is understood to be between the 2 and 4? (multiplication) What is 2 × 4? (8) How did we know to do the multiplication before the addition? (order of operations) What is 8 + 3? (11) Continue writing on the board:

\[ = 8 + 3 \]
\[ = 11 \]

(MP.2) **Exercises:** Replace the variable with the given number and then evaluate.

a) \( 3x - 1 \quad x = 2 \)  

b) \( 4h + 5 \quad h = (-1) \)  

**Bonus:** \( 3x^2 + 2x - 4 \quad x = (-2) \)

**Answers:** a) \( 3(2) - 1 = 5 \), b) \( 4(-1) + 5 = 1 \), Bonus: \( 3(-2)^2 + 2(-2) - 4 = 4 \)
(MP.2) Review function tables. SAY: Remember that a function table lets us calculate and organize the ordered pairs, given a rule or equation. Write on the board:

\[ y = 2x + 3 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

SAY: The left column in the function table contains the numbers we will use to substitute for the variable \( x \). Here, the rule for the function table is \( y = 2x + 3 \). ASK: What does this rule tell us to do to each value of \( x \)? (multiply by 2, then add 3) What are the different values we will substitute for \( x \)? (−1, 0, 1, 2) Ask for volunteers to do the substitutions and write the answers in the table. (1, 3, 5, 7)

Exercises:

(MP.2) 1. Use the rule for the function to complete the function table.

a) \( y = 5x - 1 \)  

b) \( y = -4x + 5 \)  

Bonus: \( y = \frac{2x + 6}{x - 3} \)

<table>
<thead>
<tr>
<th></th>
<th>y</th>
<th></th>
<th>y</th>
<th></th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
<td></td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
<td></td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Answers: a) −16, −11, −6, −1, 4; b) 9, 5, 1, −3, −7; Bonus: −1, −2, −10, 14, 6

(MP.3) 2. In the bonus in Exercise 1, why should we not include 3 as a value for \( x \) in the function table?

Answer: If \( x = 3 \), the denominator in the fraction would equal zero. We cannot divide by zero.

(MP.2) Review finding ordered pairs from a function table. SAY: We can create ordered pairs from a function table. If you plot the ordered pairs, you can create a graph for the function table. ASK: In an ordered pair, which variable comes first, \( x \) or \( y \)? (x) SAY: To create an ordered pair from a function table, use the number in the first column as the \( x \)-coordinate and the number in the second column as the \( y \)-coordinate. Write on the board:

\[ y = x - 1 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>
Ask for volunteers to complete the function table. \((-3, -2, -1, 0, 1)\) Demonstrate how to create an ordered pair by circling the last row of the function table. Point to the 2 and SAY: This is the \(x\)-coordinate. Point to the 1 and SAY: This is the \(y\)-coordinate. Write on the board:

\[(2, 1)\] is an ordered pair for the graph of \(y = x - 1\)

**Exercises:** Find the ordered pairs in the function table.

a) \(y = 2x + 5\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

b) \(y = -x + 4\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**Bonus:** \(y = -3x - 1\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>-7</td>
<td></td>
</tr>
</tbody>
</table>

**Answers:** a) \((-2, 1), (-1, 3), (0, 5), (1, 7), (2, 9)\); b) \((-3, 7), (-2, 6), (-1, 5), (0, 4), (1, 3)\); Bonus: \((-2, 5), (-1, 2), (0, -1), (1, -4), (2, -7)\)

**(MP.2) Finding a common ordered pair in two function tables.** SAY: Sometimes we need to find an ordered pair that shows up in two different function tables. Write on the board:

\[
y = 3x - 1
\]

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

\[
y = -x + 3
\]

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Ask for volunteers to complete the function tables and list the ordered pairs. \((-2, -7), (-1, -4), (0, -1), (1, 2), (2, 5); (-3, 6), (-2, 5), (-1, 4), (0, 3), (1, 2)\) ASK: Which ordered pair appears in both lists? \((1, 2)\). Circle the ordered pair \((1, 2)\) in each function table. SAY: The ordered pair \((1, 2)\) is the common ordered pair. It doesn’t matter that \((1, 2)\) is in different rows in the two tables.

**(MP.6) Introduce compound inequality notation.** Write on the board:

\[
x \leq 5
\]

SAY: If we are talking about integers, we read this as the “integers that are less than or equal to 5.” ASK: What are those integers? \((4, 3, 2, 1, 0, -1, …)\) Write on the board:

\[
x \geq -2
\]

ASK: How do we read this? (integers that are greater than or equal to \(-2\)) What are those integers? \((-2, -1, 0, 1, 2, 3, …)\)
Write on the board:

\[ x \geq -2 \text{ and } x \leq 5 \]

SAY: This is a pair of inequalities. Here, we are looking for the integers that are greater than or equal to \(-2\) and less than or equal to 5. ASK: What are those integers? \((-2, -1, 0, 1, 2, 3, 4, 5)\)

Write on the board:

\[ -2 \leq x \leq 5 \]

SAY: We call this notation a compound inequality. This is a short form for writing the pair of inequalities \(x \geq -2\) and \(x \leq 5\).

**Exercises:**

1. Write the pair of inequalities as a compound inequality.
   a) \(x \geq -3\) and \(x \leq 7\)  
   b) \(x \geq -1\) and \(x \leq 4\)  
   c) \(x \geq -5\) and \(x \leq -1\)
   **Answers:** a) \(-3 \leq x \leq 7\), b) \(-1 \leq x \leq 4\), c) \(-5 \leq x \leq -1\)

2. For each of the answers in Exercise 1, write the set of integers that satisfy the inequality.
   **Answers:** a) \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7\}; b) \{-1, 0, 1, 2, 3, 4\}; c) \{-5, -4, -3, -2, -1\}

3. Find the common ordered pair for the two function tables.
   a) \(y = 4x - 3\)  
   \(\begin{array}{c|c} x & y \\ \hline -2 & -11 \\ -1 & -7 \\ 0 & -3 \\ 1 & 1 \\ 2 & 5 \\ \end{array}\)  
   b) \(y = 2x - 1\)  
   \(\begin{array}{c|c} x & y \\ \hline -2 & -5 \\ -1 & -3 \\ 0 & -1 \\ 1 & 1 \\ 2 & 5 \\ \end{array}\)  
   c) \(y = -2x + 3\)  
   \(\begin{array}{c|c} x & y \\ \hline -2 & -1 \\ 0 & 1 \\ 1 & 3 \\ \end{array}\)  
   d) \(y = x - 3\)  
   \(\begin{array}{c|c} x & y \\ \hline -2 & -5 \\ 0 & -3 \\ 2 & -1 \\ \end{array}\)
   **Answers:** a) \((1, 1)\); b) \((2, -1)\)

4. Create the function tables. Then find the common ordered pair.
   a) \(y = -x + 4\), \(y = 3x + 8\)  
   Hint: Use \(-2 \leq x \leq 2\).  
   **Answers:** a) \((-1, 5)\); b) \((7, 27)\); c) \((-10, -28)\)

**MP.3** Extending the function table to find the common point. Write on the board:

\(y = 2x - 5\)  
\(\begin{array}{c|c} x & y \\ \hline -1 & -7 \\ 0 & -5 \\ 1 & -3 \\ \end{array}\)
\(y = 3x + 1\)  
\(\begin{array}{c|c} x & y \\ \hline -1 & -2 \\ 0 & 1 \\ 1 & 4 \\ \end{array}\)
ASK: Do the function tables have a point in common? (no) Have volunteers find the value of $y$ if $x = -6$ for each function table. (-17) ASK: If we extended the table to include more $x$ values, including $x = -6$, what will the common point be for the two function tables? (-6, -17) Is it easy to find the common point from function tables? (no) Why? (sample answer: sometimes you might have to extend the function tables) Even if we extended each function table to include all integers, why might it still be difficult to find the common point? (sample answer: the values of $x$ or $y$ could be fractions)

Extensions

(MP.1) 1. The common point may include non-integer values for $x$ or $y$. Complete the function tables to find the common point.

\[
y = 3x + 1 \quad y = x
\]

\[
\begin{array}{c|c}
 x & y \\
 \hline
 -\frac{3}{2} & \frac{-1}{2} \\
 -1 & -1 \\
 -\frac{1}{2} & -\frac{1}{2} \\
 0 & 0 \\
 \frac{1}{2} & \frac{1}{2} \\
 1 & 1 \\
 \frac{3}{2} & \frac{3}{2} \\
\end{array}
\]

Answer: (-1/2, -1/2)

(MP.1, MP.6) 2. When graphing a function, it is easier to use integer coordinates rather than non-integer coordinates. Complete the function tables. Choose the $x$-coordinates carefully so that the $y$-coordinate will be an integer. Then write the ordered pairs.

a) $y = \frac{3}{4}x - 2$

\[
\begin{array}{c|c}
 x & y \\
 \hline
 -8 & \\
 -4 & \\
\end{array}
\]

Sample answers: a) (-8, -8), (-4, -5), (0, -2), (4, 1), (8, 4), (12, 7); b) (-4, 5), (-2, 4), (0, 3), (2, 2), (4, 1), (6, 0); c) (0, 0), (1, 1), (4, 2), (9, 3), (16, 4), (25, 5)
(MP.1) 3. Find two common ordered pairs for the function tables:
   a) \( y = x^2 \)  
      \[ \begin{array}{c|c}
          x & y \\
          \hline
          -1 & -1 \\
          0 & 0 \\
          1 & 1 \\
          2 & 2 \\
          3 & 3 \\
          4 & 4
        \end{array} \]
   b) \( y = 3x + 4 \)  
      \[ \begin{array}{c|c}
          x & y \\
          \hline
          -2 & -1 \\
          -1 & 0 \\
          0 & 4 \\
          1 & 7 \\
          2 & 10
        \end{array} \]
   c) \( y = x^2 - 1 \)  
      \[ \begin{array}{c|c}
          x & y \\
          \hline
          -1 & -2 \\
          -1 & -2 \\
          1 & 0 \\
          2 & 2 \\
          3 & 3
        \end{array} \]
   d) \( y = 4x - x^2 - 1 \)  
      \[ \begin{array}{c|c}
          x & y \\
          \hline
          0 & -1 \\
          1 & 3 \\
          2 & 7 \\
          3 & 11
        \end{array} \]

   Answers: (MP.1) a) (-1, 1), (4, 16); b) (0, -1), (2, 3)

(MP.1, MP.3) 4. The equations for two function tables are \( y = 4x - 1 \) and \( y = -1 + 4x \).
   a) Complete the function tables using \( x \) values of -3, -2, -1, 0, 1, 2, 3.
   b) List the ordered pairs in the function tables that are common.
   c) If we continue the function tables for as many \( x \) values as possible, how many common ordered pairs would the tables have? Explain.

   Selected answers: b) (-3, -13), (-2, -9), (-1, -5), (0, -1), (1, 3), (2, 7), (3, 11); c) There will be an infinite number of common ordered pairs. The equations for the function tables are exactly the same because \( 4x - 1 \) and \( -1 + 4x \) are equivalent expressions.

(MP.1, MP.3) 5. The equations for two function tables are \( y = x \) and \( y = \frac{1}{x} \).
   a) Explain why you shouldn’t use the value of \( x = 0 \) for the function table for \( y = \frac{1}{x} \).
   b) Find two common ordered pairs for the function tables.

   Answers:
   a) If \( y = 1/x \), the denominator in the fraction would equal zero. We cannot divide by zero.
   b) (1, 1), (-1, -1)

(MP.1, MP.2, MP.4) 6. The distance around an Olympic track is 400 m. Each straightaway is 84.39 m in length. Each end is a semi-circle. Grace is standing at point \( A \) and realizes she left her hat at point \( B \). How far does she need to walk to get her hat if she cuts across the lawn? Show your work. Say what each step means in the situation.

   Answer: The circular part of the track has a circumference equal to \( 400 - 2 \times 84.39 = 231.22 \) m. If \( d \) is the diameter of the circular part, then \( \pi d = 231.22 \). Using 3.14 for \( \pi \), \( d \approx 231.22 / 3.14 \approx 73.6 \) m. Now I can use the Pythagorean theorem to find the distance Grace needs to walk. Let \( D \) be the distance that Grace needs to walk. Then \( D^2 \approx 84.4^2 + 73.6^2 \), so \( D \approx 112 \) m. Grace has to walk about 112 m to get her hat.
EE8-50  Graphing Systems of Linear Equations (Introduction)

Pages 138–140

Standards: 8.EE.C.8, 8.EE.C.8a

Goals:
Students will find the intersection point of a pair of lines by looking at the graph and by checking that the coordinates satisfy both equations simultaneously.

Prior Knowledge Required:
Can plot ordered pairs in the coordinate plane
Can check to see that an ordered pairs coordinates satisfy the equation of a line

Vocabulary: common ordered pair, intersection point, lie on a line, satisfy the equation

Materials:
The Geometer’s Sketchpad®
BLM On, Above, or Below a Line (p. P-43, see Extension 1)
BLM On, Outside, or Inside a Circle (pp. P-44–45, see Extension 3)

(MP.2, MP.3) Determining whether a point lies on a line. Write on the board:

\[ y = 2x + 1 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2</td>
<td></td>
</tr>
<tr>
<td>−1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

Ask a student to fill in the function table. (−3, −1, 1, 3, 5) Ask another student to write the ordered pairs for the function table. ((−2, −3), (−1, −1), (0, 1), (1, 3), (2, 5)) Project a coordinate grid on the board and ask a volunteer to mark the points for the ordered pairs. Then, connect the dots and label the line with the equation. (see completed graph on the next page)
ASK: Where did the coordinates of the ordered pairs come from? (the function table)
SAY: Notice all the points lie on the line. Mark the ordered pair (3, 2) on the grid. SAY: Notice this point is not on the line. ASK: How can you tell that the ordered pair (3, 2) did not come from the function table? (when you substitute 3 for \(x\) in the equation \(y = 2x + 1\), \(y\) is not 2—instead, it is 7)

SAY: If a point lies on the line, its ordered pair follows the rule that the function table shows. So, for example, if you substitute 1 for \(x\) and 3 for \(y\) into the equation \(y = 2x + 1\), the left side of the equation equals the right side of the equation—in other words, 3 = 2(1) + 1. We can say that the coordinates (1, 3) “satisfy the equation of the line” \(y = 2x + 1\), and we can say that the point “lies on the line.”

SAY: If a point does not lie on the line, its ordered pair does not follow the rule for the function table. So, for example, if you substitute 1 for \(x\) and 5 for \(y\) into the equation \(y = 2x + 1\), the left side of the equation will not equal the right side of the equation—in other words, 5 \(\neq\) 2(1) + 1. We say that the coordinates (1, 5) “do not satisfy the equation of the line.”

(MP.2) Exercises: Without graphing, determine whether the point lies on the line.

a) \(y = x + 3\); (3, 4)  
b) \(y = -2x + 4\); (-1, 6)

c) \(y = 3x - 1\); (0, -1)  
Bonus: \(2x - 5y = 10\); (-1, 1)

Answers: a) 4 \(\neq\) 3 + 3, so the point does not lie on the line; b) 6 = -2(-1) + 4, so the point lies on the line; c) -1 = 3(0) - 1, so the point lies on the line; Bonus: 2(-1) - 5(1) \(\neq\) 10, so the point does not lie on the line

(MP.2) Showing that a point lies on two lines. SAY: It is possible for more than one line to pass through a given point. Draw on the board:
SAY: Here, both \( \ell_1 \) and \( \ell_2 \) pass through point \( A \). If point \( A \) lies on \( \ell_1 \), then its coordinates have to satisfy the equation for \( \ell_1 \). If point \( A \) lies on \( \ell_2 \), then its coordinates must also satisfy the equation for \( \ell_2 \). We call \( A \) the intersection point of lines \( \ell_1 \) and \( \ell_2 \).

Write on the board:

\[
\begin{align*}
y &= 3x - 1 \\
y &= -2x + 4 \\
\text{point: (1, 2)}
\end{align*}
\]

ASK: How can we check whether the point lies on \( y = 3x - 1 \)? (see if the coordinates satisfy the equation) Does 2 equal 3(1) - 1? (yes) SAY: So, the point (1, 2) lies on \( y = 3x - 1 \). ASK: Does the point (1, 2) lie on the line \( y = -2x + 4 \)? (yes) How can you tell? (2 = -2(1) + 4) SAY: So, the point (1, 2) lies on both lines \( y = 3x - 1 \) and \( y = -2x + 4 \).

**Exercises:** Show that the point lies on both lines.

a) \( y = x + 4 \)  \quad b) \( y = 2x - 5 \)  \quad c) \( y = -5x + 1 \)

\[
\begin{align*}
y &= -x - 2 \\
y &= -3x + 5 \\
\text{point: (-3, 1)} \\
y &= -2x - 5 \\
\text{point: (2, -1)}
\end{align*}
\]

**Selected answer:** a) \( 1 = -3 + 4 \) and \( 1 = -(3) - 2 \), so (-3, 1) lies on both lines.

**MP.2** Finding an intersection point by looking at a graph. Project a coordinate grid on the board and draw the following lines:

Ask for a volunteer to draw a dot where the lines intersect. ASK: What are the coordinates of the intersection point? (\((-1, 2)\)) Using the equation of the line, how can you that tell \((-1, 2)\) lies on the line \( y = x + 3 \)? (it satisfies the equation \( 2 = -1 + 3 \)) How can we tell \((-1, 2)\) lies on the line \( y = -x + 1 \)? (it satisfies the equation \( 2 = -(1) + 1 \))
**Exercises:** Write the coordinates of the intersection point of the lines. Then, check that the intersection point satisfies the equations of both lines.

**a)**

\[ y = 2x - 1 \quad \text{and} \quad y = x - 4 \]

**b)**

\[ y = 3x - 4 \quad \text{and} \quad y = -x - 4 \]

**Answers:**

**a)** \((-1, -3), \text{ check: } -3 = 2(-1) - 1 \text{ and } -3 = -(1) - 4\)

**b)** \((1, -1), \text{ check: } -1 = 3(1) - 4 \text{ and } -1 = (1) - 2\)

**(MP.2)** Finding the intersection point by first graphing the lines using function tables.

Write on the board:

\[
\begin{array}{c|c|c}
\text{x} & \text{y} & \text{x} \\
-2 & -3 & -2 \\
-1 & -4 & -1 \\
0 & -5 & 0 \\
1 & -6 & 1 \\
2 & -7 & 2 \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\text{x} & \text{y} & \text{x} \\
-2 & 1 & -2 \\
-1 & 2 & -1 \\
0 & 3 & 0 \\
1 & 4 & 1 \\
2 & 5 & 2 \\
\end{array}
\]

Ask for volunteers to fill in the function tables. \((-5, -2, 1, 4, 7; 0, -2, -4, -6, -8\) Then, project a coordinate grid on the board and ask volunteers to draw the lines and label the lines with the equations. (see graph below)
ASK: What is the intersection point? (−1, −2)) Ask for volunteers to check on the board that the coordinates (−1, −2) satisfy the equations of both lines. (−2 = 3(−1) + 1, −2 = −2(−1) − 4) Then, ask for a volunteer to circle on the function tables the ordered pairs that are common to both.

Exercises:
1. a) For each pair of lines, complete the function tables and graph the lines. Then find the intersection point by looking at the graph.
   i) \( y = x - 2 \)
   ii) \( y = -4x + 3 \)
   iii) \( y = 2x + 3 \)
   \( y = 2x + 1 \)
   \( y = 3x - 4 \)
   \( y = -4x + 3 \)
   b) Check that the coordinates of each intersection point from part a) satisfy the equations of both lines.
   **Answers:** a) i) (−3, −5); ii) (1, −1); iii) (0, 3)
   **Sample answer:**
   b) ii) Check:
   \( -1 = -4(1) + 3, \) \( -1 = 3(-1) - 4 \)

2. For each pair of lines in Exercise 1, find the common ordered pair in the function tables.
   **Answers:** a) (−3, −5); b) (1, −1); c) (0, 3)

Activity
Have students use The Geometer’s Sketchpad® to check their answers to the previous exercises involving pairs of lines. For example, to find the intersection point of the lines \( y = 2x - 5 \) and \( y = -3x + 5 \), students should follow these steps:
   a) Under the Graph menu, use the “Show Grid” option to show a grid.
   b) Under the Graph menu, select “Plot New Function.” Enter \( 2x - 5 \) in the dialog box.
   c) Under the Graph menu, select “Plot New Function” again. Enter \( -3x + 5 \) in the dialog box.
   d) Use the Point Tool to draw a dot at the intersection point of the two lines. Label it A.
   e) Select A. Under the Measure menu, select “Coordinates” to find the coordinates of the intersection point.
   (end of activity)

Extensions
(MP.3, MP.8) 1. Distribute BLM On, Above, or Below a Line. Ask students to complete the BLM to discover how to predict whether a point lies on the line, above the line, or below the line.
   **Answers:**
<table>
<thead>
<tr>
<th>Point</th>
<th>Coordinates</th>
<th>Calculation</th>
<th>Coordinates Satisfy …</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>(−7, 3)</td>
<td>3 &gt; −7 + 2</td>
<td>( y &gt; x + 2 )</td>
<td>above the line</td>
</tr>
<tr>
<td>D</td>
<td>(−7, −2)</td>
<td>−2 &gt; −7 + 2</td>
<td>( y &gt; x + 2 )</td>
<td>above the line</td>
</tr>
<tr>
<td>E</td>
<td>(−4, −2)</td>
<td>−2 = −4 + 2</td>
<td>( y = x + 2 )</td>
<td>on the line</td>
</tr>
<tr>
<td>F</td>
<td>(2, 7)</td>
<td>7 &gt; 2 + 2</td>
<td>( y &gt; x + 2 )</td>
<td>above the line</td>
</tr>
<tr>
<td>G</td>
<td>(−4, −5)</td>
<td>−5 &lt; −4 + 2</td>
<td>( y &lt; x + 2 )</td>
<td>below the line</td>
</tr>
<tr>
<td>H</td>
<td>(2, 4)</td>
<td>4 = 2 + 2</td>
<td>( y = x + 2 )</td>
<td>on the line</td>
</tr>
<tr>
<td>I</td>
<td>(2, −2)</td>
<td>−2 &lt; 2 + 2</td>
<td>( y &lt; x + 2 )</td>
<td>below the line</td>
</tr>
</tbody>
</table>

2. When the \( y \)-coordinate is equal to \( x + 2 \), the point is on the line. When the \( y \)-coordinate is greater than \( x + 2 \), then the point \((x, y)\) is directly above the point \((x, x + 2)\) which is on the line, so \((x, y)\) is above the line. When the \( y \)-coordinate is less than \( x + 2 \), then the point \((x, y)\) is directly below \((x, x + 2)\) which is on the line, so the point \((x, y)\) is below the line.
2. If the coordinates of a point satisfy the equation \( y = 2x + 3 \), the point lies on the line. If the coordinates satisfy \( y < 2x + 3 \), the point lies below the line. If the coordinates satisfy \( y > 2x + 3 \), the point lies above the line. Does the given point lie on, below, or above the line?

a) \( y = 2x - 5; (3, -2) \)  
b) \( y = -3x - 5; (-1, 1) \)  
c) \( y = x + 3; (0, 4) \)  
d) \( y = -x; (1, -1) \)  

**Answers:** a) \(-2 < 2(3) - 5\), so the point lies below the line; b) \(1 > -3(-1) - 5\), so the point lies above the line; c) \(4 > 0 + 3\), so the point lies above the line; d) \(-1 = -(1)\), so the point lies on the line

(M.P.1, M.P.3) 3. Distribute BLM On, Outside, or Inside a Circle. Have students use the BLM to discover how to predict whether a point lies on the circle, inside the circle, or outside the circle.

**Answers:**

<table>
<thead>
<tr>
<th>Point</th>
<th>Coordinates</th>
<th>Calculation</th>
<th>Distance to Origin</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>(3, 1)</td>
<td>(d = \sqrt{3^2 - 0^2 + 1^2 - 0^2} \approx 3.2)</td>
<td>(d &lt; 5)</td>
<td>inside the circle</td>
</tr>
<tr>
<td>D</td>
<td>(-7, 4)</td>
<td>(d = \sqrt{(-7)^2 - 0^2 + 4^2} \approx 8.1)</td>
<td>(d &gt; 5)</td>
<td>outside the circle</td>
</tr>
<tr>
<td>E</td>
<td>(-4, 3)</td>
<td>(d = \sqrt{(-4)^2 - 0^2 + 3 - 0^2} = 5)</td>
<td>(d = 5)</td>
<td>on the circle</td>
</tr>
<tr>
<td>F</td>
<td>(-2, 2)</td>
<td>(d = \sqrt{(-2)^2 - 0^2 + 2 - 0^2} \approx 2.8)</td>
<td>(d &lt; 5)</td>
<td>inside the circle</td>
</tr>
<tr>
<td>G</td>
<td>(-3, -4)</td>
<td>(d = \sqrt{(-3)^2 - 0^2 + 4 - 0^2} = 5)</td>
<td>(d = 5)</td>
<td>on the circle</td>
</tr>
<tr>
<td>H</td>
<td>(6, -6)</td>
<td>(d = \sqrt{6^2 - 0^2 + (-6)^2} \approx 8.5)</td>
<td>(d &gt; 5)</td>
<td>outside the circle</td>
</tr>
<tr>
<td>I</td>
<td>(4, -3)</td>
<td>(d = \sqrt{4^2 - 0^2 + (-3)^2} = 5)</td>
<td>(d = 5)</td>
<td>on the circle</td>
</tr>
<tr>
<td>J</td>
<td>(2, -2)</td>
<td>(d = \sqrt{2 - 0^2 + (-2)^2} \approx 2.8)</td>
<td>(d &lt; 5)</td>
<td>inside the circle</td>
</tr>
<tr>
<td>K</td>
<td>(-5, -5)</td>
<td>(d = \sqrt{(-5)^2 - 0^2 + (-5)^2} \approx 7.1)</td>
<td>(d &gt; 5)</td>
<td>outside the circle</td>
</tr>
<tr>
<td>L</td>
<td>(-3, -1)</td>
<td>(d = \sqrt{(-3)^2 - 0^2 + (-1)^2} \approx 3.2)</td>
<td>(d &lt; 5)</td>
<td>inside the circle</td>
</tr>
</tbody>
</table>

2. You can determine by looking at the symbol of the distance to origin: = indicates the point will lie on the circle, > indicates outside the circle, and < indicates inside the circle.

3.  

<table>
<thead>
<tr>
<th>Point</th>
<th>Coordinates</th>
<th>Calculation</th>
<th>Coordinates Satisfy …</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>(3, 1)</td>
<td>(3^2 + 1^2 = 10)</td>
<td>(x^2 + y^2 &lt; 25)</td>
<td>inside the circle</td>
</tr>
<tr>
<td>D</td>
<td>(-7, 4)</td>
<td>((-7)^2 + 4^2 = 65)</td>
<td>(x^2 + y^2 &gt; 25)</td>
<td>outside the circle</td>
</tr>
<tr>
<td>E</td>
<td>(-4, 3)</td>
<td>((-4)^2 + 3^2 = 25)</td>
<td>(x^2 + y^2 = 25)</td>
<td>on the circle</td>
</tr>
<tr>
<td>F</td>
<td>(-2, 2)</td>
<td>((-2)^2 + 2^2 = 8)</td>
<td>(x^2 + y^2 &lt; 25)</td>
<td>inside the circle</td>
</tr>
<tr>
<td>G</td>
<td>(-3, -4)</td>
<td>((-3)^2 + (-4)^2 = 25)</td>
<td>(x^2 + y^2 = 25)</td>
<td>on the circle</td>
</tr>
<tr>
<td>H</td>
<td>(6, -6)</td>
<td>(6^2 + (-6)^2 = 72)</td>
<td>(x^2 + y^2 &gt; 25)</td>
<td>outside the circle</td>
</tr>
<tr>
<td>I</td>
<td>(4, -3)</td>
<td>(4^2 + (-3)^2 = 25)</td>
<td>(x^2 + y^2 = 25)</td>
<td>on the circle</td>
</tr>
<tr>
<td>J</td>
<td>(2, -2)</td>
<td>(2^2 + (-2)^2 = 8)</td>
<td>(x^2 + y^2 &lt; 25)</td>
<td>inside the circle</td>
</tr>
<tr>
<td>K</td>
<td>(-5, -5)</td>
<td>((-5)^2 + (-5)^2 = 50)</td>
<td>(x^2 + y^2 &gt; 25)</td>
<td>outside the circle</td>
</tr>
<tr>
<td>L</td>
<td>(-3, -1)</td>
<td>((-3)^2 + (-1)^2 = 10)</td>
<td>(x^2 + y^2 &lt; 25)</td>
<td>inside the circle</td>
</tr>
</tbody>
</table>

4. As in Question 2, you can determine by looking at the symbol in the calculation: = indicates the point will lie on the circle, > indicates outside the circle, and < indicates inside the circle.
4. The graph of the equation \( x^2 + y^2 = 9 \) is a circle with center \((0, 0)\) and radius \(\sqrt{9} = 3\). If the coordinates of a point satisfy \(x^2 + y^2 < 9\), the point lies inside the circle. If the coordinates satisfy \(x^2 + y^2 > 9\), the point lies outside the circle. If the coordinates satisfy \(x^2 + y^2 = 9\), the point lies on the circle.

Does the given point lie on, inside, or outside the circle?

a) \(x^2 + y^2 = 9\); \((1, 2)\)  
b) \(x^2 + y^2 = 16\); \((3, 5)\)  
c) \(x^2 + y^2 = 25\); \((3, -4)\)  
d) \(x^2 + y^2 = 169\); \((-5, -12)\)

**Answers:**  
a) \(1^2 + 2^2 < 9\), so the point lies inside the circle;  
b) \(3^2 + 5^2 > 16\), so the point lies outside the circle;  
c) \(3^2 + (-4)^2 = 25\), so the point lies on the circle;  
d) \((-5)^2 + (-12)^2 = 169\), so the point lies on the circle

**MP.1, MP.3, MP.6**

5. a) Prove by contradiction that \(\sqrt{2}\) is not rational: assume that it is rational and get a contradiction. Use the meaning of a rational number in your explanation.

b) In pairs, explain your reasoning from part a). Do you agree with each other? Discuss why or why not.

**Sample answers:**

• Suppose that \(\sqrt{2}\) is rational. Then it can be written as a fraction, so \(\sqrt{2} = \frac{a}{b}\). Then \(\left(\frac{a}{b}\right)^2 = 2\), so \(\frac{a^2}{b^2} = 2\). Cross multiplying, we get \(a^2 = 2b^2\). I realized that the prime factorization of \(a^2\) has one more 2 than the prime factorization of \(b^2\). That means that if \(b^2\) has an even number of 2s, then \(a^2\) has an odd number of 2s, and if \(b^2\) has an odd number of 2s, then \(b^2\) has an even number of 2s. But the prime factorization of a perfect square has an even number of 2s; the prime factorization of \(a^2\) has twice as many 2s as the prime factorization of \(a\), and the prime factorization of \(b^2\) has twice as many 2s as the prime factorization of \(b\). So it can't be that the prime factorization of \(a^2\) has one more 2 than the prime factorization of \(b^2\). That is a contradiction, so \(\sqrt{2}\) is not rational.

• I got \(a^2 = 2b^2\), but then I noticed that \(a\) is even, because its square is even. I wrote \(a = 2c\). Then \((2c)^2 = 2b^2\), so \(4c^2 = 2b^2\), so \(b^2 = 2c^2\). But then \(b\) is even. So both \(a\) and \(b\) are even. But if \(\sqrt{2}\) really can be written as a fraction, we should be able to write it as a fraction in lowest terms, where the numerator and denominator do not share a common factor. This is a contradiction, so \(\sqrt{2}\) is not rational.

Individual or small-group follow-up: You may need to get some students started by encouraging them to cross multiply, or to pretend the square root of 2 is a specific fraction with specific numbers as numerator and denominator. If students struggle to see the structure required, encourage them to look at the prime factorizations of \(a^2\) and \(2b^2\) for different numbers. Can they be the same? Since they cannot have the same prime factorization, they cannot be the same number.

**NOTE:** In part b), encourage students to ask questions to understand and challenge each other’s thinking (MP.3) and use of vocabulary (MP.6)—see p. A-49 for sample sentence and question stems.
Goals:
Students will solve a system of linear equations by graphing using the slope and y-intercept of the lines.

Prior Knowledge Required:
Can identify the slope and y-intercept from the equation of a graph
Can graph a line using the slope and y-intercept

Vocabulary: intersection point, rise, run, slope, slope-intercept form, solve a system of linear equations, system of linear equations, y-intercept

(MP.2, MP.3) Review finding the slope and y-intercept from an equation. Write on the board:

\[ y = 2x + 3 \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-3</td>
</tr>
<tr>
<td>-2</td>
<td>-1</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

Point to the function table and ask for volunteers to write the change in \( x \) in each circle for the \( x \)-column (1) and the change in \( y \) in each circle for the \( y \)-column (2). Project a coordinate grid on the board and ask a volunteer to graph \( y = 2x + 3 \) by plotting the ordered pairs from the function table. Finally, ask another volunteer to pick two adjacent dots on the graph to find and label the rise and run. (see graph below)
ASK: What is the y-intercept of the graph? (3) How could we have found the y-intercept from the function table? (from the row where \( x = 0 \)) What is the rise? (2) What is the run? (1) How could we have found the rise and run from the function table? (find the change in \( x \) and the change in \( y \)) How can we find the slope from the rise and run? (slope = rise/run) What is \( 2 \div 1 \)? (2) SAY: So, the slope is 2.

Point to the y-intercept on the graph and SAY: When we know the y-intercept, we can find other points on the graph by using 1 for the change in \( x \) and 2 for the change in \( y \). Demonstrate counting from the y-intercept 1 unit to the right and 2 units up to get the point \((1, 5)\). SAY: We could also use \(-1\) for the change in \( x \) and \(-2\) for the change in \( y \) because \((-2)/(-1) = 2\). Demonstrate counting from the y-intercept 1 unit to the left and 2 units down to get the point \((-1, 1)\). Repeat the processing counting 1 unit left and 2 units down to get the point \((-2, -1)\). Then, point to the function table and show that these same ordered pairs \((1, 5)\) and \((-2, -1)\) are on the function table.

Write on the board:

\[
y = 2x + 3
\]

ASK: How can we find the slope from the equation of the line? (look at the coefficient, or the number in front of \( x \) when the equation is in this form, so 2) Remind the students about slope-intercept form, \( y = mx + b \), where \( m \) is the slope and \( b \) is the y-intercept. ASK: How can we find the y-intercept from the equation of the line? (when the equation is in slope-intercept form, the y-intercept is the constant, \( b \), so 3) SAY: If we know the slope and the y-intercept, we can draw the graph without using a function table.

Project a coordinate grid on the board. SAY: For the graph of \( y = 2x + 3 \), we found that the slope was 2 and the y-intercept was 3. Ask a volunteer to draw a dot for the y-intercept. SAY: To draw another dot, we can move 1 unit to the right for the run and 2 units up for the rise. Notice that \( 2 \div 1 = 2 \), which is the slope. Demonstrate and draw a dot at \((1, 5)\). SAY: We could also start at the y-intercept and move 1 unit to the left and 2 units down. Notice that \((-2) \div (-1) = 2\), which is again the slope. Demonstrate and draw another dot at \((-1, 1)\) SAY: We can draw the line now by connecting the dots. (see completed graph below)
**Exercises:** State the slope and $y$-intercept, then draw the graph.

a) $y = 3x - 2$  

b) $y = -2x + 1$  

c) $y = x + 3$

**Answers:** see completed graph below; a) slope = 3, $y$-int. = -2; b) slope = -2, $y$-int. = 1;  

c) slope = 1, $y$-int. = 3

---

**(MP.2) Solving a system of equations by graphing.** SAY: A **system of linear equations** is a set of two equations. To **solve a system of linear equations** means to find the point where the two lines for the equations intersect. In other words, we need to find the point the two lines have in common. We often write a system of linear equations with one equation written below the other. Write on the board:

\[
\begin{align*}
  y &= x + 1 \\
  y &= 2x - 1
\end{align*}
\]

SAY: This is a system of linear equations. To solve it, we want to find where the graphs of the lines intersect. ASK: What are the slope and $y$-intercept of $y = x + 1$? (slope = 1, $y$-intercept = 1)  
What are the slope and $y$-intercept of $y = 2x - 1$? (slope = 2, $y$-intercept = -1) Project a coordinate grid on the board and ask volunteers to draw the lines. (see graph below)
ASK: What is the intersection point? ((2, 3)) How can we check that this is the correct solution for the system of linear equations? (check whether the coordinates satisfy the equations of both lines) Write on the board:

\[
\begin{align*}
y &= x + 1 & \text{Check: } 3 &= 2 + 1 \\
y &= 2x - 1 & \text{Check: } 3 &= 2(2) - 1
\end{align*}
\]

SAY: The coordinates (2, 3) satisfy both equations, so (2, 3) is the solution to the system of linear equations.

Exercises: Solve the system of linear equations by graphing. Check that the coordinates satisfy the equations of both lines.

a) \( y = -2x + 1 \)  
   \( y = x + 4 \)

b) \( y = -x - 3 \)  
   \( y = 3x + 1 \)

c) \( y = 3x - 2 \)  
   \( y = -2x + 8 \)

Answers: a) (-1, 3); b) (-1, -2); c) (2, 4)

(MP.1) Trying to find the intersection point of parallel lines. Write on the board:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>

Ask for volunteers to complete the function tables for the two lines. (1, 2, 3, 4, 5; 3, 4, 5, 6, 7) ASK: Do the two tables have a common ordered pair? (no) Do you think if we extended the tables we would be able to find a common ordered pair? (no) Why not? (For any \( x \)-coordinate, the \( y \)-coordinate in the second table will always be 2 greater than the corresponding \( y \)-coordinate in the first table.)

SAY: Notice that the equations for the two lines are written in slope-intercept form. ASK: What is the slope of each line? (1) What does that tell you about the lines? (they are parallel) What are the \( y \)-intercepts of the lines? (1 and 3) If you draw the lines, will they intersect? (no) Will there be an intersection point? (no)

Exercises: Which of the following systems will have no solution?

A. \( y = 3x - 1 \)  
   \( y = 3x + 4 \)

B. \( y = 2x - 3 \)  
   \( y = -3x + 1 \)

C. \( y = -x + 4 \)  
   \( y = x + 4 \)

D. \( y = -4x + 3 \)  
   \( y = -4x + 2 \)

Answers: A and D
Extensions

(MP.1) 1. The lines \( y = x + 5 \), \( y = -2x + 8 \), and \( y = -x + 3 \) intersect in three different points. The three points form a triangle. Find the perimeter of the triangle by following these steps:

a) Complete a functions table for all three lines using values of \( x \) where \(-3 \leq x \leq 7\) for each line.
b) Graph each line on a grid.
c) Find the intersection point of the lines \( y = x + 5 \) and \( y = -2x + 8 \). Label it \( A \).
d) Find the intersection point of the lines \( y = -2x + 8 \) and \( y = -x + 3 \). Label it \( B \).
e) Find the intersection point of the lines \( y = -x + 3 \) and \( y = x + 5 \). Label it \( C \).
f) Check your answers to parts c), d), and e) by looking at the functions table.
g) Find the lengths of line segments \( AB \), \( AC \), and \( BC \) to the nearest tenth.
h) Find the perimeter of \( \triangle ABC \).

Answers:

\[
\begin{array}{c|c|c|c}
 x & y = x + 5 & y = -2x + 8 & y = -x + 3 \\
 \hline
 -3 & 2 & 14 & 6 \\
 -2 & 3 & 12 & 5 \\
 -1 & 4 & 10 & 4 \\
 0 & 5 & 8 & 3 \\
 1 & 6 & 6 & 2 \\
 2 & 7 & 4 & 1 \\
 3 & 8 & 2 & 0 \\
 4 & 9 & 0 & -1 \\
 5 & 10 & -2 & -2 \\
 6 & 11 & -4 & -3 \\
 7 & 12 & -6 & -4 \\
\end{array}
\]

c) \( A (1, 6); \) d) \( B (5, -2); \) e) \( C (-1, 4); \)
g) \( d_{AB} = \sqrt{5 - 1^2 + (-2 - 6)^2} = \sqrt{80} \approx 8.9 \)
\[
\begin{align*}
&d_{BC} = \sqrt{5 - (-1)^2 + (-2 - 4)^2} = \sqrt{72} \approx 8.5 \\
&d_{AC} = \sqrt{(-1 - 1)^2 + (4 - 6)^2} = \sqrt{8} \approx 2.8 ;
\end{align*}
\]
h) perimeter \( \approx AB + BC + AC = 8.9 + 8.5 + 2.8 = 20.2 \)

(MP.2, MP.4, MP.6) 2. a) Earth’s orbit around the Sun can be approximated by a circle with radius \( 1.5 \times 10^8 \) km. It takes Earth approximately \( 365 \frac{1}{4} \) days to make one orbit. Estimate Earth’s average speed as it travels around the Sun. Say what each step means in the situation.

b) In part a), what place value did you round your answer to? If you kept your answer with more decimal places, would your answer be more accurate? Explain.

Answers: a) Let \( d \) be the distance Earth travels in one orbit. Then \( d \) is approximately the circumference of the circle with radius \( 1.5 \times 10^8 \) km. \( d = 2\pi r = 2(3.14)(1.5 \times 10^8) = 9.42 \times 10^8 \) km.

Since one orbit takes about 365.25 days, or 365.25 \times 24 = 8,766 hours, Earth’s average speed is about \( (9.42 \times 10^8)/(8.77 \times 10^3) \) km/h \( \approx 1.1 \times 10^5 \) km/h; b) I rounded the decimal part of the scientific notation to the nearest tenth. It wouldn’t be more accurate to round to the nearest hundredth because the radius is already approximated only to the nearest tenth.
**EE8-52  Solving Word Problems by Graphing**

Pages 143–144

Standards: 8.EE.C.8, 8.EE.C.8c

**Goals:**
Students will solve word problems by graphing two lines and finding the intersection point.

**Prior Knowledge Required:**
- Can complete a function table
- Can graph a line using a function table
- Can find the intersection point by looking at the graphs of two lines

**Vocabulary:** flat fee, intersection point, slope-intercept form

**Materials:**
- overhead projector
- transparency of BLM Word Problem Coordinate Grid (p. P-46)

**(MP.2) Review flat fees.** SAY: Remember, in business, the cost of something sometimes has two parts: a flat fee and a charge per unit, such as a charge per person, hour, or item. Write on the board:

Jennifer’s Banquet Hall charges $30 per guest, plus a fee of $200.

<table>
<thead>
<tr>
<th>Number of Guests (x)</th>
<th>Total Cost (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
</tr>
</tbody>
</table>

SAY: If we are renting the banquet hall and have 10 guests, we have to charge $30 per guest plus a fee of $200. ASK: What is $30 × 10? ($300) What is $300 + $200? ($500) Fill in the total cost for 10 guests in the chart, where x = 10. Ask for volunteers to fill in the total cost for 20, 30, and 40 guests. ($800; $1,100; $1,400) ASK: What is the cost for zero guests? ($200) How did you calculate that? ($30 × 0 + $200) Fill in the answer in the chart. SAY: Notice that even if we have no guests, we still have to pay $200 just to reserve the hall. Write on the board:

\[ y = 30x + 200 \]
Ask for volunteers to find the value of \( y = 30x + 200 \) for \( x = 0, 10, 20, 30, \) and \( 40 \), and write the results in the function table. (200; 500; 800; 1,100; 1,400) SAY: Notice that the equation for this function table gives the same answers as in the previous function table. Write on the board:

\[
y = 30x + 200
\]

ASK: What part of the equation tells us the charge per guest? (the coefficient of \( x \)) What part of the equation tells us the flat fee? (the constant) Point out that this number is not in front of the variable \( x \). Write on the board:

\[
y = 30x + 200 \quad \text{flat fee}
\]

charge per person

**Exercises:** Write a formula for the word problem.

a) A gravel company charges $25 per cubic yard and a delivery charge of $75.

b) A yearbook company charges $500 plus $15 per yearbook.

c) A taxi charges a flat fee of $4 and $3 per mile.

**Answers:** a) \( y = 25x + 75 \), b) \( y = 15x + 500 \), c) \( y = 3x + 4 \)

**(MP.1) Solving word problems by graphing and finding the intersection point.** Write on the board:

Electrician A charges $100 for a house call and $40 per hour. Electrician B charges only $70 for the house call, but charges $50 per hour. After how many hours of work would the two electricians charge the same amount? What will the total charge be?

ASK: What is the flat fee for electrician A? ($100) What equation can we write to calculate the total charge for electrician A? (\( y = 40x + 100 \)) What is the flat fee for electrician B? ($70) What equation can we write for electrician B? (\( y = 50x + 70 \)) Write on the board:

A. \( y = 40x + 100 \) \hspace{1cm} slope = ____ \hspace{1cm} y-intercept = ____

B. \( y = 50x + 70 \) \hspace{1cm} slope = ____ \hspace{1cm} y-intercept = ____

Ask for volunteers to fill in the slope and the \( y \)-intercept. PROMPT: We have written each equation in slope-intercept form. Which number is the slope? What number is the constant? (A: slope = 40, \( y \)-int. = 100, B: slope = 50, \( y \)-int. = 70)

Project **BLM Word Problem Coordinate Grid.** SAY: Pay close attention to the scale on the \( x \)- and \( y \)-axes. Remind students that they may have to change the scale on either or both axes to allow the graph to fit on the page. Draw a dot at (0, 100) and SAY: Here is the \( y \)-intercept of the line \( y = 40x + 100 \). ASK: How can I place another dot so the slope will be 40? (move four grid units to the right and two grid units up) PROMPT: Be careful—the units on the \( y \)-axis are going up by 20 with each square on the grid, and the units on the \( x \)-axis are going up by 1/4 with each square on the grid. SAY: Four units across is a run of 1. Two units up is a rise of 40. Note that 40 ÷ 1 = 40. Repeat to draw the graph of \( y = 50x + 70 \). (see completed graph on the next page)
ASK: What is the intersection point? ((3, 220)) After how many hours will the two electricians cost the same? (3 hours) What will the cost be? ($220) Show students how to check their answers by substituting the coordinates (3, 220) in each equation. Demonstrate with one equation on the board and ask a volunteer to check the other. (check: 220 = 40(3) + 100, check: 220 = 50(3) + 70)

(MP.3) Difficulties with trying to find the intersection point by graphing. Have a discussion about some of the difficulties you may have when trying to find the intersection point by graphing. Here are some points to consider:
- It is difficult to find the correct scale of the x- and y-axes.
- It is difficult to graph large numbers.
- It is difficult to graph an intersection point involving fractions or decimals.
- If the slopes of the graphs are close to each other, it may be difficult to find coordinates of the intersection point exactly.

Ask students to complete the word problems in the AP Book. Remind them that they may have to adjust the size of the graph and the scales of the axes.
Extensions

(MP.4) 1. Nancy is paid a salary of $600 per week, plus a bonus of $2.50 for every pair of shoes she sells. Sal is paid a salary of $400 per week, plus a bonus of $3.50 for every pair of shoes he sells. Last week, they each sold the same number of shoes and earned the same total pay. How many pairs of shoes did each sell? What was their total pay?

**Answer:** for Nancy’s pay: slope = 2.50, y-int. = 600; for Sal’s pay: slope = 3.50, y-int. = 400; intersection point = (200, 1,100), so they each sold 200 pairs of shoes and earned a total of $1,100

(MP.1, MP.6) 2. In the United States, most people measure temperature using the Fahrenheit scale. On the Fahrenheit scale, water freezes at 32°F and boils at 212°F. On the Celsius scale, water freezes at 0°C and boils at 100°C. Scientists sometimes use the Kelvin scale. On the Kelvin scale, water freezes at 273 K and boils at 373 K. The formulas below show how to convert from Fahrenheit or Celsius to Kelvin:

Celsius to Kelvin: \( y = x + 273 \)
Fahrenheit to Kelvin: \( y = \frac{5}{9}(x - 32) + 273 \)

a) Draw a graph in the second quadrant for converting Celsius to Kelvin. Use breaks in the axes to show \(-50 \leq x \leq -35\) and \(225 \leq y \leq 240\).
b) On the same grid, draw a graph for converting Fahrenheit to Kelvin.
c) Find the intersection point of the graphs.
d) What temperature in Celsius and Fahrenheit is equivalent to 233 K?
e) How cold is it when the temperature is the same on the Celsius scale and the Fahrenheit scale?

**Answers:**
a–b)

c) intersection point = \((-40, 233)\)
d) \(-40°C = 233 K, -40°F = 233 K\)
e) \(-40°C = -40°F\)
EE8-53 Solving Systems of Equations Algebraically I

Pages 145–147

Standards: 8.EE.C.8, 8.EE.C.8a

Goals:
Students will solve systems of equations algebraically where each of the equations has been written in slope-intercept form.

Prior Knowledge Required:
Can complete a function table given the equation of the line
Can rearrange an equation to solve for \( y \)
Can substitute to check that the intersection point satisfies the equations of both lines

Vocabulary: general point, intersection point, slope-intercept form

Materials:
The Geometer’s Sketchpad® (see Extension 3)

(MP.2) Finding the general point in a function table. Write on the board:

\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
1 & \quad \\
2 & \quad \\
3 & \quad \\
4 & \quad \\
\hline
\end{array}
\]

\( y = 2x - 1 \)

Ask for a volunteer to fill in the values of the function table, except for the last row. (1, 3, 5, 7)

ASK: How can we describe in words how to calculate each value? (\( x \) is the value given; for \( y \), we multiply each \( x \)-coordinate by 2 and subtract 1) SAY: We can write a general point to describe these calculations. Write on the board:

\[
\text{general point} = (x, 2x - 1)
\]

Fill in the last row of the function table by writing “2x – 1” in the \( y \)-column.
**Exercises:** Write the values for $y$ to the $y$-column. Then write the coordinates of the general point for the line.

a) $y = 3x + 2$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
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<tr>
<td>3</td>
<td></td>
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<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>$x$</td>
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</tr>
</tbody>
</table>

b) $y = -2x + 3$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>2</td>
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<td>3</td>
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<td>4</td>
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<tr>
<td>$x$</td>
<td></td>
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</tbody>
</table>

c) $y = 5x$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>$x$</td>
<td></td>
</tr>
</tbody>
</table>

**Answers:** a) 5, 8, 11, 14, general point: $(x, 3x + 2)$; b) 1, −1, −3, −5, general point: $(x, −2x + 3)$; c) 5, 10, 15, 20, general point: $(x, 5x)$

**(MP.2) Finding the general point without a function table.** SAY: Notice that we do not need to complete the function table to write the general point. We can just create an ordered pair. The first coordinate will always be $x$. The second coordinate will be the right-hand side of the equation when the equation is in slope-intercept form. Point to the first table in the previous exercises. Write the general point on the board:

$$(x, 3x + 2)$$

Point to the first $x$ and SAY: Here is the $x$-coordinate. Point to “$3x + 2$” and SAY: Here is how to calculate the $y$-coordinate. The general point is like a summary of the function table and the rule. We can pick any value we want for $x$, and then use the general point in the ordered pair to find the $y$-coordinate. For example, if $x = 4$, $y = 3(4) + 2 = 14$. The ordered pair $(4, 14)$ is a point on the line.

**Exercises:** Write the general point for the line.

a) $y = 2x + 7$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
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</thead>
<tbody>
<tr>
<td></td>
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</table>

b) $y = -4x + 3$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
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</table>

c) $y = \frac{1}{2}x - 3$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</table>

**Answers:** a) $(x, 2x + 7)$; b) $(x, -4x + 3)$; c) $x, \frac{1}{2}x - 3$

**(MP.2) Finding the $y$-coordinate in an intersection point, given the $x$-coordinate.** Remind students that the $x$- and $y$-coordinates of the intersection point for a system of linear equations must satisfy the equations of both lines. Write on the board:

$$y = 2x + 1$$
$$y = -3x + 6$$

ASK: How do we know that $(1, 3)$ is the intersection point for these two lines? (their coordinates satisfy the equations of both lines: $3 = 2(1) + 1$ and $3 = -3(1) + 6$) SAY: Notice that if we know the $x$-coordinate of the intersection point, we can substitute that value into either equation to find
the y-coordinate. The answer must be the same in both equations because the coordinates of the intersection point must satisfy the equations of both lines. Write on the board:

\[ y = -3x - 2 \]
\[ y = x + 6 \]

ASK: If the x-coordinate of the intersection points of these two lines is -2, how can we find the y-coordinate? (substitute \( x = -2 \) into either equation) Ask a volunteer to use the first equation to find the y-coordinate of the intersection point on the board. Ask another volunteer to use the second equation to find the y-coordinate of the intersection point. (first equation: \( y = -3(-2) - 2 = 4 \); second equation: \( y = (-2) + 6 = 4 \)) ASK: So, what is the y-coordinate of the intersection point? (4)

What are the coordinates of the intersection point? ((-2, 4))

(MP.2) Exercises:

a) The x-coordinate of the intersection point of the lines is given. Find the y-coordinate.

i) \( y = 3x + 1 \)
\( y = -x - 7 \)
int. point = (-2, ?)

ii) \( y = 2x + 3 \)
\( y = -3x - 2 \)
int. point = (-?, ?)

iii) \( y = x + 3 \)
\( y = -x - 7 \)
int. point = (-5, ?)

Bonus: The y-coordinate of the intersection point of the lines is given. Find the x-coordinate.
\( y = -3x - 2 \)
y = 2x + 13
int. point = (?, 7)

Answers: a) i) -5, ii) 1, iii) -2; Bonus: -3

(MP.2, MP.6) Finding the x-coordinate of the intersection point by first comparing the general points. SAY: Remember that the coordinates of the intersection point have to satisfy the equations of both lines. At the intersection point, even the general points must be equal. Project a coordinate grid on the board and draw the lines shown below:

\[ y = 2x - 1 \]
\[ y = 3x - 3 \]

ASK: From the graph, what do the coordinates of the intersection point appear to be? (2, 3)
What is the general point for each line? (\((x, 2x - 1), (x, 3x - 3)\)) Point to the intersection point, and SAY: At the intersection point, each coordinate of the general point must be the same.
We know that $x = x$. ASK: What is the $y$-coordinate for the general point of the first line? $(2x - 1)$

What is the $y$-coordinate for the general point of the second line? $(3x - 3)$

SAY: At the intersection point for the lines, the $y$-coordinates must be equal—even for the two general points. So, we can write the general points for the $y$-coordinates as an equation. Write on the board:

$$2x - 1 = 3x - 3$$

Ask for a volunteer to solve the equation for $x$. $(2x - 3x = 1 - 3, -x = -2, x = 2)$ SAY: So, we now have the $x$-coordinate at the intersection point: 2. Keep this example on the board while students complete the following exercises.

**(MP.2) Exercises:** Write an equation using the $y$-coordinates of the general points. Then, solve the equation to find the $x$-coordinate of the intersection point.

a) $y = -2x - 3$

b) $y = 5x - 1$

**Bonus:** $y = 0.2x - 3$

**Answers:** a) $-2x - 3 = -3x + 4, x = 7$; b) $5x - 1 = 2x + 8, x = 3$; Bonus: $0.2x - 3 = 0.5x + 3, x = -20$

**(MP.2, MP.6) Finding the $y$-coordinate of the intersection point by substitution.** Refer back to the example on the board. SAY: For the intersection point of these two linear equations, we solved the equation $2x - 1 = 3x - 3$ and found that $x = 2$. ASK: How can we find the $y$-coordinate? (substitute $x = 2$ into the equation of either line) Ask two volunteers to solve for the $y$-coordinate, one student using the equation of the first line and the other student using the equation of the second line. (first line: $y = 2(2) - 1 = 3$, second line: $y = 3(2) - 3 = 3$) SAY: So, the $y$-coordinate for the intersection point is 3. ASK: So, what are the coordinates of the intersection point? $(2, 3)$ Remind students that they can use either equation to find the $y$-coordinate. Point out that the answer we got using algebra matches the intersection point we saw on the graph.

**Exercises:** For each of the equations in the previous exercises, find the $y$-coordinate and state the intersection point.

**Answers:**

a) $y = -2(7) - 3 = -17$

b) $y = 2(3) + 8 = 14$

**Bonus:** $y = 0.2(-20) - 3 = -7$

**int. point = (7, -17)**

**int. point = (3, 14)**

**int. point = (-20, -7)**

**(MP.3, MP.6) Checking for errors.** Write on the board:

$$y = 3x - 1$$

$$y = 2x - 4$$

SAY: Will thinks the $x$-coordinate of the intersection point of these two equations is 5 and that the intersection point is $(5, 14)$. When he checks his answer, he finds that $(5, 14)$ satisfies the equation of the first line. Lynn thinks the $x$-coordinate of the intersection point is $-3$ and that the intersection point is $(-3, -10)$. She checks her answer and finds that $(-3, -10)$ satisfies the equation of the first line. ASK: Who is right? Allow some time for discussion. Remind students that the coordinates of the intersection point must satisfy both equations. Demonstrate that the
coordinates of Will's intersection point satisfy the first equation, but not the second. Show that the coordinates of Lynn's intersection point satisfy both equations.

**Extensions**

(MP.1) 1. The $x$-axis and two lines with equations $y = x + 3$ and $y = -2x + 6$ intersect in three different points to form a triangle. Find the area of the triangle.

**Answer:** The line $y = x + 3$ intersects the $x$-axis at the point $(-3, 0)$, and the line $y = -2x + 6$ intersects the $x$-axis at the point $(3, 0)$. The lines $y = x + 3$ and $y = -2x + 6$ intersect when $x + 3 = -2x + 6$, so $3x = 3$, so $x = 1$ and $y = 4$. The base of the triangle is 6 units long and the height of the triangle is 4 units long, so the area of the triangle is $1/2 \times 6 \times 4 = 12$ square units.

(MP.1, MP.6) 2. In $\triangle ABC$, the coordinates of the vertices are $A (2, 6)$, $B (7, 1)$, and $C (-2, 4)$. A perpendicular bisector of a line is at right angles (perpendicular) to the line and passes through its midpoint. The perpendicular bisector of $AC$ has the equation $y = -2x + 5$. The perpendicular bisector of $BC$ has the equation $y = 3x - 5$.

a) Find the intersection point $D$ of the perpendicular bisectors.
b) Show that the lengths of $AD$, $BD$, and $CD$ are equal.
c) Explain why point $D$ can be used as the center of a circle that will touch the three vertices of the triangle.

**Answers:**

a) $(2, 1)$
b) $AD = 5$, $BD = 5$, $CD = \sqrt{\text{change in } x^2 + \text{change in } y^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$
c) since the lengths of $AD$, $BD$, and $CD$ are all equal to 5 units, $D$ can be the center of a circle with radius 5 that will touch $A$, $B$, and $C$

(MP.5) 3. Use The Geometer’s Sketchpad® to show that if you translate the lines in a system of equations, the intersection point of the original system will be translated by the same vector.

a) In the Graph menu, select “Show Grid” and “Snap Points.”
b) Complete function tables for the lines $y = x + 3$ and $y = -x - 5$ on paper.
c) For each line, plot two of the points on the grid and join them with a line using the Straightedge tool. Label the lines $m$ and $n$.
d) Use the Point tool to label the intersection point of the graphs. Label the intersection point $A$.
e) Use the Measure menu to find the coordinates of the intersection point.
f) Use the Point tool to draw dots at $(2, -2)$ and $(5, 2)$. Label the points $E$ and $F$.
g) In the Transform menu, use the “Mark Vector” option to choose $EF$ as a translation arrow.
h) Select line $m$. Use the Transform menu to translate the line. Label the new line $m'$.
i) Select line $n$. Use the Transform menu to translate the line. Label the new line $n'$.
j) Use the Point tool to draw the intersection point of lines $m'$ and $n'$. Label the intersection point $B$.
k) Verify that if you translate point $A$ by the same vector, you will get point $B$. 
EE8-54 Solving Systems of Equations Algebraically II

Pages 148–149

Standards: 8.EE.C.8, 8.EE.C.8b

Goals:
Students will solve systems of linear equations algebraically by first writing each of the equations in slope-intercept form.

Prior Knowledge Required:
Can solve a system of equations algebraically where each of the equations has been written in slope-intercept form

Vocabulary: general point, intersection point, slope-intercept form, solve a system of linear equations, system of linear equations

Materials:
The Geometer’s Sketchpad® (see Extension 3)

(MP.2) Review how to find the slope and y-intercept when an equation is written in slope-intercept form. Write on the board:

\[ y = 2x - 5 \]

SAY: Remember, if the equation of a line is written in the form where the variable \( y \) is alone on one side of the equation, we say the equation is in slope-intercept form. ASK: How can we tell what the slope is when the equation is written in this form? (look at the coefficient of \( x \)) What is the slope of this line? (2) How can we tell what the y-intercept is? (look at the constant) What is the y-intercept here? (-5)

Exercises: State the slope and y-intercept.
a) \( y = -3x + 4 \)  
b) \( y = -2 + 5x \)  
**Bonus:** \( y = -x \)

Answers: a) slope = -3, y-int. = 4; b) slope = 5, y-int. = -2; Bonus: slope = -1, y-int. = 0

(MP.2) Review how to rearrange an equation so that it is in slope-intercept form. Write on the board:

\[ 3x + 2y = 4 \]

ASK: Can we easily read the slope and y-intercept of the equation of this line? (no) Why not? (the equation is not written in slope-intercept form) SAY: We want to get the variable \( y \) all by itself on one side of the equation—in other words, we need to isolate for \( y \). Ask a volunteer to move “3x” to the other side of the equation. \( 2y = -3x + 4 \) ASK: Is the equation in slope-intercept
form? (no, not yet) Why? (it says “2y” on one side of the equation; we need the form $y = mx + b$, so we have to get $y$ by itself) How do get $y$ by itself? (divide both sides by 2) Write on the board:

$$y = -\frac{3}{2}x + 2$$

ASK: What is the slope of the line? ($-3/2$) What is the $y$-intercept? (2) Project a coordinate grid on the board and draw a dot at (0, 2) for the $y$-intercept. SAY: Here is the $y$-intercept. ASK: How can we place another dot so that the slope of the line will be $-3/2$? (mark the rise = $-3$, run = 2) SAY: After we place this second point, we can draw the line for $y = -\frac{3}{2}x + 2$. Demonstrate by drawing the second dot on the graph and drawing a line through the points, as shown below:

![Graph](image)

**Exercises:** Find the slope and $y$-intercept of each line. Then graph the lines and find the intersection point.

a) $3x - 6y = 12$

$2x + y = 3$

b) $-2x + 5y = 10$

$x - 2y = -4$

**Answers:**

a) $3x - 6y = 12$: $m = 1/2$, y-int. = 2; $2x + y = 3$: $m = -2$, y-int. = 3; int. point = (2, -1)
b) $-2x + 5y = 10$: $m = 2/5$, y-int. = 2; $x - 2y = -4$: $m = 1/2$, y-int. = 2; int. point = (0, 2)

**(MP.2) Solving systems algebraically where the equations of the lines are not in slope-intercept form.** SAY: Remember that in the last lesson we solved systems of linear equations where the equations of the lines were in slope-intercept form. For example, we solved the system of linear equations for $y = 2x - 1$ and $y = 3x - 3$.

Write on the board:

$$-4x + 3y = -1$$

$$2x + y = 3$$
ASK: What should we do to these two new equations before trying to solve them algebraically? (write the equations in slope-intercept form by isolating for y) Ask volunteers to rearrange each equation into slope-intercept form. \( y = \frac{4}{3} x - \frac{1}{3}, \ y = -2x + 3 \)

ASK: What is the general point for each line? \((x, \ \frac{4}{3} x - \frac{1}{3}), \ (x, \ -2x + 3)\). SAY: Remember, to solve systems of linear equations algebraically, we must first write the equation for the y-coordinates of the general points. Write on the board:

\[
\frac{4}{3} x - \frac{1}{3} = -2x + 3
\]

SAY: Remember, it is easier to solve equations involving fractions if we multiply all the terms by the lowest common denominator of the fractions. Here, the lowest common denominator is 3. ASK: What is the equation after we multiply all terms by 3? \((4x - 1 = -6x + 9)\) Ask a volunteer to solve the equation on the board. (see solution below)

\[
\begin{align*}
10x &= 10 \\
x &= 1
\end{align*}
\]

SAY: So, the x-coordinate of the intersection point is 1. ASK: How do we find the y-coordinate? (substitute \(x = 1\) into the equation for either line) Ask half the class to find the y-coordinate using one of the equations. Ask the other half to find the y-coordinate using the other equation. (see answers below)

\[
\begin{align*}
y &= \frac{4}{3} x - \frac{1}{3} \\
y &= \frac{4}{3}(1) - \frac{1}{3} = 1
\end{align*}
\]

ASK: So, what are the coordinates of the intersection point? \((1, 1)\)

**Exercises:** Solve the system algebraically. Check your answer in both equations.

a) \(x + 3y = -2\)  \quad b) \(3x - 4y = -11\)

\[
\begin{align*}
2x - 5y &= 7 \\
2x + 5y &= 8
\end{align*}
\]

**Answers:** a) \((1, -1)\); b) \((-1, 2)\)

**Extensions**

(MP.1) 1. Find the solution to the following system:

\[
\begin{align*}
0.2x - 0.3y &= 80 \\
0.03x + 0.04y &= -5
\end{align*}
\]

**Answer:** \((100, -200)\)
(MP.1) 2. Find the values of $m$ and $n$ if the solution to the system \[ \begin{align*}
mx - ny &= 4 \\
mx + ny &= 6
\end{align*} \] is $(1, -1)$.

**Solution:** If $(1, -1)$ is the solution, then it must satisfy both equations.

So, \[ \begin{align*}
m(1) - n(-1) &= 4 \\
m(1) + n(-1) &= 6
\end{align*} \] or \[ \begin{align*}
m + n &= 4 \\
m - n &= 6
\end{align*} \]. Thus, $m = 5$, $n = -1$.

Redirecting students: Encourage students to ask themselves, What makes this problem hard? (that there are so many variables) What does it mean for the solution to be $(1, -1)$? (set $x = 1$ and $y = -1$, and then both equations will be true) PROMPT: What variables are represented by the solution $(1, -1)$? You may need to point out that $m$ and $n$ are coefficients.

(MP.1, MP.4, MP.6) 3. Tasha has quarters, dimes, and nickels worth $10.35. She has 5 more quarters than nickels. She has twice as many nickels as dimes. How many of each type of coin does she have?

**Solution:** Let $q$ be the number of quarters, $d$ the number of dimes, and $n$ the number of nickels. Then the amount Tasha’s coins are worth, in cents, is $25q + 10d + 5n$. We are told that this amount is $10.35$, or 1,035 cents, so $25q + 10d + 5n = 1,035$. Since Tasha has 5 more quarters than nickels, we can write $q = n + 5$, and since Tasha has twice as many nickels as dimes, we can write this as $n = 2d$. So we have three equations:

\[ \begin{align*}
25q + 10d + 5n &= 1,035 \\
q &= n + 5 \\
n &= 2d
\end{align*} \]

Putting $q = n + 5$ into the first equation, we get $25(n + 5) + 10d + 5n = 1,035$, which simplifies to $3n + d = 91$. So now we have two equations with two variables: $3n + d = 91$ and $n = 2d$. \[ 3(2d) + d = 91, \text{ so } 6d + d = 91, \text{ so } 7d = 91, \text{ so } d = 13, \text{ so } n = 2d = 26, \text{ and } q = n + 5 = 31. \]

Tasha has 13 dimes, 26 nickels, and 31 quarters.

I checked my answer by calculating the total value of these coins: the dimes are worth $130\,\text{¢}$, the nickels are worth $26 \times 5 = 130\,\text{¢}$, and the quarters are worth $31 \times 25 = 775\,\text{¢}$, so in total the coins are worth $775 + 130 + 130 = 1,035\,\text{¢}$, or $10.35$.

Redirecting students: ASK: What makes this problem hard? (there are three variables instead of two) Challenge students to look for a way to make it into a problem with two variables instead of three.

Look for students to say precisely what the variables they use represent (MP.6), to model the situation using equations (MP.4), to reduce the problem to a system of two equations instead of three (MP.1), and to check that their answer makes sense (MP.1).
Goals:
Students will solve word problems where the problems lead to equations of lines that are not written in slope-intercept form.

Prior Knowledge Required:
Can solve a system of linear equations algebraically where the equations are not written in slope-intercept form

Vocabulary: general point, intersection point, solve a system of linear equations, system of linear equations

(MP.3) Review key words in word problems that indicate operations to be used in the equations. Tell students that writing equations for word problems is really a translation problem. SAY: We need to translate the problem in English into numerals, mathematical operations, and symbols. Write on the board:

\[ + \quad - \quad \times \quad \div \]

Ask volunteers to write underneath each operation words that indicate the operation that should be used in the equation for the word problem. Answers may include the following:

+: sum, plus, total, and, more than
−: difference, minus, less than
×: product, times, twice, double
÷: quotient, ratio, one-half, one-third

Remind students that it is important in a word problem to find where the equal sign will appear in the equation. ASK: What words may indicate where the equal sign will go? (sample answers: is, are, was, were, will be)

Write on the board:

The sum of two numbers is 45.

SAY: The first step in translation is to find where the equal sign belongs. ASK: Which word may indicate where the equal sign belongs? (“is”) Write an equal sign under the word “is.” ASK: What operation is indicated by the word “sum”? (addition) Write “+” under the word “sum.” ASK: What
is being added? (two numbers) How many variables will we need? (2) SAY: We should explain that we are going to use variables to represent the numbers. Write on the board:

\[ \text{The sum of two numbers is 45.} \quad \text{Let } x \text{ and } y \text{ represent the numbers.} \]

\[ x + y = 45 \]

Have a volunteer write the variables in the appropriate places in the equation. \((x + y)\)

ASK: Where does the number 45 appear? (to the right of the equal sign) The final equation should look like this:

\[ x + y = 45 \]

**Exercises:** Write the word problem with values, symbols, and (if needed) variables.

a) The difference between two numbers is 36.

b) The ratio of two numbers is 3 : 2.

c) Ken has twice as many books as Blanca.

**Bonus:** The product of two numbers is 5 greater than the sum of the two numbers.

**Answers:** Let \( x \) and \( y \) represent the numbers: a) \( x - y = 36 \), b) \( x/y = 3/2 \) or \( x : y = 3 : 2 \),

c) \( x = 2y \), Bonus: \( xy = 5 + x + y \)

**(MP.2, MP.3)** **Writing word problems as two equations using two variables each.**

SAY: Some word problems contain two related sentences that need to be translated into systems of linear equations. Write on the board:

The sum of two numbers is 15. The difference between the numbers is 3.

ASK: How many unknown numbers are involved in this question? (2) SAY: Then, we need two variables. Remind students that we need to explain what the variables will represent. Write on the board:

Let \( x \) represent the first number. Let \( y \) represent the second number.

Ask volunteers to write an equation for each sentence. \((x + y = 15, \ x - y = 3)\)

**Exercises:** Write two equations that use two variables each for the word problem.

a) Raj has $5 bills and $10 bills in his wallet. He has 10 bills. The value of the bills is $65.

b) Mary has nickels and dimes in her purse. She has 10 coins. The value of the coins is $0.65.

c) The perimeter of an isosceles triangle is 22 cm. The difference between the length of one of the equal sides and the base is 5 cm.

**Answers:**

a) Let \( x \) be the number of $5 bills. Let \( y \) be the number of $10 bills. \( x + y = 10, \ 5x + 10y = 65 \)

b) Let \( x \) be the number of nickels. Let \( y \) be the number of dimes. \( x + y = 10, \ 0.05x + 0.10y = 0.65 \)

c) Let \( x \) be the length of each equal side. Let \( y \) be the length of the base. \( 2x + y = 22, \ x - y = 5 \).

**NOTE:** Point out the difference between parts a) and b) in the previous exercises. In each case, the second equation is arrived at by multiplying the number of each category by each currency’s
worth. So, for example, for $5 bills, we multiply the number of bills by 5. But, for nickels, we multiply the number of coins by 0.05 because a nickel is worth $0.05.

**MP.2) Solving equations from a word problem algebraically.** SAY: We need to write each word problem in mathematical equations. Then, we must solve a system of equations algebraically. In other words, we need to follow these steps:

Step 1: “Translate” the word problem and state what each variable represents.

Step 2: Write each equation in slope-intercept form.

Step 3: Write an equation for the $y$-coordinates of the general point at the intersection point.

Step 4: Solve the equations to find the $y$-coordinate of the intersection point.

As you write on the board, SAY: In part a) of the previous exercises, we arrived at these equations. Write on the board:

\[
\begin{align*}
x + y &= 10 \\
5x + 10y &= 65
\end{align*}
\]

SAY: $x$ represented the number of $5 bills and $y$ represented the number of $10 bills.

Ask volunteers to write each equation in slope-intercept form by isolating for the variable $y$.

\[
(y = -x + 10, \ y = \frac{-1}{2}x + \frac{13}{2}).
\]

ASK: How do we write an equation for the $y$-coordinates of the general point at the intersection point? \((-x + 10 = \frac{-1}{2}x + \frac{13}{2}).\) Ask students to solve the equation at their desks. Remind them it is easier to solve equations with fractions if we multiply every term by the lowest common denominator. After students have had enough time, take up and discuss the solution, as shown below:

\[
\begin{align*}
-x + 10 &= \frac{-1}{2}x + \frac{13}{2} \\
-2x + 20 &= -x + 13 \\
-x &= -7 \\
x &= 7
\end{align*}
\]

SAY: So, there are seven $5 bills. ASK: How can we find the number of $10 bills? (substitute $x = 7$ into either equation to find the $y$-coordinate of the general point) Ask half the class to find the value of $y$ using $y = -x + 10$ and half to find the value using $y = \frac{-1}{2}x + \frac{13}{2}$. (see solutions below)

\[
y = -(7) + 10 = 3 \quad \quad \quad y = \frac{-1}{2}(7) + \frac{13}{2} = 3
\]

SAY: We should write a concluding statement and then, check that our answers work in the original system of linear equations. Write on the board:

\[
\begin{align*}
\text{There are seven$5 bills and three$10 bills.} \\
\text{Check in } x + y &= 10 \text{ and } 5x + 10y = 65. \\
(7) + (3) &= 10, \ 5(7) + 10(3) = 65
\end{align*}
\]
Exercises: For parts b) and c) in the previous exercises, solve the system of equations created for the word problems.

Answers: b) there are 7 nickels and 3 dimes; c) the equal sides are 9 cm long, the base is 4 cm.

Extensions

(MP.1, MP.2, MP.4) 1. Alex is four times as old as Clara. In 5 years, Alex will only be three times as old as Clara. How old are Alex and Clara today?

Solution: Let \(x\) represent Clara’s age today and \(y\) represent Alex’s age today. In 5 years, Alex’s age will be \(y + 5\). Clara’s age will be \(x + 5\).

Today: \(y = 4x\)

Five years from now: \((y + 5) = 3(x + 5)\)

Isolating for \(y\) gives: \(y = 4x\)
\(y = 3x + 10\)

At the intersection point: \(4x = 3x + 10\)
\(x = 10\)

Substituting \(x = 10\) into \(y = 4x\) gives \(y = 40\).

So, today, Alex is 40 years old and Clara is 10 years old.

Check: \(y + 5 = 3(10 + 5)\), so \(y = 40\).

(MP.1, MP.2, MP.4) 2. Fred drives his car to the cottage. He travels 60 miles per hour. If he had travelled 50 miles per hour, the trip would have taken 1 hour longer. How far away is the cottage?

Solution: The formula speed = distance/time, or \(s = \frac{d}{t}\), is needed. Let \(x\) represent the time it takes Fred to drive to the cottage if he travels 60 miles per hour. Let \(y\) represent the distance to the cottage. Then, \(x + 1\) represents how long it takes Fred if he travels 50 miles per hour.

Equations:
\(60 = \frac{y}{x}\)
\(50 = \frac{y}{x + 1}\)

Isolating for \(y\) gives:
\(y = 60x\)
\(y = 50x + 50\)

At the intersection point:
\(60x = 50x + 50\)
\(10x = 50\)
\(x = 5\)

Substituting \(x = 5\) into \(y = 60x\) gives \(y = 60(5) = 300\).

So, the cottage is 300 miles away.

(MP.1, MP.2, MP.4) 3. A health food store wants to sell a mixture of cashews and almonds. Cashews sell for $9 per pound. Almonds sell for $6 per pound. The store owner plans to sell 450 pounds of the mixture. How many pounds of cashews and almonds should the owner use so that the mixture will be $7 per pound?
Solution: Let \( x \) represent the number of pounds of cashews. Let \( y \) represent the number of pounds of almonds.

Equations:
\[
\begin{align*}
x + y &= 450 \\
9x + 6y &= 450(7)
\end{align*}
\]

Isolating for \( y \) gives:
\[
\begin{align*}
y &= 450 - x \\
y &= 525 - \frac{3}{2}x
\end{align*}
\]

At the intersection point:
\[
\begin{align*}
450 - x &= 525 - \frac{3}{2}x \\
900 - 2x &= 1,050 - 3x \\
x &= 150
\end{align*}
\]

Substituting \( x = 150 \) into \( y = 450 - x \) gives \( y = 450 - 150 = 300 \).
So, the store owner should mix 150 pounds of cashews with 300 pounds of almonds.

(MP.1, MP.4) 4. At a museum, the admission fees are lower for children than for adults, but the museum requires at least one adult for every four children. Two classrooms go on a school trip to the museum. To save costs, they bring the minimum number of adult supervisors. One class of 19 children pays $241.25 and another class of 26 children pays $332.50. If Alexa goes to the museum with her two children, how much will she pay?

Answer: Let \( x \) be the cost of child tickets and let \( y \) be the cost of adult tickets. The class of 19 children has to bring 5 adults, so \( 19x + 5y = 241.25 \). The class of 26 children has to bring 7 adults, so \( 26x + 7y = 332.50 \). From \( 19x + 5y = 241.25 \), you get \( y = (241.25 - 19x) ÷ 5 \), so:
\[
\begin{align*}
26x + 7y &= 332.50 \\
26x + 7(241.25 - 19x) &= 332.50 \\
26x + 337.75 - 26.6x &= 332.50 \\
5.25 &= 0.6x \\
8.75 &= x \\
y &= (241.25 - 19(8.75)) ÷ 5 = 15
\end{align*}
\]
So each child ticket costs $8.75 and each adult ticket costs $15. I found \( 2 \times 8.75 + 15 = 32.50 \), so Alexa will pay $32.50.
EE8-56  Solving Systems of Linear Equations by Inspection

Pages 153–155

Standards: 8.EE.C.8, 8.EE.C.8b

Goals:
Students will determine whether a system of linear equations has a solution by finding the slopes and determining whether the lines are parallel.

Prior Knowledge Required:
Can find the slope of a line when the equation is written in slope-intercept form
Can rearrange the equation of a line to write the equation in slope-intercept form
Knows that parallel lines do not meet
Knows that parallel lines have the same slope
Knows that if two lines have different slopes, they are not parallel and must intersect

Vocabulary: intersection point, parallel, slope, slope-intercept form, solve a system of linear equations, system of linear equations

(MP.2) Review the meaning of parallel lines. Draw on the board:

\[ \text{A} \quad \text{B} \]

ASK: Are these line segments parallel? (A, yes; B, no). Do they intersect? (A, no; B, no). Draw on the board:

\[ \text{A} \quad \text{B} \]

ASK: What is the difference in the drawings? (in the first set of drawings, there are line segments; in the second set, there are lines) Are these lines parallel? (A, yes; B, no) Do they intersect? (A no; B, yes) SAY: In B, although the lines do not appear to intersect, if we extend the lines far enough, they will eventually intersect.
(MP.2) **Review parallel lines on a coordinate grid.** Project a coordinate grid on the board and draw the lines shown below:

![Coordinate Grid with Parallel Lines](image)

ASK: Do the lines appear to be parallel? (maybe...it's hard to tell!) What can we calculate to determine for certain whether they are parallel? (the slope of each line) Ask for volunteers to choose two points for each line so that all the coordinates will be integers. Then, have them calculate the slope of each line. (see sample calculations below)

\[
\ell_1: (-4, -2), (-2, 2) \\
m = \frac{\text{rise}}{\text{run}} = \frac{2 - (-2)}{-2 - (-4)} = \frac{4}{2} = 2
\]

\[
\ell_2: (0, -2), (2, 3) \\
m = \frac{\text{rise}}{\text{run}} = \frac{3 - (-2)}{2 - 0} = \frac{5}{2} = 2.5
\]

ASK: How do we know for certain that the lines are not parallel? (the slopes are not equal)

SAY: Remember that if lines are parallel, they have the same slope. If they are not parallel, the slopes will be different.

**Exercises:** Plot the points and draw the lines. Calculate the slopes to find whether the lines are parallel.

a) \(\ell_1: A (-1, 4), B (3, 6)\)  
   b) \(\ell_1: A (-2, 4), B (3, -1)\)  
   c) \(\ell_1: A (-3, -5), B (1, -8)\)

\(\ell_2: C (-2, 7), D (-5, 6)\)  
\(\ell_2: C (3, 4), D (5, 2)\)  
\(\ell_2: C (0, 4), D (4, 1)\)

**Answers:**

a) \(m_1 = 1/2, m_2 = 1/3\). The slopes are not equal, so the lines are not parallel.

b) \(m_1 = -1, m_2 = -1\). The slopes are equal, so the lines are parallel.

c) \(m_1 = -3/4, m_2 = -3/4\). The slopes are equal, so the lines are parallel.

(MP.2) **Determining whether a system of equations will have an intersection point from equations in slope-intercept form.** Write on the board:

\[\ell_1: y = 2x - 1\]

\[\ell_2: y = -3x + 1\]
ASK: What is the slope of each line? (ℓ₁, 2; ℓ₂, −3) Are the lines parallel? (no) How can you tell? (the slopes are not equal) Will there be an intersection point? (yes) Explain. (if the lines are not parallel, they will eventually intersect)

**Exercises:** Find the slope of each line and then state whether the pair of lines will intersect.

a) ℓ₁: \( y = 4x + 5 \)  
\( ℓ₂: y = -3x + 1 \)

b) ℓ₁: \( y = 3x - 5 \)  
\( ℓ₂: y = 3x - 2 \)

c) ℓ₁: \( y = 2 + x \)  
\( ℓ₂: y = x - 6 \)

**Answers:**

a) \( m₁ = 4, m₂ = -3 \). The slopes are not equal, so the lines are not parallel and will intersect.

b) \( m₁ = 3, m₂ = 3 \). The slopes are equal, so the lines are parallel and will not intersect.

c) \( m₁ = 1, m₂ = 1 \). The slopes are equal, so the lines are parallel and will not intersect.

(MP.2) Determining whether a system of equations will have an intersection point from equations not written in slope-intercept form. Write on the board:

\[
\begin{align*}
ℓ₁: & \quad 3x - 4y = 7 \\
ℓ₂: & \quad 4x + 3y = 2
\end{align*}
\]

ASK: Can you tell what the slope of each line is just by looking at the equation? (no) What should we do to the equations so that we can easily find the slope? (isolate for \( y \)—in other words, rearrange the equation so that it is in slope-intercept form) Ask for volunteers to rearrange each equation on the board. (see answers below)

\[
\begin{align*}
ℓ₁: & \quad y = \frac{3}{4}x - \frac{7}{4} \\
ℓ₂: & \quad y = -\frac{4}{3}x + \frac{2}{3}
\end{align*}
\]

ASK: What is the slope of the first line? (3/4) What is the slope of the second line? (−4/3) Are the lines parallel? (no) Explain. (the slopes are not equal, so the lines cannot be parallel) Will there be an intersection point? (yes, because the lines are not parallel)

**Exercises:** Rewrite the equation of each line in slope-intercept form. Then determine whether the system of equations will have an intersection point.

a) ℓ₁: \( 4x + 3y = 7 \)  
\( ℓ₂: 8x + 6y = 5 \)

b) ℓ₁: \( 2x - 3y = 1 \)  
\( ℓ₂: 4x + 6y = 8 \)

c) ℓ₁: \( -2x + 5y = 7 \)  
\( ℓ₂: 4x - 10y = 12 \)

**Answers:**

a) \( ℓ₁: y = -\frac{4}{3}x + \frac{7}{3}, ℓ₂: y = \frac{4}{3}x + \frac{5}{8}, m₁ = -\frac{4}{3}, m₂ = \frac{4}{3} \). The slopes are equal, so the lines are parallel and there is no intersection point.

b) \( ℓ₁: y = \frac{2}{3}x - \frac{1}{3}, ℓ₂: y = -\frac{2}{3}x - \frac{4}{3}, m₁ = \frac{2}{3}, m₂ = -\frac{2}{3} \). The slopes are not equal, so the lines are not parallel and there is an intersection point.

c) \( ℓ₁: y = \frac{2}{5}x + \frac{7}{5}, ℓ₂: y = \frac{2}{5}x - \frac{6}{5}, m₁ = \frac{2}{5}, m₂ = \frac{2}{5} \). The slopes are equal, so the lines are parallel and there is no intersection point.
(MP.2) Determining by inspection whether there will be an intersection point. Write on the board:

\[ 3y - 2x = -6 \]
\[ 3y - 2x = -12 \]

Ask volunteers to rewrite each equation in slope-intercept form. (\( y = \frac{2}{3}x - 2 \), \( y = \frac{2}{3}x - 4 \))

ASK: What are the slopes for both equations? (2/3) What are the \( y \)-intercepts? (−2 and −4)

SAY: If the slopes are the same, the lines are parallel. The \( y \)-intercepts are different so the lines are parallel and never intersect. Project a coordinate grid on the board and ask volunteers to graph the lines, as shown below:

![Graph of lines](graph.png)

SAY: This was a lot of work. We’d like to find an easier way to determine if there is an intersection point. Write on the board:

\[ x = 6 \]
\[ x = 7 \]

Point to the two equations, and SAY: This system of equations has no solution. For us to find an intersection point for two lines, the variable \( x \) can only represent one number. To demonstrate this, ask volunteers to plot the lines \( x = 6 \) and \( x = 7 \) on a grid. Discuss as a class that the lines with these equations are parallel, vertical lines that therefore never intersect. SAY: The variable \( x \) cannot be both 6 and 7 at the same time. Write on the board:

\[ 2x - 3y = 6 \]
\[ 2x - 3y = 12 \]

ASK: What do you notice about the left side of the equations in this system of linear equations? (they are the same) SAY: If the left sides of the equations are the same, the right sides cannot be both 6 and 12 simultaneously. So, this system has no intersection point—in other words, there is no solution to this system of linear equations.
Exercises:

(MP.2) 1. Without finding the slope or graphing the lines, find whether the system has no solution.
   a) $3x - 4y = 5$
   b) $2x - 5y = 7$
   c) $x + y = 10$

   $3x - 4y = 8$
   $-3x + 4y = 8$
   $x + y = 2$

   **Answers:** a) and c) have no solutions. In b), the slopes will be different so the lines will intersect.

(MP.3) 2. Kyle believes the system $\begin{align*} 2x - 3y &= 4 \\ -2x + 3y &= 5 \end{align*}$ has no solution.

   a) Multiply both sides of the second equation by $-1$.
   b) Explain why there is no solution.

   **Answers:**
   a) If you multiply both sides of the second equation by $-1$, you get the following system:
   $2x - 3y = 4$
   $2x - 3y = -5$
   b) The left sides of the equations are the same, but the right sides are different. A variable for an intersection point can’t be both 4 and $-5$ at the same time, so there is no solution.

Extensions

(MP.1) 1. Consider the following system:
   $2x - 3y = 6$
   $4x - 6y = 12$

   a) Find the slope and $y$-intercept of each line.
   b) Graph each line.
   c) What do you notice? How many intersection points are there?
   d) Divide both sides of the second equation by 2.
   e) What do you notice about the equation after part d)?

   **Answers:**
   a) $m = \frac{2}{3}$ and $y$-intercept = $-2$ for both equations
   c) The lines are identical. The graphs are on top of each other. There are an infinite number of intersection points or common ordered pairs.
   d) $2x - 3y = 6$
   e) The equations are now identical. That explains why the graphs are on top of each other.

(MP.3, MP.6) 2. Evaluate $\sqrt{32}$ accurately to two decimal places. Do not use a calculator. Justify your answer.

   **Answer:** I know it is between 5 and 6, because $5 = \sqrt{25} < \sqrt{32} < \sqrt{36} = 6$. So I found $5.5 \times 5.5 = 30.25 < 32$, so $\sqrt{32} > 5.5$ and $5.6 \times 5.6 = 31.36 < 32$, so $\sqrt{32} > 5.6$. Then I found $5.7 \times 5.7 = 32.49$, so $5.6 < \sqrt{32} < 5.7$. Then I found $5.65 \times 5.65 = 31.9225 < 32$ and $5.66 \times 5.66 = 32.0356 > 32$, so $5.65 < \sqrt{32} < 5.66$. $32.0356$ is closer to 32 than 31.9225, so $\sqrt{32} \approx 5.66$, to two decimal places.
On, Above, or Below a Line

1. Use the graph to complete the table.

   a) For each point on the graph, calculate whether its coordinates satisfy the equation \( y = x + 2 \), the inequality \( y > x + 2 \), or the inequality \( y < x + 2 \).

   b) State whether each point lies on the line \( y = x + 2 \), above the line \( y = x + 2 \), or below the line \( y = x + 2 \).

<table>
<thead>
<tr>
<th>Point</th>
<th>Coordinates</th>
<th>Calculation</th>
<th>Coordinates Satisfy …</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(4, 6)</td>
<td>6 = 4 + 2</td>
<td>( y = x + 2 )</td>
<td>on the line</td>
</tr>
<tr>
<td>B</td>
<td>(5, 2)</td>
<td>2 &lt; 5 + 2</td>
<td>( y &lt; x + 2 )</td>
<td>below the line</td>
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2. How can we determine by looking at the calculations whether the point lies on the line, above the line, or below the line? Explain.
On, Outside, or Inside a Circle (1)

1. Use the graph to complete the table.
   a) For each point on the graph, calculate whether the distance to the origin is equal to 5, greater than 5, or less than 5 using the formula \( d = \sqrt{\text{change in } x^2 + \text{change in } y^2} \).
   b) State whether each point lies on the circle, outside the circle, or inside the circle.

<table>
<thead>
<tr>
<th>Point</th>
<th>Coordinates</th>
<th>Calculation</th>
<th>Distance to Origin</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(3, 4)</td>
<td>( d = \sqrt{3^2 + 4^2} = 5 )</td>
<td>( d = 5 )</td>
<td>on the circle</td>
</tr>
<tr>
<td>B</td>
<td>(6, 4)</td>
<td>( d = \sqrt{6^2 + 4^2} \approx 7.2 )</td>
<td>( d &gt; 5 )</td>
<td>outside the circle</td>
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On, Outside, or Inside a Circle (2)

2. How can we determine by looking at the distances in Question 1 whether the point lies on the circle, outside the circle, or inside the circle?

3. For each point on the graph in Question 1, ...
   a) Calculate whether the coordinates of the point satisfy the equation $x^2 + y^2 = 25$, the inequality $x^2 + y^2 > 25$, or the inequality $x^2 + y^2 < 25$.
   b) State whether the point lies on the circle, outside the circle, or inside the circle.

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<tr>
<th>Point</th>
<th>Coordinates</th>
<th>Calculation</th>
<th>Coordinates Satisfy …</th>
<th>Position</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>(3, 4)</td>
<td>$3^2 + 4^2 = 25$</td>
<td>$x^2 + y^2 = 25$</td>
<td>on the circle</td>
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<tr>
<td>B</td>
<td>(6, 4)</td>
<td>$6^2 + 4^2 = 52$</td>
<td>$x^2 + y^2 &gt; 25$</td>
<td>outside the circle</td>
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4. How can we determine by looking at the calculations in Question 3 whether the point lies on the circle, outside the circle, or inside the circle?
Word Problem Coordinate Grid

Electrician Costs

Cost ($) vs Hour Worked

- Cost: $0, $40, $80, $120, $160, $200, $240, $280, $320, $360
- Hours Worked: 0, 1, 2, 3, 4
PS8-8 Using Structure II

Teach this lesson after: 8.2 Unit 5

Standards: 8.EE.A.1, 8.EE.C.7.B, 8.EE.C.8

Goals: Students will solve equations involving exponents and understand the limitations of systematic search in cases where the answer is not a whole number.

Prior Knowledge Required:
Can evaluate a power
Can multiply powers with the same base
Can divide powers with the same base
Can find the power of a power
Can write the power of a product as a product of powers
Can write the power of a quotient as a quotient of powers

Vocabulary: base, exponent, exponential equation, guess-check-revise, power, power of a power, power of a product, power of a quotient, product, product of powers, quotient, unit fraction

Materials:
BLM Exponent Rules and Powers of Whole Numbers (p. P-52)
BLM Camera Manufacturing (pp. P-54–56, see Performance Task)


a) $2^{-3} \times 2^5$  b) $(3^{-4})^2$  c) $(2 \times 5)^4$  d) $\frac{4^7}{4^2}$  e) $9^0$  f) $3^{-4}$

Answers: a) $2^2$, b) $3^{-6}$, c) $2^4 \times 5^4$, d) $4^5$, e) 1, f) $\frac{1}{3^4}$ or $\frac{1}{81}$

Solving equations with exponents. Write on the board:

$$2^{2x+1} = 128$$

SAY: This is an example of an exponential equation. We call it an exponential equation because the variable we are going to find is in an exponent. You can solve this equation using the systematic search strategy or the guess-check-revise strategy. ASK: What is $2^{2x+1}$ for $x = 1$? (8)
Draw the table below on the board, but fill in only the first row. Have a volunteer complete the table and find $x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$2^{2x+1}$</th>
<th>True?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2^{2(1)+1} = 2^3 = 8$</td>
<td>no</td>
</tr>
<tr>
<td>2</td>
<td>$2^{2(2)+1} = 2^5 = 32$</td>
<td>no</td>
</tr>
<tr>
<td>3</td>
<td>$2^{2(3)+1} = 2^7 = 128$</td>
<td>yes</td>
</tr>
</tbody>
</table>

(MP.5) SAY: Using the guess-check-revise strategy needs more steps when the answer is not a whole number. Write on the board:

$$2^{3x-2} = 8$$

ASK: What is $2^{3x-2}$ for $x = 1$? (2) SAY: So, 1 is too low. Let’s try $x = 2$. Write on the board:

$$2^{3(2)-2} = 2^4 = 16$$

SAY: 16 is too high, so the answer is between 1 and 2. The next guess would be 1 1/2 or 1.5, but calculating $2^{3x-2}$ for $x = 1.5$ is not easy, so it’s better to use another way to solve the equation. We will use algebra to solve the equation. ASK: What power of 2 is 8? (3rd power) Write on the board:

$$2^{3x-2} = 2^3$$

SAY: The bases on the sides of the equation are the same, so the exponents must be equal. Ask a volunteer to write the equation for the exponents on the board and solve it. (see answer below)

$$3x - 2 = 3$$
$$3x = 3 + 2$$
$$3x = 5$$  so  $x = \frac{5}{3}$

SAY: Knowing the powers of whole numbers is very handy, like knowing times tables. The bottom of BLM Exponent Rules and Powers of Whole Numbers has some powers of whole numbers when the result is up to 1,000, which are useful when you solve exponential equations.

(MP.7) Exercises: Solve the equation and find $x$.

a) $2^{2x-1} = 32$  b) $2^{3x+1} = 16$  c) $3^{2x+3} = 81$  d) $5^{7x-2} = 625$

Bonus:

e) $4^{2x-3} = 4^{x+1}$  f) $2^{x+6} = 8^x$

Selected solution: Bonus: e) $2x - 3 = x + 1$, so $2x - x = 1 + 3$, so $x = 4$

Answers: a) 3, b) 1, c) 1/2, d) 6/7, f) 3
Solving exponential equations with exponents in the numerator and denominator.

SAY: Remember, a power with a negative exponent is equal to a unit fraction with a positive exponent in the denominator. Write on the board:

\[ 5^{-3} = \frac{1}{5^3} = \frac{1}{125} \]

SAY: You can write a unit fraction with a negative exponent as a power with positive exponent, as well. Write on the board:

\[ \frac{1}{2^{-3}} = \frac{1}{\frac{1}{2^3}} = 1 \div \frac{1}{2^3} = 1 \times 2^3 = 2^3 \]

Exercises: Write the unit fraction as a power of a whole number.

a) \( \frac{1}{3^{-3}} \)  
b) \( \frac{1}{5^{-4}} \)  
c) \( \frac{1}{7^{-5}} \)  
d) \( \frac{1}{2^{-8}} \)  
Bonus: \( \frac{1}{(3^{-2})^4} \)

Selected solution: Bonus: \( \frac{1}{(3^{-2})^4} = \frac{1}{3^{-8}} = 3^8 \)

Answers: a) \( 3^2 \), b) \( 5^4 \), c) \( 7^5 \), d) \( 2^8 \)

Write on the board:

\( \frac{1}{2^{-3}} = 2^3 \)

ASK: What should the box be? (-3) Write on the board:

\( \frac{1}{2^{-5}} = 2^5 \)

ASK: What should the box be? (5) Write on the board:

\( \frac{1}{2^{-x}} = 2^x \)

ASK: What should the box be? (-x) SAY: If there is a unit fraction with an exponent in the denominator, then you can write it with an opposite exponent in the numerator. Write on the board:

\[ 2^{3x-4} = \frac{1}{2^{x-1}} \]

ASK: What is the exponent of 2 in the denominator on the right side of the equation? \( x - 1 \)
SAY: You can write 2 with an opposite exponent in the numerator. Write on the board:

\[
\frac{1}{2^{x-1}} = 2^{-(x-1)}
\]

SAY: Now we can rewrite the equation. Write on the board:

\[
2^{3x - 4} = 2^{-(x - 1)}
\]

Have a volunteer to solve the equation on the board. (see answer below)

\[
\begin{align*}
3x - 4 &= -(x - 1) \\
3x - 4 &= -x + 1 \\
x + 4 &= 1 + 4 \\
4x &= 5, \text{ so } x = \frac{5}{4}
\end{align*}
\]

(MP.7) Exercises: Solve the equation.

a) \(3^{2x - 2} = \frac{1}{3^x}\)

b) \(\frac{1}{5^{1 - 2x}} = 5^{x - 3}\)

c) \(2^{x - 4} = \frac{1}{2^{4x - 1}}\)  

Bonus: \(2^{2x + 5} = \frac{1}{2^{x - 1}}\)

Answers: a) 1/3, b) 1/2, c) 1, Bonus: -4/3

Solving systems of equations using structure. Write on the board:

\[
\begin{align*}
2x + y &= 19 \\
x + y &= 14
\end{align*}
\]

\[
2x + y - (x + y) = \begin{align*}
\end{align*}
\]

\[
19 - 14 = \begin{align*}
\end{align*}
\]

ASK: If you subtract the left sides, what expression do you get? \(2x + y - x - y = x\) in the first blank. ASK: If you subtract the right sides, what do you get? \(5\) in the second blank. SAY: Since both the left sides equal both the right sides, subtracting the left sides gets the same answer as subtracting the right sides. Write on the board:

So, \(x = 5\)

SAY: So, \(x = 5\), and once you know \(x\), you can find \(y\). Have a volunteer do so. \(y = 9\)

(MP.5) Exercises: Here are three systems of linear equations.

A. \(3x + y = 17\) \hspace{1cm} B. \(7x + 3y = 27\) \hspace{1cm} C. \(4x + 9y = 35\)

\[
\begin{align*}
x + y &= 13 \\
x + y &= 12 \\
4x + 5y &= 23
\end{align*}
\]

a) Solve each system of equations by isolating \(y\).

b) Solve each system of equations by subtracting the left sides and the right sides.

c) Which way is faster?
Sample solution: a) A. \( y = 17 - 3x \), and \( y = 13 - x \), so \( 17 - 3x = 13 - x \), or \( 17 - 13 = -x + 3x \), then \( 4 = 2x \), so \( x = 2 \) and \( y = 11 \)

Answers: a–b) B. \( x = 3 \), \( y = 2 \); C. \( y = 3 \), \( x = 2 \); c) subtracting the left sides and the right sides is faster

Problem Bank
1. a) Find \( n \), so that \( 10^n - 6 = 100 \).
   b) Find \( n \), so that \( 10^n - 6 = 0.001 \).
   Answers: a) 8, b) 3

2. Solve the equation and find \( x \).
   a) \( 2^{5x-1} = 27 \)
   b) \( 7^{3x-2} = 1 \)
   c) \( 4^{-2x} = 4^{3-5x} \)
   d) \( 5^{-x} \times 5^{3x} = 5^{x+3} \)
   e) \( 3^{3x} = 3 \)
   f) \( \frac{2^{7x+1}}{81^{x-2}} = 3^3 \)
   g) \( \left( \frac{1}{5} \right)^{2x+5} \times 125^{x-1} = \frac{1}{5^3} \)
   Answers: a) \( 4/5 \), b) \( 2/3 \), c) 1, d) 3, e) \( -1 \), f) \( -6/10 \) or \( -3/5 \), g) 3

(MP.7) 3. Find \( n \), so that \( 2^n \times 5^{n+1} = 50,000,000 \).
   Solution: \( 2^n \times 5^{n+1} = 2^n \times 5^n \times 5 = (2 \times 5)^n \times 5 = 5 \times 10^n \), so \( 10^n = 10,000,000 \) and \( n \) is 7.

(MP.1) 4. Find \( n \), so that \( n^n = 46,656 \).
   Answer: 6

5. a) Find \( A \), so that \( A^{A^{-5}} = 6,561 \).
   b) Find \( A \), so that \( A^{A/2} = 10^{100} \).
   Answers: a) 9, b) 100

(MP.7) 6. Find \( n \), so that \( 8^n \times 4^n \times 2 = 2^{36} \).
   Solution: \( 8^n \times 4^n \times 2 = \left( 2^3 \right)^n \times \left( 2^2 \right)^n \times 2 = 2^{3n} \times 2^{2n} \times 2 = 2^{3n+2n+1} \), so \( 3n + 2n + 1 = 36 \) and \( n \) is 7.

7. a) Solve the equation \( 9^{2x-5} = 729 \).
   b) Find \( x \) in the equation \( 5^{2x-3} = \frac{1}{5^{x-4}} \).
   Selected solution: b) \( \frac{1}{5^{x-4}} = 5^{-(x-4)} \), so \( 2x - 3 = -(x - 4) \) and \( 2x - 3 = -x + 4 \), so \( 3x = 7 \) and then \( x = 7/3 \)
   Answer: a) 4

8. a) Solve the system of equations by graphing. \( 2x + 3y = 17 \)
   \( x + y = 7 \)
   b) Graph the line \( 2x + 2y = 14 \). Are the lines \( x + y = 7 \) and \( 2x + 2y = 14 \) exactly the same lines?
   c) Solve the system of equations by subtracting the left sides and the right sides. Compare your answer with part a).
   \( 2x + 3y = 17 \)
   \( 2x + 2y = 14 \)
   Answers: a) \( x = 4 \), \( y = 3 \); b) yes; c) \( x = 4 \), \( y = 3 \). The answers are the same.
Exponent Rules and Powers of Whole Numbers

<table>
<thead>
<tr>
<th>Example</th>
<th>Power Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3^5 \times 3^2 = 3^7)</td>
<td>To multiply powers with the same base, keep the base and add the exponents.</td>
</tr>
<tr>
<td>(3^8 \div 3^2 = 3^6)</td>
<td>To divide powers with the same base, keep the base and subtract the exponents.</td>
</tr>
<tr>
<td>(3^0 = 1)</td>
<td>A power with exponent zero is equal to 1.</td>
</tr>
<tr>
<td>(3^{-2} = \frac{1}{3^2} = \frac{1}{9})</td>
<td>A power with a negative exponent is equal to a unit fraction with a positive exponent in the denominator.</td>
</tr>
<tr>
<td>((3^3)^7 = 3^{21})</td>
<td>To rewrite the power of a power, keep the base and multiply the exponents.</td>
</tr>
<tr>
<td>((3 \times 5)^4 = 3^4 \times 5^4)</td>
<td>To rewrite the power of a product, apply the exponent to each factor.</td>
</tr>
<tr>
<td>(\left(\frac{3}{5}\right)^4 = \frac{3^4}{5^4})</td>
<td>To rewrite the power of a quotient, apply the exponent to the numerator and the denominator.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(2^1 = 2)</td>
<td>(3^1 = 3)</td>
<td>(4^1 = 4)</td>
<td>(5^1 = 5)</td>
<td>(6^1 = 6)</td>
</tr>
<tr>
<td>(2^2 = 4)</td>
<td>(3^2 = 9)</td>
<td>(4^2 = 16)</td>
<td>(5^2 = 25)</td>
<td>(6^2 = 36)</td>
</tr>
<tr>
<td>(2^3 = 8)</td>
<td>(3^3 = 27)</td>
<td>(4^3 = 64)</td>
<td>(5^3 = 125)</td>
<td>(6^3 = 216)</td>
</tr>
<tr>
<td>(2^4 = 16)</td>
<td>(3^4 = 81)</td>
<td>(4^4 = 256)</td>
<td>(5^4 = 625)</td>
<td></td>
</tr>
<tr>
<td>(2^5 = 32)</td>
<td>(3^5 = 243)</td>
<td>(4^5 = 1,024)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2^6 = 64)</td>
<td>(3^6 = 729)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2^7 = 128)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2^8 = 256)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2^9 = 512)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2^{10} = 1,024)</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
Performance Task: Camera Manufacturing

Materials:
BLM Camera Manufacturing (pp. P-54–56)

Performance Task: Camera Manufacturing. Give students BLM Camera Manufacturing. The Bonus question provides students with an opportunity to use problem-solving strategies from this unit.

Selected answers:
1. a) 20; b) 15; c) (20, 0), (0, 15); e) $-15/20$ or $-3/4$; f) 15, $y = -3/4x + 15$
2. a) 18; b) $y = -x + 18$, y-intercept is 18; d) 12 5MP and 6 8MP; e) $x = 12$, $y = 6$
Bonus: $15x + 20y = 300$, 18 5MP cameras and 6 8MP cameras; no, because $x + y$ is getting smaller
Camera Manufacturing (1)

A manufacturer produces 5-megapixel (5MP) resolution and 8-megapixel (8MP) resolution cameras. The 5MP camera requires 15 hours of labor to produce, and the 8MP camera requires 20 hours. The company has 300 hours of labor available per day.

1. a) If the company only produces 5MP cameras, how many cameras can they produce in a day?

b) If the company only produces 8MP cameras, how many cameras can they produce in a day?

c) If \( x \) and \( y \) are the number of 5MP and 8MP cameras that the manufacture can produce in a day, use your answers from parts a) and b) to fill in the table.

\[
\begin{array}{c|c}
5\text{MP Cameras (}x\text{)} & 8\text{MP Cameras (}y\text{)} \\
\hline
0 & 0 \\
\end{array}
\]

d) Draw a line that passes through the points in the table in part c).
Camera Manufacturing (2)

e) Find the slope of the line in part d).

f) Determine the $y$-intercept of the line in part d) and write the equation of the line in slope-intercept form.

2. The plant's capacity is a total of 18 cameras per day.

a) If $x$ and $y$ are the number of 5MP and 8MP cameras that the manufacture can produce in a day, complete the equation for the plant's capacity.

$$x + y = \quad$$

b) Write the equation in part a) in slope-intercept form and circle the $y$-intercept.

c) Draw the line from the part b) on the same grid as Question1.d), and then find the coordinates of the intersection point of the two lines.
Camera Manufacturing (3)

d) If all the available time and capacity are to be used, how many of each type camera should be produced?

e) Solve the system of equations from Questions 1.f) and 2.b) algebraically and compare your answer with the graphical method.

Bonus ► Write the equation of the line in Question 1.f) in standard form.

\[ ____ x + ____ y = 300 \]

If the company produces one 8MP camera, how many 5MP cameras can they produce in one day?

Use the table to find how many of each type of camera could be produced.

<table>
<thead>
<tr>
<th>8MP Camera (y)</th>
<th>5MP Camera (x)</th>
<th>( x + y )</th>
<th>( \text{Is } x + y \text{ equal to } 18? )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Do you need to continue the table? _____ Why? ____________________________
Unit 6  Geometry: Volume

This Unit in Context
In Grade 2, students prepared for the future study of area by partitioning a rectangle into rows and columns of same-size squares and counting to find the total number of squares (2.G.A.2). In Grade 6, they learned how to find the area of two-dimensional shapes, as well as the volume and surface area of three-dimensional shapes. They applied the formulas $V = \ell \text{wh}$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in order to solve real world and mathematical problems (6.G.A.2).

In Grade 7, students drew, constructed, and described geometrical figures, as well as described the relationships between them. They solved problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale (7.G.A.1). Students also found the areas of rectangles, triangles, parallelograms, trapezoids, and composite shapes, found the circumference of a circle and developed, and used the formula to find the area of a circle (7.G.B.4).

In this unit, students will learn the formulas for the volumes of cones, cylinders, and spheres, and will use them to solve real world and mathematical problems (8.G.C.9).

Mathematical Practices in This Unit
In this unit, you will have the opportunity to assess MP.1 to MP.8. The MP labels in this unit flag both opportunities to develop the mathematical practice standards and opportunities to assess them. Below is a list of where we recommend assessing each standard, as well as some examples of how students can show that they have met a standard.

**MP.1:** G8-50 Extension 6, G8-51 Extension 3, G8-52 Extension 1, G8-55 Extension 5
In G8-52 Extension 1, students make sense of a non-routine problem when they put four identical triangles together to make an open-ended pyramid, recognizing that the base must be a 12 inch by 12 inch square. They persevere in solving the problem when they draw a picture and determine the height using the slant height and then complete the question by determining the volume.

**MP.2:** G8-50 Extension 2, activity in G8-52, G8-54 Extension 4, G8-55 Question 8 on AP Book 8.2 p. 175
In the activity in G8-52, students reason abstractly and quantitatively when they use the fact that they need to fill a pyramid three times to fill a cube with the same base and height to deduce the formula for the volume of a pyramid from the formula for the volume of a cube.

**MP.3:** G8-50 Extension 2

**MP.4:** G8-50 Extension 7, G8-51 Extension 3, Exercise 3 on p. Q-51 and Extension 4 in G8-54, G8-55 Extension 5
**MP.5:** G8-50 Extension 1, G8-51 Extension 3, G8-53 Extension 3
In G8-53 Extension 3, students use appropriate tools strategically when they find the intersection points of \( y = x - 1 \) with various other functions, recognizing that they need the help of dynamic geometric software for a non-linear equation but they can use paper and pencil or mental math for linear equations.

**MP.6:** G8-50 Extension 7

**MP.7:** G8-53 Extension 3

**MP.8:** G8-50 Extension 6
Unit 6  Geometry: Volume

Introduction
In this unit, students will deepen their understanding of volume. Students will develop and apply the formulas for the volumes of prisms, cylinders, pyramids, cones, and spheres. They will find the volume of solids composed of two or more 3-D shapes. They will use the Pythagorean Theorem to determine the heights of pyramids and cones, and they will use algebra to write and solve equations to find unknown dimensions. Throughout the unit, students will solve real-world and mathematical problems involving volume.

Materials. For this unit, students need to see a variety of 3-D shapes. If you do not have a commercial set of shapes, you can use the nets, noted below, to make shapes. As well, collect household items that are various 3-D shapes. For example, cereal and tissue boxes are rectangular prisms, and some candy bar boxes are triangular prisms. Painter’s pyramids and some tea bags are triangular or rectangular pyramids. Cans of food, pieces of chalk, and paper towel rolls are cylinders. For cones, you might use paper party hats, paper cone cups, and ice cream cones. Balls, oranges, and marbles are spheres.

To create (or have students create) sturdy 3-D shapes from the nets on BLM Nets of 3-D Shapes (pp. Q-59–68), photocopy the BLMs on cardstock, cut them out, and glue or tape them together. Nets are available for the following shapes:

- Right rectangular prism (p. Q-59)
- Right triangular prism (p. Q-60)
- Right trapezoidal prism (p. Q-61)
- Right pentagonal prism (p. Q-62)
- Cylinder (p. Q-63)
- Right rectangular pyramid (p. Q-64)
- Right square-based pyramid (p. Q-65)
- Right triangular pyramid (p. Q-65)
- Skew rectangular pyramid (p. Q-67)
- Cone (p. Q-68)

In addition to the BLMs provided at the end of this unit, the following Generic BLMs, found in section S, are used in Unit 6:

BLM 1 cm Grid Paper (p. S-1)

Calculators. Volume calculations, especially those involving \( \pi \), are long and difficult to calculate mentally. Allow students to use calculators in this unit.

Computer access. As noted twice in the unit’s lesson plans, you could show students applets (small computer applications) that simulate 3-D shapes; to do so, you will need a computer, Internet access, and a data projector. You should search for and load the web page in advance.

Diagrams. Diagrams in the unit’s lesson plans are not drawn to scale. When you draw on the board, let students know that the diagrams are drawn to help students visualize the shape, but the diagrams cannot be “eyeballed” for actual measurements or to get an answer.

In addition, in this unit, students will use 2-D cross sections to understand certain 3-D shapes. Provide examples to ensure students understand how cross sections work.
**Pi and rounding.** Throughout the unit, instruct students to use 3.14 for $\pi$ and to round answers to the nearest tenth unless stated otherwise.

**Fraction notation.** We show fractions in two ways in our lesson plans:

- Stacked: $\frac{1}{2}$
- Not stacked: 1/2

If you show students the non-stacked form, remember to introduce it as new notation.
G8-49 Volume of Prisms

Pages 156–159

Standards: preparation for 8.G.C.9

Goals:
Students will identify the characteristics of prisms and they will find the volume of right prisms.

Prior Knowledge Required:
Knows units of measurement for length, area, and volume
Can multiply and divide whole numbers, fractions, and decimal numbers
Can substitute numbers for variables in a formula
Can find the area of a rectangle, triangle, parallelogram, and trapezoid
Can find missing side lengths of 2-D shapes

Vocabulary: area, base, composite solid, congruent, cubic units, dimension, edge, face, height, net, oblique prism, polygon, prism, right prism, skew prism, square units, surface, three-dimensional (3-D), vertex, vertices, volume

Materials:
right prisms with a variety of bases or made from BLM Nets of 3-D Shapes (1) to (4) (pp. Q-59–62)
about 10 books of the same size (as thin as possible) for demonstration (see also Extension 2)
BLM Sorting Prisms (p. Q-69), reproduced on a transparency and/or photocopied for individual students or pairs
21 connecting cubes for demonstration
plain, 8.5 by 11-inch paper (see Extension 3)

(MP.6) Introduce prisms. SAY: Prisms are 3-D shapes that have faces, edges, vertices, and bases. Explain that the flat surfaces are called faces. Give a triangular prism (for example, one made with BLM Nets of 3-D Shapes (2)) to a student and ASK: How many faces does this prism have? (5) Tell students that two faces meet at an edge. Give the prism to another student and ASK: How many edges does this prism have? (9) Tell students that two or more edges meet at a vertex. Give the prism to a third student and ASK: How many vertices does this prism have? (6) SAY: Every prism has two bases. The bases of a prism are parallel faces, and they are congruent polygons. ASK: What type of polygon makes the base of this prism? (triangle) SAY: Prisms are named for the shape of their base, so this is a “triangular prism.” Lay the prism on its side and point out that the triangular face is still the base of the prism even if it’s not the bottom face of the shape.

Next, give a student a trapezoidal prism (BLM Nets of 3-D Shapes (3)), and ask the student to say what shape makes the base and to put the prism base-down on a desk in a central location. Repeat with a pentagonal prism (BLM Nets of 3-D Shapes (4)). ASK: For all of these prisms, what shapes are the side faces? (rectangles) Hold up a rectangular prism (BLM Nets of 3-D Shapes (1)). ASK: Which face is the base? (any face can be the base) SAY: For a rectangular
prism, it doesn’t matter which face we call the base, because for every single face, there is a parallel face that is a congruent polygon. Leave the prisms on the desk to display and use again later on in the lesson.

(MP.3, MP.7) Exercises: Look around the classroom.
a) List 5 objects that are prisms. Choose one and explain why it is a prism.
b) List 5 objects that are not prisms. Choose one and explain why it is not a prism.
Sample answers:
a) a book, tissue box, recycling bin, eraser, calculator; a book is a prism because it has two congruent rectangles as bases, the bases are parallel to each other, and the side faces are rectangles
b) a water bottle, round-faced clock, highlighter, basketball, computer mouse; a water bottle is not a prism because its base is a circle (not a polygon) and the side face is curved (not a flat rectangle)

Identifying right prisms and skew prisms. SAY: When a prism stands on one of its bases, the other base becomes the top face. Hold up one of the prisms on the display desk and SAY: This is a right prism. When placed base-down, the top face is directly above the base. Point out that the side edges are vertical and the side faces are rectangles. ASK: Are all of the prisms on the desk right prisms? (yes)

Stack some thin books of the same size on the display desk. ASK: What shape does the stack of books make? (rectangular prism) Is it a right rectangular prism? (yes) Why? (the top book is directly above the bottom book) Next, push the books so that the shape becomes skewed, as shown below:

SAY: The top book is no longer directly above the bottom book. The stack is not a right prism; instead it is a skew prism. In a skew prism, the top face is not directly above the base. Explain that “skew” means “not straight” or “crooked.” Point out that, when the bases are skewed or “askew,” the edges are diagonal. ASK: Are the side faces rectangles? (no, they are parallelograms) Tell students that skew prisms are also called oblique prisms.

For the following exercises, project BLM Sorting Prisms on a transparency and/or distribute photocopies to individual students or pairs.

Exercises:
(MP.7) 1. Sort the 3-D shapes.
   a) Right prisms   b) Skew prisms   c) Not prisms

(MP.3) 2. Choose one 3-D shape from each group in Exercise 1 and explain why it belongs in the group.
Sample answers:
a) B is a right prism because it has two congruent rectangles as its bases and they are directly above each other. Its side faces are rectangles.
b) A is a skew prism because it has two congruent hexagons as its bases but they are not directly above each other. Its side faces are parallelograms.
c) C is not a prism because it does not have two congruent bases and its side faces are trapezoids, not rectangles or parallelograms.

Identifying the height of a prism. SAY: A prism’s height is the distance between the two bases. When a prism is standing on one base, the height is how tall the shape is. Using the right triangular prism on the display desk, point to an edge that shows the height. Then tip that prism over and SAY: Now, when the prism is on its side, the height is horizontal. For right prisms, height is always measured along an edge. Match up the corner of a piece of paper with a side face to show that the height meets the base at a 90° angle. SAY: This right angle is why these prisms are called “right prisms.” Have students identify the heights of the other prisms on the desk. Point out that, in a right prism, any edge perpendicular to the base gives the height.

Developing the formula Volume = area of base × height. SAY: The space taken up by a 3-D object is called volume. Volume is measured in cubic units, such as cubic centimeters, cubic meters, cubic inches, and cubic yards. Write on the board:

Units for volume: \( \text{cm}^3, \text{m}^3, \text{in}^3, \text{yd}^3 \)

SAY: We can determine the volume of a prism by counting the number of equal-sized cubes that it takes to make or fill the prism. Use 7 connecting cubes to build the shape shown below:

![Image of a 3D shape]

ASK: What is the volume of this shape? (7 cubes) Add two more identical layers, as shown below:

![Image of a 3D shape with additional layers]

ASK: What is the volume now? (21 cubes) How do you know? (counted 21 cubes, \( 7 + 7 + 7 = 21 \), \( 7 \times 3 = 21 \)) SAY: To quickly get the volume, you can multiply the number of cubes in one layer by the number of layers. Write on the board:

\[
\text{Volume} = \text{number of cubes in one layer} \times \text{number of layers}
\]

ASK: Which face makes the base? (the face with 7 squares) Is this 3-D shape a right prism? (yes) Why? (it has two congruent bases and the side faces are rectangles) What is the area of the base? (7) What does this equal in our formula on the board? (the number of cubes in one
layer) What is the height of the prism? (3) What does this equal in our formula? (the number of layers) Continue writing on the board:

\[
\text{Volume} = \text{number of cubes in one layer} \times \text{number of layers} = \text{area of base} \times \text{height of prism}
\]

Draw on the board:

ASK: What is the area of the base of this prism? (7) PROMPT: How do we count half squares? (count them as 0.5 each; for every 2 half-squares, count 1 square) ASK: What is the height? (2) So what is the volume of the prism? (14 cubes)

SAY: Mathematicians like to use symbols in formulas. If we use \( B \) for the area of the base and \( h \) for the height of the prism, then we can write the formula like this. Write on the board:

\[
V = B \times h
\]

Remind students that we don’t always write the multiplication sign between the variables, so this formula can be written as \( V = Bh \). Point out that \( B \) should not be confused with \( b \), which is used for the base of a triangle or a parallelogram.

Using the formula \( V = Bh \). Draw on the board:

ASK: What is the area of the base? (10 cm\(^2\)) What is the height of the prism? (5 cm) Write on the board:

\[
V = Bh
\]

\[
= 10 \text{ cm}^2 \times 5 \text{ cm}
\]

SAY: It’s important to check the units because we can’t calculate volume unless the units are consistent. ASK: Are the units consistent? In other words, are we using the same unit, centimeters, for the base and for the height? (yes) SAY: Since the units are consistent, we can just substitute the numbers to calculate the volume. ASK: What is \( 10 \times 5 \)? (50) What unit do we write for the answer? (cm\(^3\)) Continue writing on the board:

\[
= 50 \text{ cm}^3
\]
Exercises: Calculate the volume of the prism.

a) A rectangular prism with
   \[ B = 8 \text{ in}^2, \ h = 5.5 \text{ in} \]

b) A triangular prism with
   \[ B = 20 \text{ cm}^2, \ h = 3.25 \text{ cm} \]

c) A hexagonal prism with

Answers: a) 45 \text{ ft}^3, \ b) 44 \text{ in}^3, \ c) 65 \text{ cm}^3

Review the area of polygons. Remind students that, while 3-D shapes have volume, 2-D shapes have area. Ask: What units do we use for area? (\text{cm}^2, \text{m}^2, \text{in}^2, \text{yd}^2) Draw a triangle, rectangle, trapezoid, and parallelogram on the board. For each shape, ask a volunteer to write the formula they would use to find the area of the shape and to label the dimensions on the drawings. (see answers below)

\[
\begin{align*}
\text{Triangle:} & \quad b \times h \div 2 \\
\text{Rectangle:} & \quad \ell \times w \\
\text{Trapezoid:} & \quad (b_1 + b_2) \times h \div 2 \\
\text{Parallelogram:} & \quad b \times h
\end{align*}
\]

NOTE: Students may not be familiar with the formula for the area of a trapezoid. If this is the case, give them the formula, label the trapezoid, and go through a few numerical examples.

Exercises: Calculate the area of the polygon.

a) \[ 3 \text{ cm} \times 6 \text{ cm} \]

b) \[ 3 \text{ in} \times 4 \text{ in} \]

c) \[ 7 \text{ m} \times 4 \text{ m} \]

d) \[ 8 \text{ ft} \times 6 \text{ ft} \]

Answers: a) 18 \text{ cm}^2, \ b) 6 \text{ in}^2, \ c) 20 \text{ m}^2, \ d) 48 \text{ ft}^2

Finding the volume of polygonal prisms. Draw on the board:

\[
\begin{align*}
\text{Volume:} & \quad B = b \times h \div 2 \\
& \quad = (10)(12) \div 2 \\
& \quad = 60 \text{ ft}^2
\end{align*}
\]
ASK: What is the height of the prism? (15 ft) Write on the board:

\[ h = 15 \text{ ft} \]

ASK: What is the volume of the prism? (900 ft\(^3\)) Write on the board:

\[ V = Bh \]
\[ = (60)(15) \]
\[ = 900 \text{ ft}^3 \]

**Exercises:**
1. Calculate the area of the base. Hint: First shade the base and circle its dimensions.
   a) [Diagram]
   b) [Diagram]
   c) [Diagram]

**Answers:**
- a) \( B = 33.25 \text{ m}^2 \)
- b) \( B = 29 \text{ cm}^2 \)
- c) if the bottom/top face is the base: \( B = 6 \frac{1}{8} \text{ in}^2 \), if the side face is the base: \( B = 11 \frac{2}{3} \text{ in}^2 \), if the front/back face is the base: \( B = 23 \frac{1}{3} \text{ in}^2 \)

2. Find the volume of the prisms in Exercise 1.
   **Answers:**
   - a) \( V = 498.75 \text{ m}^3 \)
   - b) \( V = 185.6 \text{ cm}^3 \)
   - c) \( V = 40 \frac{5}{6} \text{ in}^3 \)

**Finding the volume of composite solids.** Draw on the board:

![Diagram of a composite solid]

Point out that not all of the edges on the shape are labeled. Point to the horizontal edge with no measurement at the bottom of the picture. ASK: What is this length? (11 in) How do you know? (20 \(-\) 9 = 11) Point to the vertical edge with no measurement and ASK: What is this length? (5 in) How do you know? (11 \(-\) 6 = 5) Label the diagram with the two lengths.

SAY: A composite solid is a 3-D shape made up of two or more simpler shapes. Composite solids don’t have an easy formula for volume, but we can divide the shape into simpler shapes that do have formulas. Point to the shape on the board and ASK: How could we divide this shape? (into two rectangular prisms) Have a student show how the shape could be divided. Label the prisms \( V_1 \) and \( V_2 \)—the two possible divisions are shown below:
Write on the board:

\[ V_1 = \]
\[ V_2 = \]

Total volume =

ASK: Are the units consistent? (no, the height is measured in feet but the other measurements are in inches) SAY: We need to convert this measurement to inches before we can go any further. ASK: How many inches are in 1/4 foot? (3 inches) PROMPT: 12 in = 1 ft. Have students tell you what calculations are needed to find the volumes. The two possible solutions are shown below:

\[
V_1 = 11(11)(3) = 363 \text{ in}^3 \quad \text{or} \quad V_1 = 20(6)(3) = 360 \text{ in}^3
\]
\[
V_2 = 9(6)(3) = 162 \text{ in}^3 \quad \text{or} \quad V_2 = 5(11)(3) = 165 \text{ in}^3
\]
\[
\text{Total volume} = 363 + 162 = 525 \text{ in}^3 \quad \text{or} \quad \text{Total volume} = 360 + 165 = 525 \text{ in}^3
\]

Have a volunteer come to the board to show a different way to divide the shape and calculate the volume. ASK: Why doesn’t it matter which way we divide the shape? (Whichever way we divide up the composite solid, the solid shape is the same, so the volume is still the same.) Point out that calculating the volume in a different way allows us to check our original answer.

**(MP.7) Exercises:** Find the volume of the composite solid.

a) 

b) Hint: 1 ft = 12 in.

**(MP.1) Bonus:** Find the volume of the composite solid in part a) in a different way. Check that you get the same answer.

**Answers:** a) \( V = 171 \text{ in}^3 \), b) \( V = 10,332 \text{ in}^3 \) or \( V = 5 47/48 \text{ ft}^3 \)

**Word problems practice.** For the following word problems, work through one as an example on the board with students. Model each step. Then ask students to do the rest individually.

**Exercises:**

**(MP.2)** 1. Sketch and label two different prisms that have a volume of 300 cm\(^3\).

**Sample answers:**

10 cm

6 cm

5 cm

6 cm

5 cm

20 cm
(MP.2) 2. What is the effect on the volume of a rectangular prism when you double ...
a) one dimension? Hint: Find the volume of a rectangular prism with $\ell = 1$, $w = 1$, and $h = 1$. Then
find the volume of a rectangular prism with $\ell = 2$, $w = 1$, and $h = 1$. Compare the volumes.
b) two dimensions?
c) all three dimensions?
**Answers:** a) the volume is multiplied by 2, b) the volume is multiplied by 4, c) the volume is multiplied by 8

(MP.4) 3. a) Milly wants to fill the sandbox with 3 inches of sand. How many cubic feet of sand
does she need? Hint: 1 in = $\frac{1}{12}$ ft.

b) A 50-pound bag of sand contains 0.5 cubic feet of sand. How many bags of sand should Milly
buy to fill the sandbox?
**Answers:** a) 7.5 ft³, b) 15 bags

**Extensions**
(MP.1, MP.8) 1. Find the volume of the rectangular prism in three different ways.

a) area of top $\times$ height $= 10$ cm² $\times$ 7 cm $= 70$ cm³
b) area of front $\times$ width $= 35$ cm² $\times$ 2 cm $= 70$ cm³
c) area of right side $\times$ length $= 14$ cm² $\times$ 5 cm $= 70$ cm³; d) yes, because any face of a rectangular prism can be the base

(MP.3) 2. Stack two equal piles of thin books of the same size, as shown below:

**Stack A**

**Stack B**

ASK: Is Stack A a right prism? (yes) Is Stack B a right prism? (no, it's a skew prism) Have
students vote on whether or not they agree with the following statement: “The volumes of the
two stacks are equal.” Ask a few students to explain their vote. The discussion should lead to
the conclusion that the volumes are equal because, although the shape of the pile has changed, the size, shape, and number of books that make up the pile haven’t changed. ASK: How can you find the volume of a skew prism? (it’s the same as for a right prism: \( V = Bh \))

Have students find the volumes of the skew prisms.

\begin{align*}
\text{a)} & \quad \text{b)} & \quad \text{c)} \\
7.2 \text{ in} & \quad 15.4 \text{ in}^2 & \quad 12 \text{ ft} \\
12 \text{ ft} & \quad 8 \text{ ft} & \quad 8 \text{ cm} \\
5 \text{ ft} & \quad 5 \text{ ft} & \quad 3 \text{ cm} \\
\end{align*}

**Answers:** a) \( V = 110.88 \text{ in}^3 \), b) \( V = 480 \text{ ft}^3 \), c) \( V = 60 \text{ cm}^3 \)

**(MP.4)** 3. Have students identify a prism in the classroom, such as a book or a calculator. Ask students to measure the prism and calculate its volume. You could have each student make a poster using a plain sheet of paper with a labeled sketch of the object and the calculations for volume.

**Bonus:** Have students find a composite solid in the classroom, such as a desk or a computer, and calculate its volume.
**Volume of Cylinders**

Pages 160–162

**Standards:** 8.G.C.9

**Goals:**
Students will identify the characteristics of cylinders.
Students will develop the formula for the volume of a cylinder and use it to solve problems.

**Prior Knowledge Required:**
Can identify the diameter and the radius of a circle
Can find the radius when given the diameter and vice versa
Knows that $\pi \approx 3.14$
Knows that the area of a circle is $A = \pi r^2$
Knows that the volume of a prism is $V = Bh$
Can round to a given place value
Can solve equations

**Vocabulary:** cylinder, diameter, dimension, line segment, net, pi ($\pi$), radius

**Materials:**
a variety of cylinders, possibly including BLM Nets of 3-D Shapes (5) (p. Q-63), one per pair of students
computer access to show students an applet
grid paper or BLM 1 cm Grid Paper (p. S-1)
plain, 8.5 by 11-inch paper, 2 sheets per student (see Extension 1)
tape and rulers for each student (see Extension 1)
counting cubes (see Extension 1)
BLM Compare and Contrast Graphic Organizer (p. Q-70, see Extension 4)

Review characteristics of circles. Draw on the board:

![Circle with 15 cm line segment](image)

SAY: 15 cm is the length of a line segment that passes through the center of the circle and joins two points on the circle. ASK: What do we call this line segment? (diameter) What is the radius of a circle? (the distance of any point on the circle from the center) What is the relationship between radius and diameter? (the radius is half the diameter) So, what is the radius of this circle? (7.5 cm) What is the formula for the area of a circle? ($A = \pi r^2$) To find the area of this
circle, which measurement do we use? \((r = 7.5 \text{ cm})\) Point out that a common mistake is to use diameter rather than radius when calculating area. Write on the board:

\[
A = \pi r^2 \\
= (3.14)(7.5)^2
\]

Point out that, when we use 3.14 for \(\pi\), we are using an approximation, so we write “\(\approx\).” Ask students to use their calculators to evaluate the expression. You may want to remind them that to calculate \((7.5)^2\), they can use the \(x^2\) key or they can key in \(7.5 \times 7.5\). Continue writing on the board:

\[
= 176.625 \text{ in}^2
\]

SAY: In most cases, we will round the answer to the nearest tenth, which is one decimal place. ASK: What is the rounded area of the circle? \((\approx 176.6 \text{ in}^2)\)

**Exercises:** Find the area of the circle. Round to the nearest tenth. Hint: Are you given the radius or the diameter?

a) \(2 \text{ cm}\)  

b) \(6 \text{ in}\)  

c) \(1 \text{ m}\)

**Answers:** a) \(A \approx 12.6 \text{ cm}^2\); b) \(A \approx 28.3 \text{ in}^2\); c) \(A \approx 0.8 \text{ m}^2\) or \(A \approx 7,850 \text{ cm}^2\)

**Introduce cylinders.** Show students some cylinders, including one made from a net (for example, BLM Nets of 3-D Shapes (5)) or a commercial shape, and some that are household items. SAY: These shapes are called *cylinders*. Like prisms, cylinders have two congruent parallel bases. ASK: What shapes are the bases? (circles) SAY: Prisms have rectangular side faces. ASK: What is different about the side face of the cylinder? (the side face is curved) SAY: As with prisms, the height of a cylinder is the distance between the bases. Pass around a few different cylinders and have students point out the bases and the height of each.

To reinforce the similarities between prisms and cylinders, use an applet that can be found online by searching for the following phrase: “relation of a cylinder to a prism.” The applet allows students to see that as the number of sides of the base of the prism increases, the prism looks more and more like a cylinder.

**Identifying the dimensions of a cylinder.** Tell students that they can describe a cylinder by giving the height of the cylinder and the radius or diameter of the base. SAY: These are called the dimensions of the cylinder. Hold up two cylinders of different sizes. ASK: Which cylinder has the greater height? (the taller one) Which cylinder has the greater radius? (the one with the bigger base) Which cylinder has the smaller diameter? (the one with the smaller base)
Developing the formula Volume of a cylinder = \( Bh \).

**Activity**
Students work in pairs. Give each student a sheet of grid paper and each pair a cylinder with a base that is smaller than the paper. Have each pair follow the steps below.

**Step 1:** Set the cylinder base-down on the grid paper and trace the base. Take the cylinder off the paper.

**Step 2:** To estimate the area of the base, count the squares in the circle that you drew on the paper. Hint: Some easy ways to count the squares are: divide the circle in 4, count one section only, and multiply by 4 for the total; or count the full squares and the almost-full squares, but not the almost-empty squares. Record this number.

**Step 3:** Place the cylinder on its side. Use the grid paper to count how many squares tall the cylinder is, from base to base. Record this number.

**Step 4:** Estimate the volume of the cylinder. Compare your estimate with your partner.

**MP.3 Step 5:** How is the volume of a cylinder similar to the volume of a prism? Why does this make sense? Hint: Think about the ways cylinders are similar to prisms.

**Step 6:** Predict a formula for the volume of a cylinder in terms of the area of the base and height.

As a class, discuss students’ findings from Steps 5 and 6. Make sure that students understand that, as with prisms, the volume of a cylinder is equal to the area of the base multiplied by the height. Write on the board:

\[
\text{Volume of a cylinder} = Bh
\]

(end of activity)

**Exercises:** Find the volume of the cylinder.

a) \( V = 36 \text{ cm}^3 \)  
   ![Image a)](image)

b) \( V = 120 \text{ in}^3 \)  
   ![Image b)](image)

c) \( V = 90 \text{ ft}^3 \)  
   ![Image c)](image)

d) \( V = 323.5 \text{ m}^3 \)  
   ![Image d)](image)

**Answers:** a) \( V = 36 \text{ cm}^3 \), b) \( V = 120 \text{ in}^3 \), c) \( V = 90 \text{ ft}^3 \), d) \( V = 323.5 \text{ m}^3 \)

**Using the formula Volume = \( \pi r^2 h \).** Write on the board:

\[
V = Bh
\]

ASK: For a cylinder, how do we find the area of the base? (Area of a circle = \( \pi r^2 \)) So what is another way to write the formula for the volume of a cylinder? (\( V = \pi r^2 h \)) Write on the board:

\[
V = \pi r^2 h
\]
Draw on the board:

![Cylinder diagram](image)

**ASK:** What dimensions are you given for this cylinder? (height = 2 in, radius = 4 in) Remind students that, since the units are the same, they can just substitute the values into the formula and evaluate the expression. Make sure students know how to use their calculator to evaluate $3.14 \times 4^2 \times 2$. Write on the board:

$$V = \pi r^2 h$$

$$= (3.14)(4)^2(2)$$

$$= 100.48 \text{ in}^3$$

$$\approx 100.5 \text{ in}^3$$

**MP.2** Tell students that volume is measured in cubic units because volume measures three dimensions. SAY: For example, when we find the volume of a rectangular prism, we multiply three dimensions—length, width, and height. But we only use two dimensions—radius and height—to find the volume of a cylinder. Write on the board:

$$V \approx (3.14)(4)^2(2)$$

$$\text{in}^2 \times \text{in} = \text{in}^3$$

Point to the $(4)^2$ and say that, because the 4-inch radius is squared, the units are squared and, when we multiply that by the height, we end up with cubic units. Tell students to always check their work to make sure that they have included all relevant dimensions when substituting in the formula.

Draw on the board:

![Cylinder diagram](image)

**ASK:** What dimensions are you given for the cylinder? (height = 9 cm, diameter = 6 cm) To find the volume of the cylinder, what do we need to do first? (find the radius) What is the radius? (3 cm) Have a student calculate the volume on the board, as shown below:

$$V = \pi r^2 h$$

$$= (3.14)(3)^2(9)$$

$$= 254.34 \text{ cm}^3$$

$$\approx 254.3 \text{ cm}^3$$
NOTE: If necessary, remind students that, for the rest of the unit and unless directed otherwise, they should round their answers to the nearest tenth.

**Exercises:** Find the volume of the cylinder.

a) ![Cylinder A](image)

b) ![Cylinder B](image)

c) ![Cylinder C](image)

d) ![Cylinder D](image)

**Answers:** a) \(V \approx 1,413\) in\(^3\), b) \(V \approx 197.8\) ft\(^3\), c) \(V \approx 725.7\) m\(^3\), d) \(V \approx 190.8\) cm\(^3\)

**Word problems practice.** For the following word problems, work through one as an example on the board with students. Model each step. Then ask students to do the rest individually.

**Exercises:**

(MP.4) 1. A water wheel is a giant inflatable water toy in the shape of a hollow cylinder. The outer diameter of the toy is 49 inches and the inner diameter is 37 inches. The height is 33 inches.

a) Find the volume of the outer cylinder.

b) Find the volume of the inner cylinder.

c) Find the volume of air inside the water wheel. Hint: Volume inside water wheel = volume of outer cylinder − volume of inner cylinder.

d) An electric pump takes 1 minute to pump 3,000 cubic inches of air. How long will it take the pump to fill the toy with air?

**Answers:** a) \(V \approx 62,197.9\) in\(^3\); b) \(V \approx 35,463.9\) in\(^3\); c) \(V \approx 26,734.0\) in\(^3\); d) about 9 minutes

(MP.4) 2. Greg has three cooking pots: a saucepan, a stock pot, and a shallow pot.

a) Find the volume of each cooking pot if …

i) the saucepan has a diameter of 15 cm and a height of 12 cm;

ii) the stock pot has twice the height but the same diameter as the saucepan; and

iii) the shallow pot has twice the diameter as the saucepan but the same height.

b) Which produces a greater change in volume: doubling the height or doubling the diameter?

**Bonus:** Write the volumes as a simplified ratio in the following format:
saucepan : stock pot : shallow pot

**Answers:** a) i) \(V \approx 2,119.5\) cm\(^3\), ii) \(V \approx 4,239\) cm\(^3\), iii) \(V \approx 8,478\) cm\(^3\); b) doubling the diameter produces a greater change because that measurement is squared when calculating volume;

Bonus: \(2,119.5 : 4,239 : 8,478 = 1 : 2 : 4\)
(MP.4) 3. The treasure chest is completely filled with treasure! It contains 50% gold, 30% diamonds, and 20% rubies. Find the volume of gold, diamonds, and rubies in the chest. Hint: The treasure chest is made with a prism and half a cylinder.

Answer: Total volume of chest \( \approx 1.7 \text{ ft}^3 \), so gold \( \approx 0.9 \text{ ft}^3 \), diamonds \( \approx 0.5 \text{ ft}^3 \), rubies \( \approx 0.3 \text{ ft}^3 \)

(MP.4) 4. Each square is 3 cm by 3 cm.

a) Find the area of the large circle. Don’t round your answer.
b) Find the area of a small circle. Don’t round your answer.
c) One large cylinder and nine small cylinders with height 5 cm and bases equal to the circles shown are made. Is the volume of the large cylinder greater than, less than, or equal to the volume of the 9 smaller cylinders together? Explain.

Answers: a) \( A \approx 7.065 \text{ cm}^2 \), b) \( A \approx 0.785 \text{ cm}^2 \), c) the large cylinder has the same volume as the 9 smaller cylinders together \( (V \approx 35.3 \text{ cm}^3) \)

Solving equations to find unknown dimensions. Draw on the board:

\[
V \approx 100.48 \text{ m}^3
\]

SAY: First, let’s make a list of what we know. ASK: What information do we know? \((r = 2 \text{ m}, V \approx 100.48 \text{ m}^3)\) What do we want to know? \((h)\) Write on the board:

\[
V \approx 100.48 \text{ m}^3 \quad r = 2 \text{ m} \quad h = ?
\]

ASK: What formula do we use to find the volume of a cylinder? \((V = \pi r^2 h)\) Write on the board:

\[
V = \pi r^2 h
\]

SAY: Let’s substitute the variables we know in the formula. Write on the board:

\[
100.48 \approx (3.14)(2)^2 h
\]
SAY: Now we have an equation that we can solve. While writing on the board, walk students through each step shown below:

\[ 100.48 \approx 12.56h \]
\[ 100.48 \div 12.56 \approx h \]
\[ 8 \approx h \]

SAY: So the height must be about 8 m.

**Exercises:**
1. Write and solve an equation to find the unknown.
   a) The volume of a cylinder is about 200 ft\(^3\) and the height is 20 ft. What is the area of the base?
   b) The volume of a cylinder is about 197.8 cm\(^3\) and the radius is 3 cm. What is the height?
   c) The volume of a cylinder is about 1,271.7 in\(^3\) and the diameter is 9 in. What is the height?

   **Answers:** a) \(B \approx 10\ ft^2\), b) \(h \approx 7\ cm\), c) \(h = 20\ in\)

**Extensions**

**MP.4**
2. Two cans of chili have different sizes but the same volume.

   a) Find the volume of the first can.
   b) How tall is the second can?
   c) One serving of chili is about 250 cm\(^3\) and has about 280 calories. How many calories are in each can?

   **Answers:** a) \(V \approx 307.7\ cm^3\); b) \(h \approx 3.9\ cm\); c) \(307.7\ cm^3 \div 250\ cm^3 = 1.2\) servings per can, \(1.2 \times 280 \approx 336\) calories per can

**Extensions**

**MP.5**
1. a) Take two sheets of plain, 8.5 by 11-inch paper. Write a big letter “A” on one piece of paper. Make a cylinder by joining the two 8.5-inch sides, with as little overlap as possible, and securing them with tape.
   b) Write a big letter “B” on the other piece of paper. Make a cylinder by joining the two 11-inch sides, with as little overlap as possible, and securing them with tape.
   c) Make a prediction by completing the following sentence with “greater than,” “less than,” or “equal to.” Explain your choice.

   The volume of Cylinder A is ____________ the volume of Cylinder B.

   d) Test your prediction from part c) in any way you choose. Explain what you did to test your prediction.
   e) Was your prediction in part c) correct? Explain.

   **Selected answer:** d) Students could test the volumes very roughly by filling both cylinders with counting cubes to see which one holds more; students could measure the diameter and height to calculate the volume: Cylinder A has \(r = 1.75\ in, \ h = 8.5\ in, \ and \ V \approx 81.7\ in^3\), Cylinder B has \(r = 1.35\ in, \ h = 11\ in, \ and \ V \approx 62.9\ in^3\), so the volume of Cylinder A is about 1.3 times greater than the volume of Cylinder B.
2. Coins are stacked in two ways.

**Stack A**

**Stack B**

Both stacks are cylinders; Stack A is a right cylinder and Stack B is a skew or oblique cylinder.

a) Do you think that Stack A has the same volume as Stack B? Explain.

b) The volume of any cylinder is \( V = \pi r^2 h \). Find the volume of the cylinder to the nearest tenth.

i)  

\[
\text{Volume} \approx 197.8 \text{ in}^3
\]

ii)  

\[
\text{Volume} \approx 12.7 \text{ m}^3
\]

iii)  

\[
\text{Volume} \approx 4,534.2 \text{ cm}^3
\]

**Answers:**

a) The way the coins were stacked has changed but the coins themselves haven’t changed, so both volumes are equal to 4 coins.

b) i)  

\[
\text{Volume} \approx 197.8 \text{ in}^3
\]

ii)  

\[
\text{Volume} \approx 12.7 \text{ m}^3
\]

iii)  

\[
\text{Volume} \approx 4,534.2 \text{ cm}^3
\]

**MP.2** 3. a) Find the volume of a cube with an edge length 1 cm.

b) A cylinder fits exactly inside the cube. It has diameter 1 cm and height 1 cm. Find the volume of the cylinder. Don’t round your answer.

c) Write the volume of the cylinder as a percentage of the volume of the cube.

d) Use your answer to part c) to find the volume of a cylinder that fits exactly inside a cube with ...

i)  

\[
\text{Volume} = 50 \text{ in}^3
\]

ii)  

\[
\text{Edge length} = 12 \text{ m}
\]

e) Use your answer to part c) to find the volume of a cube that fits exactly around a cylinder with ...

i)  

\[
\text{Volume} = 100 \text{ ft}^3
\]

ii)  

\[
\text{Volume} = 14.6 \text{ in}^3
\]

**Answers:**

a) \( V = 1 \text{ cm}^3 \); b) \( V \approx 0.785 \text{ cm}^3 \); c) \( V \approx 78.5\% \); d) i) \( V \approx 39.3 \text{ in}^3 \), ii) \( V \approx 1,356.5 \text{ m}^3 \);

e) i) \( V \approx 127.4 \text{ ft}^3 \), ii) \( V \approx 18.6 \text{ in}^3 \)

**MP.7** 4. Have students use BLM Compare and Contrast Graphic Organizer to compare the characteristics of a prism and a cylinder. Have prisms and cylinders available for reference.

**Sample answers:**

Similar features: 3-D shape, two parallel congruent bases, \( V = Bh \)

Different features for prisms: prisms have vertices, the bases are polygons, the side faces are rectangles

Different features for cylinders: cylinders don’t have vertices, the bases are circles, the side face is a curved rectangle
5. To find the surface area of a 3-D shape, find the sum of the areas of each face. Drawing a net of a cylinder can help with the calculation by showing the shape and dimensions of each face. The net of a cylinder has two circles and a rectangle, as shown below:

![Diagram of a cylinder net]

The area of each circular base is $\pi r^2$. The side face, when cut and made flat, is a rectangle with a length equaling the circumference of the circle ($2\pi r$) and width equaling the height of the cylinder ($h$). So the formula to find the surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$.

a) Find the surface area of a cylinder with radius 4 cm and height 8 cm.
b) Find the surface area of a cylinder with radius 11.4 in and height 16.7 in.
c) Find the surface area of a cylinder with diameter 4.4 m and height 2.1 m.

**Answers:**
a) $SA \approx 301.4 \text{ cm}^2$, b) $SA \approx 2,011.7 \text{ in}^2$, c) $SA \approx 59.4 \text{ m}^2$

(MP.1, MP.8) 6. Describe all the values of $a$ so that the equation $x^2 + 36 = a$ has ...

a) two solutions.  
b) no solutions.  
c) exactly one solution.

**Answers:**
a) $a > 36$, because then $x^2 = a - 36 > 0$ and there are two solutions. For example, when $a = 40$, $x^2 = 4$, so $x = +2$ or $-2$.  
b) $a < 36$, because then $x^2 = a - 36 < 0$, which cannot be true for the square of a number, because when you multiply a number by itself, you are multiplying two numbers that have the same sign and so the answer is either 0 if the number you are multiplying is 0 or positive if the number you are multiplying is positive or negative.  
c) $a = 36$, because then the equation becomes $x^2 = 0$, which only has $x = 0$ as a solution.

Redirecting students: Encourage students to try different values of $a$ and to focus on the reason why the equation has two solutions, one solution, or no solution. Encourage students to look for repetition in their reasoning for different examples.

(MP.4, MP.6) 7. SAY: A boat travels faster with the current (the movement of the water) than against the current. For example, if you travel at 5 km/h and the speed of the current is 1 km/h, then if you go in the same direction as the current, you travel 6 km/h, and if you go against the current in the opposite direction, you travel only 4 km/h.

When going with the current, a boat travels 30 km in 2 hours. When returning, the boat goes against the current, but it takes 5 hours to go the same 30 km. How fast would the boat go without the current, in calm water?

**Answer:** Let $x$ be the speed of the boat in calm water and let $y$ be the speed of the current. $x + y$ is the speed of the boat with the current, which is 15 km/h (because the boat travels 30 km in 2 hours) and $x - y$ is the speed of the boat against the current, which is 6 km/h (because the boat travels 30 km in 5 hours). So I solved the system of equations $x + y = 15$ and $x - y = 6$. So $x = 10.5$ and $y = 4.5$. So the speed of the boat in calm water is 10.5 km/h.
G8-51 Pyramids

Pages 163–165

Standards: 8.G.B.7

Goals:
Students will identify the characteristics of pyramids.
Students will use the Pythagorean Theorem to find the height of a pyramid.

Prior Knowledge Required:
Can identify the bases, faces, edges, and vertices of prisms
Can identify a prism as a right prism or a skew prism
Can square numbers and find the square roots of numbers, with and without a calculator
Can use the Pythagorean Theorem to find an unknown side of a right triangle

Vocabulary: apex, cross section, cylinder, hypotenuse, lateral face, lateral height, oblique pyramid, pyramid, Pythagorean Theorem, right pyramid, right triangle, side face, skew prism, skew pyramid, slant height, vertex

Materials:
BLM Square Numbers and Square Roots (p. Q-71, optional)
pyramids with a variety of bases or shapes made from BLM Nets of 3-D Shapes (6) to (9) (pp. Q-64–67)
BLM Sorting Pyramids (p. Q-72), reproduced on a transparency and/or photocopied for individual students or pairs
two copies of BLM Nets of 3-D Shapes (9) (p. Q-67), photocopied on cardstock and assembled to make two skew pyramids; then, with sticky notes used to hold the two pyramids together to make one right pyramid

NOTE: In this lesson, students will be working with the Pythagorean Theorem. If you think students will have difficulties evaluating square numbers and square roots, have them complete BLM Square Numbers and Square Roots before the lesson. (1. a) 1, b) 36, c) 81, d) 4; 2. a) 61, b) 53, c) 48, d) 91; 3. a) 4, b) 5, c) 9, d) 6; 4. a) 9.2, b) 3.5, c) 6.4, d) 0.7; 5. a) 2.2, b) 6.2, c) 7.8, d) 9.1; 6. a) $x = 7$, b) $x = 12$, c) $x \approx 7.5$, d) $x \approx 9.1$; 7. a) $x \approx 8.6$, b) $x = 10$, c) $x = 3$, d) $x \approx 6.9$)

(MP.6) Introduce pyramids. ASK: What does a pyramid look like? (a 3-D object with triangular sides that meet in a point) What are some real-world examples of pyramids? (sample answers: the pyramids in Egypt, the roof of a house, a pile of fruit in a grocery store, the pyramid on the one-dollar bill) Give a pyramid with a rectangular base (for example, BLM Nets of 3-D Shapes (6)) to a student and ASK: How many faces does this shape have? (5) Remind students that prisms and cylinders have two congruent bases that are parallel. ASK: Does this pyramid have two parallel congruent bases? (no) SAY: Pyramids have only one face that is a base. ASK: Which face would you say is the base for this pyramid? (rectangle) SAY: Pyramids are named for the
shape of their base, so this is a “rectangular pyramid.” Lay the pyramid on its side and point out that the rectangular base doesn’t have to be the bottom face of the shape. ASK: What shapes are the side faces? (triangles) When the pyramid is base-down, are the side faces vertical, like in a prism? (no, they are slanted) Have a student point out the vertex where the side faces meet. SAY: In pyramids, this vertex has a special name—it is called an apex. Give a student a pyramid with a square base (for example, BLM Nets of 3-D Shapes (7)), and ask the student to say what shape makes the base, to point out the apex, and to put the pyramid base-down on a display desk. Hold up a triangular-based pyramid (for example, BLM Nets of 3-D Shapes (8)). ASK: Which face is the base? (any one of the four faces could be the base) SAY: For a triangular-based pyramid, it doesn’t matter which face we call the base because, for each face, there are three other triangular side faces that meet at an apex.

Identifying right pyramids and skew pyramids. Hold up a right pyramid from the display desk. SAY: In a right pyramid, the apex is directly above the center of the base. ASK: Are all of the pyramids on the desk right pyramids? (yes) Show students a skew pyramid (for example, BLM Nets of 3-D Shapes (9)) and ASK: Is this a right pyramid? (no, the apex is not directly above the center of the base; it is pushed to one side) Tell students that in a skew pyramid, the apex is not directly above the center of the base. In the same way that a skew prism looks as if the top face had been pushed sideways, the apex of a skew pyramid looks like it has been pushed to one side. Let students know that skew pyramids are also sometimes called oblique pyramids.

For the following exercises, project BLM Sorting Pyramids and/or distribute a photocopy to individual students or pairs.

(MP.7) Exercises:
1. Sort the 3-D shapes.
   a) Pyramids b) Not pyramids
   **Answers:** a) A, C, E, F, G, I, K; b) B, D, H, J, L

(MP.3) 2. Choose one 3-D shape from each group in Exercise 1 and explain why it belongs in the group.
   **Sample answers:** a) A is a pyramid because its side faces are triangles and they meet at the apex; b) B is not a pyramid because its side faces do not meet at one vertex, the apex

(MP.3) 3. a) Choose one 3-D shape in Exercise 1 that is a right pyramid. Explain why it is a right pyramid.
   b) Choose one 3-D shape in Exercise 1 that is a skew pyramid. Explain why it is a skew pyramid.
   **Sample answers:** a) A is a right pyramid because its vertex appears to be above the center of the base; b) C is a skew pyramid because its vertex does not appear to be above the center of the base

**Identifying the height of a pyramid.** SAY: When a pyramid stands on its base, the height is how tall it is. The height of a right pyramid is the vertical distance from the base to the apex. **NOTE:** You might tell students that with a skew pyramid we must think of the height as the vertical distance from the base to the level of the apex.
Point to one of the right pyramids on the display desk. ASK: Why is it hard to measure the height of this pyramid? (the height is measured inside the pyramid and not along one of the edges) Draw on the board:

![Diagram of a pyramid with height labeled](image)

Tell students that, to show the height of a pyramid in a drawing, we draw a vertical line inside the pyramid from the apex to the middle of the base, so that the line is perpendicular to the base. SAY: We show that the line is perpendicular by drawing a right angle where the line meets the base. Continue drawing on the board:

![Diagram showing height](image)

**Identifying the slant height of a pyramid.** Point to one of the right pyramids on the display desk. SAY: The side faces of a pyramid are also called *lateral faces*. Lateral means “of the side.” Remind students that the lateral faces of a pyramid are triangles. SAY: The measurement of a lateral face from base to apex is called the *lateral height* or *slant height*. The line that shows the lateral height of a triangle meets the base of the triangle at a right angle, so we draw the right-angle symbol where the slant-height line meets the base. Draw on the board:

![Diagram showing slant height](image)

Point out that the slant height is specific to the face; each different lateral face might have a different slant height.

**Differentiating dimensions of a pyramid.** Draw on the board:

![Diagram showing different measurements](image)

**(MP.6)** SAY: Three different measurements are labeled on the pyramid. Write the terms on the arrows as students answer the following questions. ASK: What measurement is arrow 1 pointing at? (slant height) Arrow 2? (height of pyramid) Arrow 3? (the length of a lateral edge) ASK: Which height is greater in a right pyramid: the height of the pyramid or the slant height of a lateral face? (the slant height) Why? (the height goes straight down from the apex to the base, but the slant height goes from the apex, down the side face, to a point further along the base) Which of the three measurements would be the greatest? (the length of a lateral edge) SAY: It’s important to identify which part of the pyramid you are measuring when giving its dimensions, because these three measurements are all different.
Identifying a right triangle inside a right pyramid. Show students the right pyramid that you made by putting the two skew pyramids from BLM Nets of 3-D Shapes (9) together. SAY: We are going to cut the pyramid in half vertically—from the apex to the base, along the slant height. When we cut a 3-D shape with a plane, such as when we cut a cake with a knife, we separate the shape into two parts, and what we see in the cut itself is called a cross section. Separate the pyramids to show the cross section. ASK: What shape is the cross section? (a triangle) What type of triangle is it? (isosceles) How do you know? (the pyramid is symmetrical so the sides of the cross section will be equal) Draw an isosceles triangle on the board.

![Diagram of right triangle inside a right pyramid]

ASK: Where is the height of the pyramid located on the cross section? (the height would be a line in the middle, from the apex to the base) Draw the line and label the drawing, as shown below:

![Diagram with height marked]

Point out that, because of symmetry, the height divides the isosceles triangle into two right triangles. ASK: How do you know the length of the base of the isosceles triangle? (it’s equal to the length of the edge of the base that is parallel to the cut) Draw on the board:

![Diagram with base length marked]

SAY: Imagine we slice the pyramid in half along the slant height, which is marked as 12 inches. Ask a student to come to the board to draw and label the isosceles triangle that would be the cross section, as shown below:

![Diagram with slant height marked]

Then, ask the student to divide the triangle into two right triangles, and then to draw and label one of the right triangles, as shown below:

![Diagram with right triangle marked]
SAY: In the next part of this lesson, we’ll see that it is very useful to be able to imagine a right triangle that is inside a pyramid.

(MP.6) Exercises: Draw and label a right triangle inside the pyramid.

a)  
\[
\begin{array}{c}
16 \text{ in} \\
17 \text{ in}
\end{array}
\]

b)  
\[
\begin{array}{c}
14 \text{ cm} \\
21 \text{ cm}
\end{array}
\]

Bonus:  
\[
\begin{array}{c}
7 \text{ m} \\
9 \text{ m}
\end{array}
\]

Answers:

a)  
\[
\begin{array}{c}
8 \text{ in} \\
17 \text{ in}
\end{array}
\]

b)  
\[
\begin{array}{c}
5 \text{ cm} \\
14 \text{ cm}
\end{array}
\]

Bonus:  
\[
\begin{array}{c}
4.5 \text{ m} \\
7 \text{ m}
\end{array}
\]

Review the Pythagorean Theorem. SAY: It’s not easy to measure the height of a pyramid, because we can’t normally cut pyramids open to see inside. But it is easy to measure the slant height. So we need to find a way of finding the height when we’re given the slant height. We’ve seen that the height and slant height of a pyramid make two sides of a right triangle inside the pyramid. Remember that we can use the Pythagorean Theorem to find the third side of a right triangle—and this will give us the pyramid’s height. Draw on the board:

\[
\begin{array}{c}
c \\
a \\
b
\end{array}
\]

SAY: Let’s review the Pythagorean Theorem. ASK: Which side is the hypotenuse? (the longest side, c) What is the relationship between the lengths of the three sides? \(c^2 = a^2 + b^2\) Add measurements to the diagram, as shown below:

\[
\begin{array}{c}
15 \text{ in} \\
9 \text{ in}
\end{array}
\]

SAY: To use the Pythagorean Theorem, substitute numbers for the variables and evaluate the expression. Go through the steps on the board as shown below.

\[
\begin{align*}
c^2 &= 15^2 + 9^2 \\
c^2 &= 225 + 81 \\
c^2 &= 306 \\
c &= \sqrt{306} \\
c &\approx 17.5 \text{ in}
\end{align*}
\]

NOTE: If necessary, remind students how to use the \(\sqrt{\text{ key on their calculators and to round answers to the nearest tenths.}}\)
Exercises: Find the hypotenuse. All measurements are in inches.

a) \[ \begin{array}{c}
12 \\
\ \ \\
5
\end{array} \]

b) \[ \begin{array}{c}
3 \\
\ \ \\
2
\end{array} \]

c) \[ \begin{array}{c}
a \\
\ \ \\
5
\end{array} \]

Answers: a) \( x = 13 \text{ in} \), b) \( n \approx 3.6 \text{ in} \), c) \( a \approx 7.1 \text{ in} \)

Draw on the board:

\[ \begin{array}{c}
10 \text{ cm} \\
\ \ \\
6 \text{ cm}
\end{array} \]

Remind students that to use the Pythagorean Theorem to find the side of a triangle that is not the hypotenuse, they have to manipulate the equation in order to isolate \( x \). Have a volunteer find the length of the unknown side. The solution is shown below.

\[
6^2 + x^2 = 10^2 \\
36 + x^2 = 100 \\
x^2 = 100 - 36 \\
x^2 = 64 \\
x = \sqrt{64} \\
x = 8 \text{ cm}
\]

Exercises: Find the unknown side. All measurements are in centimeters.

a) \[ \begin{array}{c}
8 \\
\ \ \\
17
\end{array} \]

b) \[ \begin{array}{c}
6 \\
\ \ \\
2
\end{array} \]

c) \[ \begin{array}{c}
k \\
\ \ \\
11
\end{array} \]

Answers: a) \( x = 15 \text{ cm} \), b) \( y \approx 5.7 \text{ cm} \), c) \( k \approx 9.2 \text{ cm} \)

Using the slant height and the Pythagorean Theorem to find the height of a pyramid.

SAY: Let’s go back to our earlier example with the pyramid and the right triangle and use the Pythagorean Theorem to find the height of the pyramid. Go through the steps on the board as shown in the solution below.

\[
h^2 + 8.5^2 = 12^2 \\
h^2 + 72.25 = 144 \\
h^2 = 144 - 72.25 \\
h^2 = 71.75 \\
h = \sqrt{71.75} \\
h \approx 8.5 \text{ in}
\]

Point out that we need to identify on any diagram the dimensions that are useful for the required calculation and those that are not. In the above calculations, 17 inches and 12 inches are useful, but we don’t need to use the width of the base (11 inches) in any of the calculations.
NOTE: In the following exercise, the pyramids are the same as in the earlier exercise where students drew the right triangle inside the pyramid on p. Q-25.

Exercises:
1. Use the Pythagorean Theorem to find the height of the pyramid.
   a) \( h = 15 \text{ in} \), b) \( h \approx 13.1 \text{ cm} \), c) \( h \approx 5.4 \text{ m} \)

   Answers: a) \( h = 15 \text{ in} \), b) \( h \approx 13.1 \text{ cm} \), c) \( h \approx 5.4 \text{ m} \)

(MP.4) 2. A right pyramid with a square base has a slant height of 15 feet and a base with 24-foot sides. What is the height of the pyramid? Hint: Sketch the pyramid.
   Answer: 9 feet

   SAY: If we don’t know the slant height, you can find the height of the pyramid using other triangles inside the pyramid. Draw on the board:

   SAY: As a first step for this new pyramid, we could cut the pyramid in half along a diagonal line. Draw a diagonal line across the base, as shown below:

   SAY: To find the length of the diagonal line, we should draw one of the two triangles formed in the rectangular base and use the Pythagorean Theorem to find the length of the diagonal. Have students tell you how to draw the triangle and calculate the length of the hypotenuse. (see solution below)

   \[
   x^2 = 10^2 + 8^2 \\
   x^2 = 164 \\
   x = \sqrt{164} \\
   x \approx 12.8 \text{ in}
   \]

   SAY: Next, to find the pyramid height, we can imagine a triangle inside the pyramid. Draw a line to show the pyramid height, as shown below:
SAY: If the diagonal line across the pyramid base is 12.8 inches, then what is the base of the right triangle shown inside the pyramid? (half of that, 6.4 in) SAY: Now to find the height of the pyramid, we must draw another triangle. Have students tell you how to draw the triangle and calculate the length of the side. (see solution below)

\[6.4^2 + h^2 = 12^2\]
\[40.96 + h^2 = 144\]
\[h^2 = 144 - 40.96\]
\[h^2 = 103.04\]
\[h = \sqrt{103.04}\]
\[h \approx 10.2\text{ in}\]

**Exercises:** Find the height of the pyramid to the nearest tenth.

**Answer:** The diagonal line in the base is 7.2 cm long, so the vertical right triangle inside the pyramid is as follows, and \(h \approx 14.6\text{ cm}\).

**Extensions**

**(MP.6)** 1. a) Jen measured a pyramid. Use Jen’s measurements to find the height of the pyramid.

   **Jen’s Measurements**

   \[\begin{array}{c}
   10.3\text{ in} \\
   8\text{ in} \\
   5\text{ in}
   \end{array}\]

   b) Mark measured the same pyramid. Use Mark’s measurements to find the height of the pyramid.

   **Mark’s Measurements**

   \[\begin{array}{c}
   10.8\text{ in} \\
   8\text{ in} \\
   5\text{ in}
   \end{array}\]

   c) What did Jen and Mark do differently?

   d) Jen and Mark got slightly different answers for the height of the same pyramid. Explain why.
Answers: a) $h \approx 10.0$ in; b) $h \approx 10.0$ in; c) they measured different slant heights; d) their measurements are accurate only to the nearest tenth of an inch, so the answers agree only after rounding.

(MP.3) 2. One measurement is incorrect. Find which one it is and correct it.

Answer: The height of the pyramid is incorrect, because the pyramid height can't be greater than the slant height. The correct pyramid height is 15 in.

(MP.1, MP.4, MP.5) 3. Tina has to pick her sister up at the train station. The train is scheduled to arrive at 4:00 p.m., but it is running late. Tina regularly checks online to see the train’s estimated arrival time. This is what she finds:

<table>
<thead>
<tr>
<th>Current Time</th>
<th>Estimated Arrival Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>2:00 p.m.</td>
<td>5:00 p.m.</td>
</tr>
<tr>
<td>2:15 p.m.</td>
<td>4:57 p.m.</td>
</tr>
<tr>
<td>2:30 p.m.</td>
<td>4:54 p.m.</td>
</tr>
<tr>
<td>2:45 p.m.</td>
<td>4:51 p.m.</td>
</tr>
</tbody>
</table>

Estimate the time the train will arrive. Use any tool you think will help.

Sample solutions:
- Let $x$ be the number of minutes after 2 p.m. Let $y$ be the estimated number of minutes the train will arrive before 5 p.m. Then every increase of 15 in $x$ results in an increase of 3 in $y$. So $(\text{change in } y) / (\text{change in } x) = 3/15 = 1/5$, so $y = x/5$. When the train arrives, the time equals the estimated arrival time, so $x + y = 180$:

\[
\begin{align*}
2:00 & \quad \text{actual time} \quad 5:00 \\
\rightarrow_{x} & \quad y \\
= \text{estimated arrival time}
\end{align*}
\]

So $y = 180 - x = x/5$, so $180 = 6/5 \times x$, so $x = 5/6 \times 180 = 150$, so the train arrives 30 minutes before 5:00 p.m., or at 4:30 p.m.

- I continued the table, and when I got past 4:30, I realized the train was estimated to arrive before the actual time, and that means that the train already did arrive. The train must have arrived when the estimated arrival time equals the actual time, and that happened at 4:30 p.m.

Whole-class follow-up: ASK: What assumption did you make to help you estimate? (that the train continues to make up time at the same rate) What does it mean when the estimated arrival time equals the current time? (that is the time the train arrives) How do you know? (because the people who estimate when the train arrives keep track of how close the train is to arriving)
Goals:
Students will develop the formula for the volume of a pyramid and use it to solve problems.
Students will use the Pythagorean Theorem when finding the volume of a pyramid.

Prior Knowledge Required:
Can use the Pythagorean Theorem to find the height of a pyramid given its slant height

Vocabulary: composite solid, pyramid, right pyramid, skew pyramid, slant height

Materials:
BLM Comparing Volumes: Prisms and Pyramids (pp. Q-73–76) photocopied on cardstock, one set for every 2 or 3 students
tape and scissors for every 2 or 3 students
about 1 1/4 cups of popcorn kernels for every 2 or 3 students
BLM How Many Pyramids Make a Cube? (p. Q-77) photocopied on cardstock, one per student
BLM Compare and Contrast Graphic Organizer (p. Q-70, see Extension 4)

Review pyramids. Draw on the board:

ASK: What shape is the base of this pyramid? (rectangle) Shade the base. ASK: How do we find the area of a rectangle? (length × width) What is the area of this rectangle? (75 in²) Leave this drawing on the board for later use.

Exercises: The base of the right pyramid is shaded. Find the area of the base (B).

a) \( B = 240 \text{ cm}^2 \)

b) \( B = 119 \text{ in}^2 \)

c) \( B = 44 \text{ ft}^2 \)

Answers: a) \( B = 240 \text{ cm}^2 \), b) \( B = 119 \text{ in}^2 \), c) \( B = 44 \text{ ft}^2 \)
Developing the formula Volume of a pyramid = $Bh ÷ 3$.

Activity
(MP.2) Have students work in groups of 2 or 3 to make the four open-ended shapes from **BLM Comparing Volumes: Prisms and Pyramids**. Make sure that students tape the sides of the shapes securely. Have each group match each pyramid with the prism that has the same base and the same height to make two pairs of shapes. Groups should use the pairs of shapes to follow the steps below:

**Step 1:** Take one pair of shapes. Fill the pyramid with popcorn kernels. Then pour the popcorn kernels from the pyramid into the prism until you fill the prism. How many times do you have to refill the pyramid to fill up the prism? (3 times)

**Step 2:** Repeat with the other pair of shapes. Did you get the same result? (yes, 3 times)

**Step 3:** Check with another group. Did they get the same result as your group? (yes, 3 times)

**Step 4:** Describe the relationship between the volume of a prism and the volume of a right pyramid with the same base and height. (the volume of a prism is three times the volume of a right pyramid with the same base and height)

Have students work individually to make the net from **BLM How Many Pyramids Make a Cube?**. Then have them use the resulting shapes to follow the steps below:

**Step 5:** Work with your classmates to see how many pyramids you need to put together to make a cube. (3 pyramids)

**Step 6:** Does each pyramid have the same base and height as the cube? (yes)

**Step 7:** Describe the relationship between the volume of the cube and the volume of the pyramid. (the volume of the cube is three times the volume of the pyramid with the same base and height)

As a class, discuss students’ findings from Steps 4 and 7. Make sure that students understand that the volume of a pyramid is equal to the volume of a prism with the same base and height divided by three—in other words, $V = Bh ÷ 3$. Point out that this holds true for both right pyramids (as seen in Steps 1–4) and skew pyramids (as seen in Steps 5–7). Write on the board:

$$\text{Volume of a pyramid} = Bh ÷ 3 \text{ or } \frac{1}{3}Bh$$

(end of activity)

Using the formula $V = Bh ÷ 3$. Add the height measurement to the drawing on the board:
Show students how to use the formula \( V = Bh + 3 \) to find the volume of this pyramid. First, check that the units are consistent. Then, find the area of the base and identify the height. Finally, substitute the numbers in the formula and solve. (see solution below)

\[ B = 10(7.5) = 75 \text{ in}^2 \quad V = Bh + 3 \]
\[ h = 6 \text{ in} \quad = (75)(6) + 3 \]
\[ = 150 \text{ in}^3 \]

**Exercises:** Find the volume of the pyramid.

a) \hspace{1cm} b) \hspace{1cm} c) Hint: 1 m = 100 cm

\[ \text{Answers: a) } V = 12 \text{ in}^3; \hspace{0.5cm} \text{b) } V = 80 \text{ cm}^3; \hspace{0.5cm} \text{c) } V = 1.395 \text{ m}^3 \text{ or } 1,395,000 \text{ cm}^3 \]

Draw on the board:

ASK: What shape is the base of this pyramid? (trapezoid) Have a student calculate the area of the trapezoid on the board. PROMPT: The area of a trapezoid = \((\text{base}_1 + \text{base}_2) \times \text{height} \div 2\). (see solution below)

\[ B = (3 + 5) \times 4 \div 2 \]
\[ = 16 \text{ cm}^2 \]

ASK: What is the height of the pyramid? (6 cm) Have a student calculate the volume of the prism on the board. (see solution below)

\[ V = Bh + 3 \]
\[ = 16 \times 6 + 3 \]
\[ = 32 \text{ cm}^3 \]

**Exercises:** Find the volume of the pyramid.

a) \hspace{1cm} b)

\[ \text{Answers: a) } V = 32 \text{ cm}^3, \hspace{0.5cm} \text{b) } V \approx 33.3 \text{ in}^3 \]
Solving equations to find unknown dimensions. Draw on the board:

\[ V = 168 \text{ in}^3 \]

SAY: First, let’s make a list of what we know. ASK: What information do we know? 
\((B = 9 \times 8 = 72 \text{ in}^2, V = 168 \text{ in}^3)\) What do we want to know? \((h)\) Write on the board:

\[ V = 168 \text{ in}^3 \quad B = 72 \text{ in}^2 \quad h = ? \]

ASK: What formula do we use to find the volume of a pyramid? \((V = Bh ÷ 3)\) Write on the board:

\[ V = Bh ÷ 3 \]

Remind students that they can leave the units out while they do the calculations, but they have to remember to write the answer with units. SAY: Let’s substitute the variables we know in the formula. Write on the board:

\[ 168 = (72)h ÷ 3 \]

SAY: Now we have an equation that we can solve. Write each step on the board, as shown below:

\[ 168 = (24)h \]
\[ 168 ÷ 24 = h \]
\[ 7 = h \]

SAY: So the height must be 7 inches.

**Exercises:** Write and solve an equation to find the unknown.

a) The volume of a rectangular pyramid is 180 ft\(^3\) and the height is 15 ft. What is the area of the base?

b) The volume of a rectangular pyramid is about 253.3 cm\(^3\) and the rectangular base is 5 cm by 8 cm. What is the height of the pyramid?

**Answers:** a) \(B = 36 \text{ ft}^2\), b) \(h ≈ 19 \text{ cm}\)

**Using the Pythagorean Theorem to find the volume of a pyramid.** Remind students that it is easier to measure the slant height than the height of a pyramid, so they will often have the slant height measurement and will have to work out the height of a pyramid first, before they can find the volume. Tell students that, if they are given the slant height, there are four steps to finding the volume. They already know how to do each step but now they will put it all together! Draw on the board:
SAY: The first step is to draw and label the right triangle inside the pyramid. Have a volunteer do this on the board, as shown below:

![Diagram of a right triangle with sides labeled](image)

SAY: Step 2 is to use the Pythagorean Theorem to find the height of the pyramid. Have students calculate the height in their notebooks while a volunteer works on the board, as shown below:

\[
\begin{align*}
h^2 + 9^2 &= 19^2 \\
h^2 + 81 &= 361 \\
h^2 &= 361 - 81 \\
h^2 &= 280 \\
h &\approx 16.7 \text{ in}
\end{align*}
\]

SAY: Step 3 is to find the area of the base. Have a volunteer do this on the board, as shown below:

\[B = 18(15) = 270 \text{ in}^2\]

SAY: Step 4 is to find the volume using the formula \(V = Bh/3\). Round the final answer to the nearest tenth. Have a volunteer do this on the board, as shown below:

\[
V = Bh/3 \\
\approx (270)(16.7)/3 \\
\approx 1,503 \text{ in}^3
\]

**NOTE:** Answers may vary, depending on if students round at the final answer only or at the answer for height and the final answer.

**Exercises:** Use the Pythagorean Theorem to find the height of the rectangular pyramid. Then find the volume.

- **a)** \(h \approx 17.7 \text{ in}, \ V \approx 1,156.4 \text{ in}^3\);
- **b)** \(h \approx 13.7 \text{ m}, \ V \approx 456.7 \text{ m}^3\);
- **c)** \(h \approx 2.9 \text{ ft}, \ V \approx 10.7 \text{ ft}^3\)

**Answers:** a) \(h \approx 17.7 \text{ in}, \ V \approx 1,156.4 \text{ in}^3\); b) \(h \approx 13.7 \text{ m}, \ V \approx 456.7 \text{ m}^3\); c) \(h \approx 2.9 \text{ ft}, \ V \approx 10.7 \text{ ft}^3\)

**Finding the volume of composite solids.** Draw on the board:
Remind students that a composite solid is a 3-D shape made up of two or more simpler shapes. 

ASK: How could you divide this shape into smaller shapes, each with a volume you know how to find? (divide it into a rectangular prism and a rectangular pyramid) Label the shapes $V_1$ (prism) and $V_2$ (pyramid). Write on the board:

$$V_1 =$$
$$V_2 =$$
Total volume =

Have students tell you what calculations are needed to find the volumes. (see solution below)

$$V_1 = \ell wh = 6(4)(5) = 120 \text{ cm}^3$$
$$V_2 = Bh + 3 = 6(4)(4) ÷ 3 = 32 \text{ cm}^3$$
Total volume = 120 + 32 = 152 cm$^3$

(MP.7) Exercises: Find the volume of the composite solid.

a)

b)

Answers: a) $V = 19 \text{ in}^3$; b) height of pyramid $\approx 11.2 \text{ m}$, $V = 3,746.7 \text{ m}^3$

(MP.4) Word problems practice. For the following word problems, work through one as an example on the board with students. Model each step. Then ask students to do the rest individually.

Exercises:

1. There is a vertical pole in the middle of the tent.

a) How tall is the pole to the nearest tenth of a foot?
b) What is the volume of the tent?

Answers: a) $h \approx 6.1 \text{ ft}$, b) $V \approx 128.1 \text{ ft}^3$

2. Sketch a pyramid with whole number dimensions that has a volume that is …

a) an even number 
   b) an odd number 
   c) a decimal number

Sample answers:

a)

b)

c)

$V = 12 \text{ in}^3$

$V = 9 \text{ in}^3$

$V = 2.7 \text{ in}^3$
Extensions

(MP.1) 1. Anwar puts four of the following triangles together to make an open-ended rectangular pyramid. What is the volume of the pyramid?

![Triangle diagram]

**Answers:** slant height = 8 in, height of pyramid ≈ 5.3 in, volume ≈ 254.4 in³

(MP.4) 2. The Luxor Pyramid in Las Vegas has a square base (ABCD) with sides of 646 feet. The length of the lateral edges is 575.5 feet.

a) Draw a diagonal from A to C. What shape is ΔABC?
b) Find the length of AC to the nearest tenth of a foot.
c) Draw a diagonal from B to D. Mark the intersection point of the diagonals F.
d) Find the length of AF to the nearest tenth of a foot. Explain.
e) Draw the line segments AF and FE. What shape is ΔAFE?
f) Find the height of the Luxor Pyramid to the nearest foot.

**Answers:** a) isosceles triangle, b) AC ≈ 913.6 ft, d) AF ≈ 456.8 ft, e) right triangle, f) h ≈ 350 ft

3. Sketch and label two pyramids with different bases that have a volume of 10 in³.

**Sample answers:**

![Pyramid diagrams]

(MP.7) 4. Have students use BLM Compare and Contrast Graphic Organizer to compare the characteristics of a prism and a pyramid. Have prisms and pyramids available for reference.

**Sample answers:**

Similar features: 3-D shape; base is a polygon; named for the shape of the base; have faces, edges, vertices, and height
Different features of a prism: side faces are rectangles, side edges are perpendicular to the bases, there are two congruent bases, \( V = Bh \)
Different features of a pyramid: side faces are triangles, side edges are not parallel, side faces meet at a vertex, there is one base, \( V = Bh + 3 \)
G8-53  Volume of Cones

Pages 169–171


Goals:
Students will identify the characteristics of cones.
Students will develop the formula for the volume of a cone and use it to solve problems.

Prior Knowledge Required:
Knows the volume of a cylinder

Vocabulary: apex, coefficient, cone, cylinder, pyramid, right cone, slant height

Materials:
a variety of cones, possibly including one made from BLM Nets of 3-D Shapes (10) (p. Q-68)
computer access to show students an applet
BLM Comparing Volumes: Cylinders and Cones (pp. Q-78–80) copied onto cardstock, a set
for every 2 or 3 students (optional)
tape and scissors for every 2 or 3 students (optional)
about 1 cup of popcorn kernels for every 2 or 3 students (optional)

Review finding the area of a circle. Draw on the board:

\[ A = \pi r^2 \approx (3.14)(9.5)^2 \approx 283.4 \text{ in}^2 \]

Exercises: Find the area of the circle. Hint: Are you given the radius or the diameter?

a) \[ A \approx 78.5 \text{ in}^2 \]
b) \[ A \approx 660.2 \text{ ft}^2 \]
c) \[ A \approx 69.4 \text{ cm}^2 \]
Introduce cones. SAY: This is a cone. ASK: What characteristics does a cone have? (it’s a 3-D object with a circular base and a curved side that comes to a point) What are some real-world examples of cones? (ice cream cones, party hats, traffic cones) SAY: Cones are similar to pyramids because a cone has one base and an apex, just as a pyramid does.

ASK: What shape is the base of a cone? (circle) SAY: Pyramids have triangular sides.

ASK: What is different about the side of the cone? (it’s curved) SAY: As with pyramids, the height of a cone is the vertical distance from the base to the apex. Pass around a variety of cones (for example, BLM Nets of 3-D Shapes (10)) and have students point out the base, apex, radius, diameter, and height. Tell students that when the apex of a cone is directly above the center of the base, the cone is a right cone. You might note for students that, as with pyramids, when the apex is not directly above the center of the base, the shape is skewed; in the case of a skewed cone, we must think of the height as from the base to the level of the apex.

To reinforce the similarities between cones and pyramids, show students an applet that can be found online by searching for the following phrase: “comparison of a cone and a pyramid.” The applet allows students to see that as the number of sides of the base of the pyramid increases, the pyramid looks more and more like a cone.

Developing the formula for the volume of a cone. SAY: Cones have a structure similar to pyramids, so we can suppose that the formula for volume will be the same or close. ASK: What is the volume of a pyramid? (Bh ÷ 3) So, what do you think the volume of a cone will be? (Bh ÷ 3) Write on the board:

\[ \text{Volume of a cone} = \frac{Bh}{3} \]

SAY: In fact, the volume of a cone is equal to base multiplied by height and divided by three.

ASK: What is the area of the base of a cone? (the base is a circle, so A = πr²) Write on the board:

\[ \text{Volume of a cone} = \frac{\pi r^2 h}{3} \text{ or } \frac{1}{3} \pi r^2 h \]

NOTE: The following activity demonstrates that the volume of a cylinder is three times the volume of a cone with the same base and height. It is very similar to the activity in G8-52 that showed the same relationship between the volume of a prism and a pyramid. You may choose to not do this activity if your students have grasped the argument above and in G8-52.

Activity

(MP.3) Have students work in groups of 2 or 3 to make the open-ended shapes from BLM Comparing Volumes: Cylinders and Cones. Make sure that students tape the sides of the shapes securely. Have each group match each cone with the cylinder that has the same base size and the same height to make three pairs of shapes. Groups should use the pairs of shapes to follow these steps:

Step 1: Take one pair of shapes. Fill the cone with popcorn kernels. Then pour the popcorn kernels from the cone into the cylinder until you fill the cylinder. How many times do you have to refill the cone to fill up the cylinder? (3 times)

Step 2: Repeat with the other two pairs of shapes. Did you get the same result? (yes, 3 times)
Step 3: Check with another group. Did they get the same result as your group? (yes, 3 times)

Step 4: Describe the relationship between the volume of a cone and the volume of a cylinder with the same base and height. (the volume of a cylinder is three times the volume of a cone with the same base and height)

(end of activity)

Using the formula for the volume of a cone. Draw on the board:

\[ V = \frac{1}{3} \pi r^2 h \]

ASK: What dimensions are given for this cone? (height = 6 in, radius = 2 in) ASK: Are the units consistent? (yes, both are inches) SAY: So, to find the volume of the cone, we can substitute the values into the formula and evaluate the expression. Go through the steps on the board.

(see solution below)

\[
\begin{align*}
V & = \frac{1}{3} \pi r^2 h \\
& \approx \frac{1}{3} (3.14)(2)^2(6) \\
& = \frac{1}{3} (3.14)(4)(6) \\
& \approx 25.1 \text{ in}^3
\end{align*}
\]

(MP.2) Remind students that 3.14 is an approximation for \( \pi \), so we used the \( \approx \) sign. Make sure students know how to use their calculators to evaluate \( 3.14 \times 4 \times 6 \div 3 \). Also, have students check the number of dimensions in the answer—they should note that \( r^2 \) gives us squared inches and \( h \) gives us inches, so, in the calculation, we have \( \text{in}^2 \times \text{in} = \text{in}^3 \). Thus, we have 3 dimensions, which is what we need for volume. Tell students that checking the dimensions helps them to avoid missing a number when multiplying.

**Exercises:** Find the volume of the cone. Hint: Are you given the radius or the diameter?

a) \[
\begin{align*}
V & \approx 84.8 \text{ in}^3
\end{align*}
\]

b) \[
\begin{align*}
V & \approx 212.3 \text{ m}^3
\end{align*}
\]

c) \[
\begin{align*}
V & \approx 52.3 \text{ cm}^3
\end{align*}
\]

**Answers:** a) \( V \approx 84.8 \text{ in}^3 \), b) \( V \approx 212.3 \text{ m}^3 \), c) \( V \approx 52.3 \text{ cm}^3 \)

Solving equations to find unknown dimensions. Draw on the board:

\[
V \approx 769.3 \text{ in}^3
\]
ASK: In this example, what information are we given? (volume and height) What do we want to know? (the radius) Write on the board:

\[ V \approx 769.3 \text{ in}^3 \quad h = 15 \text{ in} \quad r = ? \]

ASK: What formula can we use to write an equation? (the formula for the volume of a cone) Write on the board:

\[ V = \frac{1}{3} \pi r^2 h \]

ASK: With the information we have already, are the units consistent? (yes, we have inches and cubed inches) SAY: So, we can substitute the variables we know in the formula and solve the equation. Write on the board:

\[ 769.3 = \frac{1}{3} (3.14) (r)^2 (15) \]

Tell students that this equation looks messy, but is easy to solve once we get the right side cleaned up. SAY: Use your calculator to evaluate the coefficient of \( r^2 \). (3.14 \times 15 \div 3 \approx 15.7) Round the answer to one decimal place and write on the board:

\[ 769.3 \approx 15.7 r^2 \]

SAY: To solve the equation, we need to isolate the variable \( r \)—in other words, put it to one side of the equal sign, on its own. To do this, we must divide both sides by 15.7, the coefficient of \( r \). Write on the board:

\[ 769.3 \div 15.7 \approx r^2 \]

\[ 49 \approx r^2 \]

SAY: To find \( r \), we take the square root of both sides. ASK: What is the square root of 49? (7) Continue writing on the board:

\[ \sqrt{49} \approx r \]

\[ 7 \approx r \]

SAY: The radius is about 7 inches.

**Exercises:** Write and solve an equation to find the unknown.

a) The volume of a cone is about 100 cm³ and the radius is 5 cm. What is the height?

b) The volume of a cone is about 300 in³ and the diameter is 8 in. What is the height?

c) The volume of a cone is about 50 ft³ and the height is 11 ft. What is the radius?

d) The volume of a cone is about 66 m³ and the height is 9 m. What is the diameter?

**Answers:** a) \( h \approx 3.8 \) cm, b) \( h \approx 17.9 \) in, c) \( r \approx 2.1 \) ft, d) \( d \approx 5.3 \) m
**Using the Pythagorean Theorem to find the height of a cone.** Tell students that, like a pyramid, a cone has a slant height that is measured up the side face, from the edge of the base to the apex. Remind students that it is easier to measure the slant height than the height of a cone, so they will often have the slant height and must work out the height before they can find the volume. Draw on the board:

ASK: How can we find the height of this cone? (draw the triangle inside the cone and use the Pythagorean Theorem) Have a student draw the right triangle inside the cone on the board, as shown below:

\[ h^2 + 9^2 = 20^2 \]
\[ h^2 + 81 = 400 \]
\[ h^2 = 400 - 81 \]
\[ h^2 = 319 \]
\[ h \approx 17.9 \text{ in} \]

SAY: Now we can use the height to find the volume of the cone. (see solution below)

\[ V = \frac{1}{3} \pi r^2 h \]
\[ \approx \frac{1}{3} \times 3.14 \times (9)^2 \times (17.9) \]
\[ \approx 1,517.6 \text{ in}^3 \]

**Exercises:** Use the Pythagorean Theorem to find the height of the cone. Then find the volume.

a) ![Diagram with measurements 5 in and 9 in]

b) ![Diagram with measurements 7 cm and 4 cm]

c) ![Diagram with measurements 13 ft and 12 ft]

**Answers:** a) \( h \approx 7.5 \text{ in}, V \approx 196.3 \text{ in}^3 \); b) \( h \approx 6.7 \text{ cm}, V \approx 28.1 \text{ cm}^3 \); c) \( h \approx 10.1 \text{ ft}, V \approx 446.6 \text{ ft}^3 \)
Word problems practice. For the following word problems, work through one as an example on the board with students. Model each step. Then ask students to do the rest individually.

Exercises:
(MP.4) 1. A paper cup in the shape of a cone has a radius of 1.5 inches and height of 4 inches. What is the volume of water that the cup can hold?
Answer: \( V \approx 9.4 \text{ in}^3 \)

(MP.4) 2. A tepee is a tent in the shape of a cone. The diameter of one tepee is 30 feet and the sides are 20 feet long. How high is the tepee?
Answer: \( h \approx 13.2 \text{ ft} \)

(MP.4) 3. A farmer uses a silo to hold grain.

![Diagram of silo](image)

a) What is the volume of the silo?
b) If the silo is 60% full, how much grain is inside?
Answers: a) \( V \approx 26,166.7 \text{ ft}^3 \); b) 15,700 \( \text{ft}^3 \) of grain

(MP.4) 4. A cylinder has radius of 10 cm and height of 20 cm. A cone has the same volume and radius as the cylinder. What is the height of the cone? Explain.
Answer: The height of the cone is 60 cm; a cone has one-third of the volume of a cylinder with the same base and height, so, if the volumes and bases are equal, the cone must have three times the height of the cylinder.

Extensions
1. The net of a cone is made up of a circle and a portion of a larger circle, as shown below:

![Diagram of cone net](image)

The area of the circular base is \( \pi r^2 \). The side face is a portion of a circle with area \( \pi rs \), where \( s \) is the slant height of the cone. So, the formula to find the surface area of a cone is \( SA = \pi r^2 + \pi rs \).

a) Find the surface area of a cone with radius 4 cm and slant height 9 cm.
b) Find the surface area of a cone with radius 2.5 in and slant height 9.3 in.
Bonus: Find the surface area of a cone with diameter 9 m and height 13 m. Hint: Find the slant height.
Answers: a) \( SA \approx 163.3 \text{ cm}^2 \); b) \( SA \approx 92.6 \text{ in}^2 \); Bonus: \( s \approx 13.8 \text{ m} \), \( SA \approx 258.6 \text{ m}^2 \)
(MP.2) 2. a) Find the volume of a pyramid with a 1 cm by 1 cm square base and height of 1 cm. b) A cone fits exactly inside the pyramid. The cone has diameter 1 cm and height 1 cm. Find the volume of the cone. c) Write the volume of the cone as a percentage of the volume of the pyramid. d) Use your answer to part c) to find the volume of a cone that fits exactly inside a pyramid with … 
   i) \( V = 40 \text{ in}^3 \) 
   ii) \( V = 95 \text{ cm}^3 \) 
e) Use your answer to part c) to find the volume of a pyramid that fits exactly around a cone with … 
   i) \( V = 300 \text{ ft}^3 \) 
   ii) \( V = 4.9 \text{ in}^3 \) 

**Answers:** a) \( V = 1/3 \text{ cm}^3 \); b) \( V \approx 0.26 \text{ cm}^3 \); c) \( V \approx 78\% \); d) i) \( V \approx 31.2 \text{ in}^3 \), ii) \( V \approx 74.1 \text{ cm}^3 \); e) i) \( V \approx 384.6 \text{ ft}^3 \), ii) \( V \approx 6.3 \text{ in}^3 \)

(MP.5, MP.7) 3. Find the intersection points of the function with \( y = x - 1 \). Use mental math, paper and pencil, or dynamic geometric software. Explain your choice.

a) \( y = x^3 - 3x - 1 \)  
   b) \( y = 2x - 2 \)  
   c) \( y = x \) 

**Answers:**

a) I used dynamic geometric software because I don’t know how to solve equations that are not linear. I found the points \((2, 1)\) and \((-2, -3)\). I verified my answer by checking that both points satisfy both equations. For example, I plugged \( x = 2 \) into \( x^3 - 3x - 1 \) and got \( 2^3 - 3(2) - 1 = 8 - 6 - 1 = 1 \), and \( x - 1 = 2 - 1 = 1 \). Also, when \( x = -2 \), \( x^3 - 3x - 1 = (-2)^3 - 3(-2) - 1 = -8 + 6 - 1 = -3 \) and \( x - 1 = -2 - 1 = -3 \). 

b) I used paper and pencil and solved \( 2x - 2 = x - 1 \), or \( 2x - x = 2 - 1 \), so \( x = 1 \). Or, I used mental math and noticed that \( 2x - 2 \) is always twice as much as \( x - 1 \), so they are only equal when \( x - 1 = 0 \), which is when \( x = 1 \). So the intersection point is \((1, 0)\).

c) I used mental math, because \( x \) is never equal to \( x - 1 \), which is always 1 less than \( x \), so the function has no intersection point with \( y = x - 1 \).

Whole-class follow-up: Allow volunteers to share their solutions with the class. Point out for part a) that if someone tries to guess the answer, they might find one of the solutions, but are unlikely to find the other one. Compare using paper and pencil to using mental math for parts b) and c). ASK: Which way is faster? (using mental math) Point out that when both equations are linear, students can always use paper and pencil, but using mental math is sometimes faster.
G8-54   Volume of Spheres

Pages 172–173


Goals:
Students will know the formula for the volume of a sphere and use it to solve problems.

Prior Knowledge Required:
Can cube numbers and find the cube root of numbers, with and without a calculator
Can find the volume of a prism, cylinder, and cone

Vocabulary: composite solid, cone, cylinder, hemisphere, pyramid, sphere, spherical cap

Materials:
a hemisphere (optional)
a precut Styrofoam ball (cut off the top third of the ball, then cut the ball in half from top to bottom—hold the four resulting pieces together with tape or toothpicks

Introduce spheres. SAY: A sphere is a 3-D object where every point on the surface is the same distance from the center of the shape. ASK: What are some real-world examples of spheres? (a ball, the Earth, marbles, a perfect snowball) How can we describe the size of a sphere? (by giving the circumference, radius, or diameter)

(MP.2) Developing the formula Volume of a sphere = \( \frac{4}{3} \pi r^3 \). Tell students that the formula for the volume of a sphere has been developed by mathematicians using calculus. SAY: You may learn this in high school math, but for now we can approximate the formula using shapes we know. If possible, show a hemisphere or half sphere, then draw on the board:

\[
\text{hemisphere}
\]

SAY: This shape is half of a sphere—called a hemisphere. It will be easier to work with than a sphere because it has a flat base. ASK: What shape is its base? (circle) What other 3-D shapes have circular bases? (cylinders, cones) SAY: Let’s draw a cylinder with the same base and height as this hemisphere. Continue drawing on the board:
ASK: What is the height of the cylinder? (it’s equal to the radius) PROMPT: Draw the vertical radius. Write on the board:

\[ V (cylinder) = \pi r^2 h = \pi (r)^2 (r) \]

ASK: What is \( r^2 \times r? \) \( (r^3) \) PROMPT: \( 2^2 \times 2 = 2^{2+1} = 2^3 \). SAY: So, the volume of the cylinder around the hemisphere is \( V = \pi r^3 \). Point to the diagram and ASK: Is the volume of the hemisphere greater than or less than the volume of the cylinder? (less than) Write on the board:

\[ V (hemisphere) < \pi r^3 \]

SAY: Now, let’s find the volume of a different shape—a cone—with the same base and height as the hemisphere. Draw on the board:

![Diagram of a cone with radius r and height r]

SAY: The cone has a radius of \( r \) and a height of \( r \), so what is the volume of the cone? \( (\frac{1}{3} \pi r^3) \) Point to the diagram and ASK: Is the volume of the hemisphere greater than or less than the volume of the cone? (greater than) Write on the board:

\[ V (hemisphere) > \frac{1}{3} \pi r^3 \]

SAY: So, the volume of the hemisphere is between \( \frac{1}{3} \pi r^3 \) and \( \pi r^3 \). ASK: How would we use the volume of the hemisphere to find the volume of the sphere? (multiply by 2) So, what would the volume of the sphere be between? (between twice the volume of the cone and twice the volume of the cylinder, so between \( \frac{2}{3} \pi r^3 \) and \( 2 \pi r^3 \)) Write on the board:

\[ V (sphere) \text{ is between } \frac{2}{3} \pi r^3 \text{ and } 2 \pi r^3 \]

SAY: This is actually a good estimate, because the volume of the sphere is exactly half-way between twice the volume of the cone and twice the volume of the cylinder. ASK: What number is halfway between \( \frac{2}{3} \) and \( 2 \), or between \( \frac{2}{3} \) and \( \frac{6}{3} \)? (4/3) SAY: So, the volume of a sphere is \( \frac{4}{3} \pi r^3 \). Write on the board:

\[ \text{Volume of a sphere} = \frac{4}{3} \pi r^3 \]
Using the formula to find the volume of a sphere. Draw on the board:

![Diagram of a sphere with radius 2 cm]

Tell students that, to calculate the volume of a sphere using the formula, all they have to do is substitute the radius and evaluate. Tell students to always use 3.14 for \( \pi \) and to round answers to the nearest tenth. Write on the board:

\[
V = \frac{4}{3} \pi r^3 \\
\approx \frac{4}{3} (3.14)(2)^3
\]

**MP.1** SAY: We can’t do this calculation mentally, so let’s estimate the answer. ASK: What whole number is \( \pi \) close to? (3) What is \( 4/3 \times 3 \)? (4) What is \( 2^3 \)? (8) What is \( 4 \times 8 \)? (32) SAY: So, the volume will be about 32 cm³.

At this point, students can input the entire expression into their calculators at once: \( 4 \times 3.14 \times 2 \times \frac{\sqrt[3]{3}}{3} \ldots \), or they can cube the radius first \( (4/3 \times 3.14 \times 8) \) and then use their calculators: \( 4 \times 3.14 \times 8 \div 3 \ldots \). Point out that because multiplication and division are commutative, they can do the calculations in any order. For example, inputting \( 2 \times \sqrt[3]{3} \times 3.14 \times 4 \div 3 \ldots \) will also give the correct answer. Make sure they are all able to get the answer \( V \approx 33.5 \text{ cm}^3 \). ASK: Does the answer agree with our estimate? (yes) SAY: With so many numbers going into your calculator, it’s a good idea to check that the answer is reasonable!

Point out that, although we only use only one dimension in the formula (the radius), the dimension is cubed, and so we have three dimensions, or cubic units, in the answer.

ASK: What would you do if you were given the diameter of the sphere and wanted to find the volume? (divide the diameter by 2 to get the radius, and then use the formula with the value of the radius) Tell students that an easy mistake to make is to substitute diameter when you need to substitute radius.

**Exercises:** Find the volume of a sphere with the given measurement. Hint: Are you given the radius or the diameter?

a) \( r = 3 \text{ in} \)  
b) \( r = 12 \text{ m} \)  
c) \( d = 10 \text{ cm} \)  
d) \( d = 6.8 \text{ ft} \)

**Answers:** a) \( V \approx 113.0 \text{ in}^3 \), b) \( V \approx 7,234.6 \text{ m}^3 \), c) \( V \approx 523.3 \text{ cm}^3 \), d) \( V \approx 164.6 \text{ ft}^3 \)
**Finding the volumes of hemispheres and composite solids.** Draw on the board:

![Diagram of a hemisphere with a 7-inch diameter.]

**ASK:** How could we find the volume of a hemisphere? (divide the volume of a sphere with the same diameter by 2)

**Exercise:** Find the volume of a hemisphere with diameter 7 inches.

**Answer:** \( V \approx 89.8 \text{ in}^3 \)

Draw on the board:

![Diagram of a composite solid consisting of a cylinder and a hemisphere.]

**ASK:** How would you find the volume of this composite solid? (find the volume of the cylinder, the volume of the hemisphere, and then add them to find the total volume) Write on the board:

**Volume of cylinder:**

\[
V = \pi r^2 h \\
= (3.14)(5)^2(8) \\
= 628 \text{ m}^3
\]

**Volume of hemisphere:**

\[
V = \frac{4}{3} \pi r^3 \\
\approx \frac{4}{3}(3.14)(5)^3 \\
\approx 261.7 \text{ m}^3
\]

**ASK:** What is the total volume of the shape? (628 + 261.7 \( \approx \) 889.7 m³)

**(MP.7) Exercises:** Find the volume of the composite solid.

a) ![Diagram of a composite solid consisting of a cone and a hemisphere.]

**b) Three hemispheres with diameter 1.5 inches are removed from a prism, as shown below.**

**Hint:** Subtract the volume of the hemispheres from the volume of the prism.

**Answers:** a) \( V \approx 6.8 \text{ in}^3 \), b) \( V \approx 34.9 \text{ in}^3 \)
Solving equations to find unknown dimensions. Tell students that, if they are given the volume of a sphere, then they can write and solve an equation to find the radius. Write on the board:

\[ V \approx 6.6 \text{ cm}^3 \quad r = ? \]

ASK: What formula is useful here? (the volume of a sphere) Write the formula on the board, and substitute the values that are known, as shown below:

\[ V = \frac{4}{3} \pi r^3 \]
\[ 6.6 \approx \frac{4}{3}(3.14)r^3 \]

Tell students that this equation looks messy, but it is easy to solve once we get the right side cleaned up. SAY: Use your calculator to evaluate the coefficient of \( r^3 \), then round the answer to one decimal place. \((4 \times 3.14 \div 3 \approx 4.2)\) Write on the board:

\[ 6.6 \approx 4.2r^3 \]

Remind students that we need to isolate the variable \( r \). They should divide both sides by 4.2 (the coefficient of \( r \)) and round to one decimal place, as shown below:

\[ 6.6 \div 4.2 \approx r^3 \]
\[ 1.6 \approx r^3 \]

SAY: To find \( r \), we take the cube root of both sides. Make sure that students know how to use their calculator to do this. Continue writing on the board:

\[ \sqrt[3]{1.6} \approx r \]
\[ 1.2 \approx r \]

SAY: The radius is about 1.2 cm. ASK: What would the diameter of this sphere be? (2.4 cm)

Exercises: Write and solve an equation to find the unknown information.

a) A sphere has \( V \approx 1,436 \text{ in}^3 \). What is its radius?
b) A sphere has \( V \approx 381.5 \text{ cm}^3 \). What is its diameter?
c) A sphere has \( V \approx 4,186.7 \text{ m}^3 \). What is its diameter?

Answers: a) \( r \approx 7 \text{ in} \), b) \( d \approx 9 \text{ cm} \), c) \( d \approx 20 \text{ m} \)
Applying the Pythagorean Theorem to spheres and spherical caps.

SAY: A spherical cap is the part of a sphere that is cut off by a plane; for example, if we cut off part of a foam ball or cut off part of an orange. Demonstrate this by removing the top of a precut Styrofoam ball and identifying these parts:

 Tell students that it is hard to look inside a sphere. Remind them of the pyramid in Lesson G8-51 that was cut to create a cross section so that they could describe what they saw in two dimensions. Reattach the cap, and then separate the Styrofoam ball into its precut halves. ASK: When we look inside the sphere, what shape is the cross section? (circle)

Draw on the board:

SAY: The line segment $AB$ represents the cut made by the plane and the diameter of the spherical cap. $O$ is the center of the sphere. $OB$ is the radius of the sphere. ASK: What is the length of $OA$? (2 in) Why? (because $OA$ is also the radius) How could we find the length of $AB$? (using the Pythagorean Theorem) Have a student find $AB$ on the board. (see solution below)

$$AB^2 = 2^2 + 2^2$$
$$AB^2 = 8$$
$$AB = \sqrt{8}$$
$$\approx 2.8 \text{ in}$$

SAY: So, in the cross section, the line from $A$ to $B$ marks where the plane cut off the top portion of the sphere. The line $AB$ measures 2.8 inches. Show the spherical cap and SAY: Looking at the spherical cap, we see that its diameter is 2.8 in.

Exercises:

1. A plane slices through the points $CD$ on a sphere. What is the length of $CD$?

Answer: $CD \approx 6.6 \text{ cm}$
2. A sphere is cut through at \( EF \) to create a spherical cap. \( EF \) is 20 inches long.

\[ \text{20 in} \]

a) What is the radius of the sphere? Hint: \( 2x^2 = 20^2 \).

b) What is the volume of the sphere including the spherical cap?

**Answer:** a) \( r \approx 14.1 \text{ in} \); b) \( V \approx 11,736.2 \text{ in}^3 \)

3. What is the volume of a sphere if \( GH \) is 4.2 cm?

\[ \text{4.2 cm} \]

**Answer:** \( r \approx 3 \text{ cm}, \ V \approx 113.0 \text{ cm}^3 \)

4. The radius of a sphere is 10 in. A plane cuts off a spherical cap with height 5 in.

a) Label these measurements on the cross section.

b) Use the Pythagorean Theorem to find the radius of the spherical cap.

**Answers:** a) \( r \approx 8.7 \text{ in} \)

**Word problems practice.** For the following word problems, work through one as an example on the board with students. Model each step. Then ask students to do the rest individually.

**Exercises:**

1. The diameter of a spherical ice cube is about 1.4 inches. What is the volume of the ice cube?

**Answer:** \( V \approx 1.4 \text{ in}^3 \)

(MP.2, MP.4) 2. A sphere with radius 5 cm has a volume of about 523.3 cm\(^3\). Does a sphere with half the radius have half the volume? Explain.

**Answer:** A sphere with half the radius has \( V \approx 65.4 \text{ cm}^3 \). This volume is much less than half the volume of the larger sphere because radius is cubed; in other words, a little change in the radius will have a big impact on the volume.
3. Sally used the cone to fill the sphere with water. How many cones filled with water did she need to fill the sphere?

![Diagram of sphere and cone]

**Answer:** She needed to fill the cone 2 times.

**Extensions**
1. The surface area of a sphere is given by the formula \( \text{SA} = 4\pi r^2 \). Find the surface area of a sphere with …
   a) \( r = 5 \text{ cm} \)  
   b) \( r = 2.6 \text{ in} \)  
   c) \( d = 8 \text{ m} \)
   **Answers:** a) \( \text{SA} \approx 314 \text{ cm}^2 \), b) \( \text{SA} \approx 84.9 \text{ in}^2 \), c) \( \text{SA} \approx 201.0 \text{ m}^2 \)

2. The diameter of Earth is 12,742 km. What is the volume of the northern hemisphere? Write your answer using scientific notation.
   **Answer:** \( 5.4 \times 10^{11} \text{ km}^3 \)

3. a) The composite solid A consists of a cylinder with radius \( R \) and height \( R \), and two cones each with the radius \( R \) and with height \( \frac{R}{2} \). What is the volume of composite solid A?

   b) The sphere B has radius \( R \). What is the volume of sphere B?

   c) Look at shape C, which compares shapes A and B. What is larger, the volume of sphere B that extends outside of shape A or the volume of shape A that extends outside of sphere B?

   **Answers:** a) \( V = \frac{4}{3}\pi R^3 \); b) \( V = \frac{4}{3}\pi R^3 \); c) they are equal, since the volumes are equal

(MP.2, MP.4) 4. Jay is hanging a light from the ceiling. The light is cylindrical with a diameter of 3 feet and a height of 1 foot. The ceiling is 9 feet above the floor, and Jay wants the bottom of the light to be 5 feet above the floor. What is the total length of chain that Jay needs to buy to hang the light as shown in the picture? Say what each step means in the situation.

![Diagram of light and ceiling]

**Answer:** The radius of the light is 1.5 feet and the height from the top of the light to the ceiling is 3 feet. The length of the chain from one side of the light to the ceiling is \( \sqrt{(1.5)^2 + 3^2} \approx 3.35 \text{ ft} \), by the Pythagorean Theorem. The total length of the chain that Jay needs is \( 2 \times 3.35 = 6.7 \text{ ft} \).
**Goals:**
Students will know the formulas for the volume of cones, cylinders, and spheres, and use them to solve real-world problems.

**Prior Knowledge Required:**
Can use the formulas for the volume of prisms, pyramids, cylinders, cones, and spheres
Can use the Pythagorean Theorem to find the height of a pyramid or cone, given the slant height

**Vocabulary:** apex, cone, cylinder, lateral face, pyramid, slant height, sphere, spherical cap

**Materials:**
overhead projector and erasable markers (optional)
transparency of **BLM Frayer Model** (p. Q-81, optional)
a prism for demonstration
**BLM Frayer Model** (p. Q-81)
3-D shapes (pyramids, cylinders, cones, and spheres), one per student
**BLM Relay Race** (pp. Q-82–84), copied and cut out so that each team of 2–4 students will have a color-coded or letter-marked (for Team A, B, C, etc.) set of questions

**Review characteristics of 3-D shapes.** Copy or project **BLM Frayer Model** on the board.
Show students a prism. Go through each part of the Frayer model and have students brainstorm how to complete it. The completed model could look like this:

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Facts and Formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td>• 3-D shape</td>
<td>( V = Bh )</td>
</tr>
<tr>
<td>• has faces, edges, vertices</td>
<td>( b \times h \div 2 )</td>
</tr>
<tr>
<td>• has 2 parallel congruent bases</td>
<td>( t \times w )</td>
</tr>
<tr>
<td>• named for the polygon that makes the base</td>
<td>( (b_1 + b_2) \times h \div 2 )</td>
</tr>
</tbody>
</table>

**Examples**: 

```
6 in  
5 in 3 in
```

\( V = 5(3)(6) = 90 \) in\(^3\)

Cereal boxes
Tissue boxes

<table>
<thead>
<tr>
<th>Non-Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>cone</td>
</tr>
<tr>
<td>cylinder</td>
</tr>
<tr>
<td>sphere</td>
</tr>
<tr>
<td>pyramid</td>
</tr>
</tbody>
</table>

For the following exercises, give each student a 3-D shape (from a collection of pyramids, cylinders, cones, and spheres) and a copy of **BLM Frayer Model**.
Exercises: Complete the Frayer Model for your 3-D shape. You may look through your notes for ideas. Include a good amount of information (neither too much nor too little), be creative, and add color.

Bonus: Fill out a Frayer Model for an additional shape.

Review the concepts taught in this unit.

Activity
To review the concepts taught in this unit, ask students to work in groups of 2–4 to complete a relay race. Allow 10–20 minutes of class time for the race. The race ends when the time is up; it’s not necessary to keep track of which team completes the most questions correctly in the allotted time. Encourage students to work together as a team to answer the questions.

Using the copied and cut cards from BLM Relay Race, organize the questions in sets in a central location. After students form their groups, assign each group a set of questions, by color or letter. As well, designate one member of the team to be the “relay runner” to control movement in the classroom.

To start the race, ask the relay runner from each team to collect Question 1. In groups, students work out the answer, write it on the card, and the relay runner brings the card to you to be checked. If the answer is correct, give the student the next question from that group’s set. If not, give the group an opportunity to make a correction. Continue the relay race until the allotted time is up. (see below for answers)

1. \( C, V = \frac{4}{3} \pi r^3 \)
2.  
   \[
   \text{slant height} \quad \text{height} \quad \text{base} \quad \text{lateral face}
   \]

4. \( h \approx 7.1 \text{ ft} \)
5. \( V \approx 87.9 \text{ in}^3 \)
6. \( V \approx 57.9 \text{ m}^3 \)
7. \( h \approx 5.6 \text{ in}, V \approx 312.4 \text{ in}^3 \)
8. Sample answers: rectangular prism with dimensions 25 cm by 4 cm by 5 cm, rectangular pyramid with 25 cm by 4 cm base and height 15 cm, cylinder with height 10 cm and radius 4 cm, cone with height 10 cm and radius 6.9 cm, sphere with radius 4.9 cm
9. Volume of cylinder \( \approx 785 \text{ in}^3 \), volume of cone \( \approx 669.9 \text{ in}^3 \), so the cylinder has a greater volume
10. Sample answer: What is the volume of water in this fountain? \( (V \approx 2.1 \text{ m}^3) \)

(end of activity)
Writing exact answers when finding the volumes of cylinders, cones, and spheres.
Remind students that, when we write 3.14 for \( \pi \), we are finding an approximate answer. SAY: If we leave \( \pi \) as \( \pi \), then we can write an exact answer. Draw on the board:

![Diagram of a cylinder with dimensions 5 m and 16 m.]

On the board, write the formula for the volume of a cylinder. Then have students tell you what values to substitute, but don’t write 3.14 for \( \pi \). (see solution below)

\[
V = \pi r^2h \\
= \pi (5)^2(16)
\]

SAY: We can simplify the expression by evaluating the numerical part. (5\(^2 \times 16\)) Continue writing on the board:

\[
= 400\pi \text{ m}^3
\]

SAY: This is an exact answer for the volume of the cylinder. We can use different approximations for \( \pi \) to estimate the answer. Write on the board:

\[
V \approx 400(3) \quad V \approx 400(3.14) \quad V \approx 400(3.14159265…)
\]

Have students calculate the three products. For the third product, students should use the \( \pi \) button on their calculators. (\( V \approx 1,200 \text{ m}^3 \), \( V \approx 1,256 \text{ m}^3 \), \( V \approx 1256.6 \text{ m}^3 \)) SAY: Using 3 for \( \pi \) allows us to calculate mentally, but multiplying by 3.14 produces a better estimate than 3. Using the \( \pi \) button on the calculator gives an even better estimate.

**Exercises:** Find the exact volume.

a) ![Diagram of a cylinder with dimensions 2 in and 6 in.]

b) ![Diagram of a cone with dimensions 11 cm and 6 cm.]

c) ![Diagram of a sphere with a radius of 6 ft.]

d) A cone is removed from a cylinder.![Diagram of a cone with dimensions 12 in and 5 in.]

**Answers:** a) \( V = 6\pi \text{ in}^3 \), b) \( V = 132\pi \text{ cm}^3 \), c) \( V = 36\pi \text{ ft}^3 \), d) \( V = 120\pi \text{ in}^3 \)
Word problems practice. For the following word problems, work through one as an example on the board with students. Model each step. Then ask students to do the rest individually.

Exercises:
(MP.4) 1. A baseball is packed in a box that is a cube. The box exactly fits the baseball. The baseball has diameter \( \frac{7}{8} \) inches. What is the volume of the empty space in the box?

**Answer:**

\[ V_{\text{cube}} \approx 23.8 \text{ in}^3, \quad V_{\text{sphere}} \approx 12.4 \text{ in}^3, \quad \text{so volume of empty space} \approx 11.4 \text{ in}^3 \]

(MP.4) 2. In the pair of shapes, how many times bigger is the second shape than the first shape? Find exact answers by using \( \pi \).

![Shapes](image)

**Answers:**

a) volumes \( \pi \) and \( 2\pi \), so the second shape is 2 times bigger
b) volumes \( \frac{1}{3}\pi \) and \( \frac{8}{3}\pi \), so the second shape is 8 times bigger
c) volumes \( \frac{4}{3}\pi \) and \( \frac{108}{3}\pi \), so the second shape is 27 times bigger

(MP.4) 3. a) Each dimension of the larger can is twice the corresponding dimension of the smaller can. Write the dimensions of the larger can.

\[ r = 2 \text{ in}, \quad h = 2.5 \text{ in} \]

b) The small can costs $0.59. How much should the large can cost, if cost is proportional to volume?

**Answers:**

a) \( r = 4 \text{ in}, \quad h = 5 \text{ in} \); b) the volume of the larger can is 8 times greater than the smaller can, so the larger can should cost \( 8 \times 0.59 = $4.72 \)

4. a) Which hanging flowerpot will hold more soil?

![Flowerpots](image)

b) If soil costs $0.10 for every 35 cubic inches, how much would it cost to fill each flowerpot?
**Answers:** a) Pyramid: \( h \approx 14.1 \text{ in}, \ V \approx 470 \text{ in}^3 \), Cone: \( h \approx 14.1 \text{ in}, \ V \approx 369.0 \text{ in}^3 \), so the pyramid will hold more soil; b) Pyramid: $1.34, Cone: $1.05

5. An empty pool is being filled with water.

![Pool Diagram]

a) What is the volume of water that the pool can hold?

b) The pool is filled at a constant rate for 8 hours. Complete the chart.

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>Volume of Water (ft$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>280</td>
</tr>
<tr>
<td>4</td>
<td>700</td>
</tr>
<tr>
<td>6.5</td>
<td></td>
</tr>
</tbody>
</table>

**Bonus:**
3 hours, 15 minutes

c) After 8 hours, what percent of the volume of the pool is filled?

d) The pool should only be filled to 85% of its volume. How long will this take in total?

**Answers:**

a) \( V \approx 1,959.4 \text{ ft}^3 \)

b) 

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
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<td>700</td>
</tr>
<tr>
<td>6.5</td>
<td>910</td>
</tr>
</tbody>
</table>

**Bonus:**
3 hours, 15 minutes 455

c) After 8 hours, the volume of water is 1,120 ft$^3$. This is about 57% of the total volume.

d) 85% of the volume is 1,665.5 ft$^3$. This will take about 11.9 hours, or about 11 hours and 54 minutes.
6. The opening of a shampoo bottle is in the shape of a cylinder with both a diameter and a height of 1 inch. The design company wants to use a bottle cap in the shape of a sphere. How large must the sphere be? How much of the sphere must be cut off at the bottom to allow the cap to fit around the opening? Hint: Label the cross section with the information from the problem.

**Answer:** Using the Pythagorean Theorem, \(1^2 = r^2 + r^2\); thus, \(r \approx 0.7\) in, so the diameter of the sphere is 1.4 in. Since the height of the bottle opening (the cylinder) is 1 in, then the sphere is 0.4 in taller than the cylinder. This means there is 0.2 in of sphere above the cylinder and 0.2 in below the cylinder. So, 0.2 in would have to be cut off from the sphere at the bottom to make it fit around the bottle’s opening.

**Extensions**

1. a) Rewrite the formula for the volume of a cylinder to ...
   i) solve for \(h\).
   ii) solve for \(r\).

   b) Use the formula you developed in part a) to find the unknown.
   i) \(V \approx 1,846.3\) m\(^3\), \(r = 7\) m, \(h = ?\)
   ii) \(V = 80.5\) in\(^3\), \(r = ?, h = 4.1\) in

   **Answers:** a) i) \(h = \frac{V}{\pi r^2}\), ii) \(r = \sqrt[3]{\frac{V}{\pi h}}\); b) i) \(h \approx 12\) m, ii) \(r \approx 2.5\) m

2. Rewrite the formula for the volume of a cone to ...
   i) solve for \(h\).
   ii) solve for \(r\).

   b) Use the formula you developed in part a) to find the unknown.
   i) \(V \approx 150.72\) cm\(^3\), \(r = 6\) cm, \(h = ?\)
   ii) \(V = 340.2\) in\(^3\), \(r = ?, h = 13\) in

   **Answers:** a) i) \(h = \frac{3V}{\pi r^2}\), ii) \(r = \sqrt[3]{\frac{3V}{\pi h}}\); b) i) \(h \approx 4\) cm, ii) \(r \approx 5\) in

3. a) Rewrite the formula for the volume of a sphere by solving for \(r\).

   b) Use the formula from part a) to find the radius if the volume is ...
   i) 50 cm\(^3\)
   ii) 500 cm\(^3\)

   **Answers:** a) \(r = \sqrt[3]{\frac{3V}{4\pi}}\); b) i) \(r \approx 2.3\) cm, ii) \(r \approx 4.9\) cm
(MP.7) 4. The diagram shows three horizontal cross sections of a 3-D shape. Name the shape.

![Diagram](image)

Answers: a) square prism, b) square pyramid, c) pentagonal prism, d) triangular prism, e) pentagonal pyramid, f) rectangular pyramid, g) cylinder, h) cone or sphere

(MP.1, MP.4) 5. Gold costs $42.45 per gram and silver costs $0.53 per gram. Greg wants to make a 10 g necklace from gold and silver. He decides to buy the gold and silver he needs and sell the necklace for his cost plus 50% profit. If he wants a profit of $160, how much gold and how much silver does he need to buy? Show your work. Say what each step means in the situation.

Solution: Let $x$ be the number of grams of gold that Greg needs to buy and let $y$ be the number of grams of silver that he needs to buy. Then $x + y = 10$, since the necklace will weigh 10 g. His cost, in dollars, will be $42.45x + 0.53y$, and he wants 50% of that cost to be $160, so $(42.45x + 0.53y)(0.5) = 160$. Multiplying the equation by 2, you get $42.45x + 0.53y = 320$. So now I have two equations: $x + y = 10$ and $42.45x + 0.53y = 320$.

I substituted $y = 10 - x$ from the first equation into the second equation to get:

$42.45x + 0.53(10 - x) = 320$
$42.45x + 5.3 - 0.53x = 320$
$41.92x = 314.7$
$x = 314.7 ÷ 41.92 ≈ 7.5$

Greg should buy about 7.5 grams of gold and 2.5 grams of silver.

MP.2 Assessment Opportunity: Question 8 on AP Book 8.2 p. 175
Nets of 3-D Shapes (2)
Nets of 3-D Shapes (3)
Nets of 3-D Shapes (4)
Nets of 3-D Shapes (5)
Nets of 3-D Shapes (6)
Nets of 3-D Shapes (7)
Nets of 3-D Shapes (8)
Nets of 3-D Shapes (9)
Nets of 3-D Shapes (10)
Sorting Prisms

A. B. C.

D. E. F.

G. H. I.

J. K. L.

Blackline Master — Geometry — Teacher Resource for Grade 8

Q-69
Compare and Contrast Graphic Organizer

Object 1

Similar Features

Object 2

Different Features
Square Numbers and Square Roots

1. Square the number.
   a) $1^2 = \underline{\phantom{000}}$
   b) $6^2 = \underline{\phantom{000}}$
   c) $9^2 = \underline{\phantom{000}}$
   d) $2^2 = \underline{\phantom{000}}$

2. Evaluate the expression.
   a) $6^2 + 5^2$
   b) $7^2 + 2^2$
   c) $8^2 - 4^2$
   d) $10^2 - 3^2$

3. Find the square root.
   a) $\sqrt{16} = \underline{\phantom{000}}$
   b) $\sqrt{25} = \underline{\phantom{000}}$
   c) $\sqrt{81} = \underline{\phantom{000}}$
   d) $\sqrt{36} = \underline{\phantom{000}}$

4. Round to the nearest tenth.
   a) $9.18 \approx \underline{\phantom{000}}$
   b) $3.517 \approx \underline{\phantom{000}}$
   c) $6.3768 \approx \underline{\phantom{000}}$
   d) $0.7366 \approx \underline{\phantom{000}}$

5. Use a calculator to find the square root. Round to the nearest tenth.
   a) $\sqrt{5} \approx \underline{\phantom{000}}$
   b) $\sqrt{39} \approx \underline{\phantom{000}}$
   c) $\sqrt{61} \approx \underline{\phantom{000}}$
   d) $\sqrt{83} \approx \underline{\phantom{000}}$

6. Find the positive solution to the equation. Round to the nearest tenth.
   a) $x^2 = 49$
   b) $x^2 = 144$
   c) $x^2 = 57$
   d) $x^2 = 82$

7. Group like terms. Then find the positive solution to the equation. Round to the nearest tenth.
   a) $x^2 = 7^2 + 5^2$
   b) $x^2 = 8^2 + 6^2$
   c) $4^2 + x^2 = 5^2$
   d) $x^2 + 1^2 = 7^2$
Sorting Pyramids

A. B. C.

D. E. F.

G. H. I.

J. K. L.
Comparing Volumes: Prisms and Pyramids (1)
Comparing Volumes: Prisms and Pyramids (2)
Comparing Volumes: Prisms and Pyramids (3)
Comparing Volumes: Prisms and Pyramids (4)
How Many Pyramids Make a Cube?
Comparing Volumes: Cylinders and Cones (1)
Comparing Volumes: Cylinders and Cones (2)
Comparing Volumes: Cylinders and Cones (3)
Frayer Model

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Facts and Formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>Non-Examples</td>
</tr>
</tbody>
</table>
Relay Race (1)

Relay Race—Question 1
Which formula has an error in it?

A. \( V = \pi r^2 h \)    
B. \( V = \frac{1}{3} \pi r^2 h \)    
C. \( V = \frac{3}{4} \pi r^3 \)    
D. \( V = Bh \)

Formula _____ has an error. The correct formula is ________________.

Relay Race—Question 2
Label the diagram with the terms below.

- apex
- lateral face
- base
- slant height
- height

Relay Race—Question 3
Give two real-world examples of the 3-D shape.

a) pyramid: _________________ and _________________

b) cylinder: _________________ and _________________

c) sphere: _________________ and _________________

Relay Race—Question 4
Find the height of the pyramid to the nearest tenth.

9 ft
3 ft
11 ft
Relay Race (2)

Relay Race—Question 5
Find the volume to the nearest tenth.

Relay Race—Question 6
Find the volume to the nearest tenth.

Relay Race—Question 7
Find the volume to the nearest tenth.

Relay Race—Question 8
Draw two different 3-D shapes with a volume of about 500 cm³.
Relay Race (3)

Relay Race—Question 9
Which is greater, the volume of a cylinder with radius 5 in and height 10 in, or the volume of a cone with radius 8 in and height 10 in?

Relay Race—Question 10
Create a word problem that involves volume.
Unit 7  Statistics and Probability: Line of Best Fit and Two-Way Tables

This Unit in Context
In 8.1 Unit 7, students continued their study of statistics and probability by drawing, describing, and interpreting scatter plots. In this unit, students will investigate patterns of association in bivariate data. They will construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities (8.SP.A.1). For scatter plots that suggest a linear association, they will learn to informally fit a straight line and assess the model by judging the closeness of the data to points on the line (8.SP.A.2). They will use the equation of a linear model to solve problems in the context of bivariate measurement data, and interpret the slope and intercept (8.SP.A.3). They will investigate patterns of association that can be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table (8.SP.A.3).

Mathematical Practices in This Unit
In this unit, you will have the opportunity to assess MP.1 to MP.7. The MP labels in this unit flag both opportunities to develop the mathematical practice standards and opportunities to assess them. Below is a list of where we recommend assessing each standard, as well as some examples of how students can show that they have met a standard.

MP.1: Extension in SP8-5

MP.2: SP8-4 Extension 2, SP8-8 Question 7 on AP Book 8.2 p. 189, SP8-12 Extension 2
In SP8-8 Question 7 on AP Book 8.2 p. 189, students reason abstractly and quantitatively when they recognize from the equation of the line of best fit how much to expect a test mark to increase by if a student studies for one extra hour, and also the mark she would get if she didn’t study at all.

MP.3: SP8-6 Extension 3
In SP8-6 Extension 3, students construct an argument to explain why a sphere with a given volume will or will not fit inside a box with a given, larger, volume. Students have the opportunity to critique a partner’s argument. For example, if one student argues incorrectly that the ball will fit into the cube because 11.3 < 20, their partner will need to explain that they need to compare the diameter, not the radius, to the width of the box.

MP.4: SP8-4 Extension 2, extension in SP8-5, SP8-6 Extension 3, SP8-9 Extension 2, SP8-10 Extension 2, SP8-11 Extension 2, SP8-12 Extension 2
In SP8-12 Extension 2, students model with mathematics a novel real-world situation when they determine the percentage of the daily limit of sodium they would get if they eat an entire can of soup. Given the serving size, students determine how the serving size compares to the volume of soup in the can.

MP.5: SP8-8 Extension 3, SP8-9 Extension 2
**MP.6:** SP8-4 Extension 3, SP8-12 Question 4 on AP Book 8.2 p. 198
In SP8-12 Question 4 on AP Book 8.2 p. 198, students attend to precision when they use a clear title and clear labels to draw a scatter plot.

**MP.7:** SP8-4 Extension 3, SP8-7 Extension 3
Unit 7 Statistics and Probability: Line of Best Fit and Two-Way Tables

Introduction
In this unit, students will learn about bivariate data expressed on scatter plots, describe associations, understand that a straight line can represent a scatter plot with a linear association, and make connections between positive slope and positive association or negative slope and negative association. They will informally fit, check, and interpret the line of best fit as the line that goes through the center of the “cloud” of data points to express the trend of the data, and they will examine the effect of outliers on the line of best fit. They will also find the slope and y-intercept of the line of best fit, write the equation of the line of best fit, understand that linear models can be represented with the equation of the line of best fit, and interpret the slope and y-intercept of the line in the context of a problem.

Students will look at categorical data in Venn diagrams, two-way tables, frequency tables, and relative frequency tables. They will construct a two-way table from a Venn diagram and a Venn diagram from a two-way table. Students will also find the missing cells in two-way tables and will represent categorical data in two-way frequency or relative frequency tables. As well, students will find patterns in bivariate categorical data displayed in two-way tables and calculate relative frequencies to describe possible associations between two variables.

NOTE: Within the lesson plans and the AP Book, students will find data regarding many variables that relate to their experience, some of which they will return to again in the unit and some of which will change over the unit. For example, they will see one set of data about student ages and monthly allowances and then later see new data for the same variables. Encourage students to examine data with care each time and use any data with precision.

Terminology. Scatter plots can also be called scattergraphs, plots, or graphs. We use the term “scatter plots.” Students will see the terms “horizontal axis” and “vertical axis” and “x-axis” and “y-axis”; use all terms so that students are familiar with both ways of describing the axes of a scatter plot.

Materials. Students will often need grid paper. Additionally, for some of the lessons of this unit, you will need a pre-drawn grid on the board. If you do not have such a grid, you can photocopy BLM 1 cm Grid Paper (p. S-1) onto a transparency and project it onto the board. Similarly, when a coordinate grid is required, use a transparency of BLM Large Coordinate Grid (p. S-2). Students should be working in grid paper notebooks. If they are not working in such notebooks, provide them with grid paper or BLM 1 cm Grid Paper. To save time, you can occasionally provide students with BLM Coordinate Grids (p. S-3).

Technology: graphing software. Some extensions in this unit ask students to use Microsoft Excel® to create a table, create a scatter plot from the table, apply the line of best fit, and then add more data to see the impact on the scatter plot and trend line. NOTE: The instructions provided are for Microsoft Excel 2010. If you use a different computer program to complete these activities, the instructions provided may need to be adjusted.
Goals:
Students will understand that a straight line can represent a scatter plot with linear association. Students will understand the connection between positive slope and positive association or negative slope and negative association.

Prior Knowledge Required:
Can draw scatter plots
Can recognize and describe patterns of association in a scatter plot

Vocabulary: association, bivariate data, cluster, decreasing, dependent variable, increasing, independent variable, linear association, negative association, no association, nonlinear association, outlier, positive association, scatter plot

Review bivariate data and positive and negative association. Remind students that in bivariate data, there are two variables that can change—for example, the age and the height of Grade 8 students, or the day’s temperature and how many ice creams a store sells. SAY: Bivariate data allows us to look for relationships between two sets of data. Draw on the board:

SAY: These scatter plots show three types of associations. Point to the scatter plot on the left and ASK: As we move to the right, do the points get higher or lower? (higher) SAY: There is a positive association between two sets of data if the values in the sets of data increase together. So we can say there is a positive association between temperature and number of ice creams sold. Point to the scatter plot in the middle and SAY: There is a negative association if the values in one set of data increase as the values in the other set of data decrease. Here we can say that there is a negative association: as cars get older, car prices decrease. Point to the scatter plot on the right and ASK: As we move to the right, do the points get higher or lower? (both) SAY: Here we can say that there is no association because there is neither a positive nor a negative association between the two sets of data. Label the three scatter plots on the board with the type of association they show. (positive association, negative association, no association)
Exercises: State the type of association for the scatter plot.

a) ![Image of scatter plot]

b) ![Image of scatter plot]

c) ![Image of scatter plot]

d) ![Image of scatter plot]

Answers: a) negative association, b) no association, c) positive association, d) no association

Review clusters and outliers. Draw on the board:

![Image of clusters and outliers]

ASK: What type of association does the scatter plot show? (positive) How do you know? (the values in the sets of data increase together) Are there any groups of data? (yes, in the top right and near the origin) Remind students that data points that are grouped close to each other are called a cluster. SAY: Data can be clustered about a point (indicate the cluster near the origin) or a line (indicate the cluster in the top right). ASK: Are there points that stand out? (yes, one in the top left and one in the bottom right) Remind students that a data point that is very different from the rest of the data in the set is called an outlier. SAY: Clusters tell you about trends in the data and outliers tell you which points don’t follow the trend. Explain to students that identifying clusters and outliers gives you more information about the data.

Exercises: a) Identify and count the clusters and the outliers.

i) ![Image of scatter plot]

ii) ![Image of scatter plot]

iii) ![Image of scatter plot]

b) What type of association does each scatter plot in part a) show?

Answers: a) i) two clusters and three outliers, ii) three clusters and two outliers, iii) two clusters and two outliers; b) i) negative association, ii) positive association, iii) negative association
Review drawing scatter plots. Draw on the board:

<table>
<thead>
<tr>
<th>Hours of Study</th>
<th>4</th>
<th>7</th>
<th>2</th>
<th>5</th>
<th>4</th>
<th>1</th>
<th>3</th>
<th>6</th>
<th>7</th>
<th>3</th>
<th>5</th>
<th>8</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math Mark (%)</td>
<td>72</td>
<td>85</td>
<td>67</td>
<td>79</td>
<td>74</td>
<td>55</td>
<td>65</td>
<td>93</td>
<td>87</td>
<td>69</td>
<td>77</td>
<td>90</td>
<td>71</td>
</tr>
</tbody>
</table>

Tell students that the table shows the number of hours studied per week and the math marks for 13 students in a class. On the board, draw a horizontal axis and a vertical axis. Remind students that we put one variable on the x-axis and the other variable on the y-axis. SAY: In an equation with two variables, if one variable is an input that causes the other variable, then the input is described as the independent variable and the output is described as the dependent variable. We put the independent variable on the x-axis and the dependent variable on the y-axis.

ASK: With the two variables shown in the table, can we describe one as the independent variable? (yes) PROMPT: Can we describe one variable as causing the other variable? (yes, hours of study) Why? (because the cause of a changing math mark is studying) SAY: So we will put the variable “hours of study” on the x-axis and the variable “math mark” on the y-axis. Ask a volunteer to add labels to the axes on the board and give the graph a title.

SAY: Next, we need to make some decisions about the numbers on the scale and how we are going to show them. The data for the horizontal axis goes from 1 to 8, so we could show that with increments of 1. Have a volunteer mark the horizontal axis. SAY: The data for the vertical axis goes from 55 to 93. ASK: What is the range of this data? (93 – 55 = 38) To show this data, what number should we start at? (55) What number should we end at? (95) How should we count? (by 5s) How should we divide the vertical axis? (with nine ticks, starting at 55) Have a volunteer mark the vertical axis and plot the points. The final picture should look like this:

(MP.7) Exercises: Use the scatter plot of the hours of study and math mark to answer the following questions.

a) Is there a positive association, a negative association, or no association between hours of study and math marks?
b) How many outliers and how many clusters are in the scatter plot?

Answers: a) positive association, b) two outliers and two clusters
Ask a volunteer to circle the clusters and cross out the outliers on the graph on the board, as shown below:

![Graph of Hours of Study and Math Mark](image)

(MP.7) Linear and nonlinear association. Draw on the board:

ASK: What type of association do the three scatter plots show? (negative association) How do you know? (the values in one set of data increase as the others decrease) In which graph do the points lie more or less on a straight line? (A) Draw a line through most of the points in A, as shown below. SAY: Data points that lie more or less close to a line have linear association. Draw a line through the points in B, as shown below, so that students can see that being close to the line but not on the line is good enough to describe a linear association. ASK: Does C show a linear association? (no, it’s a curve) Draw a curve through the data points in C, as shown below.

![Scatter plots A, B, and C](images)

SAY: An association is nonlinear or not linear if the points don’t lie more or less on a straight line.
Exercises: For which scatter plot or scatter plots does the data show...

a) a positive association?  

A.  

b) a negative association?  

B.  

c) a linear association?  

C.  

d) a nonlinear association?  

D.  

Answers: a) A, D; b) B, C; c) A, B, D; d) C

Positive and negative slope. Draw on the board:

Remind students that if a line goes from the bottom left to the top right on a graph, the line is increasing, and if it goes from top left to bottom right, it is decreasing. ASK: Which scatter plot shows a positive association? (the left one) Which line is increasing? (the left one) Which line has a positive slope? (the left one) Which line has a negative slope? (the right one) Which scatter plot shows a negative association? (the right one) SAY: A positive slope in a linear association means positive association, and negative slope in a linear association means negative association.

Exercises:

1. For the scatter plots:

a) Which lines have a positive slope?  

A.  

b) Which lines have a negative slope?  

B.  

c) Which lines show a positive association?  

C.  

d) Which lines show a negative association?  

Answers: a) B; b) A, C; c) B; d) A, C
2. The table shows the age and the height of 12 students.

<table>
<thead>
<tr>
<th>Age</th>
<th>12</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>14</th>
<th>12</th>
<th>13</th>
<th>11</th>
<th>13</th>
<th>10</th>
<th>14</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (cm)</td>
<td>155</td>
<td>141</td>
<td>141</td>
<td>140</td>
<td>161</td>
<td>150</td>
<td>152</td>
<td>147</td>
<td>163</td>
<td>138</td>
<td>171</td>
<td>158</td>
</tr>
</tbody>
</table>

a) Draw a scatter plot for the data.
b) Cross out the outliers.
c) Does the graph show an association between age and height? If yes, is it positive or negative? Is it linear or nonlinear?

**Answers:**
a–b) ![Scatter plot of Age and Height]
c) yes, positive linear

**Extensions**
1. a) Decide which variable is the independent variable and put that variable on the x-axis. Then draw the scatter plot.

   i) **Weight (lb)** | 80 | 65 | 75 | 85 | 100 | 75 | 85 | 80 | 90 | 70
   | Age | 12 | 10 | 11 | 12 | 14 | 12 | 13 | 11 | 13 | 10

   ii) **Years of Experience** | 2 | 7 | 3 | 8 | 11 | 6 | 5 | 3 | 10 | 9
   | **Annual Income ($1,000)** | 25 | 39 | 28 | 44 | 55 | 39 | 35 | 30 | 48 | 47

   iii) **Engine Size (L)** | 3.6 | 3.0 | 2.0 | 4.0 | 1.6 | 2.6 | 2.4 | 4.5 | 1.8
   | **Fuel Efficiency (mpg)** | 16 | 18 | 28 | 15 | 39 | 19 | 22 | 14 | 34

b) Describe each scatter plot as linear or nonlinear.

**Answers:**
a) i) ![Scatter plot of Age and Weight]

   ![Scatter plot of Years of Experience and Annual Income]
b) i) linear, ii) linear, iii) nonlinear

(MP.2, MP.4) 2. Jack decided to build a public aquarium. He plans to build the aquarium so that people walk through a tunnel shaped like half a cylinder and see the fish all around them. The aquarium is a rectangular prism without the tunnel. The tunnel is 6 m wide, and the aquarium is 100 m long, 15 m wide and 12 m tall. Before bringing the fish in, Jack needs to fill the aquarium \(\frac{3}{4}\) full with water. About how much water does Jack need? Write what each step means in the situation. Remember: 1 cm\(^3\) = 1 mL.

**Answer:** The volume of the entire rectangular prism, including the tunnel, is \(100 \times 15 \times 12\) m\(^3\) = 18,000 m\(^3\). The volume of the tunnel is \(\frac{1}{2}\pi r^2 L\), where \(r\) is half the width of the tunnel and \(L\) is the length of the tunnel, so the volume of the tunnel is \(\frac{1}{2}\pi r^2 L = \frac{1}{2}\pi (3)^2 (100) \approx 1,413.7\) m\(^3\). So the volume of the aquarium is about 18,000 - 1,413.7 = 16,586.3 m\(^3\) \(\approx 1.7 \times 10^4\) m\(^3\) = 1.7 \times 10^{10}\) cm\(^3\) because 1 m = 100 cm, so 1 m\(^3\) = 100 \times 100 \times 100 cm\(^3\) = 1 \times 10^6 cm\(^3\). But 1 cm\(^3\) is the same as 1 mL, so the amount of water required to fill up the aquarium completely would be 1.7 \times 10^{10}\) mL = 1.7 \times 10^7\) L. But Jack needs to fill the aquarium \(\frac{3}{4}\) full, so I multiplied 0.75 \times 1.7 \times 10^7 \approx 1.3 \times 10^7\). So Jack needs to use about 1.3 \times 10^7\) L of water.

(MP.6, MP.7) 3. The circumference of a circle is 20 cm longer than its diameter. What is the area of the circle? Say what the symbols you use mean.

**Answer:** C = \(\pi d\), where C is the circumference, \(d\) is the diameter, and \(\pi\) is about 3.14. We are also given that \(C = d + 20\), so \(d + 20 = \pi d\), so \(d(\pi - 1) = 20\), so \(d = 20 ÷ (\pi - 1) \approx 20 ÷ 2.14 \approx 9.35\), so \(r = 4.68\). I calculated \(\pi r^2 \approx 3.14(4.68)^2 \approx 68.77\), so the area of the circle is about 69 cm\(^2\). I used “=” for exactly equal and “≈” for approximately equal.
Goals:
Students will understand that the best linear model is the line that is closest to most data points. Students will compare different sketches and identify the line of best fit, the line that goes through the “center” of the data points and captures the essential nature of the trend.

Prior Knowledge Required:
Can draw scatter plots
Can recognize and describe patterns of association in a scatter plot

Vocabulary: association, line of best fit, linear association, negative association, nonlinear association, outlier, positive association, slope, strong linear association, trend, weak linear association

Materials:
BLM Introduce the Line of Best Fit (p. R-56), one half per student

Review describing patterns of association. Draw on the board:

ASK: What type of association do the three scatter plots show, positive or negative? (positive) How do you know? (the values in the sets of data increase together) In which graph do the points lie on a straight line? (A) SAY: Association can be strong or weak. Scatter plot A shows a strong linear association because the points are very close to a straight line. Demonstrate this by drawing a line through most of the points in A, as shown on the next page. SAY: Scatter plot B shows a weak linear association because, if we draw a line through the data points, the points are further from the line. You can still draw a line through the points in B that will be reasonably close to most points, but not as close as in A. In other words, there is still a linear association in B, but it is weaker than in A. Demonstrate this by drawing a line through most of the points in B, as shown on the next page. ASK: Does C show a linear association? (no, it’s a curve) Draw a curved line through most of the points in C, as shown on the next page. SAY: If the points are
better approximated by a curve than a line, we call it nonlinear. Summarize the patterns of association on the board below each scatter plot, as shown below:

A. strong positive linear association
B. weak positive linear association
C. positive nonlinear association

Explain to students that someone might think that “strong positive” means greater positive slope, but it is actually a short form for a “strong linear and positive association.” Draw on the board:

SAY: The scatter plot on the left has a smaller slope than the scatter plot on the right, but the left one has a stronger association than the right one because the points are very close to a straight line.

Exercises:
1. For which scatter plot or scatter plots does the data show …
   a) a positive association?
   b) a negative association?
   c) a linear association?
   d) a nonlinear association?
   e) a strong linear association?
   f) a weak linear association?
   g) a strong linear and positive association?
   h) a weak linear and negative association?

A. 
B. 
C. 
D. 

Answers: a) A, D; b) B, C; c) A, B, D; d) C; e) A, B; f) D; g) A; h) none
2. The line on the scatter plot shows the general direction of the data:

A.  

B.  

C.  

D.  

a) Which linear associations are positive?
b) Which linear associations are negative?
c) Which positive linear association is strongest?
d) Which negative linear association is strongest?

**Answers:** a) B, D; b) A, C; c) B; d) A

**Introduce the line of best fit.** Distribute BLM *Introduce the Line of Best Fit.* SAY: All the scatter plots on the page are the same. They show the height and the arm span for 14 Grade 8 students. ASK: Looking at the scatter plot at the top of the BLM, do you see an association? (yes) Is it positive or negative? (positive) Is it linear or nonlinear? (linear) SAY: Because the data points lie more or less in a straight line, you can say there is a linear association. On the bottom row, we have three different ways of drawing a line on this scatter plot. Have students complete the BLM by counting the number of points on each side of the line. (A: 5 above, 6 below; B: 6 above, 6 below; C: 2 above, 9 below)

ASK: Which lines pass through the center of the data points? (A and B) How do you know? (the number of data points is about the same on each side of the line) Which lines show the direction of the relationship? (A and C) Does the line for scatter plot B show the direction of the points? (no) Why not? (it’s too steep; it doesn’t go in the same direction as the points) Point to A and SAY: We call this line the *line of best fit* because it shows the direction of the relationship between two variables, it passes through the center of the data points, and the points are closely packed around the line. Another name for the direction in a scatter plot is *trend* because it indicates the general tendency of the relationship. For example, as students get taller, their arm span gets longer.

**(MP.7) Exercises:** A scatter plot shows the elevation above sea level for some cities and the average temperature in those locations. The same scatter plot is shown three times below, each with a different line:

A.  

B.  

C.  

a) Count the number of points on each side of the line.
b) Which lines pass through the center of the data points?
c) Which lines show the trend of the data?
d) Which line is the line of best fit?
**Answers:** a) A: 8 above, 4 below, B: 4 above, 4 below, C: 5 above, 4 below; b) B and C; c) A and B; d) B

(MP.6) **How an outlier affects the line of best fit.** Draw on the board:

Point to the positive association on the left and SAY: When the x-value is low, the y-value is low, and when x is high, y is high. That is a key property of a positive association. The two variables are either both low or both high. Point to the negative association on the right and SAY: A negative association means that, generally, when one variable is high, the other one is low.

Draw on the board:

SAY: Here are two copies of the same scatter plot. It shows the marks in two math tests for 15 Grade 8 students. The x-axis shows the marks in test 1 and the y-axis shows the marks in test 2. ASK: Looking at the scatter plot, do you see an association? (yes) Is it positive or negative? (positive) Is it linear or nonlinear? (linear) Draw the line of best fit on the scatter plot on the left, as shown below:

SAY: Ben took both tests; his marks are not yet shown on the scatter plot. He got a very high mark on test 1, but unfortunately he got a much lower mark in test 2. ASK: Is Ben's mark on test 1 on the x-axis close to the origin or far from the origin, to the right? (far from the origin, to the right) Why? (because Ben got a high mark on test 1) What about test 2, is Ben's mark close to the origin on the y-axis or far from the origin, near the top? (close to the origin) To represent
Ben’s data, add another point to the scatter plot on the right with a relatively high value for x and a relatively low value for y, as shown below:

ASK: Looking at the scatter plot on the right, do you see an outlier? (yes, the point added for Ben’s marks, near the bottom right) Do you see an association on the scatter plot? (yes) Is it positive or negative? (positive) Is it linear or nonlinear? (linear) Explain to students that the outlier doesn’t affect the type of association, but it affects the line of best fit. Draw the line of best fit for the scatter plot on the right, as shown below:

SAY: When an outlier with a relatively high x-value and relatively low y-value is added, it makes the association less positive or more negative than it would be without the outlier. The line of best fit rotates clockwise.

**MP.6 Exercises:** The line of best fit for the scatter plot is given. Determine whether adding an outlier would rotate the line of best fit clockwise or counterclockwise.

a) $y$

\[ \begin{array}{c}
\text{Test 1} \\
\text{Test 2}
\end{array} \]

i) an outlier with a relatively low x-value and relatively high y-value
ii) an outlier with a relatively high x-value and relatively low y-value

b) $y$

\[ \begin{array}{c}
\text{Test 1} \\
\text{Test 2}
\end{array} \]

i) an outlier with a relatively low x-value and relatively low y-value
ii) an outlier with a relatively high x-value and relatively high y-value

**Bonus:** an outlier with a relatively high x-value and relatively low y-value
Answers: a) i) clockwise, ii) clockwise; b) i) counterclockwise, ii) counterclockwise; 
Bonus: little effect, because the points already have a negative association, so adding another 
point with a negative association will not change the line much, if at all

Extension 
(MP.1, MP.4) Zack is making a cake for his sister’s birthday. The recipe asks for: 
1 cup white sugar       1 1/2 cups all-purpose flour 
1/2 cup butter          1 3/4 tsp baking powder 
2 eggs                  1/2 cup milk 
2 tsp vanilla extract
The recipe says to use one 10-inch diameter round pan. Zack only has a 9-inch diameter round 
pan. He wants to make the cake have about the same depth in the pan so that he doesn’t have 
to change the cooking time or the oven temperature. He also has to use a whole number of 
eggs, so he decides to use 1 egg, which he measured to have a volume of 3/8 cup. He needs to 
keep the ratio of liquid ingredients to solid ingredients the same so he compensates by adding 
extra milk. What is the final recipe you would recommend he use? 
Answer: The ratio of the recipes is new : original = π × 4.5² : π × 5² = 4.5² : 5² = 81 : 100. He 
should make about 4/5 of the recipe. So he should use 4/5 of 1/2, or 2/5, of a cup milk, and he 
should use 4/5 of 2 eggs, or 4/5 of 3/4 cup eggs = 3/5 cup eggs. But he is only using 3/8 cup 
eggs, so he is using too little egg by 3/5 – 3/8 = (24 – 15)/40 cups or 9/40 cups of eggs. He 
needs to add 9/40 cups milk to 2/5 cup milk, so I added 9/40 + 2/5 = 9/40 + 16/40 = 25/40 
= 5/8 cup milk. To find all the other ingredients, I multiplied the original amount by 4/5. His final 
recipe should be: 
4/5 cup white sugar       1 1/5 cups all-purpose flour 
2/5 cup butter          1 2/5 tsp baking powder 
1 egg                  5/8 cup milk 
1 3/5 tsp vanilla extract 
Redirecting students: Encourage students to make a plan of the steps they need to do before 
carrying out the plan. Find the ratio of the recipes and what each ingredient would be without 
compensating for the egg, find the volume of egg he needs to make up for, find the volume of 
milk he needs to add, and then find the total amount of milk needed for the recipe. You may 
need to suggest to students to look for a ratio that is close to 81 to 100 but easier to work with.
SP8-6  Drawing the Line of Best Fit Informally

Pages 182–183

Standards: 8.SP.A.2

Goals:
Students will draw a line of best fit, paying attention to the closeness of the data points on either side of the line and comparing the number of points above and below the line.
Students will see the effect of outliers on the line of best fit.

Prior Knowledge Required:
Can draw scatter plots
Can recognize and describe patterns of association in a scatter plot

Vocabulary: association, line of best fit, linear association, negative association, nonlinear association, outlier, positive association, strong association, trend, weak association

Materials:
BLM Drawing the Line of Best Fit (p. R-57)
Microsoft Excel® (see Extension 2)
BLM Drawing the Line of Best Fit with Microsoft Excel® (p. R-58), one half per student (see Extension 2)

(MP.7) Drawing a line of best fit. SAY: Remember, the line of best fit has two important properties: it shows the trend and it passes through the center of the data points. In this lesson, we are going to draw the line of best fit based on these two properties. Draw on the board:

![Graph showing scatter plot and line of best fit]

SAY: The scatter plot shows the relationship between the amount of exercise and body fat. ASK: Looking at the scatter plot, do you see an association? (yes) Explain to students that the existence of the association is very important because, if there is no association, then drawing the line of best fit is meaningless. Point to the scatter plot and ASK: Is the association positive or negative? (negative) Is it linear or nonlinear? (linear) What is the trend? (higher exercise means lower body fat) SAY: When you find the type of association, you can draw a line through the center of all data points, starting where the data begins and ending where the data ends. Add a line of best fit to the scatter plot on the board, as shown below:
SAY: When you draw the line, you need to check it. There should be about the same number of data points on each side of the line. Ask students to count the number of data points on each side of the line. (7 above, 6 below) Explain to students that some points will be distinctly above or below the line of best fit, and some will be very close to the line, so students' counts might show some variation, especially with complex scatter plots. Draw the same scatter plot on the board beside the first one with a new line of best fit, as shown below:

Point to the scatter plot on the right and ask students to count the number of data points on each side of the line. (8 above, 5 below) SAY: Although the line on the right is not a bad approximation, the line on the left shows the trend better and passes through the center of the data, with about the same numbers of data points above and below the line. In later studies, you will find the line of best fit algebraically, but for now, we are going to approximate it by looking at the scatter plot.

**Exercises:** Complete BLM Drawing the Line of Best Fit.

**Answers:** 1. a) 8, 7; b) 8, 6; c) 8, 7; d) 7, 7; e) 8, 9; f) 5, 6; g) 9, 8; h) 9, 7; i) 6, 8; j) 8, 9; k) 10, 10; l) 10, 12

**Drawing the line of best fit for a table of data.** Draw on the board:

<table>
<thead>
<tr>
<th>Age</th>
<th>12</th>
<th>14</th>
<th>13</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>12</th>
<th>13</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly Allowance ($)</td>
<td>30</td>
<td>40</td>
<td>40</td>
<td>25</td>
<td>45</td>
<td>35</td>
<td>55</td>
<td>40</td>
<td>30</td>
</tr>
</tbody>
</table>

Tell students that the table shows the monthly allowance for 10 students between the ages of 12 and 14. Draw the x-axis and the y-axis on the board. ASK: Which variable should we put on the x-axis? (age) SAY: The data for the x-axis goes from 12 to 14, so a scale from 12 to 14 with one-year increments would show the data well. The data for the y-axis goes from 25 to 55, so we could use a scale that shows from 25 to 55 with five-dollar increments. Draw the grid on the board and ask a volunteer to plot the data set, as shown below:
ASK: Do you see an association? (yes) Is the association positive or negative? (positive) Is it linear or nonlinear? (linear) Is it weak linear or strong linear? (weak) Ask another volunteer to draw the line of best fit for the scatter plot, as shown below:

Have students count the number of data points on each side of the line. (4 above, 5 below) Leave the scatter plot on the board for later use.

(MP.7) Exercise: The table shows the number of ice creams sold at a convenience store each day in the first half of June.

<table>
<thead>
<tr>
<th>Date</th>
<th>Ice Creams Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35</td>
</tr>
<tr>
<td>2</td>
<td>38</td>
</tr>
<tr>
<td>3</td>
<td>41</td>
</tr>
<tr>
<td>4</td>
<td>42</td>
</tr>
<tr>
<td>5</td>
<td>37</td>
</tr>
<tr>
<td>6</td>
<td>39</td>
</tr>
<tr>
<td>7</td>
<td>44</td>
</tr>
<tr>
<td>8</td>
<td>49</td>
</tr>
<tr>
<td>9</td>
<td>47</td>
</tr>
<tr>
<td>10</td>
<td>45</td>
</tr>
<tr>
<td>11</td>
<td>49</td>
</tr>
<tr>
<td>12</td>
<td>51</td>
</tr>
<tr>
<td>13</td>
<td>55</td>
</tr>
<tr>
<td>14</td>
<td>58</td>
</tr>
</tbody>
</table>

Draw a scatter plot for the data and then draw the line of best fit for the scatter plot.

Answer:

**Effect of outliers on the line of best fit.** Point to the monthly allowance scatter plot from earlier and SAY: Based on the line of best fit for this scatter plot, the monthly allowance of a 14-year-old student is around $48.

**NOTE:** In the next part of the lesson, students will need to see how the line of best fit rotates when an outlier is added. In order for them to be able to compare the lines before and after the outlier is added, you might want to re-draw the scatter plot each time so they can make the comparison. Alternatively, you can add the new lines of best fit to the same scatter plot and explain that each new line replaces the old one.
Now ask a volunteer to add the point (14, 25) to the scatter plot. ASK: Can I say this new data point is an outlier? (yes) Why? (because it’s far from the other points and it’s not showing the trend of the data) SAY: When we add this outlier with a relatively high x-value and relatively low y-value, it makes the association less positive or more negative. The line of best fit rotates clockwise. Draw the line of best fit for the new scatter plot, as shown below:

![Age and Allowance Scatter Plot with One Outlier](image1)

SAY: Another 14-year-old student gets $70 for a monthly allowance, and I would like to add that data to the scatter plot. Ask a volunteer to extend the y-axis and add the point (14, 70) to the scatter plot. Explain to students that the new data point is another outlier because it’s far from other points and it’s not showing the trend of the data. SAY: The outlier has a relatively high x-value and relatively high y-value. When we expand the scatter plot and add this new data point, it makes the association more positive or less negative. The line of best fit rotates counterclockwise. Draw the line of best fit for the new scatter plot, as shown below:

![Age and Allowance Scatter Plot with Two Outliers](image2)

Ask students to compare the last scatter plot, with two outliers, and the original scatter plot, without outliers. ASK: Are the lines of best fit the same? (yes) Remind students that the line of
best fit shows an allowance of $48 for a 14-year-old student. SAY: The first outlier, (14, 25), is $23 below the line of best fit for 14-year-old students, and the second outlier, (14, 70), is $22 above the line of best fit for 14-year-old students, so these two points sort of cancel their effects on the line of best fit.

**Exercises:**

1. a) Circle any outliers on the scatter plot. Then draw the line of best fit for the scatter plot. Check by counting the number of points on each side of the line.

   ![Scatter plot with line of best fit](image)

   i) Points above: ____ Points below: ____
   ii) Points above: ____ Points below: ____
   iii) Points above: ____ Points below: ____

   b) What do you notice when you compare the lines of best fit in parts i) and iii) of part a)?

   **Answers:** 1. a) i)

   ![Scatter plot with line of best fit](image)

   Points above: 5 Points below: 5
   ii) Points above: 4 Points below: 7
   iii) Points above: 6 Points below: 6

   b) The lines are almost the same in i) and iii) because the outliers almost cancel their effects on the line of best fit.

**Extensions**

**MP.3** 1. The line on the scatter plot is not the line of best fit. Add one point (an outlier) to the scatter plot to make this the line of best fit.

   ![Scatter plot with outlier](image)

   **Answers:**

   ![Scatter plot with outlier](image)

   **Sample Solution:** a) The line is rotated counterclockwise compared to what should be the line of best fit, so there are more points below than above the line. A point with a high x-value and high y-value above the line will rotate the line counterclockwise to fix this problem.
(MP.5) 2. Complete BLM Drawing the Line of Best Fit with Microsoft Excel®.
Answers:

(a–d) [Graphs of data points and a line of best fit are shown.]

(e) [Graph of data points with a line of best fit.

(f–g) [Graph of data points with a line of best fit.]

(MP.3, MP.4) 3. a) A ball is a sphere with a volume of 6,000 cm$^3$. A box is a cube with a volume of 8,000 cm$^3$. Will the ball fit inside the box? Explain.

b) In pairs, discuss your answers to part a). Do you agree with each other? Discuss why or why not.

Sample solution: The ball has a volume of $\frac{4}{3} \pi r^3 = 6,000$ cm$^3$, so $r^3 = 6,000 \times 3 \div (4\pi) \approx 4,500 \div 3.14 \approx 1,433$ cm$^3$, so $r \approx 11.3$ cm. This means the diameter of the ball is about 22.6 cm. The box is a cube with side length 20 cm, since $20^3 = 20 \times 20 \times 20 = 8,000$. The ball is too wide to fit into the box.
Goals:
Students will find the slope and y-intercept of the line of best fit and write an equation of the line of best fit.

Prior Knowledge Required:
Can draw scatter plots
Can recognize and describe patterns of association in a scatter plot
Can draw the line of best fit informally
Can write an equation for a line in slope-intercept form

Vocabulary: line of best fit, rise, run, slope, slope-intercept form, y-intercept

Materials:
Microsoft Excel® (see Extension 2)
BLM Drawing the Line of Best Fit with Microsoft Excel® (p. R-58, see Extension 2)

Review rate of change and slope. Draw on the board:

ASK: What is the run, or change in x, from A to B? (+3) What is the rise, or change in y, from A to B? (+2) What is the slope between A and B? (2/3) Remind students that to find the run or rise, we need to know only the initial point and the final point. We don’t really need to know how we get from A to B. Write on the board:

\[
\text{run} = +3, \text{ rise} = +2, \text{ slope} = \frac{\text{rise}}{\text{run}} = \frac{2}{3}
\]
**Slopes between any two points on a straight line are equal.** Draw on the board:

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**ASK:** What is the run from $A$ to $B$? (+2) What is rise from $A$ to $B$? (-1) What is the slope of $AB$? (-1/2) Ask a volunteer to find the slope of $BC$ (run = +4, rise = -2, slope = $-2/4$ or $-1/2$) Ask another volunteer to find the slope of $AC$ (run = +6, rise = -3, slope = $-3/6$ or $-1/2$) Remind students that because the slope is equal between each set of two points on a straight line, we can say the slope of this line is $-1/2$ or $-0.5$. Mark point $D$ on the line at (4, 2.5), as shown below:

**ASK:** What is the run from $A$ to $D$? (3) What is rise from $A$ to $D$? (-1.5) Write on the board:

\[
\text{Slope of } AD = \frac{\text{rise}}{\text{run}} = \frac{-1.5}{3} = \frac{-15}{30} = -\frac{1}{2} = -0.5
\]

**ASK:** Which slope is easier to find, the slope of $AB$ or the slope of $AD$? (the slope of $AB$)

**SAY:** To find the slope of a line, choose two points with integer coordinates, if possible.

**Exercises:** Mark two points, $A$ and $B$, on the line and then find the slope.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>b)</td>
<td>c)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answers:**

- a) $A$ (1, 3) and $B$ (3, 4), run = 2, rise = 1, slope = $\frac{1}{2} = 0.5$
- b) $A$ (1, 7) and $B$ (3, 4), run = 2, rise = -3, slope = $\frac{-3}{2} = -1.5$
- c) $A$ (1, -1) and $B$ (2, 3), run = 1, rise = 4, slope = $\frac{4}{1} = 4$
Using two data points to find the slope of the line of best fit. Draw on the board:

<table>
<thead>
<tr>
<th>Age</th>
<th>12</th>
<th>14</th>
<th>13</th>
<th>12</th>
<th>13</th>
<th>13</th>
<th>14</th>
<th>12</th>
<th>13</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allowance ($)</td>
<td>30</td>
<td>40</td>
<td>40</td>
<td>25</td>
<td>45</td>
<td>35</td>
<td>55</td>
<td>40</td>
<td>30</td>
<td>50</td>
</tr>
</tbody>
</table>

Draw the x-axis, y-axis, and the grid on the board and ask a volunteer to plot the data points and draw the line of best fit, as shown below. SAY: This is the same graph as the one we drew in the previous lesson, before we added any outliers.

SAY: The line of best fit passes through or very close to two data points that we will call A (12, 30) and B (13, 40). Circle points A and B in the table, as shown below:

<table>
<thead>
<tr>
<th>Age</th>
<th>12</th>
<th>14</th>
<th>13</th>
<th>12</th>
<th>13</th>
<th>13</th>
<th>14</th>
<th>12</th>
<th>13</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allowance ($)</td>
<td>30</td>
<td>40</td>
<td>40</td>
<td>25</td>
<td>45</td>
<td>35</td>
<td>55</td>
<td>40</td>
<td>30</td>
<td>50</td>
</tr>
</tbody>
</table>

ASK: What is the run from A to B? (+1) What is the rise from A to B? (+10) Write on the board:

\[
\text{Slope of } AB \text{ (line of best fit)} = \frac{\text{rise}}{\text{run}} = \frac{10}{1} = 10
\]

SAY: So we can say that the slope is 10, and based on the line of best fit, students ages 12 to 14 have monthly allowances that increase $10 per year.

(MP.6) Exercises: The table shows the age and the height for a new group of students.

<table>
<thead>
<tr>
<th>Age</th>
<th>12</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>14</th>
<th>13</th>
<th>12</th>
<th>13</th>
<th>11</th>
<th>13</th>
<th>10</th>
<th>14</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (cm)</td>
<td>156</td>
<td>141</td>
<td>139</td>
<td>143</td>
<td>161</td>
<td>159</td>
<td>150</td>
<td>147</td>
<td>147</td>
<td>164</td>
<td>138</td>
<td>171</td>
<td>155</td>
</tr>
</tbody>
</table>

a) Draw a scatter plot for the table.
b) Use the points A (10, 138) and B (12, 150) to draw the line of best fit.
c) Use points A and B to find the slope of the line of best fit.
d) Interpret the slope of the line of best fit that you found in part c).
Answers:

a–b) [Insert graph showing Age and Height with points A, B, and C]

c) \( \text{run} = 12 - 10 = 2, \text{rise} = 150 - 138 = 12, \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{12}{2} = 6 \)

d) Based on the line of best fit, students ages 10 to 14 grow 6 cm on average per year.

NOTE: Leave the graph from the exercises on the board for later use.

Review y-intercept and slope-intercept form. Remind students that the y-intercept is where a line intersects the y-axis on a graph. Remind students that sometimes we have to extend a line to find the y-intercept. Draw on the board:

[Insert graph showing extended line]

Ask a volunteer to extend the line to find the y-intercept. (25) SAY: I would like to find the slope of the line. ASK: Do you see points with integer coordinates? (yes) How many points are there with integer coordinates? (two) Ask students to write two points with integer coordinates and find the slope of the line. \((A (10, 30) \text{ and } B (30, 40)), \text{run} = 30 - 10 = 20, \text{rise} = 40 - 30 = 10, \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{10}{20} = 0.5 \) Remind students that they can write the equation of a line in slope-intercept form. Write on the board:

\[ y = mx + b \]

SAY: In the slope-intercept form, \( m \) is the slope and \( b \) is the y-intercept of the line. Write the equation of the line from the example above in slope-intercept form on the board underneath \( y = mx + b \):

\[ y = mx + b \]
\[ y = 0.5x + 25 \]
**Exercises:** a) Extend the line to find the \( y \)-intercept.

b) Mark two points, \( A \) and \( B \), on each line and label the left point \( A \). Then find the slope.

c) Write the equation for each line in part a) in slope-intercept form: \( y = mx + b \).

![Graphs](image)

**Answers:**

i) \( y \)-intercept: 4, slope: 0.5, equation: \( y = 0.5x + 4 \)

ii) \( y \)-intercept: 7, slope: -2, equation: \( y = -2x + 7 \)

iii) \( y \)-intercept: 17.5, slope: 0.25, equation: \( y = 0.25x + 17.5 \)

**Finding \( y \)-intercept using the slope and a point.** SAY: In the previous exercises, you found the \( y \)-intercept by extending the line to cross the \( y \)-axis. Explain to students that sometimes the part of the line that you have information about is very far from the \( y \)-axis and you can't practically extend the line to find the \( y \)-intercept. Bring students' attention back to the graph on the board from the earlier exercise in this lesson:

![Graph](image)

SAY: If I extend the line on the graph as it is now, you will see that the line crosses the \( y \)-axis around 130 cm. ASK: Is it correct? (no) Why? (because of the break shown in the \( x \)-axis)
SAY: I already know the slope of the line is 6 and I would like to find the exact $y$-intercept using algebra. Write on the board:

$$y = mx + b$$

SAY: I know the slope is 6, so I can replace $m$ with 6 in the equation. Continue writing on the board:

$$y = 6x + b$$

SAY: I would like to find the $y$-intercept, $b$, but there are three unknowns in the equation: $y$, $x$, and $b$. The point $A$ (10, 138) is on the line, so we know its coordinates make the equation true. ASK: What is the $y$-coordinate of the point $A$? (138) What is the $x$-coordinate of the point $A$? (10) Write the values of the coordinates on the board:

$$y = 6x + b \quad (x = 10, \ y = 138)$$

SAY: I will substitute the coordinates of point $A$ in the equation. Continue writing on the board:

$$138 = 6(10) + b$$

ASK: Now how many unknowns are in the equation? (one) Can I solve the equation to find $b$? (yes) Solve the equation to find $b$, as shown below:

\[
\begin{align*}
y &= mx + b \\
y &= 6x + b \quad (x = 10, \ y = 138) \\
138 &= 6(10) + b \\
138 - 60 &= b \\
78 &= b
\end{align*}
\]

SAY: I used point $A$ and the slope for the line to find the $y$-intercept for the line. ASK: What will happen if I pick the point $B$ (12, 150)? Will I find the same value for the $y$-intercept—in other words, the same value for $b$ in the equation? (yes) Ask a volunteer to solve the equation and find the $y$-intercept using point $B$ (12, 150), as shown below:

\[
\begin{align*}
y &= mx + b \\
y &= 6x + b \quad (x = 12, \ y = 150) \\
150 &= 6(12) + b \\
150 - 72 &= b \\
78 &= b
\end{align*}
\]

SAY: If you have the coordinates of more than one point on the line, you can use any point to find the $y$-intercept, but it's easier to use a point with an integer coordinate. Ask a volunteer to write the equation of the line in slope-intercept form. ($y = 6x + 78$)
**MP.3** Exercises:
a) Find the y-intercept of the line with the given slope that passes through the given point. Write the equation of the line in slope-intercept form.
   i) slope = 2, A (3, 10)  
   ii) slope = -3, A (2, 4)  
   iii) slope = 0.25, A (2, 1)
b) Check your answers for the equations in part a) by substituting the coordinates of point A.

**Answers:**
a) i) y-intercept: 4, equation: $y = 2x + 4$; ii) y-intercept: 10, equation: $y = -3x + 10$; iii) y-intercept: 0.5, equation: $y = 0.25x + 0.5$

**Sample solution:**
b) iii) $y = mx + b$
   
   \[ 1 = 0.25(2) + b \]
   
   \[ 1 = 0.5 + b \]
   
   \[ b = 0.5 \checkmark \]

**Extensions**
1. Make a scatter plot of the data in the table. Draw a line of best fit. Write an equation for the line.

<table>
<thead>
<tr>
<th></th>
<th>-2</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3.5</td>
<td>2.5</td>
<td>2</td>
<td>0.5</td>
<td>1</td>
<td>-0.5</td>
<td>-2</td>
</tr>
</tbody>
</table>

**Answers:**

\[ m = \frac{\text{rise}}{\text{run}} = \frac{-2.5}{2} = -1.25, y\text{-intercept: } b = 0.75, \text{ equation: } y = -1.25x + 0.75 \]

**MP.5** 2. Have students complete parts a) to g) of BLM Drawing the Line of Best Fit with Microsoft Excel® again, or use the first saved results from the extension in lesson SP8-6, and then do the following:
a) Under the Chart Tools menu, select “Layout,” then “Trendline,” and then “More Trendline Options.” Select “Display Equation on chart,” then close the dialog window to get the equation of the line.
b) Add the data for another student (6, 85) and find the new equation of the line of best fit.
**Answers:**

a) \( y = 4.8815x + 53.913 \)

b) \( y = 4.9278x + 53.83 \)

**(MP.7)** 3. a) Find the equation \( y = mx + b \) of the line that passes through \((4, 3)\) and \((5, 2)\).

b) In pairs, explain your approaches to part a).

**Answers:**

a) \( y = -x + 7 \); b) sample explanations:

- I started by finding the slope by using rise/run = \((2 - 3)/(5 - 4) = -1/1 = -1\). Then I used \( y = -x + b \) and one point on the line to determine what the \( y \)-intercept is: \( 2 = -5 + b \), so \( b = 7 \).
- I solved the system \( 3 = 4m + b \) and \( 2 = 5m + b \) to find \( m \) and \( b \). Subtracting the first equation from the second gives \( m = -1 \). I substituted that into the first equation, so \( 3 = -4 + b \), so \( b = 7 \).
SP8-8 Applications of the Line of Best Fit

Pages 187–189

Standards: 8.SP.A.2, 8.SP.A.3, 8.F.B.4

Goals:
Students will understand that linear models can be represented with the equation of the line of best fit.
Students will interpret the slope and y-intercept of the line in the context of the problem.

Prior Knowledge Required:
Can draw scatter plots
Can recognize and describe patterns of association in a scatter plot
Can draw the line of best fit informally
Can write the equation for the line of best fit

Vocabulary: estimate, line of best fit, linear association, predict, slope, strong association, y-intercept

Materials:
BLM Application of the Line of Best Fit (p. R-59)
Microsoft Excel® (see Extension 2)
BLM Drawing the Line of Best Fit with Microsoft Excel® (p. R-58, see Extension 2)

Review finding input and output from a graph. Draw on the board:

```
<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>
```
Demonstrate how to find a missing value in the table: find \( x = 1 \) on the graph scale and then go up the \( x = 1 \) line to get to \( y = 3 \). Have volunteers fill in the rest of the table for the graph.

(see completed table below)

<table>
<thead>
<tr>
<th>Input</th>
<th>1</th>
<th>4</th>
<th>7</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

If students have trouble when the input is missing, show them how to start at the output on the \( y\)-axis and move right until you reach the line, then descending to the \( x\)-axis to find the input. Have students extend the line to find out what the next whole-number input and output would be. (13, 7)

**Exercises:** The data points make a linear graph. Graph the data points. Extend the line to find the output for \( x = 8 \).

a) (1, 6), (2, 11), (3, 16)  
b) (1, 7), (3, 15), (5, 23)  
c) (2, 5), (2.5, 6.5), (3, 8)

**Answers:** a) 41, b) 35, c) 23

**NOTE:** In the following part of the lesson, we are using terms precisely—“estimate” for interpolation for within the range of the data set, and “predict” for extrapolation for outside of the range. While students will not use the terms interpolation and extrapolation, we recommend that you use the terms estimate and predict with care for these.

**MP.4 Using line of best fit for estimation and prediction.** Draw on the board:

<table>
<thead>
<tr>
<th>Age</th>
<th>12</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>14</th>
<th>12</th>
<th>13</th>
<th>11</th>
<th>13</th>
<th>10</th>
<th>14</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (cm)</td>
<td>155</td>
<td>141</td>
<td>139</td>
<td>140</td>
<td>161</td>
<td>150</td>
<td>152</td>
<td>147</td>
<td>163</td>
<td>138</td>
<td>171</td>
<td>158</td>
</tr>
</tbody>
</table>

Tell students that the scatter plot shows the age and height for a new group of students ages 10 to 14 and the line of best fit shows the relationship. ASK: How many students are 11 years old? (2) What are their heights? (139 cm and 147 cm) Ask a volunteer to find the average height of two 11-year-old students in the scatter plot, as shown below:

\[
\text{average} = \frac{139 + 147}{2} = 143
\]
ASK: What is the output of the line of best fit for the input $x = 11$? (144) Explain to students that when there is a strong linear association between two variables, all the points are close to the line, so we can use the line of best fit to estimate the value of the second variable. SAY: The value of the line of best fit at $x = 11$ is around 144 cm, which is a good estimation for an 11-year-old student because 144 is close to both data values 139 and 147, and 144 is close to the average. Explain to students that when there is a strong linear association, they can even use the line of best fit to predict values beyond what is shown on the scatter plot. SAY: For example, I would like to predict the height of a 15-year-old student. Extend the line of best fit to find its value for $x = 15$, as shown below:

![Age and Height Graph]

ASK: What is the output, or $y$-value, of the line of best fit for the input $x = 15$? (171) SAY: So we can estimate values on the line of best fit and we can predict values by extending the line of best fit. However, we cannot use this line to predict data far from the original data. For example, to find the value of the line of best fit at $x = 25$, we’d have to make a very big graph because the output would be more than 230 cm.

**Exercise:** Complete Question 1 on BLM Application of the Line of Best Fit.

**Answers:**

a) i) almost $38$, ii) 12 years old
b) almost $57$

**Using the equation of the line of best fit for estimation and prediction.** Remind students that when they have the equation of a line, they can replace $x$ with a number to find the value for $y$ for the corresponding point on the line, or they can replace $y$ with a number to find the value of
The equation of a line is $y = 3x - 1$. Ask: What is $y$ for $x = 2$? (5) Write on the board:

$$
\begin{align*}
  y &= 3x - 1 \\
  y &= 3(2) - 1 \\
  y &= 5
\end{align*}
$$

Explain to students that to find the value of $y$, they can easily replace the number for $x$ and solve the equation to find $y$, but when they are given $y$ and must find $x$, solving the equation to find the missing value is harder. Ask: For example, with the same line, I would like to find the value of $x$ when $y = 11$. Write on the board:

$$
\begin{align*}
  y &= 3x - 1, \text{ so } 11 &= 3x - 1 \\
  11 + 1 &= 3x \\
  12 &= 3x \\
  x &= 4
\end{align*}
$$

**Exercises:**
1. Solve the equation to find the value of $x$ for each value of $y$.

   a) $y = 2x + 1$
   $$
   \begin{array}{c|c}
   y & y = 2x + 1 \\
   \hline
   1 & 3 \\
   3 & 1.5 \\
   \end{array}
   $$

   b) $y = 0.25x - 1$
   $$
   \begin{array}{c|c}
   y & y = 0.25x - 1 \\
   \hline
   1 & 3 \\
   3 & 1.5 \\
   \end{array}
   $$

   c) $y = -0.35x + 3.2$
   $$
   \begin{array}{c|c}
   y & y = -0.35x + 3.2 \\
   \hline
   1 & 3 \\
   3 & 1.5 \\
   \end{array}
   $$

**Answers:**
   a) 0, 1, 0.25; b) 8, 16, 10; c) 6.29, 0.57, 4.86

**Sample solutions:**
   a) $1 = 2x + 1$  \hspace{1cm} b) $3 = 0.25x - 1$  \hspace{1cm} c) $1.5 = -0.35x + 3.2$
   $$
   \begin{align*}
   0 &= 2x \\
   4 &= 0.25x \\
   -1.7 &= -0.35x \\
   16 &= x \\
   4.86 &\approx x
\end{align*}
   $$

(MP.4) 2. Complete Question 2 on BLM Application of the Line of Best Fit.

**Answers:** a) positive, strong, linear; b) about 64; c) about 25°C

**Solutions:**
   d) b) $y = 1.44(30) + 21$, $y = 64.2$ or 64 (because you can't have 0.2 of an ice cream);
   c) $57 = 1.44x + 21$, $36 = 1.44x$, $x = 25$

**Application of the line of best fit.** Explain to students that sometimes they cannot pick two points that are on the line of best fit. Draw on the board:

**Sample solutions:**
   d) $y = 1.44(30) + 21$, $y = 64.2$ or 64 (because you can't have 0.2 of an ice cream);
   c) $57 = 1.44x + 21$, $36 = 1.44x$, $x = 25$
Tell students that the scatter plot shows the height and the foot length of 10 students in Grade 8, along with the line of best fit for the data set. SAY: the line of best fit doesn’t pass through two points in the scatter plot. ASK: Which two points are very close to the line of best fit? ((130, 22) and (160, 24)) Mark the two points as A and B, as shown below:

![Height and Foot Length](image)

SAY: On this graph, we are finding the approximate line of best fit, so the line that passes through A and B gives a good approximation of the line of best fit. Ask students to find the slope of the line that passes through A and B and then write the equation for it in slope-intercept form, as shown below. If needed, remind students how to find the constant, or y-intercept in the equation in slope-intercept form.

\[
A (130, 22) \quad B (160, 24) \\
\text{run} = 160 - 130 = 30 \\
\text{rise} = 24 - 22 = 2 \\
slope = \frac{\text{rise}}{\text{run}} = \frac{2}{30} = \frac{1}{15} = 0.067 \\
y = mx + b \\
y = 0.067x + b \\
22 = 0.067(130) + b \\
22 - 8.71 = b \\
13.29 = b \\
so \ y = 0.067x + 13.29
\]

Explain to students that the slope 0.067 means the foot length of students with a height ranging from 130 to 170 cm increases 0.067 cm on average for every 1 cm that a student gets taller.

Summarize the application of the line of best fit. Remind students of the following:
- When the scatter plot is given, we can draw the line of best fit through the center of all data points and then check the line by counting the number of data points on each side of the line.
- By using the graph of the line of best fit and the value of one variable, we can estimate or predict the value of the second variable.
- By using two data points that the line of best fit passes through or points that are very close to the line, we can find the slope of the line and then write an equation for the line of best fit.
- When we have the equation for the line, we can use the equation and one variable for a data point to estimate or predict the value of the other variable.
(MP.3, MP.4, MP.6) Exercises: The table shows the number of hours studied and the science marks for 12 students.

<table>
<thead>
<tr>
<th>Study Time (h)</th>
<th>4</th>
<th>7</th>
<th>2</th>
<th>5</th>
<th>4</th>
<th>1</th>
<th>3</th>
<th>7</th>
<th>3</th>
<th>5</th>
<th>8</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Science Mark (%)</td>
<td>72</td>
<td>89</td>
<td>67</td>
<td>80</td>
<td>76</td>
<td>57</td>
<td>65</td>
<td>87</td>
<td>69</td>
<td>77</td>
<td>90</td>
<td>71</td>
</tr>
</tbody>
</table>

a) Draw a scatter plot for the data.
b) Describe the type of association.
c) Draw the line of best fit for the scatter plot.
d) How many points are above and how many points are below your line?
e) Use the line of best fit to estimate …
   i) the mark of a student who studies for 4.5 hours.
   ii) the number of hours that a student studies if the student gets 82 on the test.
f) Extend the line of best fit and predict the mark of a student who studies for 9 hours.
g) Use two points on the line of best fit (or very close to the line) to find the slope.
h) Use the slope and one point on the line of best fit to find the \( y \)-intercept.
i) Write an equation for the line of best fit in slope-intercept form.
j) What mark would a student get with no study at all? Where do you see this in the equation?
k) Use the equation from part i) to calculate the answers to parts e) and f).

Answers:

a) ![Scatter plot of study time vs. science marks]

b) positive linear

c) see graph above

d) 5, 5

e) i) 76, ii) 5.5 hours

f) 96

g) points (3, 69) and (7, 87), run = 4, rise = 18, slope = 18/4 = 4.5

h) using point (3, 69) and \( y = 4.5x + b \), \( b = 55.5 \);
i) \( y = 4.5x + 55.5 \)
j) 55.5, the constant term (\( y \)-intercept)
Extensions

(MP.3) 1. Discuss whether the outliers should be included when drawing the line of best fit for the following data. Why or why not?
   a) Emily records the ages and the weights (in pounds) of campers in the group she is responsible for at a summer camp: (6, 41), (7, 50), (7, 48), (6, 45), (77, 49), (6, 44), (7, 52).
   b) The panel of judges for a school talent show includes one representative from each grade. Josh is in Grade 6 and is performing in the competition and his best friend is one of the judges. Josh’s marks out of 10 from different judges are: (1, 3), (2, 4), (3, 3), (4, 5), (5, 3), (6, 10), (7, 3), (8, 3).
   c) At a school fundraiser, the following amounts of money (in dollars) were raised per grade: (1, 150), (2, 130), (3, 127), (4, 854), (5, 99), (6, 112.50), (7, 154), (8, 167).

   Answers: a) no, (77, 49) is a mistake; b) no, (6,10) is his friend and not representative; c) yes

(MP.5) 2. a) Use Microsoft Excel® and BLM Drawing the Line of Best Fit with Microsoft Excel® to find the equation of the line of best fit for the data below:

<table>
<thead>
<tr>
<th>Study Time (h)</th>
<th>4</th>
<th>7</th>
<th>2</th>
<th>5</th>
<th>4</th>
<th>1</th>
<th>3</th>
<th>7</th>
<th>3</th>
<th>5</th>
<th>8</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Science Mark (%)</td>
<td>72</td>
<td>89</td>
<td>67</td>
<td>80</td>
<td>76</td>
<td>57</td>
<td>65</td>
<td>87</td>
<td>69</td>
<td>77</td>
<td>90</td>
<td>71</td>
</tr>
</tbody>
</table>

   b) Under the Chart Tools menu, select “Layout,” then “Trendline,” and then “More Trendline Options.” Select “Display Equation on chart,” then close the dialog window to get the equation of the line. Compare the answer from part i) in the previous exercises that you found without using Microsoft Excel®. Are they about the same?

   Answers: a) \( y = 4.6382x + 54.901 \), b) The equations are about the same: \( y = 4.5x + 55.5 \) without the computer program and \( y = 4.6382x + 54.901 \) using the computer program.

(MP.5) 3. Hanna buys a house for $200,000. At the same time, Cameron buys a house for $155,000. The price of Hanna’s house goes up by $6,000 every 5 years. The price of Cameron’s house, in dollars, is given by the equation \( y = 2,100x + 155,000 \) where \( x \) is the number of years since Cameron bought the house. They both sold their houses at the same time and they got the same amount. How many years after buying their houses did they sell their houses? Use an equation, a table, or a graph.

   Sample answers:
   • I used an equation. Let \( h \) be the price of Hanna’s house, in thousands of dollars. Let \( c \) be the price of Cameron’s house, in thousands of dollars. Let \( t \) be the time, in years, since they bought their houses. Then the value of Hanna’s house is \( h = 200 + 1.2t \) (since 6 ÷ 5 = 1.2). The value of Cameron’s house is \( c = 155 + 2.1t \). If they are the same, then \( 200 + 1.2t = 155 + 2.1t \), so 45 = 0.9\( t \), so \( t = 45 ÷ 0.9 = 450 ÷ 9 = 50 \), so they sold their houses 50 years after buying them.
   • I used a table. I increased the number of years by 10; that way, Hanna’s house increased by $12,000 and Cameron’s house increased by $21,000. I found that after 50 years, their house values are both $260,000.
   • I used a graph. I drew the graphs of both house prices and saw that they intersect when \( x = 50 \) and \( y = 260 \), where \( x \) is the time in years and \( y \) is the price in thousands of dollars. So after 50 years, their house values are both $260,000.

MP.2 Assessment Opportunity: Question 7 on AP Book 8.2 p. 189
SP8-9  Venn Diagrams and Two-Way Tables

Pages 190–192

Standards: 8.SP.A.4

Goals:
Students will represent categorical data in a Venn diagram and construct a two-way table from a Venn diagram and a Venn diagram from a two-way table.
Students will learn about frequency and how to interpret and construct frequency tables.
Students will find the missing cells in a two-way table.

Prior Knowledge Required:
Is familiar with Venn diagrams

Vocabulary: category, frequency, row two-way table, two-way table, Venn diagram

Materials:
BLM Venn Diagrams and Two-Way Tables (p. R-60)

Review Venn diagrams. Remind students that they can visually organize information, objects, variables, and more using Venn diagrams. SAY: You organize objects in categories so that each category has some common fact. Remind students that a Venn diagram uses an oval for each category and shows if the categories overlap. Draw on the board:

```
Factors of 9
Factors of 12
```

SAY: I want to organize whole numbers between 1 and 12 in the ovals. ASK: What numbers are factors of 9? (1, 3, 9) What numbers are factors of 12? (1, 2, 3, 4, 6, 12) What numbers are factors of both 9 and 12? (1, 3) What numbers are not factors of 9 and not factors of 12? (5, 7, 8, 10, 11) Ask a volunteer to complete the Venn diagram. (see completed diagram below)

```
Factors of 9
9
Factors of 12
1

2
3

4
6

5
7
8

11
10
```

NOTE: Leave the diagram on the board for later use.
(MP.5) **Exercises:** a) Organize the numbers into odd and irrational numbers on a Venn diagram.

\[ 4, 1\frac{3}{7}, -5, \sqrt{4}, \sqrt{9}, \sqrt[3]{12}, 13, \sqrt{5} \]

b) Can you add a number to the Odd/Irrational region? Explain.

**Answers:**

a)

\[ \text{Odd} \quad \text{Irrational} \]

\[ -5, \sqrt{9}, 13 \quad \sqrt[3]{12}, \sqrt{5} \]

\[ \sqrt{4}, 4 \quad 1\frac{3}{7} \]

b) No, because all odd numbers are rational.

**Venn diagrams showing frequencies.** SAY: Venn diagrams can show the data, but they can also show the *frequency* in the data set as well—in other words, they can show how often a value occurs in a set of data. Point to the Venn diagram for the factors of 9 and 12 and ASK: How many numbers are factors of 12? (6) SAY: We say the frequency of factors of 12 is 6. ASK: What is the frequency of the factors of 9? (3) What is the frequency of the factors of both 9 and 12? (2) How many numbers are not factors of 9 and not factors of 12? (5) Draw on the board:

**Frequencies of Factors of 9 and of 12 in Numbers 1 to 12**

![Venn diagram showing factors of 9 and 12]

Point to the Venn diagram and SAY: There are 6 factors of 12 in the data set, but 2 of them are already in the intersection of the ovals. ASK: How many of factors of 12 are not factors of 9? (4) How many of factors of 9 are not factors of 12? (1) Complete the Venn diagram, as shown below:

**Frequencies of Factors of 9 and of 12 in Numbers 1 to 12**

![Completed Venn diagram showing factors of 9 and 12]

ASK: If you add all frequencies, what do you get? (12) SAY: This is an important property of Venn diagrams for frequencies: if you add up all numbers in the different sections, you get the total number of data.

**NOTE:** Leave this Venn diagram on the board for later use.
(MP.5) Exercise: Draw a Venn diagram to show the frequencies for odd and irrational numbers in the previous exercises.

Answer:

![Venn Diagram]

Introduce two-way tables. Explain to students that Venn diagrams are a good way to visually organize data in categories, and they give information about specific aspects of the data. Point to the Venn diagram of frequencies for factors of 9 and 12 from earlier in the lesson:

![Venn Diagram of Factors]

Frequencies of Factors of 9 and of 12 in Numbers 1 to 12

<table>
<thead>
<tr>
<th>Factors of 9</th>
<th>Factors of 12</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

ASK: What is the number of factors of 12? (6) How did you get the number 6? (by adding 2 and 4) How many numbers are there in total? (12) How do you know? (1 + 2 + 4 + 5 = 12)

SAY: I wish I had more information about these two categories—for example, the totals. Fortunately, there is another tool in statistics that provides this information. It is called a two-way table. Explain to students that a two-way table is a table that shows the observed number of each variable. The rows represent one category and the columns represent another category. SAY: Usually, the final row and the final column in the table show totals. Draw on the board:

<table>
<thead>
<tr>
<th>Factor of 12</th>
<th>Not a factor of 12</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor of 9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Not a factor of 9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Point to the top-left cell in the table and SAY: This cell on the table will show the intersection of “Factor of 9” and “Factor of 12,” so this cell must show the number of data that are factors of both 9 and 12. Point to the Venn diagram and ASK: How many numbers from 1 to 12 are factors of 9 and factors of 12? (2) Write “2” in the top-left cell. SAY: In the next cell to the right, this cell must show data for numbers that are factors of 9 but not factors of 12. ASK: How many numbers are factors of 9 and not factors of 12? (1) SAY: Now I’m going to fill in the second row, for “Not a factor of 9.” ASK: How many numbers between 1 and 12 are not factors of 9 but are
factors of 12? (4) How many numbers are not factors of 9 and not factors of 12? (5) Complete the two-way table, as shown below:

<table>
<thead>
<tr>
<th></th>
<th>Factor of 12</th>
<th>Not a factor of 12</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor of 9</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Not a factor of 9</td>
<td>4</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>Total</td>
<td>6</td>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>

SAY: By creating and filling in a table about frequencies and totals, we can use rows and columns to show different ways of looking at the data. Help students to see that the Venn diagram visually organizes information and summarizes the information, but the two-way table gives more detailed information. For example, the two-way table shows totals across (by rows) and it gives totals vertically (by columns).

Exercise: Make a two-way table to show frequency in the Venn diagram of odd and irrational numbers from the previous exercise.

Answer:

<table>
<thead>
<tr>
<th></th>
<th>Irrational</th>
<th>Not Irrational</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Odd</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Not Odd</td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Total</td>
<td>2</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

Tell students that if you delete the total row from the two-way table, you will have what is called a row two-way table because the totals are only given across—by rows—but not by columns. Here is the row two-way table for the two-way table shown above:

<table>
<thead>
<tr>
<th></th>
<th>Factor of 12</th>
<th>Not a factor of 12</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor of 9</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Not a factor of 9</td>
<td>4</td>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>

(MP.5) Drawing Venn diagrams from two-way tables. SAY: We have already made a two-way table using a Venn diagram. We can also draw a Venn diagram using a two-way table. Draw on the board:

<table>
<thead>
<tr>
<th></th>
<th>Laptop</th>
<th>No Laptop</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell Phone</td>
<td>19</td>
<td>18</td>
<td>37</td>
</tr>
<tr>
<td>No Cell Phone</td>
<td>9</td>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>Total</td>
<td>28</td>
<td>22</td>
<td>50</td>
</tr>
</tbody>
</table>
SAY: Fifty students were asked if they have a cell phone and a laptop, and the results are shown in the two-way table. Draw a Venn diagram on the board beside the two-way table:

```
<table>
<thead>
<tr>
<th></th>
<th>Cell Phone</th>
<th>Laptop</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

ASK: How many students have a cell phone and laptop? (19) Write 19 in the intersection of the ovals. Explain to students that it is better to start from the intersection of the ovals because it makes it easier to find the other parts. ASK: How many have a cell phone but no laptop? (18) How many have no cell phone but have a laptop? (9) How many don't have a cell phone and don’t have a laptop? (4) Complete the Venn diagram, as shown below:

```
<table>
<thead>
<tr>
<th></th>
<th>Cell Phone</th>
<th>Laptop</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

**Exercises:** Complete BLM Venn Diagrams and Two-Way Tables.

**Answers:**
1. a) | Factor of 10 | Not a factor of 10 | Total |
      |---------------|-------------------|-------|
Factor of 6 | 2              | 2                 | 4     |
Not a factor of 6 | 2            | 4                 | 6     |
Total | 4              | 6                 | 10    |

b) | Playing Online | Not Playing Online | Total |
   |----------------|-------------------|-------|
Reading | 7               | 11                | 18    |
Not Reading | 18            | 2                 | 20    |
Total | 25              | 13                | 38    |

c) | Skates | No Skates | Total |
    |--------|-----------|-------|
Bike | 24     | 47        | 71    |
No Bike | 11      | 18        | 29    |
Total | 35     | 65        | 100   |
Finding missing terms in a two-way table. Explain to students that we can find missing terms in a two-way table by using the row totals and column totals. SAY: Ben surveyed 100 students and asked if they are soccer fans, basketball fans, or neither. But when Ben wanted to describe his results, he found that his sister had spilled some ink on the two-way table, so only some parts of the table are still readable. Draw on the board:

<table>
<thead>
<tr>
<th>Basketball Fan</th>
<th>Not Basketball Fan</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soccer Fan</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Not Soccer Fan</td>
<td></td>
<td>68</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>57</td>
<td>100</td>
</tr>
</tbody>
</table>

SAY: Fortunately, Ben has enough information to complete the table. We will start with the columns or rows that have the most numbers in them; that will help us fill in the missing data. In the first column, two numbers are readable. In total, 57 students are basketball fans and 11 of them are also soccer fans. ASK: How many of those basketball fans are not soccer fans? (46) How did you get that number? (by subtracting 11 from 57) Write “46” in the empty cell in the first column. SAY: There are 100 students in the total column and 68 are not soccer fans. ASK: How many students are soccer fans in total? (32) How did you get that number? (by subtracting 68 from 100) Point to the first row and SAY: There are 32 soccer fans in total and 11 of them are also basketball fans. ASK: How many of the soccer fans are not basketball fans? (21) Ask a volunteer to complete the table, as shown below:

<table>
<thead>
<tr>
<th>Basketball Fan</th>
<th>Not Basketball Fan</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soccer Fan</td>
<td>11</td>
<td>32</td>
</tr>
<tr>
<td>Not Soccer Fan</td>
<td>46</td>
<td>68</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>57</td>
<td>100</td>
</tr>
</tbody>
</table>
(MP.3) **Exercises:** Find the missing terms in the two-way table or row two-way table.

### a)

<table>
<thead>
<tr>
<th>Playing Online</th>
<th>Not Playing Online</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reading</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>Not Reading</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>31</strong></td>
<td></td>
</tr>
</tbody>
</table>

### b)

<table>
<thead>
<tr>
<th>Baseball</th>
<th>No Baseball</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swimming</td>
<td>11</td>
<td>23</td>
</tr>
<tr>
<td>No Swimming</td>
<td>19</td>
<td>35</td>
</tr>
</tbody>
</table>

### c)

<table>
<thead>
<tr>
<th>Internet</th>
<th>No Internet</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>TV</td>
<td>30</td>
<td>69</td>
</tr>
<tr>
<td>No TV</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>52</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

### d)

<table>
<thead>
<tr>
<th>Hiking</th>
<th>No Hiking</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Camping</td>
<td>21</td>
<td>32</td>
</tr>
<tr>
<td>No Camping</td>
<td>9</td>
<td>19</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>30</strong></td>
<td><strong>60</strong></td>
</tr>
</tbody>
</table>

**Answers:**

### a)

<table>
<thead>
<tr>
<th>Playing Online</th>
<th>Not Playing Online</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reading</td>
<td>18</td>
<td>27</td>
</tr>
<tr>
<td>Not Reading</td>
<td>13</td>
<td>24</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>31</strong></td>
<td><strong>51</strong></td>
</tr>
</tbody>
</table>

### b)

<table>
<thead>
<tr>
<th>Baseball</th>
<th>No Baseball</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swimming</td>
<td>12</td>
<td>23</td>
</tr>
<tr>
<td>No Swimming</td>
<td>19</td>
<td>35</td>
</tr>
</tbody>
</table>

### c)

<table>
<thead>
<tr>
<th>Internet</th>
<th>No Internet</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>TV</td>
<td>39</td>
<td>69</td>
</tr>
<tr>
<td>No TV</td>
<td>13</td>
<td>18</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>52</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

### d)

<table>
<thead>
<tr>
<th>Hiking</th>
<th>No Hiking</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Camping</td>
<td>21</td>
<td>32</td>
</tr>
<tr>
<td>No Camping</td>
<td>9</td>
<td>19</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>30</strong></td>
<td><strong>60</strong></td>
</tr>
</tbody>
</table>

**Extensions**

(MP.3, MP.5) 1. A librarian records the number of visitors who borrow books or magazines and the number who borrow music or movies. In one month, 18 visitors borrow books or magazines and music or movies; 47 visitors borrow books or magazines but no music or movies. The library has 123 total visitors and a total of 39 borrow music or movies.

a) Make a two-way table for the data.
b) Draw a Venn diagram for the table.
c) How many visitors did not borrow music or movies that month?
### Answers:

a)  

<table>
<thead>
<tr>
<th></th>
<th>Music or Movies</th>
<th>No Music or Movies</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Books or Magazines</td>
<td>18</td>
<td>47</td>
<td>65</td>
</tr>
<tr>
<td>No Books or Magazines</td>
<td>21</td>
<td>37</td>
<td>58</td>
</tr>
<tr>
<td>Total</td>
<td>39</td>
<td>84</td>
<td>123</td>
</tr>
</tbody>
</table>

b)  

![Venn Diagram](image)

c) 84

(MP.4, MP.5) 2. Rob has $131 and saves $15 each week. Mindy has $224 and saves $12 each week. When they have the same amount of money, they decide to go shopping together. After how many weeks will they go shopping together? Use any tools you think will help.

**Sample answers:**

- I used equations. Let $x$ be the number of weeks that passed. The amount of money that Rob has, in dollars, is $131 + 15x$, and the amount of money that Mindy has, in dollars, is $224 + 12x$. If they have the same amount of money, then $131 + 15x = 224 + 12x$, so $15x - 12x = 224 - 131$, or $3x = 93$, so $x = 31$. So, after 31 weeks, Rob and Mindy will go shopping together.
- I used a table with headings “Rob’s money” and “Mindy’s money.” It was taking a long time, so I decided to go up by 10 weeks at first, until I got close, then I went up one week at a time. Rob and Mindy will go shopping together after 31 weeks.
- I used mental math. I noticed that Mindy starts with $93 more than Rob, and Rob saves $3 a week more, so it takes Rob 31 weeks to catch up with Mindy because $93 ÷ 3 = 31$.

Whole-class follow-up: Discuss the advantages of using equations over numbers. If you use equations, you don’t have to notice anything special that is going on—equations always work. Also discuss the advantages of using equations over a table—they are much faster.

**NOTE:** A student who uses a table by going up one at a time and changes approach after realizing that it is taking a long time is persevering to solve the problem (MP.1).
SP8-10  Two-Way Relative Frequency Tables

Pages 193–194

Standards: 8.SP.A.4

Goals:
Students will represent categorical data in a two-way frequency table or relative frequency table.

Prior Knowledge Required:
Can draw two-way tables

Vocabulary: frequency, frequency table, relative frequency table, two-way relative frequency table, two-way table

Introduce relative frequency tables with fractions. Draw on the board:

<table>
<thead>
<tr>
<th>Favorite type of movie</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of Movie</td>
</tr>
<tr>
<td>Comedy</td>
</tr>
<tr>
<td>Action</td>
</tr>
<tr>
<td>Horror</td>
</tr>
<tr>
<td>Other</td>
</tr>
</tbody>
</table>

SAY: I surveyed 80 students in Grades 7 and 8 about their favorite type of movie, and tallied the results in this table. We can use this data to create a relative frequency table. Draw on the board:

<table>
<thead>
<tr>
<th>Type of Movie</th>
<th>Number of People</th>
<th>Fraction of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comedy</td>
<td>12</td>
<td>$\frac{12}{80}$</td>
</tr>
<tr>
<td>Action</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horror</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Point out that the table shows both the number of times a data value occurs in a set (for example, comedy is the favorite for 12 people) and that number relative to all data (for example, comedy is the favorite for 12 of 80 surveyed). Have volunteers complete the table. (see completed table below)

<table>
<thead>
<tr>
<th>Type of Movie</th>
<th>Number of People</th>
<th>Fraction of People</th>
</tr>
</thead>
</table>
| Comedy        | 12               | \[
\frac{12}{80}
\] |
| Action        | 16               | \[
\frac{16}{80}
\] |
| Horror        | 32               | \[
\frac{32}{80}
\] |
| Other         | 20               | \[
\frac{20}{80}
\] |

SAY: In contrast, a frequency table would show only the number of times a data value occurs in a set. ASK: What should the numbers add to? (80) Why? (because you surveyed 80 people) What should the fractions add to? (1) Why? (because 80 out of 80 people is everyone, so the fraction is 1/1 or 1) SAY: Indeed, the numbers add to 12 + 16 + 32 + 20 = 80 and the fractions add to 12/80 + 16/80 + 32/80 + 20/80 = 80/80 = 1.

NOTE: Leave this table on the board for future use.

Review converting fractions to percentages. Tell students that people often write frequency tables using percentages instead of fractions. SAY: Adding whole numbers, or even decimal numbers, is easier than adding fractions. Writing a number as a percentage means writing it as a fraction of 100, but we leave off the “/100” part to make the number easier to work with and to compare. Remind students how to convert fractions to percentages: change the fraction to a fraction with denominator 100 and then write that fraction as a percentage. Write an example on the board:

\[
\frac{3}{20} = \frac{15}{100} = 15\%
\]

Exercises: Convert the fraction to a percentage.

a) \[
\frac{3}{10}
\]  
  b) \[
\frac{4}{5}
\]  
  c) \[
\frac{6}{25}
\]  
  d) \[
\frac{3}{50}
\]  
  e) \[
\frac{9}{20}
\]  
  f) \[
\frac{3}{4}
\]

Answers: a) 30%, b) 80%, c) 24%, d) 6%, e) 45%, f) 75%

Remind students that sometimes the denominator of the fraction does not divide evenly into 100. In such cases, students can reduce the fraction to make it have a denominator that does divide evenly into 100. Write on the board:

\[
\frac{6}{8} = \frac{3}{4} = \frac{75}{100} = 75\%
\]
Exercises: Convert the fraction to a percentage.

a) \(\frac{8}{40}\)  
b) \(\frac{2}{8}\)  
c) \(\frac{18}{75}\)  
d) \(\frac{14}{70}\)  
e) \(\frac{9}{15}\)  
f) \(\frac{36}{48}\)

Answers: a) 20%, b) 25%, c) 24%, d) 20%, e) 60%, f) 75%

SAY: All these worked out nicely to whole-number percentages, but some fractions don’t.

ASK: How would you write \(\frac{4}{15}\) as a percent? (4 ÷ 15 × 100 = 26.666… or 26.\(\frac{2}{3}\))
PROMPT: You can write \(\frac{4}{15}\) as a decimal by dividing 4 by 15, then you can write it as a percentage by multiplying by 100.

Exercises: Convert the fraction to a percentage.

a) \(\frac{47}{500}\)  
b) \(\frac{17}{200}\)  
c) \(\frac{23}{45}\)  
d) \(\frac{16}{55}\)  
e) \(\frac{13}{15}\)

Answers: a) 9.4%, b) 8.5%, c) 51.\(\frac{1}{10}\)%, d) 29.\(\frac{1}{10}\)%, e) 86.\(\frac{6}{15}\)%

Introduce relative frequency tables with percentages. Have a volunteer add another column to the relative frequency table about favorite movie types, with the heading “Percent of People,” and fill in the column. (Comedy 15%, Action 20%, Horror 40%, Other 25%)

Before completing the exercise below, review hockey terms with students. Ask a volunteer to explain what the terms “even strength,” “power play,” and “short-handed” mean in ice hockey. (“even strength” means both teams have the same number of players on the ice; “short handed” is when one team has a penalty and that team has one fewer player than the other team on the ice; the team with more players on the ice is said to have a “power play”)

Exercise: Complete the relative frequency table for 20 hockey games.

<table>
<thead>
<tr>
<th>Hockey Team’s Goals Scored</th>
<th>Number</th>
<th>Fraction</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Even Strength</td>
<td>45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power Play</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short Handed</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Answers: 

<table>
<thead>
<tr>
<th>Hockey Team’s Goals Scored</th>
<th>Number</th>
<th>Fraction</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Even Strength</td>
<td>45</td>
<td>45/60</td>
<td>75</td>
</tr>
<tr>
<td>Power Play</td>
<td>12</td>
<td>12/60</td>
<td>20</td>
</tr>
<tr>
<td>Short Handed</td>
<td>3</td>
<td>3/60</td>
<td>5</td>
</tr>
</tbody>
</table>

Interpreting relative frequency tables. Point to the relative frequency table from the exercise above and SAY: We can interpret the data by looking at the relative frequency. ASK: Do you think it was harder to score on the power play or at even strength? (power play) Why did this team score so many more goals at even strength than on the power play? (they were probably playing even strength for a much greater portion of their games) What is the average number of goals scored per game? (60 ÷ 20 = 3)
Two-way relative frequency tables. Explain to students that with the table about hockey, they made a relative frequency table for data in one category, but they can also make a relative frequency table for a two-way table, called a two-way relative frequency table. Draw on the board:

<table>
<thead>
<tr>
<th></th>
<th>Hiking</th>
<th>No Hiking</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Camping</td>
<td>21</td>
<td>11</td>
<td>32</td>
</tr>
<tr>
<td>No Camping</td>
<td>9</td>
<td>19</td>
<td>28</td>
</tr>
<tr>
<td>Total</td>
<td>30</td>
<td>30</td>
<td>60</td>
</tr>
</tbody>
</table>

Tell students that the two-way table shows the results of a survey of 60 students about who went camping and hiking last summer. Circle the number 60 in the table. ASK: How many students both camped and hiked last summer? (21) Ask a volunteer to find what percent of students both camped and hiked. \((21 \div 60 \times 100 = 35\%)\) SAY: You have to divide 21 ÷ 60 to write 21/60 as a decimal, and then multiply by 100 to write it as a percentage. ASK: How many students camped in total? (32) Write on the board:

\[
32 \div 60 \times 100 \approx 53.33\%
\]

Explain to students that usually we round percentages to the nearest whole number, so we say about 53% of students camped. ASK: How many students did not camp in total? (28) Write on the board:

\[
28 \div 60 \times 100 \approx 46.66\%
\]

SAY: By rounding to the nearest whole number, we can say about 47% of students didn’t camp last summer. Explain to students that they could also find 47% by subtracting the 53% of students who did camp from 100% of students. Complete a relative frequency table for the two-way table, as shown below:

<table>
<thead>
<tr>
<th></th>
<th>Hiking</th>
<th>No Hiking</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Camping</td>
<td>35%</td>
<td>18%</td>
<td>53%</td>
</tr>
<tr>
<td>No Camping</td>
<td>15%</td>
<td>32%</td>
<td>47%</td>
</tr>
<tr>
<td>Total</td>
<td>50%</td>
<td>50%</td>
<td>100%</td>
</tr>
</tbody>
</table>
**Exercise:** Students were surveyed about who swims and who plays baseball. Use the two-way table to complete a two-way relative frequency table.

<table>
<thead>
<tr>
<th></th>
<th>Baseball</th>
<th>No Baseball</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swimming</td>
<td>13</td>
<td>12</td>
<td>25</td>
</tr>
<tr>
<td>No Swimming</td>
<td>24</td>
<td>19</td>
<td>43</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>37</td>
<td>31</td>
<td>68</td>
</tr>
</tbody>
</table>

**Answer:**

<table>
<thead>
<tr>
<th></th>
<th>Baseball</th>
<th>No Baseball</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swimming</td>
<td>19%</td>
<td>18%</td>
<td>37%</td>
</tr>
<tr>
<td>No Swimming</td>
<td>35%</td>
<td>28%</td>
<td>63%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>54%</td>
<td>46%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Point to the two-way relative frequency table from the exercise above and ASK: What percent of students swim and play baseball? (19%) What percent of students swim and do not play baseball? (18%) SAY: If I choose a student among students who swim, the chance of that person playing baseball or not playing baseball is almost the same. ASK: What percent of students play baseball and don’t swim? (35%) What percent of students don’t play baseball and don’t swim? (28%) Point out that 35% is bigger than 18%, and that means that it’s more likely for someone to play baseball and not swim than for someone to swim and not play baseball.

**(MP.4, MP.5) Exercises:** Mike surveyed 75 students randomly at his school about being basketball fans and football fans and got the following results:

<table>
<thead>
<tr>
<th></th>
<th>Basketball</th>
<th>No Basketball</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Football</td>
<td>24</td>
<td>7</td>
<td>31</td>
</tr>
<tr>
<td>No Football</td>
<td>12</td>
<td>32</td>
<td>44</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>36</td>
<td>39</td>
<td>75</td>
</tr>
</tbody>
</table>

a) Use the two-way table to complete a two-way relative frequency table.

b) Based on the relative frequency table, in which group can you find more basketball fans: among students who are also football fans or among students who are not football fans?
**Answers:**

a) | Basketball | No Basketball | Total |
---|------------|-------------|------|
Football | 32%        | 9%          | 41%  |
No Football | 16%        | 43%         | 59%  |
**Total** | **48%**    | **52%**     | **100%** |

b) students who are also football fans (32%)

**Extensions**

**MP.3** 1. Kim surveyed 80 students at her school and found the relative frequencies in the tables below. Find the missing numbers in the table.

**a) Favorite type of television show**

<table>
<thead>
<tr>
<th>Type</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sitcom</td>
<td>25%</td>
<td></td>
</tr>
<tr>
<td>Drama</td>
<td>30%</td>
<td></td>
</tr>
<tr>
<td>Reality TV</td>
<td>15%</td>
<td></td>
</tr>
<tr>
<td>Game Show</td>
<td>10%</td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td>20%</td>
<td></td>
</tr>
</tbody>
</table>

**b) Favorite type of video games**

<table>
<thead>
<tr>
<th>Genre</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Action</td>
<td>16</td>
<td>40%</td>
</tr>
<tr>
<td>Adventure</td>
<td>16</td>
<td>20%</td>
</tr>
<tr>
<td>Role-Playing</td>
<td>20</td>
<td>25%</td>
</tr>
<tr>
<td>Strategy</td>
<td>8</td>
<td>5%</td>
</tr>
<tr>
<td>Other</td>
<td>8</td>
<td>10%</td>
</tr>
</tbody>
</table>

**Answers:**

a) **Favorite type of television show**

<table>
<thead>
<tr>
<th>Type</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sitcom</td>
<td>20</td>
<td>25%</td>
</tr>
<tr>
<td>Drama</td>
<td>24</td>
<td>30%</td>
</tr>
<tr>
<td>Reality TV</td>
<td>12</td>
<td>15%</td>
</tr>
<tr>
<td>Game Show</td>
<td>8</td>
<td>10%</td>
</tr>
<tr>
<td>Other</td>
<td>16</td>
<td>20%</td>
</tr>
</tbody>
</table>

b) **Favorite type of video games**

<table>
<thead>
<tr>
<th>Genre</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Action</td>
<td>32</td>
<td>40%</td>
</tr>
<tr>
<td>Adventure</td>
<td>16</td>
<td>20%</td>
</tr>
<tr>
<td>Role-Playing</td>
<td>20</td>
<td>25%</td>
</tr>
<tr>
<td>Strategy</td>
<td>4</td>
<td>5%</td>
</tr>
<tr>
<td>Other</td>
<td>8</td>
<td>10%</td>
</tr>
</tbody>
</table>

**MP.4** 2. Zara makes punch using grape juice and ginger ale. She mixes 8 cups of grape juice with 11 cups of ginger ale. The mixture has 2,000 calories. When she adds another 2 cups of ginger ale and 1 cup of grape juice, the mixture has 2,300 calories. Which has more calories, a cup of grape juice or a cup of ginger ale? How many more?

**Solution:** Let $x$ be the number of calories in a cup of grape juice and $y$ the number of calories in a cup of ginger ale. Then $8x + 11y = 2,000$ and $x + 2y = 300$ (or $9x + 13y = 2,300$). Solving the system of equations, I get $x = 140$ and $y = 80$. So there are 60 more calories in a cup of grape juice than in a cup of ginger ale.
SP8-11 Interpreting Two-Way Relative Frequency Tables

Pages 195–197

Standards: 8.SP.A.4

Goals:
Students will find patterns in bivariate categorical data displayed in two-way tables.
Students will calculate relative frequencies to describe possible associations between two variables.

Prior Knowledge Required:
Can draw two-way tables
Can calculate and construct two-way relative frequency tables

Vocabulary: row two-way relative frequency table, row two-way table, two-way relative frequency table, two-way table

Materials:
BLM Sports Fans (p. R-61)

Understanding two-way relative frequency tables. Remind students that when they work with a row two-way table, the total of a row is the sum of all numbers in that row. Draw on the board:

<table>
<thead>
<tr>
<th></th>
<th>Watching TV</th>
<th>Not Watching TV</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 Year Old</td>
<td>12</td>
<td>11</td>
<td>23</td>
</tr>
<tr>
<td>13 Year Old</td>
<td>19</td>
<td>16</td>
<td>35</td>
</tr>
</tbody>
</table>

SAY: To create the row two-way relative frequency table, I can divide all numbers in the first row by 23 and multiply the result by 100 to make percentages. Draw on the board:

<table>
<thead>
<tr>
<th></th>
<th>Watching TV</th>
<th>Not Watching TV</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 Year Old</td>
<td>52%</td>
<td>48%</td>
<td>100%</td>
</tr>
<tr>
<td>13 Year Old</td>
<td>54%</td>
<td>46%</td>
<td>100%</td>
</tr>
</tbody>
</table>

ASK: Can we say that 52% of students who watch TV are 10 years old? (no, the first column does not add to 100%) SAY: Actually, the relative frequency 52% comes from dividing 12 by the sum of the row, 23, which is all 10-year-old students. Explain to students that in a row two-way relative frequency table, we examine variables by rows. Point to the row two-way relative frequency table and ASK: Is there a significant difference between the two rows of the
Can I say that a greater proportion of older students watch TV than do younger students? (no) How do you know? (52% and 54% are not very different) SAY: If the relative frequencies are about the same for the different rows of data, you can say there is no association between the two variables. Draw on the board:

<table>
<thead>
<tr>
<th></th>
<th>Online Gaming</th>
<th>No Online Gaming</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 Year Old</td>
<td>32%</td>
<td>68%</td>
<td>100%</td>
</tr>
<tr>
<td>13 Year Old</td>
<td>57%</td>
<td>43%</td>
<td>100%</td>
</tr>
</tbody>
</table>

ASK: Can I say that a greater proportion of older students play online games than do younger students? (yes) How do you know? (57% is much more than 32%) SAY: Between the ages 10 and 13, the percentage of students who play online increases and the percentage of students who don’t play online decreases. That means there is a positive association between playing online and age.

(MP.7) Exercises: The row two-way table shows the results of a survey of Grade 8 students about reading books and playing online games.

<table>
<thead>
<tr>
<th></th>
<th>Playing Online</th>
<th>Not Playing Online</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reading</td>
<td>7</td>
<td>11</td>
<td>18</td>
</tr>
<tr>
<td>Not Reading</td>
<td>18</td>
<td>6</td>
<td>24</td>
</tr>
</tbody>
</table>

a) Complete a row two-way relative frequency table for the row two-way table.
b) Who is more likely to play online—someone who reads books or someone who doesn’t read books?  
c) If the number of students who read books increases, would you expect the percentage of students who play online games to increase, decrease, or not be affected? Why?  
d) Is there any association between reading and playing online? If so, is it a positive association or a negative association?  

Answers:
a)  

<table>
<thead>
<tr>
<th></th>
<th>Playing Online</th>
<th>Not Playing Online</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reading</td>
<td>39%</td>
<td>61%</td>
<td>100%</td>
</tr>
<tr>
<td>Not Reading</td>
<td>75%</td>
<td>25%</td>
<td>100%</td>
</tr>
</tbody>
</table>

b) someone who doesn’t read books, c) decrease, because the percentage of students who read and play online is less, d) yes, there is a negative association

Two-way tables with three or more columns. Explain to students that it’s not necessary to always have two rows and two columns. SAY: The number of rows and the number of columns can be more than two.
Exercises: Complete BLM Sports Fans.

Answers:

<table>
<thead>
<tr>
<th></th>
<th>Basketball</th>
<th>Baseball</th>
<th>Football</th>
<th>Other</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 6</td>
<td>17%</td>
<td>22%</td>
<td>35%</td>
<td>26%</td>
<td>100%</td>
</tr>
<tr>
<td>Grade 8</td>
<td>19%</td>
<td>21%</td>
<td>46%</td>
<td>14%</td>
<td>100%</td>
</tr>
</tbody>
</table>

b) i) other, baseball, ii) football, basketball; c) no, basketball and baseball are about the same, so there is no association; d) yes, positive association; e) yes, between grade level and being a fan of other sports

Extensions

(MP.3, MP.4) 1. A phone company keeps track of which customers make late payments. The company keeps track of late payments for landlines and mobile phones, and if the payments are late for the first time or repeatedly late. The company uses a table to keep track of the results.

a) In one year, 993 mobile customers and 1,235 landline customers were late for the first time. In that same year, 261 mobile customers and 352 landline customers were repeated late customers. Make a row two-way table for the data.

b) How many landline customers did not pay on time this year?

c) Make a row two-way relative frequency table for the data.

d) Based on the data, is there an association between the type of phone and repeated lateness? Explain.

Answers:

<table>
<thead>
<tr>
<th></th>
<th>First-Time</th>
<th>Repeat</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Landline</td>
<td>1235</td>
<td>352</td>
<td>1587</td>
</tr>
<tr>
<td>Mobile</td>
<td>993</td>
<td>261</td>
<td>1254</td>
</tr>
</tbody>
</table>

b) 1,587

c) |               | First-Time | Repeat | Total |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Landline</td>
<td>78%</td>
<td>22%</td>
<td>100%</td>
</tr>
<tr>
<td>Mobile</td>
<td>79%</td>
<td>21%</td>
<td>100%</td>
</tr>
</tbody>
</table>

d) The relative frequencies are about the same for the rows of data, so there is no association between the two variables.

(MP.4) 2. The ratio of mass to volume is constant for any material. For pure gold, the ratio is 19.32 g : 1 cm$^3$. Tess has a coin that looks like gold, but she’s not sure if it is pure gold or not. Her coin is a cylinder with diameter 2 cm and height 0.4 cm. It weighs 21.35 g. Is her coin pure gold? Explain.

Answer: The volume of the coin is $\pi r^2 h$, where $r$ is the radius and $h$ is the height of the coin. So the volume is about $(3.14)(1 \text{ cm})^2(0.4 \text{ cm}) = 1.256 \text{ cm}^3$. The coin weighs 21.35 g, so the ratio of mass to volume is 21.35 g : 1.256 cm$^3 \approx 17.0$ g : 1 cm$^3$. This is not the ratio for pure gold, so her coin is not pure gold.
SP8-12 Cumulative Review

Pages 198–200

Standards: 8.SP.A.1, 8.SP.A.2, 8.SP.A.3, 8.SP.A.4

Goals:
Students will review and work with scatter plots, the line of best fit and its application, and two-way tables and Venn diagrams.
Students will construct and interpret frequency tables, relative frequency tables, and two-way relative frequency tables.

Prior Knowledge Required:
Can draw scatter plots
Can recognize and describe patterns of association in a scatter plot
Can draw the line of best fit informally
Can write the equation for the line of best fit
Is familiar with Venn diagrams
Can draw two-way tables
Can draw two-way relative frequency tables

Vocabulary: line of best fit, row two-way relative frequency table, two-way table

Review. Lesson SP8-12 on AP Book 8.2 p. 198–200 offers a cumulative review of Grade 8 statistics and can be used as consolidating review.

Exercises:
(MP.3, MP.4, MP.6) 1. The table shows the weight of a dog at various times from birth to 48 weeks of age.

<table>
<thead>
<tr>
<th>Weeks</th>
<th>0</th>
<th>8</th>
<th>16</th>
<th>22</th>
<th>28</th>
<th>37</th>
<th>42</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (pounds)</td>
<td>4</td>
<td>7</td>
<td>10</td>
<td>12</td>
<td>13.5</td>
<td>15</td>
<td>16</td>
<td>16.5</td>
</tr>
</tbody>
</table>

a) Make a scatter plot of the data and draw the line of best fit.
b) Find the slope of the line of best fit and describe what it represents.
c) Write an equation for the line of best fit.
d) What is the difference between the y-intercept for the line of best fit and the weight of the dog at birth?
e) Use the equation of the line to predict the weight of this dog when it is 52 weeks old.
Answers:

a) 

b) using (8, 7) and (37, 15), slope = 0.28 and it represents the weight that the dog gains per week; 
c) \( y = 0.28x + 4.76 \); d) \( 4.76 - 4 = 0.76 \) pounds; e) \( y = 0.28(52) + 4.76 = 19.32 \) pounds

(MP.3, MP.4) 2. A hairdresser recorded all the customers she had for cuts and other procedures for the last month, and if the customer was blonde or brunette. She had 19 blonde customers for cuts and 41 blonde customers for other procedures. She had 61 total brunette customers and a total of 39 customers for cuts.

a) Make a two-way table for the data.

b) Draw a Venn diagram for the table.

c) How many customers in the month did not have cuts?

Answers:

b) 

c) 82

3. a) Make a row two-way relative frequency table for the data in Exercise 2.

b) Based on the data, is there an association between hair color and the procedure for the customers?

Answers:

a) 

b) The relative frequencies are about the same for the rows of data, so there is no association between the two variables.
Extensions
1. Make a row two-way relative frequency table for the row two-way table. Based on the data, is there any association between the variables?
   a) Glasses |
   No Glasses |
   **Total**
   | 10 Year Old | 5 | 24 | 29 |
   | 12 Year Old | 15 | 70 | 85 |
   b) Glasses |
   No Glasses |
   **Total**
   | 20 Year Old | 23 | 61 | 84 |
   | 45 Year Old | 72 | 25 | 97 |

Selected solution:
   b) Glasses |
   No Glasses |
   **Total**
   | 20 Year Old | 27% | 73% | 100% |
   | 45 Year Old | 74% | 26% | 100% |

Yes, there is a positive association between age and wearing glasses.
Answer: a) No, there is no association.

(MP.2, MP.4) 2. A can of soup contains 1,080 mg of sodium per 240 mL serving, or 45% of the recommended daily limit. The soup can has diameter 8.5 cm and height 10.7 cm.
a) About what percentage of the daily limit of sodium would you get if you ate the entire can of soup?
b) What fact did you not need to use? Explain.

Answers: a) The volume of the soup in the can is \(\pi r^2 h\), where \(r\) is the radius and \(h\) is the height of the soup can. So the volume is about \((3.14)(4.25 \text{ cm})^2(10.7 \text{ cm}) \approx 607 \text{ cm}^3 = 607 \text{ mL}\). To find the percentage of the recommended daily limit, I used the proportion \(\text{mL} : \% = 240 : 45 \approx 607 : x\), so \(240x = 45(607)\), \(x \approx 45(607) / 240 \approx 114\). So, if I ate the whole can of soup, I would get about 114% of the daily limit.
b) I didn’t need to use the amount of sodium because the question was only asking for the percentage of the daily limit.

Whole-class follow-up: Have students discuss with a partner what it means that the answer they got is more than 100%. (It means that they would get more than the daily limit of sodium if they ate the whole can of soup)

MP.6 Assessment Opportunity: Question 4 on AP Book 8.2 p. 198
Introduce the Line of Best Fit

1. The scatter plot shows the height and arm span for 14 Grade 8 students.

Count the number of points above and below the line.

A. 

Points above: _____  
Points below: _____

B. 

Points above: _____  
Points below: _____

C. 

Points above: _____  
Points below: _____
Drawing the Line of Best Fit

1. Draw the line of best fit for the scatter plot. Count the number of points above and below the line.

   a) Points above: _____  Points below: _____
   b) Points above: _____  Points below: _____
   c) Points above: _____  Points below: _____

   d) Points above: _____  Points below: _____
   e) Points above: _____  Points below: _____
   f) Points above: _____  Points below: _____

   g) Points above: _____  Points below: _____
   h) Points above: _____  Points below: _____
   i) Points above: _____  Points below: _____

   j) Points above: _____  Points below: _____
   k) Points above: _____  Points below: _____
   l) Points above: _____  Points below: _____
Use Microsoft Excel® for the following exercise.

1. The table shows how many hours students study and the math marks for 13 students in a class.

<table>
<thead>
<tr>
<th>Study (hours)</th>
<th>4</th>
<th>7</th>
<th>2</th>
<th>5</th>
<th>4</th>
<th>1</th>
<th>3</th>
<th>6</th>
<th>7</th>
<th>3</th>
<th>5</th>
<th>8</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math Mark</td>
<td>72</td>
<td>85</td>
<td>67</td>
<td>79</td>
<td>74</td>
<td>55</td>
<td>65</td>
<td>93</td>
<td>87</td>
<td>69</td>
<td>77</td>
<td>90</td>
<td>71</td>
</tr>
</tbody>
</table>

a) In the first row of column A, type “Study (hours).” In the first row of column B, type “Math mark.”

b) Starting from the second row, enter data to columns A and B (for example, for the first data point, enter “4” in column A and “72” in column B. The last data point will be in the 14th row.

c) Select all data, including the first row. (All data points turn blue when you select them.)

d) Under the Insert menu, select “Charts” and then “Scatter.” This will create the scatter plot.

e) Under the Chart Tools menu, select “Layout” and then “Trendline” to see the line of best fit. Save your file to use later.

f) In the 15th row of the table, enter data for an additional student: (6, 85).

g) Repeat parts c) to e) to see the new scatter plot and its line of best fit. Save your file with a different name to use later.
Application of the Line of Best Fit

1. The scatter plot shows data for the ages and monthly allowances for 12 students, and the line of best fit.
   a) Use the line of best fit to estimate …
      i) the allowance of a 13-year-old student. _____
      ii) the age of a student who gets a $30 allowance. _____
   b) Extend the line of best fit to predict the allowance of a 15-year-old student. _____

2. The scatter plot shows the daily temperature and the number of ice creams sold in a convenience store.
   a) Describe the association between temperature and number of ice creams sold. _______________
   b) Use the line of best fit to estimate the number of ice creams that the store can sell when the temperature is 30°C. _____
   c) Use the line of best fit to estimate the temperature on a day when the store sells 57 ice creams. _____
   d) The equation of the line of best fit is \( y = 1.44x + 21 \), where \( x \) is the temperature in Celsius and \( y \) is the number of ice creams sold. Answer parts b) and c) using the equation.
Venn Diagrams and Two-Way Tables

1. Complete the two-way relative frequency table using the Venn diagram.
   a) Factors of 6 Factors of 10
      \[\begin{array}{ccc}
      & \text{Factor of 10} & \text{Not a Factor of 10} \\
      \text{Factor of 6} & 2 & 2 \\
      \text{Not a Factor of 6} & 2 & 2 \\
      \text{Total} & 4 & 4 \\
      \end{array}\]

   b) Reading Playing Online
      \[\begin{array}{ccc}
      & \text{Playing Online} \\
      \text{Reading} & 11 & 7 \\
      \text{Total} & 18 & 14 \\
      \end{array}\]

   c) Have Bike Have Skates
      \[\begin{array}{ccc}
      & \text{Have Bike} & \text{Have Skates} \\
      \text{Have Bike} & 47 & 24 \\
      \text{Total} & 18 & 11 \\
      \end{array}\]

2. Complete the Venn diagram using the two-way table.
   a) Baseball No Basketball Total
      \[\begin{array}{ccc}
      \text{Baseball} & 7 & 22 & 29 \\
      \text{No Baseball} & 16 & 18 & 34 \\
      \text{Total} & 23 & 40 & 63 \\
      \end{array}\]

   b) Games or TV No Games or TV Total
      \[\begin{array}{ccc}
      \text{Homework} & 13 & 14 & 27 \\
      \text{No Homework} & 6 & 10 & 16 \\
      \text{Total} & 19 & 24 & 43 \\
      \end{array}\]
Sports Fans

1. Sam wants to find if there is a relationship between student grade level and being a fan of basketball, baseball, football, or other sports in his school. Sam surveys Grade 6 and Grade 8 students. If a student is both a basketball and baseball fan, then she or he is shown in both the basketball and baseball columns. Sam uses the “other” column for students who are fans of another sport. Here are the results of his survey:

<table>
<thead>
<tr>
<th></th>
<th>Basketball</th>
<th>Baseball</th>
<th>Football</th>
<th>Other</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 6</td>
<td>19</td>
<td>24</td>
<td>38</td>
<td>29</td>
<td>110</td>
</tr>
<tr>
<td>Grade 8</td>
<td>29</td>
<td>32</td>
<td>69</td>
<td>21</td>
<td>151</td>
</tr>
</tbody>
</table>

a) Use the row two-way table to complete the two-way relative frequency table.

<table>
<thead>
<tr>
<th></th>
<th>Basketball</th>
<th>Baseball</th>
<th>Football</th>
<th>Other</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade 8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Based on the relative frequency table …

i) from Grade 6 to 8, the rate of ________ and ________ fans decrease.

ii) from Grade 6 to 8, the rate of ________ and ________ fans increase.

c) Is there a difference between Grade 6 and Grade 8 students in their preference for basketball and baseball? _____ Explain. ____________________________

_________________________

d) Is there any association between grade level and being a football fan? _____

If yes, is it a positive association or negative association?

_________________________

e) Based on the data, is there any negative association? _____

If yes, between which variables? ____________________________
PS8-9  Choosing Strategies

Teach this lesson after: 8.2 Unit 7

Standards: 8.EE.C.8b, 8.G.B.7, 8.NS.A.2

Goals:
Students will choose between different problem-solving strategies learned this past year.

Prior Knowledge Required:
Can apply the Pythagorean Theorem
Can apply congruence rules for triangles
Can recognize alternating and corresponding angles
Knows that alternate and corresponding angles are equal when and only when lines are parallel
Can find the area of a circle
Can compare irrational numbers
Can find the percent of a number
Can solve systems of linear equations

Vocabulary: congruent, corresponding and alternating angles, corresponding sides and angles in congruent triangles, formula, hypotenuse, parallelogram, percent increase, perpendicular, radius, ratio, square root, surface area

Materials:
BLM TV Price (pp. R-70–72, see Performance Task)

NOTE: The following Problem Bank questions reflect the problem-solving strategies used in the problem-solving lessons for Grade 8. Choose among the following questions based on which problem-solving lessons you have taught.

Problem Bank
(MP.1, MP.3, MP.5) 1. Which diagonal in the parallelogram is longer? Explain how you know.

Answer: Draw perpendiculars from the top vertices to the base, and label the points as shown:

AE is the hypotenuse of the right triangle ADE, and BC is the hypotenuse of the right triangle BCF. Since CF is longer than DE and AD = BF, BC is longer than AE.
2. In both pictures, the circle has radius 1 cm. Is a greater fraction of the circle shaded or is a greater fraction of the square shaded?

**Solution:** The fraction of the circle that is shaded is \(2/\pi\). The fraction of the square that is shaded is \(\pi/4\). The question is asking which is greater, \(2/\pi\) or \(\pi/4\)? Multiplying both numbers by \(\pi/2\), this is like asking which is greater, \(\pi^2/8\) or 1. We know \(\pi^2 > 8\) because \(\pi > 3\), so in fact \(\pi^2 > 9\). So, \(\pi/4 > 2/\pi\), which tells us that a greater fraction of the square is shaded than the circle.

3. The size of a TV screen is displayed as the distance across its diagonal.
   a) A traditional TV screen has width to height ratio equal to 4 : 3. What is the width and height of a traditional TV screen with a diagonal measurement of 50 inches?
   b) A wide-screen TV has width to height ratio equal to 16 to 9. What is the width and height of a wide-screen TV screen with a diagonal measurement of 50 inches?
   c) The ideal viewing distance is four times the height of the screen. How far away should you sit for each TV in part a) and b)?

**Selected solution:**

b)

By the Pythagorean Theorem, \(50^2 = (16x)^2 + (9x)^2 = 256x^2 + 81x^2 = 337x^2\), so \(x^2 = 2500/337\), so \(x \approx 2.72\), so the height is about 24.5 inches and the width is about 43.5 inches

**Answers:** a) 40 inches wide and 30 inches high, c) 120 inches or 10 feet for the traditional screen TV and 98 inches or 8 feet 2 inches for the wide-screen TV

4. On a building, an overhang has its top edge 8 ft above the ground and it goes out 6 ft from the side of the building. A ladder must reach the top of the building, which is 20 ft high. How long does the ladder have to be?

**Solution:** The portion of the building above the overhang is \(20 - 8 = 12\) ft. To find how far the bottom of the ladder is from the building use \(12 : 6 = 20 : x\). The bottom of the ladder is 10 ft from the building, so the ladder needs to be \(\sqrt{10^2 + 20^2} = \sqrt{500} \approx 22.36\) or 22.4 ft long.

5. Find all whole numbers with a cube that is a three-digit number.
Answer: 5, 6, 7, 8, 9

6. How many whole numbers have a two-digit difference between their cube and their square?
Answer: two numbers (3 and 4)

(MP.5) 7. For how many whole numbers is the cube of its square root less than 100?
Answer: 22 numbers (0 through 21)

8. For how many whole numbers is the square of its cube root less than 100?
Answer: 1,000 numbers (0 through 999)

(MP.1) 9. On an analog clock, the hour hand is 5 inches long and the minute hand is 7 inches long. How many times faster is the tip of the minute hand moving than the tip of the hour hand? Hint: How much distance does each cover in an hour?
Solution: The tip of the minute hand travels 7/5 of the distance in 1/60th of the time as the tip of the hour hand does, so it is 1.4 × 60 = 84 times faster.

10. Three years ago, Cathy’s age was 9 times as much as her nephew’s age, but today, Cathy’s age is 5 times as much as her nephew’s age. How old is Cathy?
Answer: Cathy is 30 years old.

(MP.1, MP.5, MP.7) 11. If \(x : y = 3 : 2\) and \(2x + y = 40\), what are \(x\) and \(y\)?
Solution: Let \(x = 3z\) and \(y = 2z\), then \(2x + y = 6z + 2z = 8z = 40\), so \(z = 5\), \(x = 15\) and \(y = 10\)

(MP.1, MP.3) 12. Squares are drawn on the sides of a right triangle. Show that all four triangles in this picture have the same area.

Solution: \(B\) and \(C\) are congruent, so they have the same area. Now, rotate \(B\) inside the square twice as shown below, and call the resulting triangles \(E\) and \(F\):
Triangles $A$ and $E$ have equal bases and the same height, so they have equal areas. Triangles $D$ and $F$ have equal bases and the same height, so they have equal areas.

(MP.1, MP.5) 13. a) Prove that the shape shown is a square.

b) How many squares could you draw on each size of dot paper? Hint: How many different sizes of squares can you draw? How many of each size can you draw?

i) 3 by 3 dot paper
ii) 4 by 4 dot paper
iii) 5 by 5 dot paper

Answers:

a) Draw four surrounding right triangles:

They are all congruent and the two acute angles add to 90°, so the angles in the shape are right angles ($180 - 90 = 90$). The sides in the shape are all equal to the hypotenuse of the right triangle, so the shape is a square.

b) i) 3 different sizes:

There are $4 + 1 + 1 = 6$ squares altogether.

ii) 5 different sizes:

There are $9 + 4 + 1 + 4 + 2 = 20$ squares altogether.

iii) 8 different sizes:
(MP.1, MP.5, MP.7) 14. Remember, a percent increase is the percentage of the original number that the change in the amount represents.

a) A rubber band is 10 cm long. When stretched, it becomes 13 cm. What was the percent increase in length?
b) A rubber band is 8 in long. When stretched, its length increased by 50%. What is the new length of the elastic?
c) A square is cut in half across the diagonal. By what percentage did the total perimeter increase? Write your answer to the nearest whole number percentage.
d) A cube is cut in half vertically. By what percentage, to the nearest whole number, did the total surface area increase?

Answers: a) 30%; b) 12 in; c) If the side length is $x$, the perimeter increased by $2 \times \sqrt{2x^2}$, since each hypotenuse has length $\sqrt{2x^2}$. The original perimeter is $4x$, so the increase as a fraction of the original perimeter is $2\sqrt{2}/4 \approx 0.71$ or 71%; d) If the side length is $x$, the original surface area is $6x^2$ and the new surface area is $8x^2$. So, the surface area increased by $2x^2$, which is one-third of the original value, and the percent increase is about 33%.

(MP.1, MP.3, MP.5) 15. Raj left his bike at school for the weekend. On Monday, he walked to school at a speed of 5 km/hr. After school, he biked home at a speed of 15 km/hr, taking the same route. Will Raj’s average speed be more than or less than 10 km/hr? Hint: Assume a given distance, such as 5 km or 15 km, and figure out the amount of time spent at each speed.

Solution: If the distance from home to school is 5 km, Raj took 1 hour to get to school and 20 minutes to get home. His average speed is (total distance) ÷ (total time) = 10 km ÷ 4/3 hours = 30/4 km/h = 7.5 km/hour. This is closer to 5 km/h, which makes sense, because he spent more time walking 5 km/h than he did biking 15 km/h.

(MP.7) 16. Sara copied the formula below from the Internet, but part of the formula got smeared.

$$1^4 + 2^4 + 3^4 + \ldots + n^4 = \frac{6n^5 + an^4 + bn^3 - n}{30}$$, for all positive integers $n$.

a) Evaluate both sides of the equation for $n = 1$ and $n = 2$.
b) Find $a$ and $b$.
c) Check if the formula works for $n = 3$.

Answers: 

a) For $n = 1$, the left side is 1 and the right side is $(6 + a + b - 1)/30 = (a + b + 5)/30$, and for $n = 2$, the left side is 17 and the right side is $(6 \times 32 + 16a + 8b - 2)/30$.
b) Setting both sides of the equations equal, the equation for $n = 1$ simplifies to $a + b = 25$ and the equation for $n = 2$ simplifies to $2a + b = 40$. Solving these, you get $a = 15$ and $b = 10$. 
c) For \( n = 3 \), the left side is \( 1 + 16 + 81 = 98 \) and the right side is \( (6 \times 243 + 15 \times 81 + 10 \times 27 - 3)/30 = 98 \), so the formula works for \( n = 3 \).

**MP.3, MP.5** 17. In the star, \( \angle BEC = \angle EBD \), \( \angle ADB = \angle ACE \), and \( BD = CE \).

![Star Diagram]

Show that \( AC = AD \).

**Solution:** Label the unnamed corners of the star, as shown below:

![Labelled Star Diagram]

\( \triangle BDF \equiv \triangle ECG \) by ASA, so \( FD = GC \) and \( \angle EGC = \angle BFD \), since they are corresponding sides and angles of congruent triangles. Then \( \angle AFG = \angle AGF \), since they are supplementary to equal angles. Then \( \triangle AFG \) is isosceles and so \( AF = AG \); but, \( GC = FD \) and so \( AD = AC \).

**MP.1, MP.5** 18. A point is located inside a rectangle, and the distances to each corner of the rectangle are measured. Going in clockwise order around the rectangle, the distances are \( a \), \( b \), \( c \), and \( d \).

a) Show that \( a^2 + c^2 = b^2 + d^2 \).

b) If the point were located outside the rectangle instead, would the same result hold?

**Answers:**

a) Draw a picture:

![Rectangle Diagram]

From the picture, \( a^2 + c^2 = m^2 + n^2 + p^2 + q^2 \) and \( b^2 + d^2 = n^2 + p^2 + m^2 + q^2 \), so indeed \( a^2 + c^2 = b^2 + d^2 \).

b) Yes
Performance Task: TV Price

Vocabulary: scatter plot

Materials:
BLM TV Price (pp. R-70–72)

Performance Task: TV Price. Give students BLM TV Price. The Bonus question provides students with an opportunity to use problem-solving strategies in this unit.

Selected answers:
  c) 15
  d) $450
  e) $-20/100 or $-0.2$
  f) 165
  g) $y = -0.2x + 165$, $x$ stands for TV price and $y$ stands for number of TV sales
  h) 6
  i) the same as c) and d)
  j) ($150, $6,750), ($350, $1,750), ($100, $5,900), ($200, $5,800), ($250, $6,250)
  k) $600$

Bonus: ($610, $6,880), ($620, $6,970), ($630, $7,020), price at $630 has total profit $7,020
TV Price (1)

An electronics store has a new model of a TV to sell. In the table below you will see data for the TV price and the number of TVs sold in five stores.

<table>
<thead>
<tr>
<th></th>
<th>Store 1</th>
<th>Store 2</th>
<th>Store 3</th>
<th>Store 4</th>
<th>Store 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>TV Price ($)</td>
<td>600</td>
<td>800</td>
<td>550</td>
<td>650</td>
<td>700</td>
</tr>
<tr>
<td>TV Sales</td>
<td>45</td>
<td>5</td>
<td>59</td>
<td>29</td>
<td>25</td>
</tr>
</tbody>
</table>

1. a) Draw a scatter plot for the data.

b) Draw the line of best fit for the scatter plot.

c) Use the line of best fit to estimate how many TVs will sell if the TV is priced at $750.

d) Extend the line of best fit and predict at what price the store can sell 75 TVs.
e) Use two points on the line of best fit (or very close to the line) to find the slope.

f) Use the slope and one point on the line of best fit to find the $y$-intercept.

g) Write an equation for the line of best fit in slope-intercept form. What do $x$ and $y$ stand for in the equation of the line?

h) The table says that a store sells 29 TVs priced at $650. Use your equation from part g) to find $y$ for $x = 650$. How much is your prediction off by?

i) Answer parts c) and d) using the equation of the line.
j) The store’s cost is $450 per TV. Complete the table to find the total profit for each price. Hint: To find the total profit multiply the number of TVs sold by the amount of profit per TV.

<table>
<thead>
<tr>
<th>TV Price</th>
<th>Profit per TV (TV Price – 450)</th>
<th>Number of TVs Sold</th>
<th>Total Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td></td>
<td>45</td>
<td></td>
</tr>
<tr>
<td>800</td>
<td></td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>550</td>
<td></td>
<td>59</td>
<td></td>
</tr>
<tr>
<td>650</td>
<td></td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>700</td>
<td></td>
<td>25</td>
<td></td>
</tr>
</tbody>
</table>

k) At what price in the table does the store have the most profit?

**Bonus** The store manager wants to know if he can have even more total profit than the table above shows, so he sets the goal for more than $7,000 (greater than $7,000 and not equal to $7,000). Use the guess-check-revise strategy and complete the table to find out at what price the total profit would be $7,030. Hint: The number of TVs sold can be calculated using the equation in part g).

<table>
<thead>
<tr>
<th>TV Price (x)</th>
<th>Profit per TV (x – 450)</th>
<th>Number of TVs Sold</th>
<th>Total Profit</th>
<th>Is Total Profit Greater than $7,000?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
Coordinate Grids

Blackline Master — Generic — Teacher Resource for Grade 8
1. A: \(4 \times 3 \div 2 = 6\)
   B: \(7 \times 5 \div 2 = 17.5\)
   C: \(6 \times 2 \div 2 = 6\)
   D: \(4 \times 4 \div 2 = 8\)

2. a)  
   b)  
   c)  
   d)  
   e)  
   f)  

3. A: \(6 \times 3 \div 2 = 9\)
   B: \(4 \times 5 \div 2 = 10\)
   C: \(3 \times 5 \div 2 = 7.5\)
   D: \(4 \times 8 \div 2 = 16\)

4. A: \(5 \times 4 \div 2 = 10\)
   B: \(5 \times 3 \div 2 = 7.5\)
   C: \(4 \times 6 \div 2 = 12\)
   D: \(6 \times 4 \div 2 = 12\)

5. A. base = 4  
   height = 4  
   Area = \(4 \times 4 \div 2 = 8\)

   B. base = 5  
   height = 3.2  
   Area = \(5 \times 3.2 \div 2 = 8\)

6. a) The base and height measurements are different but the areas are the same.
   b) no
   c) Answers will vary. Teacher to check.

7. a)  
   b)  
   c)  

8. a) \(4 \times 2.5 = 10\)
   b) \(3 \times 2 = 6\)

9. \(AB\) as base:  
   \(2 \text{ cm} \times 2 \text{ cm} = 4 \text{ cm}^2\)
   
   \(BC\) as base:  
   \(2.5 \text{ cm} \times 1.6 \text{ cm} = 4 \text{ cm}^2\)
   
   The answers are equal because the shape (and therefore the area) doesn't change if you pick a different side to use as a base.

10. a) \(5 \times 4 = 20\)
    \[4 \times 2 \div 2 = 8\]
    \[20 + 8 = 28\]
    b) \(4 \times 5 = 20\)
    \[2 \times 5 \div 2 = 5\]
    \[20 + 5 = 25\]

11. a) \(A = \pi r^2\)
    \[r = 2 \text{ cm}\]
    \[A = 3.14 \times 10 \times 10 = 314 \text{ m}^2\]
    b) \(A = \pi r^2\)
    \[r = 3 \text{ cm}\]
    \[A = 3.14 \times 8 \times 8 = 200.96 \text{ ft}^2\]
    c) \(A = \pi r^2\)
    \[r = 15 \text{ m}\]
    \[A = 3.14 \times 15 \times 15 = 706.5 \text{ m}^2\]
    d) \(A = \pi r^2\)
    \[r = 8 \text{ in}\]
    \[A = 3.14 \times 8 \times 8 = 200.96 \text{ in}^2\]
b) medium pizza:
113.04 in$^2$ + 14
= 8.07 in$^2$ per dollar
large pizza:
200.96 in$^2$ + 18
= 11.16 in$^2$ per dollar
c) large pizza

6. a) 8, 4
d) 3 rotations:
20.56 cm
18.84 cm
113.04 in
52.76 cm
15,826.6 cm
1 hour:
15,826.6 x 60
= 949,536 cm

AP Book G8-42
page 119
1. a) 12, 6
c) 4, 2
2. Area of B
= 4 x 5 + 9 = 29
Area of C
= 4 x 2 + 9 = 17
Area of D
= 4 x 4.5 + 0 = 18
Area of E
= 4 x 6 + 1 = 25
Area of F
= 4 x 7.5 + 4 = 34
3. \(A = \ell^2\)
25 = \(\ell^2\)
\(\sqrt{25} = \sqrt{\ell^2}\)
\(5 = \ell\)
4. Side length of B
= \(\sqrt{4 x 24 + 4}\)
= \(\sqrt{100}\)
= 10
Side length of C
= \(\sqrt{4 x 30 + 49}\)
= \(\sqrt{169}\)
= 13
5. a) ii) \(a^2 \cdot 5 \times 5 = 25\)
\(b^2 \cdot 2 \times 2 = 4\)
\(c^2 \cdot 4 \times 5 + 9 = 29\)
iii) \(a^2 \cdot 6 \times 6 = 36\)
\(b^2 \cdot 1 \times 1 = 1\)
\(c^2 \cdot 4 \times 3 + 25 = 37\)
iv) \(a^2 \cdot 3 \times 3 = 9\)
\(b^2 \cdot 2 \times 2 = 4\)
\(c^2 \cdot 4 \times 3 + 1 = 13\)

AP Book G8-43
page 122
1. b) \(\frac{d}{\sqrt{b^2}}\)
\(\frac{d}{\sqrt{b^2}} = \sqrt{25}\)
\(d = 5\)
2. a) \(m^2 = n^2 + m^2\)
b) \(r^2 = l^2 + s^2\)
c) \(x^2 = w^2 + t^2\)
d) \(b^2 = a^2 + c^2\)

3. b) \(r^2 = 12^2 + 5^2\)
\(r^2 = 144 + 25\)
\(r = \sqrt{169} = 13\)

(continued)
### Geometry: Pythagorean Theorem – AP Book 8.2: Unit 4

**Page 124**

1. a) 3
   b) 4
   c) yes
   
   - They have the same side lengths.
   - P: $3^2$ Q: $4^2$
   - R: $3 \times 4$
   - $S: 4 \times 3$
   - $3^2 + 4^2 + 4 \times 3\times4 = 3^2 + 4^2 + 3 \times 4 + 4 \times 3$

2. a) $3 \times 4$
   b) $4 \times 3$
   c) $c$
   d) $c^2$
   e) $3 \times 4 + 4 \times 3 + c^2$

3. a) 7, 7
   b) 7, 7
   c) the areas are equal
   
   - $3^2 + 4^2 + 3 \times 4 + 4 \times 3$
   - $3^2 + 4^2 + 4 \times 3 + 3 \times 4$
   - $3^2 + 4^2 = 25$

4. a) $a$
   b) $b$
   c) yes
   
   - They have the same side lengths.
   - P: $a^2$
   - Q: $b^2$
   - R: $a \times b$
   - $S: b \times a$
   - $a^2 + b^2 + a \times b + b \times a$

5. a) $a \times b$
   b) $b \times a$
   c) $c$
   d) $c^2$

6. a) $7, 7$
   b) $7, 7$
   c) the areas are equal
   
   - $a^2 + b^2 + a \times b + b \times a = a \times b + b \times a + c^2$
   - $a^2 + b^2 + b \times a + c^2$

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>i)</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>ii)</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>iii)</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

### AP Book G8-45

**Page 126**

1. a) $a \times b + b \times a + c^2$
   
<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
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<td>25</td>
</tr>
<tr>
<td>iii)</td>
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</table>

### Continued

- **Geometry:** Right angle?
- **Algebra:** Obtuse, or a right angle?

<table>
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<tr>
<th>a</th>
<th>b</th>
<th>c</th>
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<tr>
<td>i)</td>
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**Table:**

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<td>$a^2 = a^2 + b^2$</td>
<td></td>
</tr>
<tr>
<td>ii)</td>
<td>$c^2 &gt; a^2 + b^2$</td>
<td></td>
</tr>
<tr>
<td>iii)</td>
<td>$c^2 &gt; a^2 + b^2$</td>
<td></td>
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</table>

- **Right Angle:** Right
- **Obtuse:** Obtuse
- **Teacher to check.**

<table>
<thead>
<tr>
<th>a^2 + b^2</th>
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<td>ii)</td>
<td>$c^2 = a^2 + b^2$</td>
</tr>
<tr>
<td>iii)</td>
<td>$c^2 = a^2 + b^2$</td>
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</tbody>
</table>

- **Right:** Right
- **Obtuse:** Obtuse
- **Acute:** Acute

**Table:**

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<th>a^2 + b^2</th>
<th>c^2</th>
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<tbody>
<tr>
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<tr>
<td>ii)</td>
<td>25</td>
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<tr>
<td>iii)</td>
<td>289</td>
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</tbody>
</table>

### Answer Keys for AP Book 8.2

- **Right:** Right
- **Obtuse:** Obtuse
- **Acute:** Acute
d) right triangle

e) \( y^2 = a^2 + b^2 \)

f) \( c^2 = y^2 \)

g) \( c = y \)

h) side-side-side

i) \( \triangle ABC \)

j) 90°

k) right

4.

a)

\[
\begin{align*}
33 \text{ ft} &< 32 \text{ ft} \\
32 + 4 &< 8 \\
\text{Total fencing needed:} &< 200 \\
\end{align*}
\]

b)

\[
\begin{align*}
32 + 4 &< 8 \\
\text{See diagram above.} &< \\
\end{align*}
\]

5.

a) \( AD = 10 + 2 = 5 \text{ cm} \)

b) \( BD^2 + 5^2 < 8^2 \)

\[
\begin{align*}
BD &< 25 \\
BD &< 25 - 64 \\
BD &< 39 \\
BD &< 6.2 \text{ cm} \\
\end{align*}
\]

c) \( A = b \times h + 2 \\
= 10 \times 6.2 + 2 \\
= 62 \text{ in} \)

6. base of the triangle

\[
\begin{align*}
17 - 8 &< 9 \text{ cm} \\
h^2 + 9^2 &< 15^2 \\
h^2 + 81 &< 225 \\
h^2 &< 225 - 81 \\
h^2 &< 144 \\
h &< \sqrt{144} \\
h &< 12 \text{ cm} \\
\end{align*}
\]

7. shortcut:

\[
\begin{align*}
d^2 &< 2^2 + 1^2 \\
d^2 &< 4 + 1 \\
d^2 &< 5 \\
d &< \sqrt{5} \\
d &< 2.2 \text{ km} \\
\text{usual trip:} &< 2 + 1 = 3 \text{ km} \\
3 - 2.2 &< 0.8 \text{ km shorter} \\
\end{align*}
\]

8. a)

\[
\begin{align*}
d^2 &< 90^2 + 90^2 \\
d^2 &< 8,100 + 8,100 \\
d^2 &< 16,200 \\
d &< \sqrt{16,200} \\
d &< 127.3 \text{ ft} \\
\end{align*}
\]

b) \( 127.3 + 2 = 63.7 \text{ ft} \)

Since the midpoint is 63.7, the pitcher is closer to home plate than second base.
7. a) \(8 \times 8 = 64 \text{ ft}^2\)  
\(3 \times 12 = 36 \text{ ft}^2\)  
\(6 \times 8 = 48 \text{ ft}^2\)

b) \(s^2 = 16^2 + 8^2 = 320\)  
\(s = \sqrt{320} = 17.91\) ft

c) \(s = \sqrt{16^2 + 8^2} = 10\) ft

d) \(s = \sqrt{16^2 + 8^2} = 10\) ft

8. a) See diagram above.

b) right triangle

c) \(AC\)

d) \(AC^2 = 20^2 + 60^2 = 4000\)
\(AC = \sqrt{4000} = 63.2\) cm

e) \(AF^2 = 63.2^2 + 80^2 = 10,394.2\)
\(AF = \sqrt{10,394.2} = 102\) cm

f) 102 cm

9. a) \( \sqrt{b^2 + c^2} = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10\)

b) \( \sqrt{b^2 + c^2} = \sqrt{7^2 + 3^2} = \sqrt{49 + 9} = \sqrt{58} = 7.6\)

c) \( \sqrt{a^2 + b^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10\)

d) \( \sqrt{a^2 + b^2} = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13\)

e) \( \sqrt{a^2 + b^2} = \sqrt{3^2 + 15^2} = \sqrt{9 + 225} = \sqrt{234} = 15.3\)

f) \( \sqrt{a^2 + b^2} = \sqrt{5^2 + 2^2} = \sqrt{25 + 4} = \sqrt{29} = 5.4\)
7. a) \[ d_{AB} = \sqrt{(-6 - 0)^2 + (3 - 3)^2} \]
\[= \sqrt{36 + 0} \]
\[= \sqrt{36} \]
\[= 6 \]
\[d_{BC} = \sqrt{(-6 - (-6))^2 + (-5 - 3)^2} \]
\[= \sqrt{0^2 + 8^2} \]
\[= \sqrt{64} \]
\[= 8 \]
\[d_{AC} = \sqrt{(-6 - 0)^2 + (-5 - 3)^2} \]
\[= \sqrt{36 + 64} \]
\[= \sqrt{100} \]
\[= 10 \]

b) \[6 + 8 + 10 = 24\]

c) If \(\triangle ABC\) is a right triangle, then
\[6^2 + 8^2 = 10^2\]
\[36 + 64 = 100\]
\[100 = 100\]

d) Area = \[8 \times 6 \div 2\]
\[= 24\]
Expressions and Equations:
Systems of Linear Equations – AP Book 8.2: Unit 5

1. b) \(2(-3) + 8\)
   \[= -6 + 8\]
   \[= 2\]

   c) \(-3(-2) - 4\)
   \[= 6 - 4\]
   \[= 2\]

   d) \(4(0) - 3\)
   \[= 0 - 3\]
   \[= -3\]

   e) \(-5(0.2) + 2\)
   \[= -1 + 2\]
   \[= 1\]

   f) \(-\frac{3}{4} + \frac{3}{4}\)
   \[= \frac{-9}{16} + \frac{3}{4}\]
   \[= \frac{-9 + 12}{16}\]
   \[= \frac{3}{16}\]

2. a) \(y\)
   \[3\]
   \[5\]
   \[7\]
   \[9\]
   \[11\]

   b) \(y\)
   \[1\]
   \[-2\]
   \[-5\]
   \[-8\]
   \[-11\]

   c) \(y\)
   \[6\]
   \[7\]
   \[8\]
   \[9\]
   \[10\]

3. b) \((-1, 2)\)

4. a) \(y\)
   \[-5\]
   \[-4\]
   \[-3\]
   \[-2\]
   \[-1\]

   b) \(y\)
   \[9\]
   \[8\]
   \[7\]
   \[6\]
   \[5\]

   c) \(y\)
   \[-4\]
   \[-3\]
   \[-2\]
   \[-1\]
   \[0\]

   d) \(x\)
   \[-10\]
   \[-9\]
   \[-8\]
   \[-7\]
   \[-6\]

   e) \(x\)
   \[-10\]
   \[-9\]
   \[-8\]
   \[-7\]
   \[-6\]

   f) \(x\)
   \[2\]
   \[3\]
   \[4\]
   \[5\]
   \[6\]

5. a) \(y\)
   \[-3\]
   \[-2\]
   \[-1\]
   \[0\]
   \[1\]

   b) no

   c) \(x\)
   \[3\]
   \[2\]
   \[1\]

   d) No, Cathy was not correct. You cannot always tell since the common ordered pair may not fall in the values for \(x\) in the tables.
Expressions and Equations:
Systems of Linear Equations – AP Book 8.2: Unit 5

1. Point Calc. On line?
   (0, 1) \[ \frac{1}{-2(0) + 1} \] yes
   (1, -3) \[ \frac{-3}{-2(1) + 1} \] no
   (1, -1) \[ \frac{-1}{-2(1) + 1} \] yes
   (-2, 2) \[ \frac{2}{-2(-2) + 1} \] no

2. a) \[ -3 \neq 3(2) - 4 \]
o no
b) \[ -1 = -2(3) + 5 \]
yes
c) \[ 4 = -1 + 5 \]
yes
d) \[ 5 \neq -3 - 2 \]

3. b) \[ 1 = 3(1) - 2 \]
   (1, 1) lies on the line \[ y = 3x - 2 \].
   \[ 1 = 5(1) + 6 \]
   So (1, 1) lies on both lines.
   c) \[ 3 = -2(-1) + 1 \]
   (-1, 3) lies on the line \[ y = -2x + 1 \].
   \[ 3 = -6(-1) - 3 \]
   (-1, 3) lies on the line \[ y = -6x - 3 \].
   So (-1, 3) lies on both lines.

4. a) (2, 1)
b) (-1, 2)
c) (-1, -2)
d) (2, 0)

5. b) (-1, 2)
   \[ 2 = -1 + 3 \]
   \[ 2 = -2(-1) \]
   c) (-1, -2)
   \[ -2 = -(1) - 3 \]
   \[ -2 = -1 - 1 \]
   d) (2, 0)
   \[ 0 = 2 - 2 \]
   \[ 0 = -2(2) + 4 \]

6. \[ -1 \neq -(3) + 4 \]
   \[ -1 \neq 2(3) - 3 \]
   Jennifer is not correct.
   (3, -1) does not lie on either line, so it cannot be the intersection point.

7. a) i) \[ y \]
   \[ \begin{array}{c|c|c}
   & -2 & -1 \\
   \hline
   1 & 0 & 2 \\
   \hline
   2 & 4 & \\
   \end{array} \]
   (2, -1)

   ii) \[ y \]
   \[ \begin{array}{c|c|c}
   & -4 & -2 \\
   \hline
   1 & 0 & \\
   \hline
   2 & -1 & 4 \\
   \end{array} \]
   (1, 0)

   iii) \[ y \]
   \[ \begin{array}{c|c|c}
   & -3 & -2 \\
   \hline
   1 & -1 & 0 \\
   \hline
   2 & -4 & 2 \\
   \end{array} \]
   (1, -1)

8. a) \[ y = 6x + 32 \]
   \[ y = 4x + 38 \]
   b) \[ y = 50x + 330 \]
   \[ y = 60x + 250 \]
   c) \[ y = 5x + 150 \]
   \[ y = 3x + 240 \]

b) \[ m = 2, m = 2 \]
The slopes are equal.

2. Teacher to check graphs.
a) \[ P: y = 24x + 50 \]
   A: \[ y = 20x + 70 \]
   5 hours
### Systems of Linear Equations – AP Book 8.2: Unit 5

**Expressions and Equations:**

- **Determining an exact value:**
  - **Decimals values fall in:**

**Train A will catch up about 102 km from the station.**

**The repair cost would be about $142.50 from both companies after about 75 minutes.**

**Jay and Helen will earn the same amount after about 20 years.**

---

**AP Book EE8-53**

**page 145**

**1. a)**

\[ y = 2x - 1 \]

- 1
- 3
- 5
- 7
- 2x - 1

\( (x, 2x - 1) \)

**b)**

\[ y = 3x - 4 \]

<table>
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<tr>
<th>( x )</th>
<th>( -1 )</th>
<th>( 2 )</th>
<th>( 5 )</th>
<th>( 8 )</th>
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<tbody>
<tr>
<td>( 3x - 4 )</td>
<td></td>
<td></td>
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</table>

\( (x, 3x - 4) \)

**c)**

\[ y = -2x + 5 \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 3 )</th>
<th>( 1 )</th>
<th>( -1 )</th>
<th>( -3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( -2x + 5 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

\( (x, -2x + 5) \)

**d)**

\[ 4x - 8 = -3x + 13 \]

- \( 4x + 3x = 13 + 8 \)
- \( 7x = 21 \)
- \( 7x + 7 = 21 + 7 \)
- \( x = 3 \)

\[ 2x - 1 = -3x - 11 \]

- \( 2x + 3x = -11 + 1 \)
- \( 5x = -10 \)
- \( 5x + 5 = -10 + 5 \)
- \( x = -2 \)

\[ 2x - 2 = 4 + 2 \]

- \( x = -2 \)

**e)**

**f)**

At the int. point:

- \( y = -2 + 3 = 5 \)
- \( So \ int. \ point = (-2, 5) \)

**Checks:**

- \( 5 = (-2) + 3 \)
- \( 5 = -2 + 7 \)

**2. b)**

\( (x, x - 5) \)

**c)**

\( (x, -x + 3) \)

**3. b)**

\( y = -2(1) + 1 = -1 \)

- or

\( y = 2(1) - 3 = -1 \)

- \( So \ int. \ point = (1, -1) \)

**c)**

\( y = 2(-2) + 4 = 0 \)

- or

\( y = -(-2) - 2 = 0 \)

- \( So \ int. \ point = (-2, 0) \)

**4. b)**

At the int. point:

- \( 3x - 4 = 2x + 1 \)

**c)**

At the int. point:

- \( x - 4 = -3x + 8 \)

**d)**

At the int. point:

- \( 4x - 8 = -3x + 13 \)

**e)**

At the int. point:

- \( 2x - 1 = -3x - 11 \)

**f)**

At the int. point:

- \( 6x - 17 = -3x + 19 \)

**5. b)**

\( -3x - 4 = 2x + 1 \)

- \( -3x - 2x = 1 + 4 \)
- \( -5x = 5 \)
- \( -5x + 5 = 5 + 5 \)
- \( x = -1 \)

**c)**

\( x - 4 = -3x + 8 \)

- \( x + 3x = 8 + 4 \)
- \( 4x = 12 \)
- \( 4x + 4 = 12 + 4 \)
- \( x = 3 \)

**6. b)**

\( x = -1 \)

**c)**

\( y = -3(-1) - 4 = -1 \)

- \( y = 2(-1) + 1 = -1 \)
- \( So \ int. \ point = (-1, -1) \)

**d)**

\( x = 3 \)

**e)**

\( x = -2 \)

**f)**

\( x = 4 \)

**7. b)**

At the int. point:

- \( x + 3 = x + 7 \)
- \( -x - x = 7 - 3 \)
- \( -2x = 4 \)
- \( -2x - 2 = 4 + 2 \)
- \( x = -2 \)

At the int. point:

- \( y = -2 + 3 = 5 \)
- \( So \ int. \ point = (-2, 5) \)

**Checks:**

- \( 5 = (-2) + 3 \)
- \( 5 = -2 + 7 \)
Expressions and Equations: Systems of Linear Equations – AP Book 8.2: Unit 5

1. a) \(3x + y - 3x = 5 - 3x\)
   \(y = -3x + 5\)
   \(m = -3\)
   \(y\)-intercept = 5

b) \(x - 3y - x = 1 - x\)
   \(-3y = -x + 1\)
   \(y = \frac{1}{3}x - \frac{1}{3}\)
   \(m = \frac{1}{3}\)
   \(y\)-intercept = \(-\frac{1}{3}\)

c) \(2x + 3y - 2x = 4 - 2x\)
   \(3y = -2x + 4\)
   \(y = \frac{2}{3}x + \frac{4}{3}\)
   \(m = -\frac{2}{3}\)
   \(y\)-intercept = \(\frac{4}{3}\)

2. a) \(y = -x + 4\)
    \(m = -1\)
    \(y\)-intercept = \(4\)

b) \(y = 2x + 1\)
    \(m = 2\)
    \(y\)-intercept = \(1\)

c) \(y = -x - 2\)
    \(m = -1\)
    \(y\)-intercept = \(-2\)

3. a) \(y = x - 3, y = -x + 7\)
   \(x = 3\)
   \(y = 7\)
   \(x + y = 10\)
   \(x = 10\)

b) \(y = 2x - 5,\)
   \(y = -3x + 10\)
   \(2x - 5 = -3x + 10\)
   \(2x + 3x = 10 + 5\)
   \(x = 5\)

At the intersection point:
\(2x - y = 5\)
\(2(3) - y = 5\)
\(y = 6 - 5 = 1\)
So int. point = \((3, 1)\)
Checks: \(2(3) - 1 = 5\)
\(3(3) + 1 = 10\)

4. Let \(x\) be the number of \$5 bills.
   Let \(y\) be the number of \$1 bills.
   \(x + y = 20, 5x + y = 64\)
   \(y = 20 - x, y = -5x + 64\)
   At the intersection point:
   \(20 - x = 5x + 64\)
   \(-x + 5x = 64 - 20\)
   \(4x = 44\)
   \(x = 11\)
   \(x + y = 20\)
   \(y = 20 - 11 = 9\)
   John has 11 \$5 bills.
   Checks: \(11 + 9 = 20\)
   \(5(11) + 9 = 64\)
5. Let $x$ be the number of stamps that Clara has.
   Let $y$ be the number of stamps that Anwar has.
   
   \[ x - 5 = y, x + 6 = 2y \]
   
   At the intersection point:
   \[ x - 5 = \frac{1}{2} x + 3 \]
   \[ 2x - 10 = x + 6 \]
   \[ 2x - x = 6 + 10 \]
   \[ x = 16 \]
   \[ x - 5 = y \]
   \[ 16 - 5 = y \]
   \[ y = 11 \]
   
   Clara has 16 stamps.
   Anwar has 11 stamps.
   
   Checks: \[ 16 - 5 = 11 \]
   \[ 16 + 6 = 2(11) \]

6. Let $x$ be the larger part.
   Let $y$ be the smaller part.
   
   \[ x + y = 36, x = 3y \]
   \[ y = 36 - x, y = \frac{1}{3} x \]
   
   At the intersection point:
   \[ 36 - x = \frac{1}{3} x \]
   \[ 108 - 3x = x \]
   \[ 108 = 4x \]
   \[ x = 27 \]
   \[ x + y = 36 \]
   \[ 27 + y = 36 \]
   \[ y = 9 \]
   
   The larger part is 27 units.
   The smaller part is 9 units.
   
   Checks: \[ 27 + 9 = 36 \]
   \[ 27 = 3(9) \]

7. Let $x$ be the measurement of $\angle A$.
   Let $y$ be the measurement of $\angle B$.
   
   \[ x + y = 90, x = y + 18 \]
   \[ y = 90 - x, y = x - 18 \]
   
   At the intersection point:
   \[ 90 - x = x - 18 \]
   \[ 90 + 18 = x + x \]
   \[ 108 = 2x \]
   \[ x = 54 \]
   \[ x + y = 90 \]
   \[ 54 + y = 90 \]
   \[ y = 36 \]
   
   \[ \angle A = 54^\circ \text{ and } \angle B = 36^\circ \]
   
   Checks: \[ 54 + 36 = 90 \]
   \[ 54 = 36 + 18 \]

8. Let $x$ be Bo’s current age.
   Let $y$ be Blanca’s current age.
   
   \[ x = y + 5 \]
   \[ (x + 3) + (y + 3) = 25 \]
   \[ y = x - 5 \]
   \[ y = 19 - x \]
   
   At the intersection point:
   \[ x - 5 = 19 - x \]
   \[ x + x = 19 + 5 \]
   \[ 2x = 24 \]
   \[ x = 12 \]
   \[ x = y + 5 \]
   \[ 12 = y + 5 \]
   \[ y = 7 \]
   
   Bo is 12 years old.
   Blanca is 7 years old.
   
   Checks:
   \[ 12 = 7 + 5 \]
   \[ (12 + 3) + (7 + 3) = 25 \]

9. Let $x$ be the number of chickens.
   Let $y$ be the number of cows.
   
   \[ x + y = 13, 2x + 4y = 40 \]
   \[ y = 13 - x, y = -\frac{1}{2} x + 10 \]
   
   At the intersection point:
   \[ 13 - x = -\frac{1}{2} x + 10 \]
   \[ 26 - 2x = -x + 20 \]
   \[ -2x + x = 20 - 26 \]
   \[ -x = -6 \]
   \[ x = 6 \]
   \[ x + y = 13 \]
   \[ 6 + y = 13 \]
   \[ y = 7 \]
   
   There are 6 chickens and 7 cows.
   
   Checks: \[ 6 + 7 = 13 \]
   \[ 2(6) + 4(7) = 40 \]

10. Let $x$ be the numeric value of the tens digit.
    Let $y$ be the numeric value of the ones digit.
    
    \[ x + y = 7, \]
    \[ (10x + y) - (10y + x) = 45 \]
    \[ y = 7 - x, y = x - 5 \]
    
    At the intersection point:
    \[ 7 - x = x - 5 \]
    \[ 7 + 5 = x + x \]
    \[ 12 = 2x \]
    \[ x = 6 \]
    \[ x + y = 7 \]
    \[ 6 + y = 7 \]
    \[ y = 1 \]
    
    The original number is
    \[ 10x + y = 10(6) + 1 = 61 \]
    The reversed number is
    \[ 10y + x = 10(1) + 6 = 16 \]
    
    Checks:
    \[ 6 + 1 = 7 \]
    \[ (10(6) + 1) - (10(1) + 6) = 45 \]
Expressions and Equations:
Systems of Linear Equations – AP Book 8.2: Unit 5

5. a) \(4x - 2 = -2x + 6\)
   \[6x = 8\]
   \[x = \frac{4}{3}\]
   \[y = 4x - 2\]
   \[y = 4 \left(\frac{4}{3}\right) - 2\]
   \[y = \frac{16}{3} - 2\]
   \[y = \frac{10}{3}\]
   \[\left\{\frac{4}{3}, \frac{10}{3}\right\}\]

   d) \(5x + 7 = -2x\)
   \[7x = -7\]
   \[x = -1\]
   \[y = 5(-1) + 7\]
   \[y = -5 + 7\]
   \[y = 2\]
   \((-1, 2)\)

6. a) parallel: no
    solution: yes

   b) \(y = x - 5, m = 1\)
    \[y = x - 2, m = 1\]
    parallel: yes
    solution: no

   c) \(y = 2x - 8, m = 2\)
    \[y = -3x + 4, m = -3\]
    parallel: no
    solution: yes

   d) \(y = -3x + 1, m = -3\)
    \[y = -3x + 7, m = -3\]
    parallel: yes
    solution: no

   e) \(y = 3x + 1, m = 3\)
    \[y = -3x + 7, m = -3\]
    parallel: no
    solution: yes

7. circle A and E

8. Convert to slope intercept form:
   \[y = \frac{3}{2}x - 1, y = \frac{3}{2}x + 2\]
   At the intersection point:
   \[\frac{3}{2}x - 1 = \frac{3}{2}x + 2\]
   \[\frac{3}{2}x - \frac{3}{2}x = 2 + 1\]
   \[0 \neq 3\]
   Since \(0 \neq 3\), there is no solution and Emma is correct.
1. a) ii) triangle
   iii) trapezoid

b) rectangles

2. a) Right prisms: A, B, F
Skew prisms: D
Not prisms: C, E

b) Answers will vary.
   Sample answer:
   C is not a prism
   because the two parallel faces are not congruent polygons.

3. b) 3 in
   c) 16 m

4. a) ii) $36 \text{ in}^3$
   iii) $24 \text{ in}^3$

b) ii) $12 \text{ in}^3$, 3 in, $36 \text{ in}^3$
   iii) $6 \text{ in}^3$, 4 in, $24 \text{ in}^3$

5. b) $(7 \text{ ft})^2(4 \text{ ft})$
   = $28 \text{ ft}^3$

6. $600 = (B)(15)$
   $B = 600 + 15$
   $B = 40 \text{ in}^2$

7. a) $48 = (B)(3)$
   $B = 48 + 3$
   $B = 16 \text{ in}^2$
   $t' = 16 \text{ in}$, $w = 1 \text{ in}$;
   $t' = 8 \text{ in}$, $w = 2 \text{ in}$;
   $t' = 4 \text{ in}$, $w = 4 \text{ in}$

b) $(\text{base} \times \text{height}) + 2$
   = $(1.5 \text{ cm})(2 \text{ cm}) + 2$
   = $1.5 \text{ cm}^2$

c) $(\text{base}_1 + \text{base}_2) \times \text{height} + 2$
   = $(4 \text{ m} + 6 \text{ m})(5.1 \text{ m})$
   + 2
   = $25.5 \text{ m}^2$

d) base $\times$ height
   $= \left[\frac{4}{2} \right] \left[\frac{1}{3} \right]$
   $= \left[\frac{9}{2} \right] \left[\frac{7}{3} \right]$
   $= 63 \text{ in}^2$
   $= 10 \frac{1}{2} \text{ in}^2$

9. b) $V = Bh$
   = $(1.5 \text{ cm}^2)(1.8 \text{ cm})$
   = $2.7 \text{ cm}^3$

c) $V = Bh$
   = $(25.5 \text{ m}^3)(5 \text{ m})$
   = $127.5 \text{ m}^3$

d) $V = Bh$
   = $(\frac{10}{2} \text{ in}^2)(\frac{3}{4} \text{ in})$
   $= \frac{21}{2} \text{ in}^2$, $\frac{13}{4} \text{ in}^2$
   $= 273 \text{ in}^3$
   $= 34 \frac{1}{6} \text{ in}^3$

10. Answers will vary.
    Sample answers:
    a) $V_1 = 14 \times 9 \times 3$
       = $378 \text{ in}^3$

    b) $V_2 = 3 \times 6 \times 4$
       = $72 \text{ in}^3$

    Total volume
    $= 378 + 72$
    $= 450 \text{ in}^3$

    c) $V_1 = 5 \times 7 \times 8$
       = $280 \text{ in}^3$

    $V_2 = (2 + 5) \times 3 + 2$
    $= 84 \text{ in}^3$

    Total volume
    $= 280 + 84$
    $= 364 \text{ in}^3$

11. $\frac{5}{16} = \frac{3}{4} \times \frac{5}{6} \times w$
    $\frac{5}{16} = 15 \times w$
    $w = \frac{5}{16} \div 15$
    $w = \frac{5}{24}$

    $w = \frac{5}{24}$

    $w = \frac{5}{16} \times 24$

    $w = \frac{24}{240}$

    $w = \frac{1}{2} \text{ ft}$

12. Answers will vary.
    Sample answers:
    a) 3 in, 4 in, 1 in;
       2 in, 2 in, 3 in

    b) 2 cm, 4 cm, 1 cm;
       2 cm, 2 cm, 2 cm

    c) 2 ft, $\frac{1}{4}$ ft, 1 ft;
       $\frac{1}{2}$ ft, 1 ft, 1 ft

    d) $\frac{3}{4}$ m, 1 m, 1 m;
       $\frac{1}{2}$ m, $\frac{3}{2}$ m, 1 m

13. a) $A = (40 \times 25)$
    $A = 1,000 - 160$
    $A = 840 \text{ ft}^2$

    $V = 840 \times \frac{1}{3}$
    $V = 280 \text{ ft}^3$

    Ben needs 280 ft$^3$ of topsoil.

    b) $280 + 70 = 4$
       $4 \times 149 = 596$

    Ben will pay $596 for the topsoil he needs.

    Bonus
    $840 \times \frac{3}{8}$
    $= 315 \text{ ft}^3$

    $315 + 70 = 45$

    $4.5 \times 149 = 670.50$

    $670.5 - 596 = $74.50

    It would cost $74.50 more.
10. a) \[ V = \pi r^2 h \]
\[ = 3.14(5)^2(4) \]
\[ = 314 \text{ ft}^3 \]

**Bonus**

\[ V_A = \pi r^2 h \]
\[ = 3.14(5)^2(8) \]
\[ = 628 \text{ in}^3 \]
\[ V_B = \pi r^2 h \]
\[ = 3.14(10)^2(8) \]
\[ = 2,512 \text{ in}^3 \]

Volume A : Volume B
\[ = 628 : 2,512 \]
\[ = 1 : 4 \]

8. a) \[ V_A = \pi r^2 h \]
\[ = 3.14(1.5)^2(5) \]
\[ = 35.33 \text{ in}^3 \]

\[ V_B = \pi r^2 h \]
\[ = 3.14(2)^2(3) \]
\[ = 37.68 \text{ in}^3 \]

Can B holds more.

b) Can A:
\[ 1.98 + 35.3 \text{ in}^3 \]
\[ = 37.3 \text{ in}^3 \]
\[ = 0.056/\text{in}^3 \]

Can B:
\[ 2.29 + 37.7 \text{ in}^3 \]
\[ = 0.061/\text{in}^3 \]

Can A costs less.

9. a) \[ V_1 = \pi r^2 h \]
\[ = 3.14(3.5)^2(14) \]
\[ = 538.5 \text{ cm}^3 \]

\[ V_2 = \pi r^2 h \]
\[ = 3.14(2.5)^2(4) \]
\[ = 78.5 \text{ cm}^3 \]

\[ V_{total} = 538.5 + 78.5 \]
\[ = 617 \text{ cm}^3 \]

b) \[ 617 + 250 = 2.5 \text{ g} \]

c) \[ 2.5 \times 4 = 10 \text{ calories} \]

10. a) \[ V_{outer} = \pi r^2 h \]
\[ = 3.14(1.27)^2(2.44) \]
\[ = 12.4 \text{ m}^3 \]

\[ V_{inner} = \pi r^2 h \]
\[ = 3.14(1.05)^2(2.44) \]
\[ = 8.4 \text{ m}^3 \]

\[ V_{outer} - V_{inner} \]
\[ = 12.4 - 8.4 = 4.0 \text{ m}^3 \]

b) \[ 4.0 \times 2,406 \]
\[ = 9,624.0 \text{ kg} \]

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**AP Book G8-51**

**Page 163**

1. b) \[ \text{trapezoid} \]

2. b) \[ 4 \text{ ft} \]

3. a) Right pyramids: B, E
Skew pyramids: D, F
Not pyramids: A, C

b) Answers will vary.
Sample answer:
A is not a pyramid because it does not have an apex.

4. a) \[ s^2 = b^2 + 8^2 \]
\[ = 64 + 64 \]
\[ = 128 \]
\[ a = \sqrt{100} \]
\[ a = 10 \text{ cm} \]

b) \[ r^2 = 4^2 + 12^2 \]
\[ r^2 = 16 + 144 \]
\[ r^2 = 160 \]
\[ r = \sqrt{160} \]
\[ r = 12.6 \text{ cm} \]

5. a) \[ 12^2 + z^2 = 15^2 \]
\[ 144 + z^2 = 225 \]
\[ z^2 = 225 - 144 \]
\[ z^2 = 81 \]
\[ z = \sqrt{81} \]
\[ z = 9 \text{ in} \]

b) \[ 3^2 + n^2 = 4^2 \]
\[ 9 + n^2 = 16 \]
\[ n^2 = 16 - 9 \]
\[ n^2 = 7 \]
\[ n = \sqrt{7} \]
\[ n = 2.6 \text{ in} \]

6. a) \[ h \]

b) The slant height is the hypotenuse of a right triangle and the height is the side of the right triangle. The hypotenuse is always greater than a side in a right triangle.

**Bonus**

The heights are the same because they are the distance from the base to the apex.
Since we don’t know the dimensions of the base, the lateral faces and therefore the slant heights may be different.
11. a) \[ \triangle ABC \]
\[ A = 9 \text{ in} \]
\[ B = 12 \text{ in} \]
\[ C = \sqrt{9^2 + 12^2} \]
\[ AC = 9 + 12 = 21 \text{ in} \]
\[ AC^2 = 81 + 144 \]
\[ AC = 3 \text{ in} \]
\[ a) \quad V_{	ext{prism}} = 15 \times 3 \]
\[ = 45 \text{ in}^3 \]
\[ b) \quad A = 6 \times 5 \]
\[ = 30 \text{ in}^2 \]
\[ h = 4 \text{ in} \]
\[ V_{	ext{prism}} = 30 \times 4 \]
\[ = 120 \text{ in}^3 \]
\[ c) \quad A = 9 \times 4 \times 2 \]
\[ = 18 \text{ in}^2 \]
\[ h = 10 \text{ in} \]
\[ V_{	ext{prism}} = 18 \times 10 \]
\[ = 180 \text{ in}^3 \]
\[ b) \quad V = Bh + 3 \]
\[ V = (3 \times 4)(5) + 3 \]
\[ V = 20 \text{ m}^3 \]
\[ V = Bh + 3 \]
\[ V = (6 \times 8)(10) + 3 \]
\[ V = 160 \text{ m}^3 \]
\[ b) \quad The volume is 8 times greater because doubling each dimension is 2 \times 2 = 8 times as much. \]
\[ b) \quad V = Bh + 3 \]
\[ V = 15 \text{ in} \]
\[ = 20 \text{ in} \]
\[ V = Bh + 3 \]
\[ V = (10 \times 9)(6.2) + 3 \]
\[ V = 166 \text{ in}^3 \]
\[ c) \quad 4^2 + h^2 = 12^2 \]
\[ 16 + h^2 = 144 \]
\[ h^2 = 128 \]
\[ h = \sqrt{128} \]
\[ h = 11.3 \text{ cm} \]
\[ V = Bh + 3 \]
\[ V = (18 \times 8)(11.3) + 3 \]
\[ V = 542.4 \text{ cm}^3 \]
\[ d) \quad 3.5^2 + h^2 = 11^2 \]
\[ 12.25 + h^2 = 121 \]
\[ h^2 = 108.75 \]
\[ h = \sqrt{108.75} \]
\[ h = 10.4 \text{ ft} \]
\[ V = Bh + 3 \]
\[ V = (9 \times 7)(10.4) + 3 \]
\[ V = 218.4 \text{ ft}^3 \]
\[ e) \quad 3^2 + h^2 = 10^2 \]
\[ 9 + h^2 = 100 \]
\[ h^2 = 91 \]
\[ h = \sqrt{91} \]
\[ h = 9.5 \text{ cm} \]
\[ V = Bh + 3 \]
\[ V = (6 \times 4)(9.5) + 3 \]
\[ V = 76 \text{ cm}^3 \]
\[ f) \quad 4.15^2 + h^2 = 9.7^2 \]
\[ 17.2 + h^2 = 94.1 \]
\[ h^2 = 76.9 \]
\[ h = \sqrt{76.9} \]
\[ h = 8.8 \text{ in} \]
\[ V = Bh + 3 \]
\[ V = (8.3 \times 5.2)(8.8) + 3 \]
\[ V = 126.6 \text{ in}^3 \]
## Answer Keys for AP Book 8.2

<table>
<thead>
<tr>
<th>Page 169</th>
</tr>
</thead>
</table>
| **9.** a) $V_1 = \pi \times w \times h$
  $= 26 \times 22 \times 16$
  $= 9,152$ in$^3$

$V_2 = Bh + 3$
  $= (26 \times 22)(18) + 3$
  $= 3,432$ in$^3$

$V_1 + V_2 = 9,152 + 3,432$
  $= 12,584$ in$^3$

b) $V_1 = Bh$
  $= (6 \times 9 + 2)(5)$
  $= 135$ in$^3$

$V_2 = Bh + 3$
  $= (6 \times 9 + 2)(7.5) + 3$
  $= 67.5$ in$^3$

$V_1 + V_2 = 135 + 67.5$
  $= 202.5$ in$^3$

**Bonus**

$V_1 = \pi r^2h + 2$
  $= 3.14(0.75)^2(4)$
  $+ 2$
  $= 3.5$ ft$^3$

$V_2 = Bh + 3$
  $= (1.5 \times 4)(3) + 3$
  $= 6$ ft$^3$

$V_1 + V_2 = 3.5 + 6$
  $= 9.5$ ft$^3$

| **10.** | $B_1 = [(4.5 + 9) \times 3.9 + 2]$
  $\times 2$
  $= 52.65$ ft$^2$

$V = Bh + 3$
  $= (52.65)(2) + 3$
  $= 35.1$ ft$^3$

| **11.** | $600^2 + d^2 = 768^2$
  $360,000 + d^2 = 589,824$
  $d^2 = 589,824 - 360,000$
  $d^2 = 229,824$
  $d = \sqrt{229,824}$
  $d = 479$ m

**AP Book G8-53**

<table>
<thead>
<tr>
<th>Page 169</th>
</tr>
</thead>
</table>
| **1.** b) $r = 4$ cm
  $d = 8$ cm
  $h = 12$ cm

c) $r = 2$ in
  $d = 4$ in
  $h = 5$ in

---

**Answer Keys for AP Book 8.2**

T-45
### Geometry: Volume – AP Book 8.2: Unit 6

#### (continued)

<table>
<thead>
<tr>
<th>Question</th>
<th>Formula</th>
<th>Calculation</th>
<th>Notice</th>
</tr>
</thead>
<tbody>
<tr>
<td>12. a) $8^2 + h^2 = 17^2$</td>
<td>$64 + h^2 = 289$</td>
<td>$h^2 = 289 - 64$</td>
<td>$h = \sqrt{225}$</td>
</tr>
<tr>
<td>$V = \frac{1}{3} \pi r^2 h$</td>
<td>$V = \frac{1}{3} \pi r^2 h$</td>
<td>$V = \frac{1}{3} (3.14)(8)^2(15)$</td>
<td>$V = 100.8 m^3$</td>
</tr>
<tr>
<td>b) $2.5^2 + h^2 = 6^2$</td>
<td>$6.25 + h^2 = 36$</td>
<td>$h^2 = 36 - 6.25$</td>
<td>$h = \sqrt{29.8}$</td>
</tr>
<tr>
<td>$V = \frac{1}{3} \pi r^2 h$</td>
<td>$V = \frac{1}{3} \pi r^2 h$</td>
<td>$V = \frac{1}{3} (3.14)(2.5)^2(5.5)$</td>
<td>$V = 36.0 m^3$</td>
</tr>
<tr>
<td>c) $10^2 + h^2 = 13.5^2$</td>
<td>$100 + h^2 = 182.3$</td>
<td>$h^2 = 182.3 - 100$</td>
<td>$h = \sqrt{82.3}$</td>
</tr>
<tr>
<td>$V = \frac{1}{3} \pi r^2 h$</td>
<td>$V = \frac{1}{3} \pi r^2 h$</td>
<td>$V = \frac{1}{3} (3.14)(10)^2(9.1)$</td>
<td>$V = 952.5 in^3$</td>
</tr>
</tbody>
</table>

### Bonus

<table>
<thead>
<tr>
<th>Question</th>
<th>Formula</th>
<th>Calculation</th>
<th>Notice</th>
</tr>
</thead>
<tbody>
<tr>
<td>13. a) $V_1 = \frac{1}{3} \pi r^2 h$</td>
<td>$V_1 = \frac{1}{3} \pi r^2 h$</td>
<td>$V_1 = \frac{1}{3} (3.14)(6)^2(18)$</td>
<td>$V_1 = 678.2 in^3$</td>
</tr>
<tr>
<td>$V_2 = 20 \times 20 \times 1$</td>
<td>$V_2 = 20 \times 20 \times 1$</td>
<td>$V_2 = 400 in^3$</td>
<td>$V_1 + V_2 = 718.2 in^3$</td>
</tr>
</tbody>
</table>

### Page 172

<table>
<thead>
<tr>
<th>Question</th>
<th>Formula</th>
<th>Calculation</th>
<th>Notice</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. a) 2 inches</td>
<td>$V = \frac{3}{2} \pi r^3$</td>
<td>$V = \frac{3}{2} (3.14)(1.95)^3$</td>
<td>$V = 400 in^3$</td>
</tr>
<tr>
<td>b) $V = \pi r^2 h$</td>
<td>$V = \pi (1)^2(2)$</td>
<td>$V = 2\pi$</td>
<td>$V = 6.28 in^3$</td>
</tr>
<tr>
<td>c) less</td>
<td>$V_{sphere} &lt; V_{cylinder}$</td>
<td>$V_{sphere} &lt; V_{cylinder}$</td>
<td>$V_{sphere} &lt; V_{cylinder}$</td>
</tr>
<tr>
<td>d) $V_{sphere} &lt; V_{cylinder}$</td>
<td>$V_{sphere} &lt; V_{cylinder}$</td>
<td>$V_{sphere} &lt; V_{cylinder}$</td>
<td>$V_{sphere} &lt; V_{cylinder}$</td>
</tr>
<tr>
<td>e) Yes, because $\frac{4}{3} \pi$ is less than $2\pi$.</td>
<td>$V = \frac{4}{3} \pi r^3$</td>
<td>$V = \frac{4}{3} (3.14)(11)^3$</td>
<td>$V = 5,572.5 cm^3$</td>
</tr>
</tbody>
</table>
b) \[ V_{cylinder} = \pi r^2 h \]
\[ = \frac{1}{3} (3.14)(4)^2(15) \]
\[ = \frac{1}{3} (3.14)(16)(15) \]
\[ = 251.2 \text{ cm}^3 \]
\[ V_{hemisphere} = \frac{4}{3} \pi r^3 + 2 \]
\[ = \frac{4}{3} (3.14)(4)^3 + 2 \]
\[ = \frac{4}{3} (3.14)(64) + 2 \]
\[ = 134.0 \text{ cm}^3 \]
\[ V_{cone} + V_{hemisphere} \]
\[ = 251.2 + 134.0 \]
\[ = 385.2 \text{ cm}^3 \]

c) \[ V_{cylinder} = \pi r^2 h \]
\[ = (3.14) (2.5)^2(3) \]
\[ = (3.14)(6.25)(3) \]
\[ = 58.9 \text{ m}^3 \]
\[ V_{hemisphere} = \frac{4}{3} \pi r^3 + 2 \]
\[ = \frac{4}{3} (3.14)(2.5)^3 + 2 \]
\[ = \frac{4}{3} (3.14)(15.6) + 2 \]
\[ = 32.7 \text{ m}^3 \]
\[ V_{cylinder} - V_{hemisphere} \]
\[ = 58.9 - 32.7 \]
\[ = 26.2 \text{ m}^3 \]

5. \[ V_{golf ball} = \frac{4}{3} \pi r^3 \]
\[ = \frac{4}{3} (3.14)(0.85)^3 \]
\[ = \frac{4}{3} (3.14)(0.6) \]
\[ = 2.5 \text{ in}^3 \]
\[ V_{box} = 7.3 \times 5.5 \times 1.9 \]
\[ = 76.3 \text{ in}^3 \]
\[ V_{box} - (V_{golf ball} \times 12) \]
\[ = 76.3 - (2.5 \times 12) \]
\[ = 46.3 \text{ in}^3 \text{ of space} \]

6. a) \[ V = \frac{4}{3} \pi r^3 \]
\[ 2,143.6 = \frac{4}{3} (3.14)r^3 \]
\[ 2,143.6 = 4.2r^3 \]
\[ 2,143.6 + 4.2 = r^3 \]
\[ 510.4 = r^3 \]
\[ \sqrt[3]{510.4} = r \]
\[ 8 \text{ in} = r \]

b) \[ V = \frac{4}{3} \pi r^3 \]
\[ 65.4 = \frac{4}{3} (3.14)r^3 \]
\[ 65.4 = 4.2r^3 \]
\[ 65.4 + 4.2 = r^3 \]
\[ 15.6 = r^3 \]
\[ \sqrt[3]{15.6} = r \]
\[ 2.5 \text{ cm} = r \]

7. \[ V = \frac{4}{3} \pi r^3 \]
\[ 14,130 = \frac{4}{3} (3.14)r^3 \]
\[ 14,130 = 4.2r^3 \]
\[ 14,130 + 4.2 = r^3 \]
\[ 3,364.3 = r^3 \]
\[ \sqrt[3]{3,364.3} = r \]
\[ 15.0 \text{ ft} = r \]
\[ d = 15.0 \times 2 = 30 \text{ in} \]

8. \[ 3^2 + r^2 = 5^2 \]
\[ 9 + r^2 = 25 \]
\[ r^2 = 25 - 9 \]
\[ r^2 = 16 \]
\[ r = \sqrt{16} \]
\[ r = 4 \text{ cm} \]

AP Book G8-55 page 174

1. Real-world examples will vary. Sample answers below.
   a) rectangular prism
   
   shoe box, building
   
   \[ V = l \times w \times h \]

   b) cylinder
   
   can, toilet paper roll
   
   \[ V = \pi r^2 h \]

   c) pyramid
   
   tent, block of cheese
   
   \[ V = Bh + \frac{1}{3} \pi r^2 h \]

   d) cone
   
   pylon, ice cream cone
   
   \[ V = \frac{1}{3} \pi r^2 h \]

   e) sphere
   
   basketball, marble
   
   \[ V = \frac{4}{3} \pi r^3 \]

2. a) i) \[ V = \frac{4}{3} \pi r^3 \]

   ii) \[ V = \frac{1}{3} \pi r^3 \]

   iii) \[ V = \frac{4}{3} \pi r^3 \]

   b) \[ \text{cylinder: } 372 = \pi r^3 \]

   i) \[ V_{cone} = \frac{1}{3} \pi r^3 \]

   ii) \[ V_{hemisphere} = \frac{4}{3} \pi r^3 \]

   ii) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   iii) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   iv) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   v) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   vi) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   vii) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   viii) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   ix) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   x) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xi) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   xii) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xiii) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   xiv) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xv) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   xvi) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xvii) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   xviii) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xix) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   xx) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xxi) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   xii) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xiii) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   xiv) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xv) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   xvi) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xvii) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   xviii) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xix) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   xx) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xxi) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   xii) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xiii) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   xiv) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xv) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   xvi) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xvii) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   xviii) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xix) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   xx) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xxi) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   xii) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xiii) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   xiv) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xv) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   xvi) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xvii) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   xviii) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xix) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]

   xx) \[ V_{cylinder} = \frac{4}{3} \pi r^3 \]

   xxi) \[ V_{sphere} = \frac{4}{3} \pi r^3 \]
b) \[ 3^2 + h^2 = 8^2 \]
\[ 9 + h^2 = 64 \]
\[ h^2 = 64 - 9 \]
\[ h^2 = 55 \]
\[ h = \sqrt{55} \]
\[ V = Bh + 3 \]
\[ = (15 \times 6)(7.4) + 3 \]
\[ = 222 \text{ in}^3 \]

c) \[ 2^2 + h^2 = 9^2 \]
\[ 4 + h^2 = 81 \]
\[ h^2 = 81 - 4 \]
\[ h^2 = 77 \]
\[ h = \sqrt{77} \]
\[ V = \frac{1}{3} \pi r^2 h \]
\[ = \frac{1}{3} (3.14)(2)^2(8.8) \]
\[ = \frac{1}{3} (3.14)(4)(8.8) \]
\[ = 36.8 \text{ cm}^3 \]

6. a) \[ V_1 = 4 \times 10 \times 15 \]
\[ = 600 \text{ m}^3 \]
\[ 5^2 + h^2 = 7.8^2 \]
\[ 25 + h^2 = 60.8 \]
\[ h^2 = 60.8 - 25 \]
\[ h^2 = 35.8 \]
\[ h = \sqrt{35.8} \]
\[ h = 6.0 \text{ m} \]
\[ V_2 = Bh + 3 \]
\[ = (10 \times 15)(6.0) + 3 \]
\[ = 300 \text{ m}^3 \]
\[ V_1 + V_2 = 600 + 300 \]
\[ = 900 \text{ m}^3 \]

b) \[ V_1 = 10 \times 12 \times 21 \]
\[ = 2,520 \text{ in}^3 \]
\[ V_2 = \pi r^2 h + 2 \]
\[ = (3.14)(21)^2 + 2 \]
\[ = (3.14)(441) + 2 \]
\[ = 1,186.9 \text{ in}^3 \]
\[ V_1 + V_2 \]
\[ = 2,520 + 1,186.9 \]
\[ = 3,706.9 \text{ in}^3 \]

7. Answers will vary.
Sample answers:

8. a) \[ V_{\text{gumball}} = \frac{4}{3} \pi r^3 \]
\[ = \frac{4}{3} (3.14)(0.5)^3 \]
\[ = \frac{4}{3} (3.14)(0.125) \]
\[ = 0.5 \text{ in}^3 \]
\[ V_{\text{machine}} = \frac{4}{3} \pi r^3 \]
\[ = \frac{4}{3} (3.14)(4)^3 \]
\[ = \frac{4}{3} (3.14)(64) \]
\[ = 267.9 \text{ in}^3 \]
\[ 267.9 + 0.5 \]
\[ = 535.8 \text{ gumballs} \]

b) Less than because there will be empty space in between the gumballs.
1. a) i) 
   b) no association 
   c) positive association 

2. a) i) 
   ii) 
   iii) 
   b) i) positive association 
   ii) negative association 
   iii) positive association 

3. a) (68, 67), (72, 69), (85, 84), (68, 77), (69, 71), (84, 82), (87, 78), (67, 70), (70, 68), (72, 69), (81, 82), (79, 80), (71, 70), (72, 72) 
   b) See graph above. 
   c) no association 

4. a) ii) positive, nonlinear 
   iii) no association 
   b) no 
   c) no 
   d) nonlinear 

5. a) i) negative, linear 
   ii) negative, nonlinear 
   iii) negative, nonlinear 
   iv) nonlinear 
   v) positive, linear 
   vi) nonlinear 
   b) i), ii), and v) 

6. a) 
   b) See graph above. 
   c) negative association 

AP Book SP8-5 

1. a) negative 
   b) linear 
   c) B 
   d) B 
   e) A 

2. a) B and D 
   b) A and C 
   c) D 
   d) C 

3. a) A and B 
   b) B 
   c) B 

4. a) C 
   b) B: 5 
   c) C: 5 

5. a) A 
   b) B 
   c) yes 

6. a) i) counterclockwise 
   ii) clockwise 
   b) no 

AP Book SP8-6 

1. Teacher to check lines. 
   Sample answers: 
   b) 7 
   c) 7 
   d) 7 
   e) 7 
   f) 8 
   g) 7 

2. a) Olympic Gold Medals Won by US Athletes by Year 
   b) positive 
   c) See graph above. 

3. a) Height and Foot Length 
   b) positive 
   c) See graph above. 

4. a) 
   b) positive 
   c) See graph above. 

5. a) 
   b) positive 
   c) See graph above. 

AP Book SP8-7 

1. a) 3, 2 
   b) 2 
   c) 1 
   d) 2 

2. a) positive 
   b) 
   c) run = 15 
   d) They are the same.
3. a) ii) 0
   iii) 42.5
b) i) A (3, 2)
    B (5, 1)
    run = 2
    rise = -1
    slope = $-\frac{1}{2}$
ii) A (1, 2)
    B (2, 4)
    run = 1
    rise = 2
    slope = $\frac{2}{1}$
iii) A (10, 40)
    B (50, 30)
    run = 40
    rise = -10
    slope = $-\frac{10}{40}$
    $= -\frac{1}{4}$
c) i) $y = -\frac{1}{2}x + 3.5$
ii) $y = 2x + 0$
iii) $y = -\frac{1}{4}x + 45$

4. b) $y = mx + b$
    5 = -2(1) + b
    5 = -2 + b
    7 = b
c) $y = mx + b$
    1 = 0.4(1) + b
    1 = 0.4 + b
    0.6 = b
d) $y = mx + b$
    1 = $\frac{1}{2}(2) + b$
    1 = $\frac{1}{2} + b$
    0 = b
e) $y = mx + b$
    9 = $\frac{2}{3}(5) + b$
    9 = $\frac{10}{3} + b$
    $\frac{27}{3} - \frac{10}{3} = b$
    $\frac{17}{3} = b$

f) $y = mx + b$
    $7 = -\frac{1}{2}(2) + b$
    $7 = -1 + b$
    $8 = b$
5. b) $y = -2x + 7$
    5 = -2(1) + 7
    5 = 5
    c) $y = 0.4x + 0.6$
    1 = 0.4(1) + 0.6
    1 = 1 ✔
d) $y = \frac{1}{2}x + 0$
    1 = $\frac{1}{2}(2) + 0$
    1 = 1 ✔
e) $y = \frac{2}{3}x + \frac{17}{3}$
    9 = $\frac{2}{3}(5) + \frac{17}{3}$
    9 = $\frac{10}{3} + \frac{17}{3}$
    9 = $\frac{27}{3}$
    $= 9$ ✔
f) $y = -\frac{1}{2}x + 8$
    7 = $-\frac{1}{2}(2) + 8$
    7 = 7 ✔
6. a) yes
   positive
b) run = 80 - 20 = 60
   rise = 362 - 204 = 158
   slope = $\frac{158}{60} \div \frac{79}{30}$
c) $y = mx + b$
    204 = $\frac{79}{30}(20) + b$
    204 = $\frac{158}{3} + b$
    612 - 158 = $b$
    454 = $b$
    $\frac{151}{3} = b$
7. a) i) 168 cm
   ii) 173 cm
b) i) 190 cm
   ii) 190 cm
8. a) positive, linear
   b) $y = 2.09(9) + 0.82$
   $y = 19.63$
9. a) $2 = 2x - 1$
   $3 = 2x$
   $\frac{3}{2} = x$
   $1.5 = 2x - 1$
   $\frac{5}{2} = 2x$
   $\frac{5}{4} = x$
   b) $2 = 0.75x + 2$
   $0 = 0.75x$
   $0 = x$
   $1.5 = 0.75x + 2$
   $-0.5 = 0.75x$
   $-0.67 = x$
   c) $2 = -0.55x + 8.1$
   $-6.1 = -0.55x$
   $11.09 = x$
   $1.5 = -0.55x + 8.1$
   $-6.6 = -0.55x$
   $12 = x$
6. a) negative, linear
   b) 32 mpg
   c) $37 = -8.18x + 53.18$
   $\therefore 16.18 = -8.18x$
   $x = 1.98 L = x$

7. a) 5%
   The slope of the line of best fit is 5.

   b) 50%
   The y-intercept of the line of best fit is 50.

8. a) 5%
   b) 50%

   Answers for parts c) to k) may vary.

   Sample answers:
   c) See graph above.
   d) 2
   e) 2
   f) i) 72%
      ii) 85%
   g) i) $y = \frac{5}{8}(95)$
      + $\frac{28}{3} \frac{3}{4}$
      $y = 88.125$
   g) ii) $90 = \frac{5}{8}x$
      $+ \frac{28}{3} \frac{3}{4}$
      $61.25 = \frac{5}{8}x$
      $x = 98$

   Bonus
   a) $30 - 2 = 28$

   AP Book SP8-9
   page 190
   1. a) i) Even: 12
      Prime: 7, $\sqrt{19}$
      Neither: $\sqrt{16}, \frac{5}{3}$
      ii) Rational: 7, $\sqrt{9}$
      $\frac{12, \frac{2}{3}}{3}$
      Irrational: $\sqrt{2}, \frac{16}{19}$
   b) Teacher to check.
   c) Yes. The number 2 is both even and prime.
   d) No. A number is either rational or irrational.
   2. a) $> 3: 13, \frac{41}{4}, \frac{25}{2}, 12.3$
      $< 10: -3, \sqrt{5}, \sqrt{8}$
      Overlap: $4 \frac{2}{5}, \sqrt{75}$
   b) Teacher to check.
   c) 6
   d) 5
   e) 2
   f) 4
   g) 3
   h) $< 10 | < 10 | T$
      $> 3 | 2 | 4 | 6$
      $> 3 | 3 | 0 | 3$
      T | 5 | 4 | 9

   3. a) 7
      b) 15
      c) 6
      d) 2
      f) $13 + 17 = 30$
      g) 30

   Bonus
   30 - 2 = 28

   AP Book SP8-10
   page 193
   1. 14 + 75 = 19%
      11 + 75 = 15%
      8 + 75 = 11%
      19 + 75 = 25%

   AP Book SP8-11
   page 195
   1. Cathy
2. a) LH | RH  
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<tbody>
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<td>32</td>
<td>268</td>
<td>300</td>
<td>300</td>
<td>11%</td>
<td>89%</td>
<td>53</td>
<td>447</td>
</tr>
<tr>
<td>300</td>
<td>300</td>
<td>11%</td>
<td>89%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>500</td>
<td>11%</td>
<td>89%</td>
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</table>

b) about the same  
c) Stay the same, because as the number of people who live in the city increases, the number of left-handed people should increase relative to the population increase.

d) no

3. a) SF | Not SF  
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<tr>
<td>24%</td>
<td>76%</td>
<td>100%</td>
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<tr>
<td>24%</td>
<td>76%</td>
<td>100%</td>
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b) No. Both relative frequencies are 24%.

4. a) H | No H  
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<tbody>
<tr>
<td>25</td>
<td>75</td>
<td>1000</td>
<td>1000</td>
<td>25%</td>
<td>75%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>275</td>
<td>305</td>
<td>305</td>
<td>10%</td>
<td>90%</td>
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</table>

b) country  
c) not be affected  
As the number of people who live in the city increases, the number of people who have ridden a horse should increase relative to the population size.

d) yes  
e) negative association  
f) 405  
g) Add all the numbers inside and outside of the circle.

5. a) N | C | O | T  
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<tbody>
<tr>
<td>18%</td>
<td>62%</td>
<td>20%</td>
<td>100%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>37%</td>
<td>30%</td>
<td>33%</td>
<td>100%</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

b) i) comics  
ii) novels and other  
iii) yes

6. a) Ph | No Ph | T  
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<tr>
<td>47</td>
<td>23</td>
<td>70</td>
<td>67%</td>
<td>33%</td>
<td>100%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>18</td>
<td>30</td>
<td>80%</td>
<td>20%</td>
<td>100%</td>
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</table>

b) yes  
The greatest percentages occur when people have both a TV and phone or neither.

c) TV | No TV | T  
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<tr>
<td>47</td>
<td>12</td>
<td>59</td>
<td>80%</td>
<td>20%</td>
<td>100%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>18</td>
<td>30</td>
<td>40%</td>
<td>60%</td>
<td>100%</td>
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</table>

Bonus
Yes, because the difference between percentages is larger between people with phones than with people with TVs.

AP Book SP8-12  
page 198  
1. a) B, C, D  
b) A  
c) A, B, D  
d) C  
e) A, D  
f) B  
2. D, because the farther away the student lives, the longer it takes. Assuming that students walk at the same rate, it should be a strong linear association.

3. a) stays the same  
b) increases  
c) decreases  
4. a) Hours Playing Sports and Watching TV  
5. a) N | C | O | T  
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<tr>
<td>18%</td>
<td>62%</td>
<td>20%</td>
<td>100%</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>37%</td>
<td>30%</td>
<td>33%</td>
<td>100%</td>
<td></td>
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</tr>
</tbody>
</table>

b) negative  
The greatest percentages occur when people have both a TV and phone or neither.

c) TV | No TV | T  
<table>
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</thead>
<tbody>
<tr>
<td>30</td>
<td>305</td>
<td>64%</td>
<td>36%</td>
<td>100%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>275</td>
<td>83 bpm</td>
<td>17</td>
<td>45 bpm</td>
<td></td>
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</tbody>
</table>

h) Sample answer:  
A (2, 75), B (6, 60)  
rise | run | slope = \( -\frac{15}{4} = -3.75 \)  
i) \( y = -3.75x + b \)  
75 = \(-3.75(2) + b\)  
75 = \(-7.5 + b\)  
82.5 = b  
j) \( y = -3.75x + 82.5 \)
iii) | P | F | T  |
    |   |   |    |
    | 23%| 18%| 40% |
    | 49%| 11%| 60% |
    | 72%| 28%|100% |
iv) | B | No B | T  |
    |   |      |    |
    | 24%| 29%| 53% |
    | 19%| 29%| 47% |
    | 42%| 58%|100% |

c) i) no
   ii) no
   iii) yes
   iv) no

7. b) 15
c) 26
d) | B | No B |
   |   |      |
   | 17 | 26   |
   | 13 | 15   |
e) | B | No B |
   |   |      |
   | 17 | 26   |
   | 13 | 15   |
   | 43 | 43   |
   | 40%| 60%  |
   | 28 | 28   |
   | 46%| 54%  |
f) Sam is not correct. He needs to look at relative frequencies, not the number of men and women.
1. Count squares to find the base and height. Then use the formula to find the area of the triangle.

   a) ![Triangle A]
   b) ![Triangle B]
   c) ![Triangle C]

   Area = Area = Area = 

2. Mark and measure the height and base to the nearest tenth of a centimeter. Then find the area.

   a) ![Parallelogram A]
   b) ![Parallelogram B]

   Area = Area =

3. a) Use a compass to construct a circle with center $O$ and radius $AB$.

   ![Circle with center O and radius AB]

   b) Find the diameter of the circle in part a) to the nearest tenth of a centimeter. __________

   __________
4. The diameter of a circle is 10 cm.
   a) Find the radius of the circle. ___________

   b) Find the approximate circumference of the circle. Use 3.14 for \( \pi \).

   c) Find the approximate area of the circle. Use 3.14 for \( \pi \).

5. Find the total area of the shape.

   ![Diagram of a shape with dimensions 6 in and 10 in]

**Bonus** The circumference of a circle is approximately 25.12 m. Find the radius of the circle.
Unit 4: Geometry
Quiz (Lessons 39 to 41)

1. a) Area = \(7 \times 6 \div 2\)
   = 21 units\(^2\)

   b) Area = \(4 \times 5 \div 2\)
   = 10 units\(^2\)

   c) Area = \(6 \times 5 \div 2\)
   = 15 units\(^2\)

2. a) Area = \(4 \times 2.2\)
   = 8.8 cm\(^2\)

   b) Area = \(2.5 \times 3.2\)
   = 8.0 cm\(^2\)

3. a) Teacher to check.

   b) 3.6 cm

4. a) 5 cm

   b) \(C = 2\pi r\)
      = \((2)(3.14)(5)\)
      = 31.4 cm

   c) \(A = \pi r^2\)
      = \((3.14)(5^2)\)
      = 78.5 cm\(^2\)

5. \(A = (b \times h) + (\pi r^2 \div 2)\)
   = \((6 \times 10) + (3.14 \times 3^2 \div 2)\)
   = (60) + (14.13)
   = 74.13 in\(^2\)

   **Bonus**

   \(2\pi r = 25.12\)
   \((2)(3.14)(r) = 25.12\)
   \(r = 25.12 + 2 + 3.14\)
   \(r = 4 m\)
Unit 4: Geometry

Quiz (Lessons 42 to 44)

1. a) Find the area of the large square by finding the area of the 4 right triangles and what is left.

   Area = 4 × ____________ + ________

   = _____________________

   = ________

   b) Find the side length of the large square.

   Side length = ________

   ≈ ________

2. a) Find the area of the squares with side lengths $a$, $b$, and $c$.

   Area of square with side length $a = _____ × _____ = _____$

   Area of square with side length $b = _____ × _____ = _____$

   Area of square with side length $c = 4 × _____ + _____ = _____$

   b) Check that $c^2 = a^2 + b^2$.

3. Use the Pythagorean Theorem to write an equation for the triangle.

   a) ____________

   b) ____________

   c) ____________
4. Use the Pythagorean Theorem to find the hypotenuse.

   a) 
   
   b) 

5. Find the missing side of the triangle.

   a) 
   
   b) 

**Bonus** ▶ Draw a right triangle that is also an isosceles triangle.
1. a) \[ 4 \times (6 \times 9 + 2) + (3 \times 3) \]
    \[ = 4 \times 27 + 9 \]
    \[ = 117 \]
    b) \[ \sqrt{117} \cdot 10.8 \]

2. a) \[ a = 4 \times 4 = 16 \]
    \[ b = 3 \times 3 = 9 \]
    \[ c = 4 \times 6 + 1 = 25 \]
    b) \[ 25 = 16 + 9 \checkmark \]

3. a) \[ m^2 = w^2 + r^2 \]
    b) \[ a^2 = b^2 + c^2 \]
    c) \[ f^2 = g^2 + h^2 \]

4. a) \[ d^2 = 5^2 + 12^2 \]
    \[ d^2 = 25 + 144 \]
    \[ d^2 = 169 \]
    \[ d = \sqrt{169} \]
    \[ d = 13 \]
    b) \[ w^2 = 2^2 + (\sqrt{21})^2 \]
    \[ w^2 = 4 + 21 \]
    \[ w^2 = 25 \]
    \[ w = \sqrt{25} \]
    \[ w = 5 \]

5. a) \[ 15^2 = c^2 + 12^2 \]
    \[ 225 = c^2 + 144 \]
    \[ 225 - 144 = c^2 \]
    \[ 81 = c^2 \]
    \[ 9 = c \]
    b) \[ 8^2 = p^2 + (\sqrt{15})^2 \]
    \[ 64 = p^2 + 15 \]
    \[ 64 - 15 = p^2 \]
    \[ 49 = p^2 \]
    \[ 7 = p \]

Bonus

![Triangle Diagram]
1. Label the missing sides or angles with the correct letter.

   a)  
   b)  

2. For each triangle in Question 1, calculate $a^2$, $b^2$, and $c^2$. Then perform a comparison and state whether $\angle A$ is obtuse, acute, or a right angle.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>$a^2$</th>
<th>$b^2$</th>
<th>$c^2$</th>
<th>Calculation</th>
<th>$\angle A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>9</td>
<td>6</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td>17</td>
<td>8</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c)</td>
<td>14</td>
<td>5</td>
<td>12</td>
<td></td>
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</tr>
</tbody>
</table>

3. A jogger runs 8 km south and 5 km east. What is the shortest distance she must travel to return to her starting point, rounded to the nearest tenth of a kilometer?

4. The equal sides of an isosceles triangle are 8 cm long. The other side is 6 cm long. Find the height of the triangle to the nearest tenth of a centimeter.
5. Find the length of $BC$ by following these steps:

a) Find the $|\text{change in } x|$.  \[ \text{ } = \text{ } \]

b) Find the $|\text{change in } y|$.  \[ \text{ } = \text{ } \]

c) Use the distance formula to find the length of $BC$.

**Bonus**> Find the coordinates of 4 points that are 5 units away from the origin.
Unit 4: Geometry

Quiz (Lessons 45 to 48)

1. a) $\triangle ABC$

   ![Triangle ABC](image1)

   b) $\triangle GEF$

   ![Triangle GEF](image2)

2. a) 81, 36, 64,
   \(81 < 36 + 64\), acute

   b) 289, 64, 225,
   \(289 = 64 + 255\),
   right angle

   c) 196, 25, 144,
   \(196 > 25 + 144\),
   obtuse

3. \(c^2 = 8^2 + 5^2\)
   \(c^2 = 64 + 25\)
   \(c^2 = 89\)
   \(c = \sqrt{89}\)
   \(c = 9.4\) km

4. \(h^2 + 3^2 = 8^2\)
   \(h^2 + 9 = 64\)
   \(h^2 = 55\)
   \(h = \sqrt{55}\)
   \(h = 7.4\) cm

5. a) \(|-1 - 3| = 4\)

   b) \(|1 - (-2)| = 3\)

   c) \(d = \sqrt{|-1 - 3| + |1 - (-2)|}\)
   \(d = \sqrt{4 + 3^2}\)
   \(d = \sqrt{25}\)
   \(d = 5\)

Bonus

Sample answers:
(5, 0), (−5, 0), (0, 5),
(0, −5), (3, 4), (4, 3),
(−3, 4), (4, −3), (3, −4),
(−4, 3), (−4, −3), (−3, −4)
Unit 4: Geometry

Test (Lessons 39 to 48)

1. Find the area of the figure.

2. Two circles with radius 0.75 cm and centers O and P are shown below. The circles intersect at Q.
   a) What is the length of OP? ______
   b) What is the length of OQ? ______
   c) What is the length of QP? ______
   d) If you draw △OPQ, what type of triangle is it?
   __________________________________________

3. a) Measure the radius of the circle with center O to the nearest tenth of an inch. ______
   
   b) Find the area of the circle to the nearest tenth of a square inch. Use 3.14 for π.

   c) Find the circumference of the circle to the nearest tenth of an inch. Use 3.14 for π.

   d) Find the diameter of the circle.
Unit 4: Geometry

Test (Lessons 39 to 48)

4. Find the length of the missing side in the triangle to the nearest tenth.

a)\[\begin{array}{c}
18 \\
30 \\
w
\end{array}\]

b)\[\begin{array}{c}
4 \\
\sqrt{8} \\
r
\end{array}\]

5. In \(\triangle ABC\), \(a = 10\), \(b = 7\), and \(c = 4\). Prove that \(\angle B\) is acute.

6. An engineer stands 200 m away from the base of a building. The distance to the top of the building is 350 m.

a) What is the height of the building to the nearest tenth of a meter?

b) If the engineer stands 100 m from the base of the building, what will be the distance to the top of the building to the nearest tenth of a meter?
7. \( \Delta ABC \) has vertices \( A (-4, 1), B (-2, 4), \) and \( C (2, 1) \).

a) Find the length of \( AB \) to the nearest tenth.

b) Find the exact length of \( BC \).

c) Find the exact length of \( AC \).

d) Find the perimeter of the triangle to the nearest tenth.

**Bonus** Explain why \( A (4, 7) \) and \( B (6, 3) \) could be points on a circle with center \( O (1, 3) \).
1. \[ A = (7 \times 4) + (7 \times 3 \div 2) \]
\[ = 28 + 10.5 \]
\[ = 38.5 \text{ units}^2 \]

2. a) 0.75 cm
b) 0.75 cm
c) 0.75 cm
d) equilateral

3. a) 0.7 in
b) \[ A = \pi r^2 \]
\[ = (3.14)(0.7^2) \]
\[ = (3.14)(0.49) \]
\[ = 1.5 \text{ in}^2 \]
c) \[ C = 2\pi r \]
\[ = (2)(3.14)(0.7) \]
\[ = 4.4 \text{ in} \]
d) \[ d = 2r \]
\[ = (2)(0.7) \]
\[ = 1.4 \text{ in} \]

4. a) \[ 30^2 = w^2 + 18^2 \]
\[ 900 = w^2 + 324 \]
\[ 900 - 324 = w^2 \]
\[ \sqrt{576} = w \]
\[ 24 = w \]
b) \[ i^2 = 4^2 + (\sqrt{8})^2 \]
\[ i^2 = 16 + 8 \]
\[ i^2 = 24 \]
\[ i = 4.9 \]

5. \[ a^2 = 100, b^2 = 49, c^2 = 16 \]
\[ 49 < 100 + 16 \]
so \[ b^2 < a^2 + c^2 \]
\[ \angle B \] is acute.

6. a) \[ 350^2 = h^2 + 200^2 \]
\[ 122,500 = h^2 + 40,000 \]
\[ 122,500 - 40,000 = h^2 \]
\[ 82,500 = h^2 \]
\[ \sqrt{82500} = h \]
\[ 287.2 \text{ m} = h \]
b) \[ r^2 = 100^2 + 287.2^2 \]
\[ r^2 = 10,000 + 82,485.44 \]
\[ r^2 = 92,485.44 \]
\[ r = \sqrt{92485.44} \]
\[ r = 304.1 \text{ m} \]

7. a) \[ d = \sqrt{|4 - (-4)| + |a - i|} \]
\[ = \sqrt{13} \]
\[ = 3.6 \]

Bonus
\[ d_{OA} = \sqrt{|a - 4| + |b - (-4)|} \]
\[ = \sqrt{25} \]
\[ = 5 \]
\[ d_{OB} = |1 - 6| \]
\[ = 5 \]

The distance from O to A is the same as the distance from O to B. So O could be the center of the circle.
1. Replace the variable with the given number, then evaluate.
   a) \(2w - 7, \quad w = 3\)  
   b) \(-3m + 2, \quad m = -4\)

2. Use the rule for the function to complete each function table. Then find the ordered pair that appears in both tables.

   \[y = x + 4\]
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -2 & \\
   -1 & \\
   0 & \\
   1 & \\
   2 & \\
   \end{array}
   \]

   \[y = 2x + 3\]
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -2 & \\
   -1 & \\
   0 & \\
   1 & \\
   2 & \\
   \end{array}
   \]

   common ordered pair: \(\quad\)

3. Find whether the point lies on the line.
   a) \(y = -3x + 2\)  
      \((2, 8)\)
   b) \(y = 4x + 10\)  
      \((-2, 2)\)

4. Write the coordinates of the intersection point.
   a) \(\quad\)
   b) \(\quad\)
5. Charlie thinks that (2, −1) is the intersection point for the lines $y = 3x - 7$ and $y = -2x + 3$. Is he correct? Explain.

6. a) Complete the function tables. Then graph the lines.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = 2x + 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = -3x - 3$</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Find the coordinates of the intersection point. ______________

**Bonus**► Find the missing numbers so that (−1, 3) is the intersection point of the lines.

$$y = 2x + _____$$
$$y = _____ x - 1$$
1. a) \(2(3) - 7 = -1\)
   b) \(-3(-4) + 2 = 14\)

2. \(y = x + 4\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

\(y = 2x + 3\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-1</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>

\((1, 5)\)

3. a) \(8 \neq -3(2) + 2\), so \((2, 8)\) is not on the line.
   b) \(2 = 4(-2) + 10\), so \((-2, 2)\) is on the line.

4. a) \((1, 2)\)
   b) \((-2, -1)\)

5. \(-1 = 3(2) - 7\), so \((2, -1)\) lies on \(y = 3x - 7\).
   \(-1 = -2(2) + 3\), so \((2, -1)\) lies on \(y = -2x + 3\).
   Charlie is correct.

6. a) \(y = 2x + 2\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

\(y = -3x - 3\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>6</td>
</tr>
<tr>
<td>-2</td>
<td>3</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>1</td>
<td>-6</td>
</tr>
</tbody>
</table>

b) \((-1, 0)\)

**Bonus**

\[y = 2x + 5\]
\[y = -4x - 1\]
1. a) Find the slope \((m)\) and the \(y\)-intercept \((y\text{-int.})\) for the equation.

   i) \(y = 2x - 5\)  
   \[ m = \quad y\text{-int.} = \quad \]

   ii) \(y = -x\)  
   \[ m = \quad y\text{-int.} = \quad \]

b) Draw the graph of the line in part a).

   i) 
   ![Graph of y = 2x - 5]

   ii) 
   ![Graph of y = -x]

2. Graph each line by first finding its slope and \(y\)-intercept. Then solve the system of equations.

   \[ y = -3x + 2 \quad m = \quad y\text{-int.} = \quad \]

   \[ y = x - 2 \quad m = \quad y\text{-int.} = \quad \]

   intersection point: __________
Unit 5: Expressions and Equations

Quiz (Lessons 51 to 52)

3. Samantha’s Snow Removal charges a fee using the formula \( y = 25x + 55 \), where \( x \) is the number of hours worked. Steve’s Shoveling Services uses the formula \( y = 30x + 40 \).

   a) Find the slope and \( y \)-intercept for each formula. 
      \( y = 25x + 55 \) \( m = \) _____ \( y \)-int. = _____
      
      \( y = 30x + 40 \) \( m = \) _____ \( y \)-int. = _____

   b) Use the grid below to graph the line for each equation.

   c) Find the intersection point of the lines. __________

   d) After how many hours would the snow removal cost the same for both companies? __________

      What would the cost be for that many hours? __________

Bonus\(\textsuperscript{★} \)  Diane and Darryl both work in sales. Diane gets paid a base salary of $200 per week and $10 per hour. Darryl gets paid a base salary of $250.

   a) How much does Diane earn if she works 20 hours in one week?

   b) If Darryl earns the same total in 20 hours, how much does he earn per hour?
### Unit 5: Expressions and Equations

**Quiz (Lessons 51 to 52)**

<table>
<thead>
<tr>
<th>Question</th>
<th>Part</th>
<th>Solution</th>
</tr>
</thead>
</table>
| 1. a) i) 2, -5  
   ii) -1, 0 | b) i) | ![Graph](image1.png)  
   ![Graph](image2.png) |
| 2. -3, 2  
  1, -2 |          | ![Graph](image3.png)  
  ![Graph](image4.png) |
| 3. a) 25, 55  
  30, 40 | b) | ![Graph](image5.png) |
|          | c) (3, 130)  
   d) 3 hours, $130 |          |          |
| **Bonus** | a) | $y = 200 + 10x$  
  $y = 200 + 10(20)$  
  $y = 400$  
  Diane earns $400. |
Unit 5: Expressions and Equations

Name: ______________________

Test (Lessons 49 to 52)

Date: ________________

1. a) Use the rule for the function to complete each function table. Then find the ordered pair that appears in both tables.

\[
\begin{array}{c|c}
 x & y \\
\hline
-2 & \ \\
-1 & \ \\
0 & \ \\
1 & \ \\
2 & \ \\
\end{array}
\quad \begin{array}{c|c}
 x & y \\
\hline
-3 & \ \\
-2 & \ \\
-1 & \ \\
0 & \ \\
1 & \ \\
\end{array}
\]

\[
y = 2x - 1
\]

\[
y = 4x + 1
\]

common ordered pair: ___________

b) How do you know that (1, 5) lies on the graph of \( y = 4x + 1 \), but not on the graph of \( y = 2x - 1 \)?

c) Graph the lines in part a). Then find the intersection point. Verify that the intersection point is the same as in part a).

intersection point: ___________

2. Graph each line by first finding the slope and \( y \)-intercept. Then solve the system of equations.

\[
y = -x \quad m = \underline{\quad} \quad y\text{-int} = \underline{\quad}
\]

\[
y = 2x - 3 \quad m = \underline{\quad} \quad y\text{-int} = \underline{\quad}
\]

intersection point: ___________
Unit 5: Expressions and Equations
Test (Lessons 49 to 52)

3. Car Rental Company A charges a flat fee of $20 plus $0.01 per kilometer. Car Rental Company B charges a flat fee of $10 and $0.02 per kilometer. Let \( y \) be the total cost of renting a car. Let \( x \) be the number of kilometers traveled. Write a formula for the cost of renting from each car company.

4. Two ants are on a number line where the integers are 1 centimeter apart. The red ant is on the number \(-1\) and is moving right at a rate of 3 centimeters per second. The black ant is on the number \(+3\) and is moving right at a rate of 2 centimeters per second. Let \( x \) be the number of seconds traveled. Let \( y \) be the position of the ant.

   a) Write a formula for the position of the red ant. _______________________________

   b) Write a formula for the position of the black ant. _______________________________

   c) Find the slope and \( y \)-intercept for each line.

      i) \( m = \) _____    \( y \)-int = _____   ii) \( m = \) _____    \( y \)-int = _____

   d) Graph the lines on the grid.

   e) Find the intersection point of the lines.

      ______________________

   f) How long will it take the ants to be at the same position?

      ______________________

   g) Where on the number line will they be when they are at the same position?

      ______________________
5. Use the rule for the function to complete each function table. Then find two common ordered pairs.

\[
y = x^2
\]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
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</table>

\[
y = x + 2
\]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
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<tbody>
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<td>-3</td>
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</tr>
</tbody>
</table>

common ordered pairs: __________


Bonus► The function tables for \( y = x + 1 \) and \( y = x + 3 \) are shown below. Use the function tables to explain why there are no common ordered pairs.

\[
y = x + 1
\]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-2</td>
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<td>-2</td>
<td>-1</td>
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<td>3</td>
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<td>3</td>
<td>4</td>
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</tbody>
</table>

\[
y = x + 3
\]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
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</thead>
<tbody>
<tr>
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<td>0</td>
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<td>5</td>
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<tr>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>
Unit 5: Expressions and Equations

Test (Lessons 49 to 52)

1. a) \( y = 2x - 1 \)

\[
\begin{array}{c|c}
 x & y \\
-2 & -5 \\
-1 & -3 \\
0 & -1 \\
1 & 1 \\
2 & 3 \\
\end{array}
\]

\( y = 4x + 1 \)

\[
\begin{array}{c|c}
 x & y \\
-3 & -11 \\
-2 & -7 \\
-1 & -3 \\
0 & 1 \\
1 & 5 \\
\end{array}
\]

\((-1, -3)\)

b) (1, 5) satisfies the equation \( y = 4x + 1 \) because 5 = 4(1) + 1.

(1, 5) does not satisfy the equation \( y = 2x - 1 \) because 5 ≠ 2(1) - 1.

c) \[
\begin{array}{c|c}
 x & y \\
-3 & -11 \\
-2 & -7 \\
-1 & -3 \\
0 & 1 \\
1 & 5 \\
\end{array}
\]

(int. point: (−1, −3))

d) \[
\begin{array}{c|c}
 x & y \\
-3 & 9 \\
-2 & 4 \\
-1 & 1 \\
0 & 0 \\
1 & 1 \\
2 & 4 \\
3 & 9 \\
\end{array}
\]

\( y = x^2 \)

\[
\begin{array}{c|c}
 x & y \\
-3 & -1 \\
-2 & 0 \\
-1 & 1 \\
0 & 2 \\
1 & 3 \\
2 & 4 \\
3 & 5 \\
\end{array}
\]

(−1, 1), (2, 4)

5. \( y = x^2 \)

\( y = x + 2 \)

\[
\begin{array}{c|c}
 x & y \\
-3 & -1 \\
-2 & 0 \\
-1 & 1 \\
0 & 2 \\
1 & 3 \\
2 & 4 \\
3 & 5 \\
\end{array}
\]

Bonus

There are no common ordered pairs because, for any value of \( x \), the \( y \)-coordinate in the second table is always 2 greater than the \( y \)-coordinate in the first table.
1. Complete the table. Write the coordinates of the general point.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = -2x - 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2$</td>
<td>$y = 1$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$y = 1$</td>
</tr>
<tr>
<td>0</td>
<td>$y = -3$</td>
</tr>
<tr>
<td>1</td>
<td>$y = -5$</td>
</tr>
<tr>
<td>$x$</td>
<td>$y = -2x - 3$</td>
</tr>
</tbody>
</table>

general point: $(x, y)$

2. The equations for two lines that intersect and the $x$-coordinate of the intersection point are given. Use the $x$-coordinate in the equation of either line to find the $y$-coordinate of the intersection point.

   a) $y = x - 4$ 
      \[ y = 3x + 2 \]
      int. point = ($-3$, ?)

   b) $y = 2x - 8$ 
      \[ y = 3x - 12 \]
      int. point = ($4$, ?)

3. Find the intersection point of the system \[ y = 3x + 1 \]
   \[ y = 4x - 2 \] algebraically.
Unit 5: Expressions and Equations

Quiz (Lessons 53 to 54)

4. Solve the system \[\begin{align*}
2x + y &= 1 \\
x + 2y &= -4
\end{align*}\] algebraically. Check that your answer works in both equations.

Bonus► The intersection point of the system \[\begin{align*}
x + ky &= 1 \\
x - y &= 3
\end{align*}\] is \((2, -1)\). Find the value of \(k\).
1. 

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$y = -2x - 3$</td>
</tr>
<tr>
<td>$-2$</td>
<td>$1$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$0$</td>
<td>$-3$</td>
</tr>
<tr>
<td>$1$</td>
<td>$-5$</td>
</tr>
<tr>
<td>$x$</td>
<td>$-2x - 3$</td>
</tr>
</tbody>
</table>

$(x, -2x - 3)$

2. a) $y = (-3) - 4$

   $y = -7$, or

   $y = 3(-3) + 2$

   $y = -7$

b) $y = 2(4) - 8$

   $y = 0$, or

   $y = 3(4) - 12$

   $y = 0$

3. $3x + 1 = 4x - 2$

   $3x - 4x = -2 - 1$

   $-x = -3$

   $x = 3$

   At the int. point:

   $y = 3(3) + 1$

   $y = 10$

   So int. point = $(3, 10)$

4. $y = -2x + 1, y = \frac{-1}{2} x - 2$

   $-2x + 1 = \frac{-1}{2} x - 2$

   $-4x + 2 = -x - 4$

   $-4x + x = -4 - 2$

   $-3x = -6$

   $x = 2$

   At the int. point:

   $2(2) + y = 1$

   $y = 1 - 4$

   $y = -3$

   So int. point = $(2, -3)$

**Bonus**

$x + ky = 1$

$2 + k(-1) = 1$

$-k = -1$

$k = 1$
Unit 5: Expressions and Equations

Quiz (Lessons 55 to 56)

1. Write two equations for the word problem.
   a) The sum of two numbers is 16.
      ________________________
      The difference between the numbers is 2.
      ________________________
   b) John has 12 bills in his wallet made up of $1 bills and $5 bills.
      ________________________
      The total value of the bills is $28.
      ________________________

2. Emma sold a total of 20 drinks at her lemonade stand. The drinks were either medium sized or large sized. The medium drinks cost $2 each. The large drinks cost $3 each. If Emma sold $48 worth of drinks, how many medium-sized drinks did she sell? State what each variable represents.

3. Jane wants to know whether the system \( y = 2x - 3 \)
   \( y = -3x + 7 \) has a solution.
   a) Find the slope of each line.
      \( m_1 = \ldots \quad m_2 = \ldots \)
   b) Without solving, use the slopes to predict whether the system has a solution. Explain your answer.

Bonus: Create a system of equations that has no solution. How can you tell there is no solution without graphing or using algebra?
Unit 5: Expressions and Equations

Quiz (Lessons 55 to 56)

1. a) \[ x + y = 16 \]
   \[ x - y = 2 \]
   b) \[ x + y = 12 \]
   \[ x + 5y = 28 \]

2. Let \( x \) be the number of medium-sized drinks.
   Let \( y \) be the number of large-sized drinks.
   \[ x + y = 20, \ 2x + 3y = 48 \]
   \[ y = -x + 20, \ y = \frac{-2}{3}x + 16 \]

   At the int. point:
   \[ -x + 20 = \frac{-2}{3}x + 16 \]
   \[ -3x + 60 = -2x + 48 \]
   \[ -3x + 2x = 48 - 60 \]
   \[ -x = -12 \]
   \[ x = 12 \]
   Emma sold 12 medium-sized drinks.

3. a) \( m_1 = 2 \)
   \( m_2 = -3 \)
   b) There is a solution.
      Since the slopes are not equal, the lines must intersect.

Bonus

Sample answer:
\[ y = 3x - 1 \]
\[ y = 3x - 2 \]

This system has no solution because the lines are parallel with different \( y \)-intercepts, so they won’t intersect.
1. The general point for a line is \((x, 3x - 5)\). Find the coordinates of any two points on the line.

2. The \(y\)-coordinate of the intersection point of the system of equations \(y = x + 4\) and \(y = 2x - 1\) is 9. Find the \(x\)-coordinate of the intersection point.

3. Solve the system \(y = -x + 2\) and \(y = 2x - 4\) algebraically. Check that your answer works in both equations.

4. Use the rise and run to find the slope of each line. Then circle whether the system of equations will have a solution or no solution.

\[ m_1 = \\
\text{solution} \\
\] \[ m_2 = \\
\text{no solution} \\
\]
5. The sum of two numbers is 18. The difference between them is 8. Find the numbers.

6. In high school basketball, a successful free throw is awarded 1 point. A successful shot closer than 19 ft 9 in from the basket is awarded 2 points. A successful shot farther than 19 ft 9 in from the basket is awarded 3 points.

Abdul made 8 successful shots, none of which were free throws. He scored a total of 19 points. How many successful shots of each type did Abdul make?
**Unit 5: Expressions and Equations**

*Test (Lessons 53 to 56)*

**Bonus**  Graph the lines $y = 2$ and $x = 3$. Find the coordinates of the intersection point.

**intersection point: ___________**
Unit 5: Expressions and Equations

Test (Lessons 53 to 56)

Answer Key

1. Sample answer:
   Let \( x = 1 \)
   \( y = 3(1) - 5 \)
   \( y = -2 \)
   So \( (1, -2) \) is a point on the line.

2. At the int. point, \( y = 9 \)
   \( 9 = x + 4 \)
   \( 9 - 4 = x \)
   \( 5 = x \)

3. \(-x + 2 = 2x - 4\)
   \(-x - 2x = -4 - 2\)
   \(-3x = -6\)
   \(x = 2\)
   At the int. point:
   \( y = -2 + 2\)
   \( y = 0\)
   So int. point = \( (2, 0) \)
   Checks: \( 0 = -2 + 2\)
   \( 0 = 2(2) - 4\)

4. a) \( m_1 = 1, m_2 = 3/2\)
    solution
    b) \( m_1 = -1, m_2 = -1\)
    no solution

5. Let \( x \) be the larger number.
   Let \( y \) be the smaller number.
   \( x + y = 18, x - y = 8 \)
   \( y = -x + 18, y = x - 8 \)
   At the int. point:
   \(-x + 18 = x - 8\)
   \(-x - x = -8 - 18\)
   \(-2x = -26\)
   \(x = 13\)
   \(y = 13 - 8\)
   \(y = 5\)
   The two numbers are 13 and 5.

6. Let \( x \) be the number of successful 2-point shots.
   Let \( y \) be the number of successful 3-point shots.
   \( x + y = 8, 2x + 3y = 19 \)
   \( y = -x + 8, y = \frac{2}{3}x + \frac{19}{3} \)
   At the int. point:
   \(-x + 8 = \frac{2}{3}x + \frac{19}{3}\)
   \(-3x + 24 = -2x + 19\)
   \(-3x + 2x = 19 - 24\)
   \(-x = -5\)
   \(x = 5\)
   \(y = -x + 8\)
   \(y = -5 + 8\)
   \(y = 3\)
   Abdul made 5 successful 2-point shots and 3 successful 3-point shots.

Bonus

(2, 3)
Unit 6: Geometry

Quiz (Lessons 49 to 52)

In all questions below, you may use a calculator. Use \( 3.14 \) for \( \pi \) and round your answer to the nearest tenth. Include the units in all your answers.

1. Fill in the table.

<table>
<thead>
<tr>
<th>Name of 3-D Shape</th>
<th>Formula for Volume</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Find the volume of the composite shape.

3. Use the Pythagorean Theorem to find the height of the pyramid to one decimal place.
   Hint: Label the triangle.
4. The volume of a cylinder is about 226 in$^3$ and the radius is 3 in. Write and solve an equation to find the height.

$$V = 226 \text{ in}^3$$

**Bonus**► A rectangular prism and a rectangular pyramid have the same base and the same volume. How do their heights compare?
Unit 6: Geometry

Quiz (Lessons 49 to 52)

1. \(a\) cylinder
   \[ V = \pi r^2 h \]
   \[ V = 3.14 \times (3.5)^2 \times 8 \]
   \[ V \approx 307.7 \text{ in}^3 \]

   \(b\) (triangular) pyramid
   \[ V = \frac{1}{3} Bh \]
   \[ V = \frac{1}{3} \times \frac{1}{2} \times 4 \times 7 \]
   \[ V = 9.3 \text{ ft}^3 \]

2. \[ V_1 = \frac{1}{3} Bh \]
   \[ = \frac{1}{3} \times 12 \times 5 \times 6 \]
   \[ = 120 \text{ m}^3 \]

   \[ V_2 = \ell \times wh \]
   \[ = 12 \times 5 \times 7 \]
   \[ = 420 \text{ m}^3 \]

   The volume is 120 + 420 = 540 \text{ m}^3.

3. \[
\begin{align*}
2^2 + h^2 &= 7^2 \\
4 + h^2 &= 49 \\
h^2 &= 49 - 4 \\
h^2 &= 45 \\
h &= \sqrt{45} \\
h &\approx 6.7 \text{ in}
\end{align*}
\]

4. \[ V = \pi r^2 h \]
   \[ 226 = 3.14 (3)^2 h \]
   \[ 226 = 28.26 h \]
   \[ 226 + 28.26 = h \]
   \[ 8 \text{ in} = h \]

Bonus

The height of the pyramid is 3 times the height of the prism.

Prism: \( V = Bh \)

Pyramid: \( V = \frac{1}{3} Bh \)

For the volumes to be equal, the height of the pyramid must be 3 times greater.
Unit 6: Geometry
Quiz (Lessons 53 to 55)

In all questions below, you may use a calculator. Use 3.14 for π and round your answer to the nearest tenth. Include the units in all your answers.

1. a) What formula do you use to find the volume of the sphere? _______________________
   
   b) Find the volume of a sphere with a diameter of 5 feet.

   ![Diagram of a sphere with a diameter of 5 feet]

2. a) Use the Pythagorean Theorem to find the height of the cone. Hint: Label the triangle.

   ![Diagram of a cone with a height of 6 inches, a radius of 2 inches]

   b) Find the volume of the cone.

   ![Diagram of a right triangle]
3. Find the volume of the composite shape.

**Bonus** Lucy has a small ball and a medium ball. The volume of the medium ball is 8 times the volume of the small ball. How do the radii of the two balls compare?
Unit 6: Geometry
Quiz (Lessons 53 to 55)

1. a) \[ V = \frac{4}{3} \pi r^3 \]
   b) \[ V = \frac{4}{3} (3.14)(2.5)^3 = 65.4 \text{ ft}^3 \]

2. a) \[
\begin{align*}
& h^2 + 2^2 = 6^2 \\
& h^2 + 4 = 36 \\
& h^2 = 36 - 4 \\
& h^2 = 32 \\
& h = \sqrt{32} \\
& h = 5.7 \text{ in}
\end{align*}
\]
   b) \[ V = \frac{1}{3} \pi r^2 h \\
= \frac{1}{3} (3.14)(2)^2(5.7) \\
= 23.9 \text{ in}^3 \]

3. \[ V_1 = \frac{4}{3} \pi r^3 + 2 \\
= \frac{4}{3} (3.14)(2)^3 + 2 \\
= 16.7 \text{ in}^3 \]
   \[ V_2 = \pi r^2 h \\
= 3.14(2)^2(6) \\
= 75.4 \text{ in}^3 \]
   \[ V_1 + V_2 = 16.7 + 75.4 = 92.1 \text{ in}^3 \]

Bonus
The radius of the medium ball is 2 times the radius of the small ball. Or, the small ball has a radius that is half the radius of the medium ball.

Small ball: \[ V = \frac{4}{3} \pi r^3 \]

Medium ball: \[ V = \frac{4}{3} \pi (2r)^3 \\
= 8 \left( \frac{4}{3} \pi r^3 \right) \]
In all questions below, you may use a calculator. Use 3.14 for π and round your answer to the nearest tenth. Include the units in all your answers.

1. Fill in the table.

<table>
<thead>
<tr>
<th>3-D Shape</th>
<th>Name of 3-D Shape</th>
<th>Formula for Volume</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>6.2 ft 5.5 ft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td>8 cm 6 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c)</td>
<td>3 in</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. a) The capsule below holds medicine. Find the volume of medicine that the capsule can hold.

b) 1 mm$^3$ is equal to 0.001 mL. How many milliliters of medicine can the capsule hold?
3. The volume of a can of soup is about 463.6 cm³.
   a) Find the height of the can.
   
   b) A grocery store shelf is 30 cm high. How many cans can be stacked to fit on the shelf?

4. Ivan rolls two perfect snowballs to build a snowman.
   a) The radius of the larger snowball, for the body, is 25 cm. Find the volume of the larger snowball.
   
   b) The diameter of the smaller snowball, for the head, is 10 cm. Find the volume of the smaller snowball.

   c) Complete the sentences with a number.
      
      i) The radius of the larger snowball is ________ times the radius of the smaller snowball.
      
      ii) The volume of the larger snowball is ________ times the volume of the smaller snowball.

   d) Explain how the two numbers in part c) are related.
5. The Great Pyramid of Giza has a square base with sides that are 230 meters long. The distance from the edge of the base to the apex, along a side face, is 180 meters. What is the height of the pyramid?

6. A foam cylinder has diameter 40 cm and height 30 cm. The cylinder is made hollow by removing a cylinder with diameter 20 cm.

   a) What is the volume of the hollow cylinder?

   b) What percentage of the cylinder was removed to make it hollow?

**Bonus** Some different shapes each have a volume of 500 cm$^3$. Sketch and label 3 possible shapes that fit this description.
1. a) cylinder
   \[ V = \pi r^2 h \]
   \[ V = (3.14)(6.2)^2(5.5) \approx 663.9 \text{ ft}^3 \]
   b) cone
   \[ V = \frac{1}{3} \pi r^2 h \]
   \[ V = \frac{1}{3}(3.14)(8)^2(6) \approx 401.9 \text{ cm}^3 \]
   c) sphere
   \[ V = \frac{4}{3} \pi r^3 \]
   \[ V = \frac{4}{3}(3.14)(1.5)^3 \approx 14.1 \text{ in}^3 \]

2. a) \[ V_1 = \pi r^2 h \]
    \[ = 3.14(3)^2(5) \approx 141.3 \text{ mm}^3 \]
    \[ V_2 = \frac{4}{3} \pi r^3 \]
    \[ = \frac{4}{3}(3.14)(3)^3 \approx 113.0 \text{ mm}^3 \]
    \[ V_1 + V_2 = 141.3 + 113 = 254.3 \text{ mm}^3 \]
   b) \[ 254.3 \times 0.001 = 0.2543 \text{ mL} \]

3. a) \[ r = 3.75 \]
    \[ V = \pi r^2 h \]
    \[ 463.6 = 3.14(3.75)^2h \]
    \[ 463.6 = 44.15625h \]
    \[ 463.6 + 44.15625 = h \]
    \[ 10.5 \text{ cm} = h \]
   b) 2 cans

4. a) \[ V = \frac{4}{3} \pi r^3 \]
    \[ = \frac{4}{3}(3.14)(25)^3 \]
    \[ = 65,416.7 \text{ cm}^3 \]
   b) \[ r = 5 \]
    \[ V = \frac{4}{3} \pi r^3 \]
    \[ = \frac{4}{3}(3.14)(5)^3 \]
    \[ = 523.3 \text{ cm}^3 \]
   c) i) 5

ii) 125
   d) Since radius is cubed in the volume formula, a radius that is 5 times bigger will give a volume that is \(5^3 = 125\) times bigger.

5. \[ 180^2 = h^2 + 115^2 \]
   \[ 32,400 = h^2 + 13,225 \]
   \[ 32,400 - 13,225 = h^2 \]
   \[ 19,175 = h^2 \]
   \[ \sqrt{19,175} = h \]
   \[ 138.5 \text{ m} = h \]

6. a) \[ V_1 = \pi r^2 h \]
    \[ = 3.14(20)^2(30) \approx 37,680 \text{ cm}^3 \]
    \[ V_2 = \pi r^2 h \]
    \[ = 3.14(10)^2(30) \approx 9,420 \text{ cm}^3 \]
    \[ V = V_1 - V_2 \]
    \[ = 37,680 - 9,420 \]
    \[ = 28,260 \text{ cm}^3 \]
   b) \[ \frac{9,420}{37,680} = 0.25 \]
    25% was removed.

**Bonus**

Answers will vary.

Sample answers:

- **Cylinder**
  - Radius: 4 cm
  - Height: 10 cm

- **Cone**
  - Height: 20 cm
  - Radius: 5 cm

- **Cube**
  - Side length: 5 cm
1. Consider the scatter plots.

A. 
B. 
C. 

a) Which scatter plots are linear? ____________
b) Which straight lines in part a) are increasing? ____________
c) Which straight lines in part a) have a positive slope? ____________
d) How can you tell from the slope if an association is positive or negative? ____________

2. a) The graph shows the line of best fit for the data set. Would the line rotate clockwise or counter-clockwise if you added an outlier with …
   i) a low x-value and low y-value? ____________
   ii) a high x-value and high y-value? ____________

b) Add a point far from the others, but on the line of best fit. Will it change the line of best fit? ____________

3. The table shows data for ten students.

| Height (in) | 64 | 66 | 67 | 62 | 65 | 68 | 58 | 63 | 65 | 60 |
| Arm Span (in) | 63 | 65 | 68 | 62 | 66 | 67 | 60 | 64 | 65 | 61 |

a) Draw a scatter plot for the data.
b) Is the association positive or negative? ____________
c) Draw the line of best fit for the scatter plot.
d) How many points are above the line? _______
e) How many points are below the line? _______
Unit 7: Statistics and Probability

Quiz (Lessons 4 to 6)

1. a) A, C
   b) C
   c) C
   d) If the slope is positive, the association is positive.

2. a) i) counter-clockwise
    ii) counter-clockwise
   b) no

3. a)

   ![Graph](image)

   b) positive

   c)

   ![Graph](image)

   d) 5
   e) 4
Unit 7: Statistics and Probability

Quiz (Lessons 7 to 8)

1. The table shows data for eight students.

<table>
<thead>
<tr>
<th>TV per Week (h)</th>
<th>17</th>
<th>20</th>
<th>5</th>
<th>11</th>
<th>10</th>
<th>23</th>
<th>15</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math Mark (%)</td>
<td>70</td>
<td>58</td>
<td>90</td>
<td>72</td>
<td>80</td>
<td>53</td>
<td>64</td>
<td>88</td>
</tr>
</tbody>
</table>

a) The scatter plot shows the data in the table.

Is the association positive or negative? _____________

b) Circle points A and B in the table.

c) Use points A and B to find the slope of the line of best fit.

2. The table shows data for eleven students.

<table>
<thead>
<tr>
<th>Height (in)</th>
<th>64</th>
<th>66</th>
<th>67</th>
<th>62</th>
<th>65</th>
<th>68</th>
<th>58</th>
<th>63</th>
<th>68</th>
<th>59</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shoe size</td>
<td>7.5</td>
<td>8</td>
<td>9</td>
<td>6.5</td>
<td>7.5</td>
<td>9</td>
<td>6</td>
<td>7</td>
<td>8.5</td>
<td>6</td>
<td>6.5</td>
</tr>
</tbody>
</table>

a) Draw a scatter plot for the data.

b) Is the association positive or negative? _____________

c) Draw the line of best fit for the scatter plot.

d) How many points are above the line? _______

e) How many points are below the line? _______

f) Use two points on the line of best fit (or very close to the line) to find the slope.

g) Use the slope and one point on the line of best fit to find the \( y \)-intercept.

h) Write an equation for the line of best fit in slope-intercept form. \( y = \) _____________

**Bonus**▶ Use the equation to predict the shoe size of a student who is 70 inches tall.
1. a) negative
   b) Teacher to check.
   c) $A (10, 80)$
   \hspace{1em} $B (20, 58)$
   \hspace{1em} run = -10
   \hspace{1em} rise = 22
   \hspace{1em} slope = \frac{22}{-10} = -2.2

2. a) positive
   b) positive
   c) \hspace{1em}

   d) 5
   e) 4
   f) $A (59, 6)$
   \hspace{1em} $B (66, 8)$
   \hspace{1em} run = 7
   \hspace{1em} rise = 2
   \hspace{1em} slope = \frac{2}{7}

   g) $y = mx + b$
   \hspace{1em} $y = \frac{2}{7}x + b$
   \hspace{1em} $8 = \frac{2}{7}(66) + b$
   \hspace{1em} $\frac{56}{7} = \frac{132}{7} + b$
   \hspace{1em} $\frac{76}{7} = b$

   h) $y = \frac{2}{7}x - \frac{76}{7}$

Bonus
\hspace{2em} $y = \frac{2}{7}x - \frac{76}{7}$
\hspace{2em} $y = \frac{2}{7}(70) - \frac{76}{7}$
\hspace{2em} $y = \frac{140}{7} - \frac{76}{7}$
\hspace{2em} $y = \frac{64}{7}$
\hspace{2em} $y = 9 \frac{1}{7}$

The student’s shoe size would be approximately 9.5.
Unit 7: Statistics and Probability

Quiz (Lessons 9 to 12)

1. Find the missing terms in the two-way table.

   a) TV  No TV  Total
       Laptop  5
       No Laptop 17  20
       Total  56

   b) MP3  No MP3  Total
       Cellphone 8  37
       No Cellphone 11
       Total  15

2. Kevin surveyed 100 students at his school about being fans of football and baseball. The table shows the results.

   a) Use the two-way table to complete the two-way relative frequency table.

   b) Based on the relative frequency table, in which group can you find the most baseball fans, among students who like football or students who don’t like football?

   ____________________

   Bonus► Why? ____________________________________________________________________

3. The Venn diagram shows the results of a survey of 50 customers of a supermarket about whether they bought milk and eggs.

   a) Complete the row two-way table and the row two-way relative frequency table using the Venn diagram.

   Bonus► Is there an association between buying milk and eggs? ________________

   If so, is it a positive or negative association? __________ Explain why. ________________
1. a) TV | No TV | T  
   31 | 5    | 36  
   17 | 3    | 20  
   48 | 8    | 56  

   b) MP3 | No MP3 | T  
   8    | 29    | 37  
   7    | 11    | 18  
   15   | 40    | 55  

2. a) Ba | No Ba | T  
   27%  | 19%   | 46% 
   11%  | 43%   | 54% 
   38%  | 62%   | 100% 

   b) among students who like football  
   Bonus  
   27% is greater than 11%. 

3. a) Mi | No Mi | T  
   12   | 5     | 17  
   11   | 22    | 33  

   Bonus  
   yes positive  
   Because most of the people purchased both milk and eggs (71%) or purchased neither (67%).
1. The graph shows the line of best fit for the data set. Would the line rotate clockwise or counter-clockwise if you added an outlier with …

a) a low x-value and low y-value? ___________________________

b) a high x-value and high y-value? ___________________________

2. The table shows the number of hours ten students studied and their math marks.

<table>
<thead>
<tr>
<th>Study Time (h)</th>
<th>3</th>
<th>5</th>
<th>4</th>
<th>1</th>
<th>3</th>
<th>7</th>
<th>5</th>
<th>7</th>
<th>4</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math Mark (%)</td>
<td>68</td>
<td>80</td>
<td>76</td>
<td>57</td>
<td>65</td>
<td>89</td>
<td>78</td>
<td>85</td>
<td>70</td>
<td>67</td>
</tr>
</tbody>
</table>

a) Draw a scatter plot for the data. Is the association positive or negative? ____________________

b) Draw the line of best fit for the scatter plot.

c) Use the line of best fit to estimate the mark of

a student who studies for 6 hours. _______

d) Extend the line of best fit and predict the mark of

a student who studies for 8 hours. _______

e) Use two points on the line of best fit (or very close to the line) to find the slope.

f) Use the slope and one point on the line of best fit to find the y-intercept and then write the equation.

g) Use the equation from part f) to calculate the answers to parts c) and d).

**Bonus** ► What mark would a student get if they didn’t study at all? ________  Where do you see this in the equation? ________________
3. Find the missing terms in the two-way table and then make a two-way relative frequency table.

<table>
<thead>
<tr>
<th>Boys</th>
<th>Girls</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 6</td>
<td>18</td>
<td>17</td>
</tr>
<tr>
<td>Grade 8</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>60</td>
<td></td>
</tr>
</tbody>
</table>

4. The Venn diagram shows the results of a survey of 50 students about whether they have a curfew on school nights and assigned chores at home.

a) Complete the row two-way table and the row two-way relative frequency table using the Venn diagram.

50 Students

<table>
<thead>
<tr>
<th>Curfew</th>
<th>Chores</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>21</td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

b) Is there an association between having a curfew and having chores at home? _________

If so, is it a positive or negative association? __________ Explain why. ___________________

______________________________________________________________________________

c) Switch the rows with the columns of the two-way table in part a) and complete the row two-way relative frequency table.

<table>
<thead>
<tr>
<th>Chores</th>
<th>No Chores</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curfew</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No Curfew</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Bonus** Can you say that students with curfews are more likely to have chores than students with chores are likely to have curfews?

______________________________________________________________________________

______________________________________________________________________________
1. a) counter-clockwise  
   b) counter-clockwise
2. a)  
   ![Graph](image1)  
   positive  
   ![Graph](image2)  
   c) 83  
   d) 93  
   e) $A = (7, 88)$
   $B = (1, 58)$
   run = 6
   rise = 30
   slope = $\frac{30}{6} = 5$
   f) $y = mx + b$
   $y = 5x + b$
   $58 = 5(1) + b$
   $58 - 5 = b$
   $53 = b$
   equation:
   $y = 5x + 53$
   g) c): $y = 5x + 53$
   $y = 5(6) + 53$
   $y = 30 + 53$
   $y = 83$
   d): $y = 5x + 53$
   $y = 5(8) + 53$
   $y = 40 + 53$
   $y = 93$
3. | B | G | T |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>17</td>
<td>35</td>
</tr>
<tr>
<td>12</td>
<td>13</td>
<td>25</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>60</td>
</tr>
<tr>
<td>B</td>
<td>G</td>
<td>T</td>
</tr>
<tr>
<td>30%</td>
<td>28%</td>
<td>58%</td>
</tr>
<tr>
<td>20%</td>
<td>22%</td>
<td>42%</td>
</tr>
<tr>
<td>50%</td>
<td>50%</td>
<td>100%</td>
</tr>
</tbody>
</table>
4. a)  
   ![Graph](image3)  
   b) yes  
   positive  
   Students who have a curfew tend to have chores, and students who do not have a curfew tend to have no chores.
   c)  
   ![Graph](image4)  
   Bonus  
   No. There is not a significant difference because students with curfews have chores 78% of the time and students with chores have curfews 75% of the time.
### Scoring Guides for Sample Unit Quizzes and Tests
#### Unit 4: Geometry

**Quiz (Lessons 39 to 41), p. U-57**

Common Core State Standards Emphasized: 8.NS.A.2

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Correctly measures base and height</td>
<td>7 units, 6 units</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correctly uses formula</td>
<td>7 × 6 ÷ 2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>= 21 units²</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b) Correctly measures base and height</td>
<td>4 units, 5 units</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correctly uses formula</td>
<td>4 × 5 ÷ 2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>= 10 units²</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>c) Correctly measures base and height</td>
<td>6 units, 5 units</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correctly uses formula</td>
<td>6 × 5 ÷ 2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>= 15 units²</td>
<td>1</td>
<td>/9</td>
</tr>
<tr>
<td>2</td>
<td>a) Correctly marks and measures base and height</td>
<td>4 cm, 2.2 cm</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correctly uses formula</td>
<td>4 × 2.2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>= 8.8 cm²</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b) Correctly marks and measures base and height</td>
<td>2.5 cm, 3.2 cm</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correctly uses formula</td>
<td>2.5 × 3.2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>= 8.0 cm²</td>
<td>1</td>
<td>/6</td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>Teacher to check.</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer with units</td>
<td>3.6 cm</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>4</td>
<td>a) Gives correct answer with units</td>
<td>5 cm</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b) Correctly uses formula</td>
<td>2 × 3.14 × 5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>= 31.4 cm</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>c) Correctly uses formula</td>
<td>3.14 × 5²</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>= 78.5 cm²</td>
<td>1</td>
<td>/5</td>
</tr>
<tr>
<td>5</td>
<td>Gives correct formula</td>
<td>[A = (b \times h) + (\pi r^2 ÷ 2)]</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correctly substitutes numbers</td>
<td>[(6 \times 10) + (3.14 \times 3^2 ÷ 2)]</td>
<td>2</td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>Makes one substitution error</td>
<td>[60 + 14.13]</td>
<td>1</td>
<td>/4</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>[= 74.13 in^2]</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct answer with units</td>
<td>4 m</td>
<td>yes / no</td>
<td>1</td>
</tr>
</tbody>
</table>

**Total Points /26**
### Scoring Guides for Sample Unit Quizzes and Tests

**Unit 4: Geometry**

#### Quiz (Lessons 42 to 44), p. U-60

**Common Core State Standards Emphasized:** 8.G.B.6, 8.G.B.7

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong></td>
<td>a) Gives correct calculation on the first line</td>
<td>$4 \times (6 \times 9 + 2)$ + $(3 \times 3)$ = $4 \times 27 + 9$ = 117</td>
<td>2 (1)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Makes one error on the first line</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Follows correct order of operations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) Gives correct calculation</td>
<td>$\sqrt{117}$ = 10.8</td>
<td>1</td>
<td>1</td>
<td>/6</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>2</strong></td>
<td>a) Gives correct area of square with side length $a$</td>
<td>$4 \times 4 = 16$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct area of square with side length $b$</td>
<td>$3 \times 3 = 9$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct area of square with side length $c$</td>
<td>$4 \times 6 + 1 = 25$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>b) Gives correct answer</td>
<td>$16 + 9 = 25$</td>
<td>1</td>
<td>/4</td>
<td></td>
</tr>
<tr>
<td><strong>3</strong></td>
<td>a) Gives correct answer</td>
<td>$m^2 = w^2 + r^2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$a^2 = b^2 + c^2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>$f^2 = g^2 + h^2$</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td><strong>4</strong></td>
<td>a) Gives correct equation</td>
<td>$d^2 = 5^2 + 12^2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>$d^2 = 25 + 144$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>$d^2 = 169$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d = 13</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) Gives correct equation</td>
<td>$w^2 = (\sqrt{21})^2 + 2^2$</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>$w^2 = 21 + 4$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>$w^2 = 25$</td>
<td>1</td>
<td>/6</td>
</tr>
<tr>
<td></td>
<td>w = 5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>5</strong></td>
<td>a) Gives correct equation</td>
<td>$15^2 = c^2 + 12^2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>$225 = c^2 + 144$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>$81 = c^2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c = 9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) Gives correct equation</td>
<td>$8^2 = (\sqrt{15})^2 + p^2$</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>$64 = 15 + p^2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>$49 = p^2$</td>
<td>1</td>
<td>/6</td>
</tr>
<tr>
<td></td>
<td>7 = p</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>yes / no</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Total Points** /25
### Scoring Guides for Sample Unit Quizzes and Tests

#### Unit 4: Geometry

(continued)

**Quiz (Lessons 45 to 48), p. U-63**


<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Correctly labels all three sides</td>
<td></td>
<td>1</td>
<td>(0.5)</td>
</tr>
<tr>
<td></td>
<td>Correctly labels one or two sides</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Correctly labels all three angles</td>
<td></td>
<td>1</td>
<td>(0.5)</td>
</tr>
<tr>
<td></td>
<td>Correctly labels one or two angles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>/2</td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer for (a^2, b^2, c^2)</td>
<td>81, 36, 64</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct calculation</td>
<td>81 &lt; 36 + 64</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Draws correct conclusion</td>
<td>acute</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer for (a^2, b^2, c^2)</td>
<td>289, 64, 225</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct calculation</td>
<td>289 = 64 + 225</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Draws correct conclusion</td>
<td>right</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer for (a^2, b^2, c^2)</td>
<td>196, 25, 144</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct calculation</td>
<td>196 &gt; 25 + 144</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Draws correct conclusion</td>
<td>obtuse</td>
<td>1</td>
<td>/9</td>
</tr>
<tr>
<td>3</td>
<td>Draws correct diagram</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct equation</td>
<td>(d^2 = 8^2 + 5^2)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>(d^2 = 64 + 25)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>(d^2 = 89)</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(d = \sqrt{89})</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer with units</td>
<td>(d = 9.4) km</td>
<td>1</td>
<td>/5</td>
</tr>
<tr>
<td>4</td>
<td>Draws correct diagram</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct equation</td>
<td>(8^2 = h^2 = 3^2)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>(64 = h^2 + 9)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>(55 = h^2)</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\sqrt{55} = h)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer with units</td>
<td>(7.4) cm (\approx h)</td>
<td>1</td>
<td>/5</td>
</tr>
</tbody>
</table>
## Scoring Guides for Sample Unit Quizzes and Tests
### Unit 4: Geometry

(continued)

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>a) Gives correct calculation</td>
<td>$</td>
<td>3 - (-1)</td>
<td>$</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>$=</td>
<td>4</td>
<td>= 4$</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct calculation</td>
<td>$</td>
<td>1 - (-2)</td>
<td>$</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>$=</td>
<td>3</td>
<td>= 3$</td>
</tr>
<tr>
<td></td>
<td>c) Gives correct equation</td>
<td>$d = \sqrt{4^2 + 3^2}$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>$d = \sqrt{25}$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>$d = 5$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>Sample answers: $(5, 0), (0, 5), (-5, 0), (0, -5), (3, 4), (4, 3), (-3, 4), (-4, 3), (-3, -4), (-4, -3), (4, -3), (3, -4)$</td>
<td>yes / no</td>
<td>/7</td>
</tr>
</tbody>
</table>

**Total Points** /28
### Scoring Guides for Sample Unit Quizzes and Tests
#### Unit 4: Geometry (continued)

**Common Core State Standards Emphasized:** 8.G.B.6, 8.G.B.7, 8.G.B.8, 8.NS.A.2

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct calculation</td>
<td>((7 \times 4) + (7 \times 3 ÷ 2))</td>
<td>(28 + 10.5)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Makes one error in calculation</td>
<td>((7 \times 3 + 2))</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Follows correct order of operations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>(= 38.5) units²</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/4</td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>0.75 cm</td>
<td>1</td>
<td>1/4</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>0.75 cm</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>0.75 cm</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>equilateral</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>0.7 in</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Correctly uses formula</td>
<td>(A \approx 3.14 \times 0.7^2)</td>
<td>(= 3.14 \times 0.49)</td>
<td>(= 1.5) in²</td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer with units</td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>c) Correctly uses formula</td>
<td>(C \approx 2 \times 3.14 \times 0.7)</td>
<td>(= 4.4) in</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer with units</td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>d) Correctly uses formula</td>
<td>(d \approx 2 \times 0.7)</td>
<td>(= 1.4) in</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer with units</td>
<td></td>
<td></td>
<td>1/7</td>
</tr>
<tr>
<td>4</td>
<td>a) Gives correct equation</td>
<td>(w^2 + 18^2 = 30^2)</td>
<td>(w^2 + 324 = 900)</td>
<td>(w^2 = 726)</td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>(w^2 = 726)</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>(w = 24)</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct equation</td>
<td>(r^2 = 4^2 + (\sqrt{8})^2)</td>
<td>(r^2 = 16 + 8)</td>
<td>(r^2 = 24)</td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>(r^2 = 16 + 8)</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>(r^2 = 24)</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer</td>
<td>(r = 4.9)</td>
<td></td>
<td>1/8</td>
</tr>
<tr>
<td>5</td>
<td>Gives correct answer for (b^2)</td>
<td>(b^2 = 7^2 = 49)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for (a^2 + c^2)</td>
<td>(a^2 + c^2 = 10^2 + 4^2 = 116)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>(\angle B) is acute</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct explanation</td>
<td>(b^2 &lt; a^2 + c^2)</td>
<td>1</td>
<td>1/4</td>
</tr>
</tbody>
</table>
### Scoring Guides for Sample Unit Quizzes and Tests
#### Unit 4: Geometry

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>a) Gives correct equation</td>
<td>$h^2 + 400^2 = 350^2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>$h^2 + 40,000 = 122,500$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>$h^2 = 82,500$</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer with units</td>
<td>$h = \sqrt{82,500}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$h \approx 287.2 \text{ m}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct equation</td>
<td>$r^2 = 100^2 + 287.2^2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>$r^2 = 10,000 + 82,483.84$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>$r^2 = 92,483.84$</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer with units</td>
<td>$r = \sqrt{92,483.84}$</td>
<td>1</td>
<td>/8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$r \approx 304.1 \text{ m}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>a) Gives correct equation</td>
<td>$d = \sqrt{3^2 + 2^2}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>$d = \sqrt{13}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer</td>
<td>$d \approx 3.6$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct equation</td>
<td>$d = \sqrt{3^2 + 4^2}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>$d = \sqrt{25}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer</td>
<td>$d = 5$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>$d = 2 - (−4)$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d = 6$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>$P = 3.6 + 5 + 6$</td>
<td>1</td>
<td>/8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= 14.6$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>$d_{OA} = 5, \quad d_{OB} = 5$</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Since $O$ is the same distance from $A$ and $B$, it could be the center of a circle with points $A$ and $B$ on the circle.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Rubric for Unit 4: Geometry

Test (Lessons 39 to 48), p. U-66

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s)</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>8.G.B.6</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Explain a proof of the Pythagorean Theorem and its converse.</em></td>
<td></td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>8.G.B.7</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.</em></td>
<td></td>
<td>4, 6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>8.G.B.8</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</em></td>
<td></td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Comments**
### Scoring Guides for Sample Unit Quizzes and Tests
#### Unit 5: Expressions and Equations

**Quiz (Lessons 49 to 50), p. U-70**

**Common Core State Standards Emphasized:** 8.EE.C.8, 8.EE.C.8a

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
</table>
| 1 | a) Correctly replaces \( w \) with 3  
   Correctly evaluates expression | \( 2(3) - 7 \)  
   \( = 6 - 7 = -1 \) | 1 | 1 |
| | b) Correctly replaces \( m \) with \(-4\)  
   Correctly evaluates expression | \( -3(-4) + 2 \)  
   \( = 12 + 2 = 14 \) | 1 | 1/4 |
| 2 | Correctly completes first table  
   Correctly completes second table  
   Gives correct answer | | | |
| 3 | a) Checks if the coordinates satisfy the equation  
   Gives correct answer | \(-3(2) + 2 \neq 8 \)  
   (2, 8) does not lie on the line | 1 | 1 |
| | b) Checks if the coordinates satisfy the equation  
   Gives correct answer | \( 4(-2) + 10 = 2 \)  
   (-2, 2) lies on the line | 1 | 1/4 |
| 4 | a) Gives correct answer | (1, 2) | 1 | |
| | b) Gives correct answer | (-2, -1) | 1 | 1/2 |
| 5 | Checks if the coordinates satisfy the equation  
   \( y = 3x - 7 \)  
   Checks if the coordinates satisfy the equation  
   \( y = -2x + 3 \)  
   Gives correct answer  
   Gives correct explanation | \(-1 = 3(2) - 7, \) so (2, -1) lies on \( y = 3x - 7 \).  
   \(-1 = -2(2) + 3, \) so (2, -1) lies on \( y = -2x + 3 \).  
   Charlie is correct.  
   (2, -1) is the intersection point since it lies on both lines. | 1 | 1 |

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### Question 6

<table>
<thead>
<tr>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Correctly completes first table</td>
<td>−2, −2</td>
<td>2</td>
<td>(1)</td>
</tr>
<tr>
<td>Makes one computational error</td>
<td>−1, 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0, 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1, 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2, 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correctly completes second table</td>
<td>−3, 6</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Makes one computational error</td>
<td>−2, 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>−1, 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0, −3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1, −6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correctly graphs coordinates from first table</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Correctly graphs coordinates from second table</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>b) Gives correct answer</td>
<td>(−1, 0)</td>
<td>1</td>
<td>/7</td>
</tr>
<tr>
<td>Bonus</td>
<td>5, −4</td>
<td>yes / no</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points** /24
### Scoring Guides for Sample Unit Quizzes and Tests

**Unit 5: Expressions and Equations**

**Quiz (Lessons 51 to 52), p. U-73**

Common Core State Standards Emphasized: 8.EE.C.8, 8.EE.C.8a, 8.EE.C.8c

<table>
<thead>
<tr>
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<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) i) Gives correct answer</td>
<td>2, −5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a) ii) Gives correct answer</td>
<td>−1, 0</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
|          | b) i) Correctly plots $y$-intercept  
Correctly uses slope to plot more points  
Uses a ruler to join points in a line |  | 1   | 1   |
|          | b) ii) Correctly plots $y$-intercept  
Correctly uses slope to plot more points  
Uses a ruler to join points in a line |  | 1   | 1   |
| 2        | Gives correct answer for first equation  
Gives correct answer for second equation  
Correctly graphs first equation  
Correctly graphs second equation | −3, 2  | 1               | 1   |
|          |                              | 1, −2  | 1               | 1   |
|          |                              | (1, −1) | 1               | 1   |
| 3        | a) Gives correct answer for first equation  
Gives correct answer for second equation | 25, 55 | 1               | 1   |
|          | b) Correctly plots $y$-intercept of first equation  
Correctly uses slope to draw a line  
Correctly plots $y$-intercept of second equation  
Correctly uses slope to draw a line |  | 1   | 1   |
|          | c) Gives correct answer | (3, 130) | 1             | 1   |
|          | d) Gives correct answer | 3 hours, $130 | 1         | 1   |
| Bonus    | a) Gives correct answer | $400  | yes / no       |              |
|          | b) Gives correct answer | $7.50 per hour | yes / no  |              |

Total Points /21
<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Correctly completes first table</td>
<td>-5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly completes second table</td>
<td>-11</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-7</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>(-1, -3)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td>Shows that the coordinates satisfy the equation</td>
<td>(1, 5) lies on</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y = 4x + 1$</td>
<td>$y = 4x + 1$</td>
<td>because $5 = 4(1) + 1$.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shows that the coordinates do not satisfy the</td>
<td>(1, 5) does not</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>equation $y = 2x - 1$</td>
<td>lie on $y = 2x - 1$</td>
<td>because $5 \neq 2(1) - 1$.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Correctly graphs $y = 2x - 1$</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly graphs $y = 4x + 1$</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>(-1, -3)</td>
<td>1</td>
<td>/8</td>
</tr>
<tr>
<td>2</td>
<td>Gives correct answer for first equation</td>
<td>-1, 0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for second equation</td>
<td>2, -3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly graphs first equation</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly graphs second equation</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>(1, -1)</td>
<td>1</td>
<td>/5</td>
</tr>
<tr>
<td>3</td>
<td>Gives correct formula for Company A</td>
<td>$y = 0.01x + 20$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct formula for Company B</td>
<td>$y = 0.02x + 10$</td>
<td>1</td>
<td>/2</td>
</tr>
</tbody>
</table>
### Scoring Guides for Sample Unit Quizzes and Tests

#### Unit 5: Expressions and Equations

<table>
<thead>
<tr>
<th>Question</th>
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<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>a) Gives correct answer</td>
<td>$y = -1 + 3x$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$y = 3 + 2x$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) i) Gives correct answer</td>
<td>3, $-1$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) ii) Gives correct answer</td>
<td>2, 3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Correctly graphs first equation</td>
<td><img src="image" alt="Graph" /></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correctly graphs second equation</td>
<td><img src="image" alt="Graph" /></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e) Gives correct answer</td>
<td>(4, 11)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>f) Gives correct answer</td>
<td>4 s</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>g) Gives correct answer</td>
<td>11 cm</td>
<td>1</td>
<td>/9</td>
</tr>
<tr>
<td>5</td>
<td>Correctly completes first table</td>
<td>9, 4, 1, 0, 1, 4, 9</td>
<td>2 (1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td><img src="image" alt="Table" /></td>
<td>2 (1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly completes second table</td>
<td>$-1, 0, 1, 2, 3, 4, 5$</td>
<td>2 (1)</td>
<td>/6</td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td><img src="image" alt="Table" /></td>
<td>2 (1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives two correct answers</td>
<td>$(-1, 1), (2, 4)$</td>
<td>2 (1)</td>
<td>/6</td>
</tr>
<tr>
<td></td>
<td>Gives one correct answer</td>
<td><img src="image" alt="Table" /></td>
<td>2 (1)</td>
<td></td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>For every value of $x$ in both tables, the $y$-coordinate for $y = x + 3$ will be 2 greater than the $y$-coordinate for $y = x + 1$.</td>
<td>yes / no</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points** /30
# Rubric for Unit 5: Expressions and Equations

**Test (Lessons 49 to 52), p. U-76**

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.EE.C.8</td>
<td>1, 2, 3, 4, 5</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
<tr>
<td><strong>8.EE.C.8a</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</em></td>
<td>1, 2, 3, 4, 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.EE.C.8c</td>
<td>1, 2, 3, 4, 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Comments**
Scoring Guides for Sample Unit Quizzes and Tests  
Unit 5: Expressions and Equations  
(continued)

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Correctly completes first four rows of table</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly completes last row of table</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>(x, −2x − 3)</td>
<td>1 /3</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Substitutes ( x = −3 ) in one equation</td>
<td>Sample answer:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( y = −3 − 4 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( = −7 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td></td>
<td>1 /4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Substitutes ( x = 4 ) in one equation</td>
<td>Sample answer:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( y = 3(4) − 12 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( = 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Writes an equation using the y-coordinates of the general points</td>
<td>3x + 1 = 4x − 2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>3x − 4x = −2 − 1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( −x = −3 )</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( x = (−3) + (−1) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( x = 3 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Finds y-coordinate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>int. point = (3, 10)</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Common Core State Standards Emphasized: 8.EE.C.8, 8.EE.C.8a, 8.EE.C.8b
### Scoring Guides for Sample Unit Quizzes and Tests
#### Unit 5: Expressions and Equations

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Rearranges the first equation so that it is in slope-intercept form</td>
<td>$2x + y = 1$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Rearranges the second equation so that it is in slope-intercept form</td>
<td>$y = -2x + 1$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x + 2y = -4$</td>
<td>$2y = -x - 4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y = -\frac{1}{2}x - 2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Writes an equation using the y-coordinates of the general points</td>
<td>$-2x + 1$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$=-\frac{1}{2}x - 2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>$-4x + 2 = -x - 4$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>$-4x + x = -4 - 2$</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-3x = -6$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$x = (-6) + (-3)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= 2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Finds the y-coordinate</td>
<td>$y = -2(2) + 1$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= -3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>int. point $= (2, -3)$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Checks answer in both lines</td>
<td>Check:</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$2(2) - 3 = 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$2 + 2(-3) = -4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus</td>
<td>Gives correct answer</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k = 1$</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total Points</td>
<td></td>
<td></td>
<td>/20</td>
</tr>
<tr>
<td>Question</td>
<td>How to Score</td>
<td>Answer</td>
<td>Number of Points</td>
<td>Total Points</td>
</tr>
<tr>
<td>----------</td>
<td>--------------</td>
<td>--------</td>
<td>-----------------</td>
<td>--------------</td>
</tr>
<tr>
<td>1</td>
<td>a) Gives correct equation for first problem</td>
<td>( x + y = 16 )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct equation for second problem</td>
<td>( x - y = 2 )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct equation for first problem</td>
<td>( x + y = 12 )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct equation for second problem</td>
<td>( 5x + y = 28 )</td>
<td>1</td>
<td>/4</td>
</tr>
<tr>
<td>2</td>
<td>States what the first variable represents</td>
<td>Let ( x ) be the number of medium-sized drinks.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>States what the second variable represents</td>
<td>Let ( y ) be the number of large-sized drinks.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Writes one correct equation</td>
<td>( x + y = 20 )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rearranges equation to slope-intercept form</td>
<td>( y = 20 - x )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Writes a second correct equation</td>
<td>( 2x + 3y = 48 )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rearranges equation to slope-intercept form</td>
<td>( 3y = 48 - 2x )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Writes an equation using the y-coordinates of the general points</td>
<td>( y = 16 - \frac{2}{3}x )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>( 60 - 3x = 48 - 2x )</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>( -3x + 2x = 48 - 60 )</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>( -x = -12 )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( x = 12 )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>2, (-3)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>There is a solution.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct explanation</td>
<td>Since the slopes are different, the lines are not parallel. They will intersect.</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td>Question</td>
<td>How to Score</td>
<td>Answer</td>
<td>Number of Points</td>
<td>Total Points</td>
</tr>
<tr>
<td>----------</td>
<td>-------------</td>
<td>--------</td>
<td>-----------------</td>
<td>--------------</td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>Sample answer: $y = 2x + 1$ and $y = 2x + 3$ have the same slope but different $y$-intercepts. They are parallel and never intersect. So there is no solution.</td>
<td>yes / no</td>
<td>/17</td>
</tr>
</tbody>
</table>

Total Points /17
## Scoring Guides for Sample Unit Quizzes and Tests
### Unit 5: Expressions and Equations

**Test (Lessons 53 to 56), p. U-85**

**Common Core State Standards Emphasized:** 8.EE.C.8, 8.EE.C.8a, 8.EE.C.8b, 8.EE.C.8c

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Gives two correct answers</td>
<td>Sample answer: $x = 1: (1, -2)$ $x = 2: (2, 1)$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives one correct answer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Substitutes $x = 9$ in one equation</td>
<td>Sample answer: $9 = x + 4$ $x = 5$</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Writes an equation using the $y$-coordinates of the general points</td>
<td>$-x + 2 = 2x - 4$</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>$-x - 2x = -4 - 2$</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>$-3x = -6$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = 2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Finds $y$-coordinate</td>
<td>$y = -(2) + 2 = 0$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>int. point = (2, 0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Checks answer in both lines</td>
<td>$0 = -2 + 2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0 = 2(2) - 4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>a) Gives correct answer for $m_1$</td>
<td>$m_1 = 1$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for $m_2$</td>
<td>$m_2 = \frac{3}{2}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Circles correct answer</td>
<td>solution</td>
<td>1</td>
<td>1/6</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer for $m_1$</td>
<td>$m_1 = -1$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for $m_2$</td>
<td>$m_2 = -1$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Circles correct answer</td>
<td>no solution</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>States what the first variable represents</td>
<td>Let $x$ be the larger number.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>States what the second variable represents</td>
<td>Let $y$ be the smaller number.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Writes one correct equation</td>
<td>$x + y = 18$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rearranges equation to slope-intercept form</td>
<td>$y = 18 - x$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Writes a second correct equation</td>
<td>$x - y = 8$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rearranges equation to slope-intercept form</td>
<td>$y = x - 8$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Writes an equation using the $y$-coordinates of the general points</td>
<td>$18 - x = x - 8$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>$-x - x = -8 - 18$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>$-2x = -26$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = 13$</td>
<td></td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Finds $y$-coordinate</td>
<td>$y = 18 - 13 = 5$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>The numbers are 13 and 5.</td>
<td>1</td>
<td>1/11</td>
</tr>
</tbody>
</table>

---

**Scoring Guides and Rubrics to accompany Sample Unit Quizzes and Tests for AP Book 8.2**

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<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>States what one variable represents</td>
<td>Let ( x ) be the number of 2-point shots.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>States what the second variable represents</td>
<td>Let ( y ) be the number of 3-point shots.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Writes one correct equation</td>
<td>( x + y = 8 )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rearranges equation to slope-intercept form</td>
<td>( y = 8 - x )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Writes a second correct equation</td>
<td>( 2x + 3y = 19 )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rearranges equation to slope-intercept form</td>
<td>( 3y = 19 - 2x )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Writes an equation using the ( y )-coordinates of the general points</td>
<td>( 8 - x = \frac{19}{3} - \frac{2}{3}x )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>( 24 - 3x = 19 - 2x )</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>( -3x + 2x = 19 - 24 )</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( -x = -5 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( x = 5 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Finds ( y )-coordinate</td>
<td>( y = 8 - (5) = 3 )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>Abdul made 5 2-point shots and 3 3-point shots.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>int. point = (2, 3)</td>
</tr>
</tbody>
</table>

Total Points /38
### Common Core State Standard

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.</td>
<td>2, 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.EE.C.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</td>
<td>1, 2, 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.EE.C.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.EE.C.8c Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.</td>
<td>5, 6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Comments

Name: ____________________
### Scoring Guides for Sample Unit Quizzes and Tests
#### Unit 6: Geometry

**Quiz (Lessons 49 to 52), p. U-89**

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct name</td>
<td>cylinder</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct formula</td>
<td>$V = \pi r^2 h$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correctly uses formula</td>
<td>$V \approx 3.14 \times 3.5^2 \times 8$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer with units</td>
<td>$\approx 307.7 \text{ in}^3$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct name</td>
<td>triangle-based pyramid</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct formula</td>
<td>$V = \frac{1}{3} Bh$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly uses formula</td>
<td>$V \approx (4 \times 2) \div 2 \times 7 \div 3$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer with units</td>
<td>$\approx 9.3 \text{ ft}^3$</td>
<td>1</td>
<td>/8</td>
</tr>
<tr>
<td>2</td>
<td>Correctly calculates the volume of the prism</td>
<td>$V_1 = 12 \times 5 \times 7$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>= 420</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly calculates the volume of the rectangular pyramid</td>
<td>$V_2 = 12 \times 5 \times 6 \div 3$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>= 120</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives one total volume with units</td>
<td>Total volume</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= 420 + 120$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= 540 \text{ m}^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Correctly labels triangle</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct equation</td>
<td>$h^2 + 2^2 = 7^2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>$h^2 + 4 = 49$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>$h^2 = 45$</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer with units</td>
<td>$h \approx 6.7 \text{ in}$</td>
<td>1</td>
<td>/5</td>
</tr>
<tr>
<td>4</td>
<td>Gives correct equation</td>
<td>$\pi r^2 h = 226$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly substitutes numbers into equation</td>
<td>$3.14 \times 3^2 \times h \approx 226$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>$h \approx 226 \div 3.14 \div 9$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer with units</td>
<td>$h \approx 8 \text{ in}$</td>
<td>1</td>
<td>/4</td>
</tr>
</tbody>
</table>
### Scoring Guides for Sample Unit Quizzes and Tests
#### Unit 6: Geometry

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>The volume of the rectangular pyramid is ( \frac{1}{3} ) of the volume of the rectangular prism with the same base. If the volumes are the same, then the height of the pyramid must be 3 times the height of the prism.</td>
<td>yes / no</td>
<td>/22</td>
</tr>
</tbody>
</table>

Total Points /22
### Scoring Guides for Sample Unit Quizzes and Tests

#### Unit 6: Geometry (continued)

**Quiz (Lessons 53 to 55), p. U-92**

**Common Core State Standards Emphasized:** 8.G.B.7, 8.G.C.9

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>$V = \frac{4}{3} \pi r^3$</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>b) Correctly substitutes numbers in formula</td>
<td>$V \approx 4 \times 3.14 \times 2.5^3 \div 3$</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer with units</td>
<td>$= 65.4 \text{ ft}^3$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Correctly labels triangle</td>
<td>$h \text{ in}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct equation</td>
<td>$h^2 + 2^2 = 6^2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>$h^2 + 4 = 36$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>$h^2 = 32$</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$h = \sqrt{32}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer with units</td>
<td>$h \approx 5.7 \text{ in}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>b) Gives correct formula</td>
<td>$V = \frac{1}{3} \pi r^2 h$</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly substitutes numbers in formula</td>
<td>$\approx 3.14 \times 2^2 \times \frac{5.7}{3}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer with units</td>
<td>$\approx 23.9 \text{ in}^3$</td>
<td>1</td>
<td>/8</td>
</tr>
<tr>
<td>3</td>
<td>Correctly calculates the volume of the hemisphere</td>
<td>$V_1 = \frac{4}{3} \pi r^3 + 2$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>$\approx 4 \times 3.14 \times 2^3 + 2 \div 3$</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly calculates the volume of the cylinder</td>
<td>$V_2 = \pi r^2 h$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>$\approx 3.14 \times 2^2 \times 6$</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct total volume with units</td>
<td>$V_T = 16.7 + 75.4 = 92.1 \text{ in}^3$</td>
<td>1</td>
<td>/5</td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td></td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>In the formula for $V$, $r$ is cubed. Multiplying $r$ by 2 multiplies $V$ by $2^3 = 8$. So $r$ of the medium ball is twice the $r$ of the small ball.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Total Points** | /16 |
## Scoring Guides for Sample Unit Quizzes and Tests
### Unit 6: Geometry

*Test (Lessons 49 to 55), p. U-95*

**Common Core State Standards Emphasized:** 8.G.B.7, 8.G.C.9

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct name</td>
<td>cylinder</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct formula</td>
<td>$V = \pi r^2 h$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly uses formula</td>
<td>$V \approx 3.14 \times 6.2^2 \times 5.5$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer with units</td>
<td>$\approx 663.9 \text{ ft}^3$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct name</td>
<td>cone</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct formula</td>
<td>$V = \frac{1}{3} \pi r^2 h$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly uses formula</td>
<td>$V \approx 3.14 \times 8^2 \times 6 \div 3$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer with units</td>
<td>$\approx 401.9 \text{ cm}^3$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct name</td>
<td>sphere</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct formula</td>
<td>$V = \frac{4}{3} \pi r^3$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly uses formula</td>
<td>$V \approx 3.14 \times 4 \times 1.5^3 \div 3$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer with units</td>
<td>$\approx 14.1 \text{ in}^3$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Correctly calculates the volume of the cylinder</td>
<td>$V_1 \approx 3.14 \times 3^2 \times 5$</td>
<td>2</td>
<td>/12</td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>$= 141.3 \text{ mm}^2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly calculates the volume of the sphere</td>
<td>$V_2 = 4 \times 3.14 \times 3^3 \div 3$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>$= 113.0 \text{ mm}^2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct total volume with units</td>
<td>Total volume</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= 141.3 + 113.0$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= 254.3 \text{ mm}^3$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct calculation</td>
<td>$0.001 \times 254.3$</td>
<td>1</td>
<td>/7</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>$= 0.2543 \text{ mL}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct equation</td>
<td>$V = \pi r^2 h$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly substitutes numbers into equation</td>
<td>$463.6 \approx 3.14 \times 3.75^2 \times h$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>$463.6 \div 3.14 + 3.75^2 \approx h$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer with units</td>
<td>$10.5 \text{ cm} \approx h$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Question</td>
<td>How to Score</td>
<td>Answer</td>
<td>Number of Points</td>
<td>Total Points</td>
</tr>
<tr>
<td>----------</td>
<td>--------------</td>
<td>--------</td>
<td>-----------------</td>
<td>--------------</td>
</tr>
<tr>
<td>3</td>
<td>b) Gives correct answer</td>
<td>10.5 × 3 = 31.5 cm 10.5 × 2 = 21 cm Only 2 cans can be stacked.</td>
<td>1</td>
<td>/5</td>
</tr>
<tr>
<td>4</td>
<td>a) Correctly uses formula</td>
<td>(V \approx 4 \times 3.14 \div 3 \times 25^3) (\approx 65,416.7 \text{ cm}^3)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer with units</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Correctly uses formula</td>
<td>(V \approx 4 \times 3.14 \div 3 \times 5^3) (\approx 523.3 \text{ cm}^3)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct rounded answer with units</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) i) Gives correct answer</td>
<td>5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) ii) Gives correct answer</td>
<td>125</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>(5^3 = 125)</td>
<td>1</td>
<td>/7</td>
</tr>
<tr>
<td>5</td>
<td>Gives correct equation</td>
<td>(h^2 + 115^2 = 180^2)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>(h^2 + 13,225 = 32,400)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>(h^2 = 19,175)</td>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(h = \sqrt{19,175})</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(h = 138.5 \text{ m})</td>
<td>1</td>
<td>/4</td>
</tr>
<tr>
<td>6</td>
<td>a) Correctly calculates the volume of the whole cylinder</td>
<td>(V_1 \approx 3.14 \times 20^2 \times 30)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>(= 37,680 \text{ cm}^3)</td>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>Correctly calculates the volume of the center cylinder</td>
<td>(V_2 \approx 3.14 \times 10^2 \times 30)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>(= 9,420 \text{ cm}^3)</td>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>Correctly calculates the total volume</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(= V_1 - V_2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(= 28,260 \text{ cm}^3)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct total volume with units</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct calculation</td>
<td>(\frac{9,420}{28,260} = 0.25)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td></td>
<td>1</td>
<td>/8</td>
</tr>
</tbody>
</table>
## Scoring Guides for Sample Unit Quizzes and Tests
### Unit 6: Geometry

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>Sample answers: <img src="image" alt="Diagram" /></td>
<td>yes / no</td>
<td>/43</td>
</tr>
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</table>

| Total Points | /43 |
# Rubric for Unit 6: Geometry

**Test (Lessons 49 to 55), p. U-95**

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.G.B.7</td>
<td><strong>Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.</strong></td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.G.C.9</td>
<td><strong>Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.</strong></td>
<td>1, 2, 3, 4, 6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Comments**

Name: ____________________
# Scoring Guides for Sample Unit Quizzes and Tests
Unit 7: Statistics and Probability

## Quiz (Lessons 4 to 6), p. U-99

Common Core State Standards Emphasized: 8.SP.A.1, 8.SP.A.2

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives both correct answers</td>
<td>A, C</td>
<td>1</td>
<td>(0.5)</td>
</tr>
<tr>
<td></td>
<td>Gives one correct answer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>C</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>C</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>If ( m &gt; 0 ), the association is positive.</td>
<td>1</td>
<td>/4</td>
</tr>
<tr>
<td>2</td>
<td>a) i) Gives correct answer</td>
<td>CCW</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ii) Gives correct answer</td>
<td>CCW</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>no</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td>3</td>
<td>a) Correctly plots all ten points</td>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly plots seven to nine points</td>
<td></td>
<td>(1.5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly plots four to six points</td>
<td></td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly plots one to three points</td>
<td></td>
<td>(0.5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>positive</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e) Gives correct answer</td>
<td>4</td>
<td>1</td>
<td>/6</td>
</tr>
<tr>
<td></td>
<td><strong>Total Points</strong></td>
<td></td>
<td>/13</td>
<td></td>
</tr>
</tbody>
</table>
Scoring Guides and Rubrics to accompany Sample Unit Quizzes and Tests
Unit 7: Statistics and Probability

(continued)

### Quiz (Lessons 7 to 8), p. U-101

**Common Core State Standards Emphasized:** 8.SP.A.2, 8.SP.A.3, 8.F.B.4

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>negative</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Circles correct column for A</td>
<td>(10, 80)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Circles correct column for B</td>
<td>(20, 58)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct calculation</td>
<td>( \frac{80 - 58}{10 - 20} = -2.2 )</td>
<td>1</td>
<td>/5</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>( \frac{80 - 58}{10 - 20} = -2.2 )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Correctly plots all eleven points</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly plots eight to ten points</td>
<td>(1.5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly plots four to seven points</td>
<td>(1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly plots one to three points</td>
<td>(0.5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>positive</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e) Gives correct answer</td>
<td>4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>f) Identifies two points</td>
<td>((66, 8), (59, 6))</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct calculation</td>
<td>( m = \frac{8 - 6}{66 - 59} )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>( m = \frac{2}{7} )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>g) Gives correct equation</td>
<td>( y = \frac{2}{7} x + b )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Identifies a point</td>
<td>If ((59, 6)) lies on the line, then</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly substitutes point into equation</td>
<td>( 6 = \frac{2}{7} (59) + b )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>( \frac{42}{7} - \frac{118}{7} = b )</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>( -\frac{76}{7} = b )</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>h) Gives correct answer</td>
<td>( y = \frac{2}{7} x - \frac{76}{7} )</td>
<td>1</td>
<td>/15</td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>about 9.5</td>
<td>yes / no</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points** /20
### Scoring Guides for Sample Unit Quizzes and Tests
### Unit 7: Statistics and Probability

**Quiz (Lessons 9 to 12),** p. U-103

Common Core State Standards Emphasized: 8.SP.A.1, 8.SP.A.2, 8.SP.A.3, 8.SP.A.4

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives all five correct answers</td>
<td>31</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Gives three or four correct answers</td>
<td>17</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Gives one or two correct answers</td>
<td>48</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b) Gives all five correct answers</td>
<td>8</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Gives three or four correct answers</td>
<td>7</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Gives one or two correct answers</td>
<td>15</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>27%</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11%</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>38%</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>among students who like football</td>
<td>yes/no</td>
<td>yes/no</td>
</tr>
<tr>
<td>Bonus: Gives correct answer</td>
<td>27% is greater than 11%</td>
<td>yes/no</td>
<td>yes/no</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct row two-way table</td>
<td>12</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Makes one or two errors</td>
<td>11</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct row two-way relative frequency table</td>
<td>71%</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Makes one or two errors</td>
<td>33%</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Bonus: Gives correct answer</td>
<td>Yes. positive</td>
<td>yes/no</td>
<td>yes/no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct type of association</td>
<td>The percentage of people who buy milk and eggs is high. At the same time, the percentage of people who buy neither is high.</td>
<td>yes/no</td>
<td>yes/no</td>
</tr>
</tbody>
</table>

Total Points /12
### Scoring Guides for Sample Unit Quizzes and Tests

**Unit 7: Statistics and Probability**

*Test (Lessons 4 to 12), p. U-105*

**Common Core State Standards Emphasized:** 8.SP.A.1, 8.SP.A.2, 8.SP.A.3, 8.SP.A.4, 8.F.B.4

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>CCW</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>CCW</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>2</td>
<td>a) Correctly plots all ten points</td>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly plots seven to nine points</td>
<td></td>
<td>(1.5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly plots four to six points</td>
<td></td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly plots one to three points</td>
<td></td>
<td>(0.5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>positive</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>b) Gives correct answer</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) Gives correct answer</td>
<td>approx. 83</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) Gives correct answer</td>
<td>approx. 93</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e) Identifies two points</td>
<td>(3,68), (7,88)</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct calculation</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>m = ( \frac{88-68}{7-3} )</td>
<td></td>
<td>( \frac{20}{4} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>m = 5</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f) Gives correct equation</td>
<td>y = 5x + b</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Identifies a point</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>If (7, 88) lies on the line, then</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>88 = 5(7) + b</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>88 – 35 = b</td>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>53 = b</td>
<td></td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>y = 5x + 53</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>g) Gives correct answer for c)</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for d)</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Bonus: Gives correct answer</td>
<td></td>
<td>53</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct explanation</td>
<td></td>
<td>yes / no</td>
<td>/17</td>
</tr>
</tbody>
</table>
### Scoring Guides for Sample Unit Quizzes and Tests
#### Unit 7: Statistics and Probability

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Gives all five correct answers in two-way table</td>
<td>18 17 35</td>
<td>3</td>
<td>(2) 3</td>
</tr>
<tr>
<td></td>
<td>Gives three or four correct answers</td>
<td>12 13 25</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives one or two correct answers</td>
<td>30 30 60</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct two-way relative frequency table</td>
<td>30% 28% 58%</td>
<td>2</td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>Makes one or two computational errors</td>
<td>20% 22% 42%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one or two computational errors</td>
<td>50% 50% 100%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| 4        | a) Gives correct row two-way table | 21 6 27 | 2 | (1) |
|          | Makes one or two errors | 7 16 23 | |
|          | Gives correct row two-way relative frequency table | 78% 22% 100% | 2 |
|          | Makes one or two errors | 30% 70% 100% | |
|          | b) Gives correct answer | Yes. | 1 |
|          | Gives correct type of association | positive | 1 |
|          | Gives correct explanation | Most students who have a curfew have chores. Most students who have no curfew do not have chores. | 1 |
|          | c) Gives correct row two-way table | 21 7 28 | 2 | (1) |
|          | Makes one or two errors | 6 16 22 | |
|          | Gives correct row two-way relative frequency table | 75% 25% 100% | 2 | |
|          | Makes one or two errors | 27% 73% 100% | |
|          | Bonus: Gives correct answer | Yes / No | yes / no |

| Total Points | /35 |

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Scoring Guides and Rubrics to accompany Sample Unit Quizzes and Tests for AP Book 8.2 U-181
### Rubric for Unit 7: Statistics and Probability

**Test (Lessons 4 to 12), p. U-105**

<table>
<thead>
<tr>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Common Core State Standard</strong></th>
<th><strong>Assessed by Question(s) …</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>8.SP.A.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.</td>
<td>1, 2</td>
</tr>
<tr>
<td>8.SP.A.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.</td>
<td>1, 2</td>
</tr>
<tr>
<td>8.SP.A.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.</td>
<td>1, 2</td>
</tr>
</tbody>
</table>
### Scoring Guides and Rubrics to accompany Sample Unit Quizzes and Tests for AP Book 8.2

### Rubric for Unit 7: Statistics and Probability

Test (Lessons 4 to 12), p. U-105

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s)</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.SP.A.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?</td>
<td>3, 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.F.B.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</td>
<td>1, 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Comments
### Grade 8 Common Core State Standards Curriculum Correlations

**NOTE:** The italicized gray JUMP Math lessons contain prerequisite material for the Common Core standards.

#### Domain

- **RP** Ratios and Proportional Relationships
- **NS** The Number System
- **EE** Expressions and Equations
- **G** Geometry
- **SP** Statistics and Probability

#### Cluster

**8.NS** **D** The Number System

**8.NS.A** **C** Know that there are numbers that are not rational, and approximate them by rational numbers.

<table>
<thead>
<tr>
<th><strong>8.NS.A</strong></th>
<th><strong>JUMP Math Grade 8 Lessons</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>8.NS.A.1</strong></td>
<td>Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.</td>
</tr>
<tr>
<td><strong>Part</strong></td>
<td><strong>Unit</strong></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

| **8.NS.A.2** | Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., \( \pi \)). For example, by truncating the decimal expansion of \( \sqrt{2} \), show that \( \sqrt{2} \) is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations. |
| **Part** | **Unit** | **Lessons** |
| 2 | 3 | NS8-9, 10 |
| 2 | 4 | G8-40, 41 |
### 8.EE.1 Work with radicals and integer exponents.

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<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>EE8-16 to 18, 24 to 26</td>
</tr>
</tbody>
</table>

#### 8.EE.A.1
Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

#### 8.EE.A.2
Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.

#### 8.EE.A.3
Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 times $10^8$ and the population of the world as 7 times $10^9$, and determine that the world population is more than 20 times larger.

#### 8.EE.A.4
Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

### 8.EE.2 Understand the connections between proportional relationships, lines, and linear equations.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
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</thead>
<tbody>
<tr>
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<td>1</td>
<td>EE8-4 to 9</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>NS8-1, 2</td>
</tr>
</tbody>
</table>

#### 8.EE.B.5
Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

#### 8.EE.B.6
Use similar triangles to explain why the slope $m$ is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $b$. 

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
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</thead>
<tbody>
<tr>
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<td>5</td>
<td>EE8-44</td>
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<tr>
<td>2</td>
<td>1</td>
<td>F8-15, 16</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>G8-37, 38</td>
</tr>
</tbody>
</table>
### 8.EE (Expressions and Equations)

#### 8.EE.C (Analyze and solve linear equations and pairs of simultaneous linear equations)

<table>
<thead>
<tr>
<th>8.EE.C.7</th>
<th>Solve linear equations in one variable.</th>
<th>JUMP Math Grade 8 Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
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<tr>
<td>1</td>
<td>1</td>
<td>EE8-1 to 9, 13, 14</td>
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<tr>
<td>1</td>
<td>4</td>
<td>EE8-27, 28, EE8-29 to 31</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>EE8-46</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8.EE.C.7a</th>
<th>Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where $a$ and $b$ are different numbers).</th>
<th>JUMP Math Grade 8 Lessons</th>
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</thead>
<tbody>
<tr>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>EE8-37</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8.EE.C.7b</th>
<th>Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.</th>
<th>JUMP Math Grade 8 Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
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<tr>
<td>1</td>
<td>4</td>
<td>EE8-32 to 36, 38</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8.EE.C.8</th>
<th>Analyze and solve pairs of simultaneous linear equations.</th>
<th>JUMP Math Grade 8 Lessons</th>
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</thead>
<tbody>
<tr>
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<td>Unit</td>
<td>Lessons</td>
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<tr>
<td>2</td>
<td>5</td>
<td>EE8-49 to 56</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8.EE.C.8a</th>
<th>Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</th>
<th>JUMP Math Grade 8 Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
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<tr>
<td>2</td>
<td>5</td>
<td>EE8-49 to 51, 53</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8.EE.C.8b</th>
<th>Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.</th>
<th>JUMP Math Grade 8 Lessons</th>
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</thead>
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<tr>
<td>Part</td>
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<td>Lessons</td>
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<td>2</td>
<td>5</td>
<td>EE8-54, 56</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>8.EE.C.8c</th>
<th>Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.</th>
<th>JUMP Math Grade 8 Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
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<tr>
<td>2</td>
<td>5</td>
<td>EE8-52, 55</td>
</tr>
</tbody>
</table>
### 8.F  Functions

#### 8.F.A Define, evaluate, and compare functions.

<table>
<thead>
<tr>
<th>8.F.A.1</th>
<th>Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.</th>
<th>JUMP Math Grade 8 Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
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<tr>
<td>1</td>
<td>6</td>
<td>F8-1 to 6</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>F8-23</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8.F.A.2</th>
<th>Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.</th>
<th>JUMP Math Grade 8 Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
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<tr>
<td>1</td>
<td>6</td>
<td>F8-7 to 12</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>F8-20, 23</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8.F.A.3</th>
<th>Interpret the equation ( y = mx + b ) as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function ( A = s^2 ) giving the area of a square as a function of its side length is not linear because its graph contains the points ((1, 1), (2, 4)) and ((3, 9)), which are not on a straight line.</th>
<th>JUMP Math Grade 8 Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
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<tr>
<td>1</td>
<td>1</td>
<td>EE8-1 to 3, 13, 14</td>
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<tr>
<td>1</td>
<td>6</td>
<td>F8-7, 9 to 12</td>
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<tr>
<td>2</td>
<td>1</td>
<td>F8-15 to 19</td>
</tr>
</tbody>
</table>

### 8.F  Functions

#### 8.F.B Use functions to model relationships between quantities.

<table>
<thead>
<tr>
<th>8.F.B.4</th>
<th>Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two ((x, y)) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</th>
<th>JUMP Math Grade 8 Lessons</th>
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</thead>
<tbody>
<tr>
<td>Part</td>
<td>Unit</td>
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<td>2</td>
<td>1</td>
<td>F8-13, 14 F8-15 to 19, 21, 23</td>
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<tr>
<td>2</td>
<td>7</td>
<td>SP8-7, 8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8.F.B.5</th>
<th>Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</th>
<th>JUMP Math Grade 8 Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
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<td>2</td>
<td>1</td>
<td>F8-22, 23</td>
</tr>
</tbody>
</table>
### 8.G. Geometry

**8.G.A** Understand congruence and similarity using physical models, transparencies, or geometry software.

<table>
<thead>
<tr>
<th><strong>8.G.A.1</strong></th>
<th>Verify experimentally the properties of rotations, reflections, and translations:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>8.G.A.1a</strong></td>
<td>Lines are taken to lines, and line segments to line segments of the same length.</td>
</tr>
<tr>
<td><strong>8.G.A.1b</strong></td>
<td>Angles are taken to angles of the same measure.</td>
</tr>
<tr>
<td><strong>8.G.A.1c</strong></td>
<td>Parallel lines are taken to parallel lines.</td>
</tr>
<tr>
<td><strong>8.G.A.2</strong></td>
<td>Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.</td>
</tr>
<tr>
<td><strong>8.G.A.3</strong></td>
<td>Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.</td>
</tr>
<tr>
<td><strong>8.G.A.4</strong></td>
<td>Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.</td>
</tr>
<tr>
<td><strong>8.G.A.5</strong></td>
<td>Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. <em>For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.</em></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
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<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>G8-4</td>
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<td>F8-13</td>
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<td>G8-19, 21, 25, 28</td>
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<td>G8-10</td>
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<td>G8-19, 21</td>
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<td>G8-20, 23, 28</td>
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<td>G8-9, 11, 12, 16</td>
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<td>G8-19, 21, 22, 24, 25, 29 to 31</td>
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<td>G8-32, 33, 35, 36</td>
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<td>1</td>
<td>3</td>
<td>G8-1 to 4, 8 G8-5 to 7, 10, 12 to 16</td>
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<tr>
<td>2</td>
<td>2</td>
<td>G8-36</td>
</tr>
</tbody>
</table>
### 8.G. Geometry

#### 8.G.B. Understand and apply the Pythagorean Theorem.

<table>
<thead>
<tr>
<th>8.G.B.6</th>
<th>Explain a proof of the Pythagorean Theorem and its converse.</th>
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</thead>
<tbody>
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<td>Unit</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>8.G.B.7</th>
<th>Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
<td>Unit</td>
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<td>4</td>
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<tr>
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<td>6</td>
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</table>

<table>
<thead>
<tr>
<th>8.G.B.8</th>
<th>Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
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</table>

#### 8.G.C. Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

<table>
<thead>
<tr>
<th>8.G.C.9</th>
<th>Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>
# 8.SP  Statistics and Probability

## 8.SPA  Investigate patterns of association in bivariate data.

### 8.SPA.1  Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

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<tbody>
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<tr>
<td>2</td>
<td>7</td>
<td>SP8-4, 12</td>
</tr>
</tbody>
</table>

### 8.SPA.2  Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

<table>
<thead>
<tr>
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<th>Unit</th>
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<tr>
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<td>7</td>
<td>SP8-4 to 8, 12</td>
</tr>
</tbody>
</table>

### 8.SPA.3  Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

<table>
<thead>
<tr>
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<th>Unit</th>
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<tr>
<td>2</td>
<td>7</td>
<td>SP8-8, 12</td>
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</tbody>
</table>

### 8.SPA.4  Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

<table>
<thead>
<tr>
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<tr>
<td>2</td>
<td>7</td>
<td>SP8-9 to 12</td>
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