Teacher’s Guide: Workbook 5

JUMP Math

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Introduction

1. Key Components of the JUMP Math Resources

Student Workbooks Parts 1 and 2, Grades 1–8
- Solid foundation for each of the strands in the curriculum at grade level
- Extensive review going back up to two grades
- All strands complete the curriculum at grade level

Teacher’s Guides, Grades 1–8
Overview of JUMP Math
Mental Math Unit
Detailed Table of Contents (Parts 1 and 2)
Lesson plans provide clear explanations and explicit guidance on how to
- Introduce one concept at a time
- Explore concepts and make connections in a variety of ways
- Assess students quickly
- Extend learning with extra bonus questions and activities
- Develop problem solving skills
- Support material for each strand

BLMs (extra worksheets, games, manipulatives, etc.)
Answer Keys (for Workbooks and Unit Tests), Grades 3–8
Unit Tests, Grades 3–8
Curriculum Correlations (WNCP, ON)

2. The Myth of Ability

There is a prevalent myth in our society that some people are born with mathematical talent—and others simply do not have the ability to succeed.

Recent discoveries in cognitive science are challenging this myth of ability. The brain is not hard-wired; it continues to change and develop throughout life. Steady, incremental learning can result in the emergence of new abilities. The brain, even when damaged, is able to rewire itself and learn new functions through rigorous instruction. As Philip E. Ross points out in his 2006 Scientific American article “The Expert Mind,” this fact has profound implications for education:

The preponderance of psychological evidence indicates that experts are made not born. What is more, the demonstrated ability to turn a child quickly into an expert—in chess, music, and a host of other subjects—sets a clear challenge before the schools. Can educators find ways to encourage the kind of effortful study that will improve their reading and math skills? Instead of perpetually pondering the question, “Why can’t Johnny read?” perhaps educators should ask, “Why should there be anything in the world that he can’t learn to do?”

JUMP Math builds on the belief that every child can be successful at mathematics by
• promoting positive learning environments and building confidence through praise and encouragement;
• maintaining a balanced approach to mathematics by concurrently addressing conceptual and procedural learning, explicit and inquiry-based learning;
• achieving understanding and mastery by breaking mathematics down into sequential, scaffolded steps, while still allowing students to make discoveries;
• keeping all students engaged and attentive by “raising the bar” incrementally;
• guiding students strategically to explore and discover the beauty of mathematics as a symbolic language connected to the real world; and,
• using continuous assessment to ensure all students are engaged and none are left behind.

JUMP Math is an approach to teaching mathematics that has been developed by John Mighton and a team of mathematicians and educators who are dedicated to excellence in mathematics teaching and learning.

3. About JUMP Math

Nine years ago I was looking for a way to give something back to my local community. It occurred to me that I should try to help kids who needed help with math. Mathematicians don’t always make the best teachers because mathematics has become obvious to them; they can have trouble seeing why their students are having trouble. But because I had struggled with math myself, I wasn’t inclined to blame my students if they couldn’t move forward.
— John Mighton

John Mighton is a mathematician, bestselling author, award-winning playwright, and the founder of JUMP Math, a national charity dedicated to improving mathematical literacy.

JUMP Math grew out of John’s work with a core group of volunteers in a “tutoring club” held in his apartment to meet the needs of the most challenged students from local schools. Over three years, John developed the early material—simple handouts for the tutors to use. This period was one of experimentation in developing the JUMP Math method through countless hours of one-on-one tutoring. Eventually, John began to work in local inner-city schools, placing tutors in classrooms. This led to the next period of innovation—working through the JUMP Math method in classrooms.

Teachers responded enthusiastically to their students’ success and wanted to adapt the method for classroom use. John and a group of volunteers and teachers developed workbooks to meet teachers’ needs for curriculum-based resources. These started out as a series of three remedial books with limited accompanying teacher materials, released in fall of 2003. The effectiveness of these workbooks led quickly to the development of grade-specific, curriculum-based workbooks and teacher’s guides, first released in 2004.

John documented his experience in two national bestselling books, The Myth of Ability (2003) and The End of Ignorance (2007). As a playwright, he has won several national awards, including the Governor General’s Literary Award for Drama, the Dora Award, the Chalmers Award, and the Siminovitch Prize. John was granted a prestigious Ashoka Fellowship as a social entrepreneur for his work in fostering mathematical literacy by building students’ self-confidence and competence through JUMP Math. In 2010 John was appointed an Officer of the Order of Canada.

In only ten years, JUMP Math has grown from a gathering around John’s kitchen table to a thriving organization reaching more than 50,000 students with high-quality learning resources and training for 3,000 teachers. JUMP Math is working in hundreds of schools across Canada and internationally, and has established a network of dedicated teachers who are mentoring and training teachers new to the program. As well, JUMP Math supports community organizations in reaching struggling students through homework clubs and after-school programs. Through the generous support of our sponsors, JUMP Math donates learning resources to classrooms and homework clubs across Canada. JUMP Math has inspired thousands of community volunteers and teachers to reach out to struggling students by donating their time as tutors, mentors, and trainers.
4. JUMP Math Works

JUMP Math is a learning organization committed to evaluation and evidence-based practice. JUMP Math is a leader in encouraging and supporting third-party research to study the efficacy of the JUMP Math approach to mathematics teaching and learning.

Hospital for Sick Children 2008–2010
Cognitive scientists from The Hospital for Sick Children in Toronto conducted a randomized-controlled study of the effectiveness of the JUMP Math program. Studies of such scientific rigour remain relatively rare in mathematics education research in North America. The results showed that, on well-established measures of math achievement, students who received JUMP instruction outperformed students who received the methods of instruction their teachers would normally use.

Researchers from OISE at the University of Toronto, led by Dr. Joan Moss, completed a one-year study on the efficacy of JUMP Math. Preliminary data indicate that
• JUMP Math’s Grade 5 resources for the curriculum in multiplication provide a greater variety of representations of concepts and more practice than provincially recommended programs; and
• JUMP Math significantly improves the conceptual understanding of math for struggling students.

Vancouver School Board 2006–2007
Over the school year, 68 JUMP Math teachers were surveyed. The teachers indicated that
• the JUMP Math methodology enhances student retention and transfer, promotes independent thinking, and creates excitement and curiosity; and
• JUMP Math develops teacher confidence and self-efficacy.

Borough of Lambeth (London, UK) 2006–2009
During the summer of 2006, 24 public schools in Lambeth participated in a pilot study on JUMP Math. As a result of the pilot’s impact on student behaviour, confidence, and achievement—as well as teachers’ strong reaction to JUMP Math as an effective teaching tool—a total of 35 local schools adopted the program for the 2006–2007 school year. In this second implementation, 69% of the students who initially performed two years below age-related expectations moved up multiple levels after using JUMP Math, and either reached or surpassed their desired level by the end of the school year. In 2008–2009, it was shown that 57% of students who were initially at grade level progressed three years ahead of national expectations for their age.

After using JUMP Math as their exclusive math program during a 5-month pilot study, a class of Grade 3 students in British Columbia showed
• an increase in math achievement equivalent to 9 months of instruction (80% more than the expected achievement in 5 months); and
• a statistically significant decrease in math anxiety and a statistically significant increase in positive attitude toward math.

In 2010, JUMP Math was recommended by the Canadian Language and Literacy Research Network and the Canadian Child Care Federation in a report entitled “Foundations for Numeracy: An Evidence-Based Toolkit for the Effective Mathematics Teacher.”
In 2009, JUMP Math was recognized by the UK Government in the document “What Works for Children with Mathematical Difficulties?”

See the JUMP Math website, www.jumpmath.org, for more details and updates on research.

5. What Is the JUMP Math Approach?

JUMP Math is a balanced approach to teaching mathematics that supports differentiated instruction. JUMP Math covers the full curriculum for both Ontario and Western Canada through student workbooks, teacher’s guides, and a range of support materials.

The JUMP Math student workbooks are not intended to be used without instruction. Teachers should use the workbooks and accompanying lesson plans in the teacher’s guides for dynamic lessons in which students are allowed to discover and explore ideas on their own. The careful scaffolding of the mathematics in the student workbooks make them an excellent tool for teachers to use for guided practice and continuous assessment.

The JUMP Math approach to teaching mathematics emphasizes:

- Confidence-building
- Guided practice
- Guided discovery
- Continuous assessment
- Rigorously scaffolded instruction
- Mental math
- Deep conceptual understanding

Confidence-building

JUMP Math recognizes that math anxiety is a significant barrier to learning for many students. The JUMP approach has been shown to reduce math anxiety by building on success in small steps. Raising the bar incrementally—a key component of the JUMP Math materials—encourages engagement and confidence. The research in cognition that shows the brain is plastic also shows that the brain can’t register the effects of training if it is not attentive. However, a child’s brain can’t be truly attentive unless the child is confident and excited and believes that there is a point in being engaged in the work. When students who are struggling become convinced that they cannot keep up with the rest of the class, their brains begin to work less efficiently, as they are never attentive enough to consolidate new skills or develop new neural pathways. That is why it is so important to give students the skills they need to take part in lessons and to give them opportunities to show off by answering questions in front of their classmates. To do this, try to constantly assess what struggling students know.

Guided practice

Research in psychology has shown that our brains are extremely fallible: our working memories are poor, we are easily overwhelmed by too much new information, and we require a good deal of practice to consolidate skills and concepts. Repetition and practice are essential. Even mathematicians need constant practice to consolidate and remember skills and concepts. New research in cognition shows how important it is to practise and build component skills before students can understand the big picture. The workbooks are designed to provide guided practice when used with the lesson plans. The multiple representations of mathematics in JUMP Math combine with guided practice in small steps to promote mastery and understanding of key concepts.
Guided discovery

In “Students Need Challenge, Not Easy Success,” Margaret Clifford makes the case that students benefit when they are given prompt, specific feedback on their work and when they are allowed to take “moderate risks.” JUMP Math provides moderate risks by providing tasks that are within students’ grasp. As students’ confidence grows, their risk tolerance grows as well, and they are ready to take more and more steps by themselves.

The lesson plans in the JUMP Math teacher’s guides show you how to build lessons around the material in the workbooks by creating tasks and questions similar to the ones on the worksheets. As much as possible, when students are ready, allow them to think about and work on these questions and tasks independently rather than teaching them explicitly. When you feel your students have sufficient confidence and the necessary basic skills, let them explore more challenging or open-ended questions.

Students are more likely to become flexible and independent thinkers in math if you guide discovery through well-designed lessons. Hence, in creating discovery-based lessons it is important to balance independent work with practice and explicit hints and instruction. According to Philip E. Ross, research in cognition shows that to become an expert in a game like chess it is not enough to play without guidance or instruction. The kind of training in which chess experts engage, which includes playing small sets of moves over and over, memorizing positions, and studying the techniques of master players, appears to play a greater role in the development of ability than the actual playing of the game.

Continuous assessment

JUMP Math workbooks are designed to allow for continuous formative assessment—the books show teachers how to break material into steps and assess component skills and concepts in every area of the curriculum.

The point of constantly assigning tasks and quizzes is not to rank students or to encourage them to work harder by making them feel inadequate. Quizzes should instead be treated as opportunities for students to show off what they know, to become more engaged in their work by meeting incremental challenges, and to experience the collective excitement that can sweep through a class when students experience success together. Continuous assessment allows the teacher to differentiate instruction with small individual interventions.

Rigorously scaffolded instruction

Consistent with emerging brain research, JUMP Math provides materials and methods that minimize differences among students, allowing teachers to more effectively improve student performance in mathematics. In “Why Minimal Guidance During Instruction Does Not Work,” Paul Kirschner, John Sweller, and Richard Clark argue that evidence from controlled studies almost uniformly supports direct, strong instructional guidance. Even for students with considerable prior knowledge, strong guidance while learning helps take into account the limitations of a student’s working memory: the mind can only retain so much of new information or so many component steps at one time.

Even in discovery-based lessons, in which there is little direct instruction, it is important to introduce new ideas through a series of well-designed tasks and explorations in which each new concept follows from the last; students are more likely to make discoveries if the progression of ideas makes sense to them and does not overwhelm them.
Mental math
Mental math is the foundation for all further study in mathematics. Students who cannot see number patterns often become frustrated and disillusioned with their work. Consistent practice in mental math allows students to become familiar with the way numbers interact, enabling them to make simple calculations quickly and effectively without always having to recall their number facts.

To solve problems, students must be able to see patterns in numbers and make estimates and predictions about numbers. It is a serious mistake to think that students who don’t know number facts can get by in mathematics using a calculator or other aids. Students can certainly perform operations and produce numbers on a calculator, but unless they have number sense, they will not be able to tell if their answers are correct, nor be able to develop a talent for solving mathematical problems.

Deep conceptual understanding
JUMP Math scaffolds mathematical concepts rigorously and completely. JUMP Math materials were designed by a team of mathematicians and educators who have a deep understanding of, and a love for, mathematics. JUMP Math teaches symbolic and concrete understanding simultaneously, using a variety of approaches. JUMP Math materials offer multiple symbolic and concrete representations for all key mathematical concepts, and provide guided practice for mastery, allowing students to master and understand each representation completely before moving on.

JUMP Math shows teachers how to see the big ideas of mathematics in even the smallest steps, how to make sense of the individual steps in a mathematical procedure or problem, and how to relate them to the wider concept. JUMP Math teaches fundamental rules, algorithms, and procedures of mathematics for mastery, but students are enabled to discover those procedures themselves (as well as being encouraged to develop their own approaches) and are guided to understand the concepts underlying the procedures fully.

6. Building Confidence with the Introductory Unit
In the twenty years that I have been teaching mathematics to children, I have never met an educator who would say that students who lack confidence in their intellectual or academic abilities are likely to do well in school. Our introductory unit has been carefully designed and tested with thousands of students to boost confidence. It has proven to be an extremely effective tool for convincing even the most challenged student that they can do well in mathematics.

—John Mighton, in conversation

In recent years, research has shown that students are more likely to do well in subjects when they believe they are capable of doing well. It seems obvious, then, that any math program that aims to harness the potential of every student would start with an exercise that builds the confidence of every student. Getting Ready for JUMP Math: Introductory Unit Using Fractions, which can be downloaded from www.jumpmath.org, was designed for just this purpose. The Introductory Unit does not teach fractions in depth: you will find a more comprehensive approach to teaching fractions in the relevant JUMP Math workbooks and teacher’s guides. We recommend that teachers only use the unit for several weeks, preferably at the beginning of the school year.

The individual steps that teachers will follow in teaching the unit are extremely small, so even students who struggle most needn’t be left behind. Throughout the unit, students are expected to

• discover or extend patterns or rules on their own,
• see what changes and what stays the same in sequences of mathematical expressions, and
• apply what they have learned to new situations.
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Students become very excited at making these discoveries and meeting these challenges as they learn the material. For many, it is the first time they have ever been motivated to pay attention to mathematical rules and patterns or to try to extend their knowledge in new cases.

The Introductory Unit Using Fractions, which consists of students worksheets and a short teacher’s guide, has been specifically designed to build confidence by:

- Requiring that students possess only a few simple skills — These skills can be taught to even the most challenged students in a very short amount of time. To achieve a perfect score on the final test in the unit, students need only possess three skills: they must be able to skip count on their fingers, add one-digit numbers, and subtract one-digit numbers.

- Eliminating heavy use of language — Mathematics functions as its own symbolic language. Since the vast majority of children are able to perform the most basic operations (counting and grouping objects into sets) long before they become expert readers, mathematics is the lone subject in which the vast majority of children are naturally equipped to excel at an early age. By removing language as a barrier, students can realize their full potential in mathematics.

- Allowing you to continually provide feedback — In the Introductory Unit, the mathematics are broken down into small steps so that you can quickly identify difficulties and help as soon as they arise.

- Keeping students engaged through the excitement of small victories — Children respond more quickly to praise and success than to criticism and threats. If students are encouraged, they feel an incentive to learn. Students enjoy exercising their minds and showing off to a caring adult.

Since the Introductory Unit is about building confidence, work with your students to ensure that they are successful. Celebrate every correct answer. Take your time. Encourage your students. Point out that fractions are considered to be one of the most difficult topics in mathematics. Have fun!

7. Using JUMP Math in the Classroom

JUMP Math supports a balanced approach to teaching mathematics. In the teacher’s guides you will find lesson plans which include everything from group work to explorations. Below are some recommendations for using JUMP Math.

Teach at regular intervals.
Build a lesson around the material on a particular worksheet by creating questions or exercises that are similar to the ones on the worksheet. Discuss one or two skills or concepts at a time with the whole class, allowing students to develop ideas by themselves, but giving hints and guidance where necessary. Ask questions in several different ways and allow students time to think before you solicit an answer, so that every student can put their hand up and so that students can discover the ideas for themselves. After presenting a particular concept, do not go on until all of the students are assessed and show a readiness to move ahead.

Give mini-quizzes.
Each time you cover a concept or skill, assign a mini-quiz consisting of several questions or a straightforward task to see exactly what students have understood or misunderstood. Write questions or instructions on the board and let students work independently in a separate notebook (or with concrete materials when indicated by the teacher’s guide). Depending upon the topic you are working on, assign questions from the workbooks only after going through several cycles of explanations (or explorations) followed by mini-quizzes. Check the work of students who might
need extra help first, then take up the answers to the quiz at the board with the entire class. If any of your students finish a quiz early, assign extra questions.

Assess continuously.
The secret to bringing an entire class along at the same pace is to use “continuous assessment.” When students are not able to keep up in a lesson it is usually because they are lacking one or two basic skills that are needed for that lesson, or because they are being held back by a simple misconception that is not difficult to correct. Make an effort to spot mistakes or misunderstandings right away. If you wait too long to correct an error, mistakes pile upon mistakes so that it becomes impossible to know exactly where a student is going wrong. To spot mistakes, it helps to break material into small steps or separate concepts, so the worksheets are an ideal tool for assessing mistakes and misunderstandings.

Prepare bonus questions.
Be ready to write bonus questions on the board from time to time during the lesson for students who finish their quizzes or tasks early. Bonus questions and extensions are included in most of the lesson plans. While students who finish quickly are occupied with these questions, circulate around the class doing spot checks on the work of students who are struggling. The bonus questions you create should generally be simple extensions of the material (see How to Create Bonus Questions below).

If a student doesn’t understand your explanation, assume there is something lacking in your explanation, not in your student.
Rephrase or reword explanations if a student doesn’t understand. Sometimes lessons go too fast for a student or steps are inadvertently skipped. Taking time to reflect on what worked and didn’t work in a lesson can help you reach even the most challenged students. When students are struggling always ask, “How could I have improved the lesson?”

In mathematics, it is always possible to make a step easier.
The exercises in the JUMP Math workbooks break concepts and skills into small steps and in a coherent order that students will find easy to master. The lesson plans in the teacher’s guides provide many examples of extra questions that can be used to fill in a missing step in the development of an idea if a problem occurs.

Introduce one piece of information at a time.
Teachers often inadvertently introduce too many new pieces of information at the same time. In trying to comprehend the final item, students can lose all memory and understanding of the material that came before, even though they may have appeared to understand this material completely as it was being explained.

According to Herb Simon, who won the Nobel Prize for his work on the brain, research in cognition shows that, “… a learner who is having difficulty with components can easily be overwhelmed by the processing demands of a complex task. Further, to the extent that many components are well mastered, the student wastes much less time repeating these mastered operations to get an opportunity to practice the few components that need additional effort.”

John Mighton once observed an intern from teachers’ college who was trying to teach a boy in a Grade 7 remedial class how to draw mixed fractions. The boy was getting very frustrated as the intern kept asking him to carry out several steps at the same time. Here is how John separated the steps to facilitate understanding and success:
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I asked the boy to simply draw a picture showing the number of whole pies in the fraction 2 1/2. He drew and shaded two whole pies cut into halves. I then asked him to draw the number of whole pies in 3 1/2, 4 1/2 and 5 1/2 pies. He was very excited when he completed the work I had assigned him, and I could see that he was making more of an effort to concentrate. I asked him to draw the whole number of pies in 2 1/4, 2 3/4, 3 1/4, 4 1/4 pies, then in 2 1/3, 2 2/3, 3 1/3 pies, and so on. (I started with quarters rather than thirds because they are easier to draw.) When the boy could draw the whole number of pies in any mixed fraction, I showed him how to draw the fractional part. Within a few minutes he was able to draw any mixed fraction. If I hadn’t broken the skill into two steps (i.e., drawing the number of whole pies then drawing the fractional part) and allowed him to practise each step separately, he might never have learned the concept.

Before you assign workbook pages, verify that all students have the skills they need to complete the work.

Before assigning a question from one of the JUMP workbooks, it is important to verify that all of your students are prepared to answer the question without your help (or with minimal help).

Never allow students to work ahead in the workbook on material you haven’t covered with the class.

Students who finish a worksheet early should be assigned bonus questions similar to the questions on the worksheet or extension questions from the teacher’s guide. Write the bonus questions on the board or have extra worksheets prepared and ask students to answer the questions in their notebooks or on the worksheets. While students are working independently on the bonus questions, you can spend extra time with anyone who needs help.

Raise the bar incrementally.

When a student has mastered a skill or concept, raise the bar slightly by challenging them to answer a question that is only incrementally more difficult or complex than the questions previously assigned.

Praise students’ efforts.

We’ve found the JUMP program works best when teachers give their students a great deal of encouragement. Because the lessons are laid out in steps that any student can master, you’ll find that you won’t be giving false encouragement. One of the reasons that kids love the program so much is that it’s a thrill to be doing well at math!

Teach the number facts.

It is a serious mistake to think that students who don’t know their number facts can always get by in mathematics using a calculator or other aids. Trying to do mathematics without knowing basic number facts is like trying to play the piano without knowing where the notes are. (See the Mental Math section of this guide for strategies to help students learn their number facts.)

Create excitement about math.

Engaging the entire class in lessons is not simply a matter of fairness; it is also a matter of efficiency. While the idea may seem counterintuitive, teachers will enable students who learn more quickly to go further if they take care of the students who struggle. Teachers can create a real sense of excitement about math in the classroom simply by convincing struggling students that they can do well in the subject. The class will cover far more material in the year and students who excel will no longer have to hide their love of math for fear of appearing strange or different.
8. How to Create Bonus Questions

Students love to show off to their teachers by solving difficult-looking puzzles and surmounting challenges, and they also love to succeed in front of their peers. You can make math lessons more exciting (and also make time to check the work of students who need extra time) if you know how to create engaging bonus questions. Bonus questions generally shouldn’t be based on new concepts and they don’t have to be extremely difficult to capture the attention of students. Here are some strategies you can use to create questions that will look hard enough to interest students who work quickly, but that all of your students can aspire to answer.

1. Make the numbers in a problem larger or introduce several new terms or elements without introducing any new skills or concepts.

This is the simplest way to create bonus questions. Kids of all ages love showing off with larger numbers or with more-challenging looking rules and procedures. If your students know how to add a pair of three digit numbers without regrouping, let them impress you by adding a pair of five or six digit numbers. If they can add two fractions let them add three or four. You will be amazed at how excited students become when they can apply their skills with larger numbers or more difficult-looking calculations. You can use this strategy in almost any lesson.

Some things to bear in mind: First, bonus questions shouldn’t look tedious. You don’t want to give students an endless series of calculations that appear to have no purpose. It helps if you are excited when you assign bonus questions and if you assign only a few questions at a time. Students should feel they are involved in a quest, faced with a series of increasingly more difficult challenges that they believe they can meet.

Students will not necessarily gain a deeper conceptual understanding of a particular mathematical idea when they work on bonus questions that involve larger numbers or that have more terms or elements. But they will still make important conceptual gains. In addition to generalizing from smaller to larger numbers, they will, for instance, develop the ability to hold more material in their working memory, to follow a series of steps in a procedure, to stay on task, and to see patterns and apply rules in increasingly complex situations. They will also consolidate their understanding and commit the material memory. Their behaviour, confidence, and level of engagement will also likely improve.

2. Make a mistake and ask your students to correct it.

Students love correcting a teacher’s mistakes—and you can find a way to make mistakes in any lesson! For instance, if you are teaching T-tables, you might draw the following T-table on the board:

<table>
<thead>
<tr>
<th>INPUT</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
</tbody>
</table>

Tell your students you created the table by adding the same number repeatedly to the initial number, but you think you made a mistake. Ask them to find the mistake and explain where you went wrong.
3. Leave out several terms or elements in a sequence and ask your students to say what is missing.

For instance, you might ask students to say what numbers are missing in the following T-tables, assuming the tables were made by adding the same number repeatedly. **NOTE:** The problem on the right is harder than the one on the left. Students might solve the problem on the right by guessing and checking or using a number line.

\[
\begin{array}{|c|c|}
\hline
\text{INPUT} & \text{OUTPUT} \\
1 & 8 \\
2 & \\
3 & 14 \\
4 & 17 \\
\hline
\end{array}
\quad
\begin{array}{|c|c|}
\hline
\text{INPUT} & \text{OUTPUT} \\
1 & 7 \\
2 & \\
3 & 15 \\
\hline
\end{array}
\]

When you create bonus questions, use number facts that your students are likely to know and give clear and concise instructions. Rather than giving lengthy explanations to struggling students, try to assign tasks that students can immediately see how to do. For instance, if the numbers in the charts above are too difficult for your students, use different numbers:

\[
\begin{array}{|c|c|}
\hline
\text{INPUT} & \text{OUTPUT} \\
1 & 2 \\
2 & 4 \\
3 & \\
4 & 8 \\
5 & 10 \\
\hline
\end{array}
\quad
\begin{array}{|c|c|}
\hline
\text{INPUT} & \text{OUTPUT} \\
1 & 5 \\
2 & \\
3 & 20 \\
4 & 25 \\
\hline
\end{array}
\]

Most students know how to count by twos or fives, so every student should see which numbers are missing from these charts. Once struggling students have succeeded with easier tasks, they will be more willing to take risks and to guess and check to solve more difficult problems.

4. Vary the task or problem slightly.

You could use more challenging patterns in the output column, such as decreasing patterns (as shown on the left) or patterns with a gap that changes (as shown on the right).

\[
\begin{array}{|c|c|}
\hline
\text{INPUT} & \text{OUTPUT} \\
1 & 23 \\
2 & 20 \\
3 & 17 \\
4 & 14 \\
\hline
\end{array}
\quad
\begin{array}{|c|c|}
\hline
\text{INPUT} & \text{OUTPUT} \\
1 & 5 \\
2 & 7 \\
3 & 10 \\
4 & 14 \\
\hline
\end{array}
\]

You could also add an extra column to the table, as you might do if you were following a recipe with three quantities that vary with each other. You could ask students to say how much of one ingredient they would need if they had a certain amount of another ingredient.

\[
\begin{array}{|c|c|c|}
\hline
\text{Number of pies} & \text{Number of cups of flour} & \text{Number of cups of cherries} \\
1 & 2 & 3 \\
2 & 4 & 6 \\
3 & 6 & 9 \\
4 & 8 & 12 \\
\hline
\end{array}
\]
5. Look for applications of the concepts.
You might tell students that the numbers in the output column represent a student’s savings and ask them to predict how long it will take to save a certain amount of money. Or you could give them a T-table and ask them to draw a picture or describe a pattern that might be represented by the table.

6. Look for patterns in any work you assign, and ask students to describe the patterns.
For example, if you ask students to find and state the rule for the T-table below, you can ask students who finish early to describe the pattern in the ones and the tens digits of the numbers in the right-hand column.

<table>
<thead>
<tr>
<th>INPUT</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>17</td>
</tr>
<tr>
<td>4</td>
<td>22</td>
</tr>
<tr>
<td>5</td>
<td>27</td>
</tr>
</tbody>
</table>

(Answer: the ones digit repeats every second term (7, 2, 7, 2, 7,...) and the tens digit increases by 1 every second term (0, 1, 1, 2, 2,...). This happens because the numbers increase by 5 each time, so after two steps they have increased by 10.)

This kind of exercise can keep students who finish their work early occupied while you do a spot check on other students’ work.

7. Use extension questions from the teacher’s guides.
As your students become more confident you will want to create questions that challenge them more and that extend the ideas in the lesson. Our lesson plans contain many suggestions for creating extension questions for your students.

9. Features of the JUMP Math Materials

Diversity of representation
What makes sense to one student does not always make sense to another. Students learn in different ways. Multiple representations help to reach a broader number of students. Students who see multiple representations of a concept also develop a deeper understanding of that concept and are better able to explain it. They can then begin to make connections between the various representations. The more ways you as a teacher know how to teach a concept, the better you will understand it yourself and the more able you will be to teach it in ways that meet the varying needs of your students.

Differentiated Instruction
Because the math is broken into steps in the workbooks, the workbooks allow you to teach to the whole class while still addressing the needs of individual students (not leaving anyone behind and challenging those who are ready to move ahead). Part of the philosophy of JUMP Math is to teach to the collective—to ensure that the class meets success as a whole, thereby increasing excitement and momentum. On the JUMP Math worksheets, concepts and skills are introduced one step at a
time, with lots of opportunities for practice. Struggling students can complete all of the questions on a worksheet while students who excel can skip some questions and do extra work provided in the teacher’s guides.

Text and layout
In the workbooks, we tried to reduce the number of words per page and to use clear, simple language. This ensures that all students have equal access to the materials, regardless of their reading level. (However, in the teacher’s guides you will find many exercises that combine mathematics with language learning and communication.) Layout is simple and uncluttered to avoid distraction, which is particularly helpful for children with learning disabilities. Visual elements such as boxes, figures, background shading, and bold text emphasize changes in content to help students learn new steps or new ideas.

Review in the workbooks
At the beginning of the school year, teachers often find that many of their students have forgotten material taught in the previous year. That’s why the workbooks for each grade level (after Grade 1) provide review that goes back one or two years in the curriculum. Research in education has shown that if teachers assess their students and start at the right level, they cover more curriculum in the year.

10. Hints for Helping Students Who Have Fallen Behind

Teach the number facts.
Struggling students often have a weak grasp of number facts. They have trouble solving problems or doing calculations because their working memories are overwhelmed trying to recall number facts. Research shows that automatic recall of number facts helps enormously in the learning of math.

Give cumulative reviews.
Even mathematicians constantly forget new material, including material they once understood completely. Giving reviews doesn’t have to create a lot of extra work for a teacher. John Mighton recommends that, once a month, you copy a selection of questions from the workbook for units already covered onto a single sheet and make copies for the class. Students rarely complain about doing questions they already did a month or more ago (and quite often they won’t even remember that they did those particular questions).

Make mathematical terms part of your spelling lessons.
In some areas of math (e.g., geometry) the greatest difficulty that students face is in learning the terminology. If you include mathematical terms in your spelling lessons, students will find it easier to remember the terms and to communicate about their work.

Set aside five minutes every few days to give extra review to struggling students.
Spending five minutes with a small group of students who need extra help can make the teaching component of lessons more productive. Ensuring that struggling students have the prerequisite skills and knowledge to participate fully in the lesson will enhance their learning and contribute to a positive learning environment for all your students.
Allow wait time.
Wait time is the time between asking the question and soliciting a response. Wait time gives students a chance to think about their answer and leads to longer and clearer explanations. It is particularly helpful for more timid students, students who are slower to process information, and students who are learning English as a second language. Studies about the benefits of increasing wait time to three seconds or longer confirm that there are increases in student participation, better quality of responses, better overall classroom performance, more questions asked by students, and more frequent and unsolicited contributions.

11. Other Features of the JUMP Math Program

1. Professional Learning

Teachers
Training sessions offered across Canada give teachers opportunities to:
• Learn about JUMP Math’s philosophy and guiding principles
• Consider their own comfort levels with mathematics and math instruction
• Watch a video of a lesson and discuss specific instructional techniques
• Learn how to use all of the JUMP Math resources
• Take home practical ideas on how to support all students’ learning in the classroom

Community Volunteers
A new JUMP Math Essentials tutor program (Grades 3–8) is now available. There are three resource packages: Grades 3 & 4, Grades 5 & 6, and Grades 7 & 8. Each package includes a detailed tutor’s guide and student worksheets. Detailed information is available at www.jumpmath.org. Training sessions for tutors include instruction on how to:
• Plan for 25 weeks of lessons
• Assess student requirements
• Engage students and get them excited about math
• Use the support materials

2. National Book Fund
JUMP Math is committed to supporting vulnerable communities in schools. We provide free resources and supports through the National Book Fund.
References


Sample Problem Solving Lesson

As much as possible, allow your students to extend and discover ideas on their own (without pushing them so far that they become discouraged). It is not hard to develop problem solving lessons (where your students can make discoveries in steps) using the material on the worksheets. The following lesson plan shows how you can create a rigorously scaffolded lesson that allows students to work independently and discover mathematical concepts on their own. The material is taken from several pages in the grade 6 workbooks.

1. Warm-up
   Review the notion of perimeter from the worksheets. Draw the following diagram on a grid on the board and ask your students how they would determine the perimeter. Tell students that each edge on the shape represents 1 unit (each edge might, for instance, represent a centimetre).
   
   Allow your students to demonstrate their method (EXAMPLE: counting the line segments, or adding the lengths of each side).

   Develop the Idea
   Draw some additional shapes and ask your students to copy them onto grid paper and to determine the perimeter of each.
   
   Check Bonus Try Again?
   The perimeters of the shapes above are 10 cm, 10 cm and 12 cm respectively.
   Have your students make a picture of a letter from their name on graph paper by colouring in squares. Then ask them to find the perimeter and record their answer in words. Ensure students only use vertical and horizontal edges.
   Students may need to use some kind of system to keep them from missing sides. Suggest that your students write the length of the sides on the shape.

   3. Go Further
   Draw a simple rectangle on the board and ask students to again find the perimeter.
   Add a square to the shape and ask students how the perimeter changes.
   Draw the following polygons on the board and ask students to copy the four polygons on their grid paper.

   Sometimes the big idea will only emerge at the end of the lesson, after you have developed the idea out of smaller skills and concepts. Always teach with the big idea in mind, but avoid overwhelming students with too much information.
4. Another Step

Draw the following shape on the board and ask your students, “How can you add a square to the following shape so the perimeter decreases?”

<table>
<thead>
<tr>
<th>Check</th>
<th>Bonus</th>
<th>Try Again?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Have your students demonstrate where they added the squares and how they found the perimeter.</td>
<td>Ask your students to calculate the greatest amount the perimeter can increase by when you add a single square.</td>
<td>Ask students to draw a single square on their grid paper and find the perimeter (4 cm). Then have them add a square and find the perimeter of the resulting rectangle. Have them repeat this exercise a few times and then follow the same procedure with the original (or bonus) questions.</td>
</tr>
<tr>
<td>Ask your students to discuss why they think the perimeter remains constant when the square is added in the corner (as in the fourth polygon above).</td>
<td>Ask them to add 2 (or 3) squares to the shape below and examine how the perimeter changes.</td>
<td>• Ask them to predict whether they could create a shape with odd perimeter by adding squares.</td>
</tr>
<tr>
<td>• Ask them to create a T-table where the two columns are labelled “Number of Squares” in the polygon and “Perimeter” of the polygon. Ask them to predict whether they could create a shape with odd perimeter by adding squares.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

When teaching a skill or concept to the whole class, give lots of hints and guidance. Ask each question in several different ways, and allow students time to think before soliciting an answer, so that every student can put their hand up and so that students can discover ideas for themselves.

TIP 2

Encourage students to communicate their understanding.

Guide students in small steps to discover ideas for themselves.

Repetition that is not tedious, subtle variations keep the task interesting.

Bonus appears harder but requires no new explanation; allows teacher to attend to struggling students.

Scaffold when necessary.
5. Develop the Idea

Hold up a photograph that you’ve selected and ask your students how you would go about selecting a frame for it. What kinds of measurements would you need to know about the photograph in order to get the right sized frame? You might also want to show your students a CD case and ask them how they would measure the paper to create an insert for a CD/CD-ROM.

Show your students how the perimeter of a rectangle can be solved with an addition statement (Example: Perimeter = 14 cm is the sum of 3 + 3 + 4 + 4). Explain that the rectangle is made up of two pairs of equal lines and that, because of this, we only need two numbers to find the perimeter of a given rectangle.

3 cm

? Perimeter = ______ cm

4 cm

Show your students that there are two ways to find this:

a) Create an addition statement by writing each number twice: 3 cm + 3 cm + 4 cm + 4 cm = 14 cm

b) Add the numbers together and multiply the sum by 2: 3 cm + 4 cm = 7 cm; 7 cm × 2 = 14 cm

Ask your students to find the perimeters of the following rectangles (not drawn to scale).

1 cm

1 cm

3 cm

3 cm

5 cm

4 cm

7 cm

6. Go Further

Demonstrate on grid paper that two different rectangles can both have a perimeter of 10 cm.

Check

Try Again?

Take up the questions (the perimeters of the rectangles above, from left to right, are 8 cm, 16 cm and 22 cm).

Continue creating questions in this format for your students and gradually increase the size of the numbers.

Have students draw a copy of the rectangle in a notebook and copy the measurements onto all four sides. Have them create an addition statement by copying one number at a time and then crossing out the measurement:

Using larger numbers makes the problem appear harder and builds excitement

In mathematics, it is always possible to make a step easier

TIP 3

Make connections explicit (Example: between math and the real world, between strands or between math and other subjects)

Assign small sets of questions or problems that test students’ understanding of each step before the next one is introduced

TIP 4

Use Extensions from the lesson plans for extra challenges. Avoid singling out students who work on extensions as geniuses and allow all of your students to try these questions (with hints if necessary).
TIP 5

Only assign questions from the workbook after going through several cycles of explanations followed by mini-quizzes. Do not allow any student to work ahead of others in the workbooks.

Ask your students to draw all the rectangles they can with a perimeter of 12 cm.

<table>
<thead>
<tr>
<th>Check</th>
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<th>Try Again?</th>
</tr>
</thead>
<tbody>
<tr>
<td>After your students have finished, ask them whether they were able to find one rectangle, then two rectangles, then three rectangles.</td>
<td>Ask students to find (and draw) all the rectangles with a perimeter of 18 cm. After they have completed this, they can repeat the same exercise for rectangles of 24 cm or 36 cm.</td>
<td>If students find only one (or zero) rectangles, they should be shown a systematic method of finding the answer and then given the chance to practise the original question.</td>
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<td>On grid paper, have students draw a pair of lines with lengths of 1 and 2 cm each.</td>
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<td></td>
<td>1 cm</td>
<td>2 cm</td>
</tr>
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<td>Ask them to draw the other three sides of each rectangle so that the final perimeter will be 12 cm for each rectangle, guessing and checking the lengths of the other sides. Let them try this method on one of the bonus questions once they accomplish this.</td>
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<td></td>
</tr>
</tbody>
</table>

7. Raise the Bar

Draw the following rectangle and measurements on paper:

```
   ?
1 cm ? Perimeter = 6 cm
   ?
```

Ask students how they would calculate the length of the missing sides. After they have given some input, explain to them how the side opposite the one measured will always have the same measurement. Demonstrate how the given length can be subtracted twice (or multiplied by two and then subtracted) from the perimeter. The remainder, divided by two, will be the length of each of the two remaining sides.

Draw a second rectangle and ask students to find the lengths of the missing sides using the methods just discussed.

```
   ?
2 cm ? Perimeter = 14 cm
   ?
```

Ask students to present their answers in a way that allows you to spot mistakes easily. You might say: “I can’t always keep up with you, so I need you to show me your steps occasionally” or “Because you’re so clever, you may want to help a friend with math one day, so you’ll need to know how to explain the steps.”

TIP 6

Introduction

Only assign questions from the workbook after going through several cycles of explanations followed by mini-quizzes. Do not allow any student to work ahead of others in the workbooks.

Ask your students to draw all the rectangles they can with a perimeter of 12 cm.

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<td></td>
<td>1 cm</td>
<td>2 cm</td>
</tr>
<tr>
<td>Ask them to draw the other three sides of each rectangle so that the final perimeter will be 12 cm for each rectangle, guessing and checking the lengths of the other sides. Let them try this method on one of the bonus questions once they accomplish this.</td>
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</tbody>
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7. Raise the Bar

Draw the following rectangle and measurements on paper:

```
   ?
1 cm ? Perimeter = 6 cm
   ?
```

Ask students how they would calculate the length of the missing sides. After they have given some input, explain to them how the side opposite the one measured will always have the same measurement. Demonstrate how the given length can be subtracted twice (or multiplied by two and then subtracted) from the perimeter. The remainder, divided by two, will be the length of each of the two remaining sides.

Draw a second rectangle and ask students to find the lengths of the missing sides using the methods just discussed.

```
   ?
2 cm ? Perimeter = 14 cm
   ?
```

Ask students to present their answers in a way that allows you to spot mistakes easily. You might say: “I can’t always keep up with you, so I need you to show me your steps occasionally” or “Because you’re so clever, you may want to help a friend with math one day, so you’ll need to know how to explain the steps.”
8. Assessment

Draw the following diagrams of rectangles and perimeter statements, and ask students to complete the missing measurements on each rectangle.

<table>
<thead>
<tr>
<th>Check</th>
<th>Bonus</th>
<th>Try Again?</th>
</tr>
</thead>
</table>
| Check that students can calculate the length of the sides (2 cm, 2 cm, 5 cm and 5 cm). | Give students more problems like above. For example:
- Side = 5 cm; Perimeter = 20 cm
- Side = 10 cm; Perimeter = 50 cm
- Side = 20 cm; Perimeter = 100 cm
- Side = 65 cm; Perimeter = 250 cm
  Be sure to raise the numbers incrementally on bonus questions. | Give students a simple problem to try (similar to the first demonstration question).
- Side = 1 cm; Perimeter = 8 cm
  Provide them with eight toothpicks (or a similar object) and have them create the rectangle and then measure the length of each side. Have them repeat this with more questions. |

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
<th>c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 cm</td>
<td>6 cm</td>
<td>?</td>
</tr>
<tr>
<td>2 cm</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

Perimeter = 12 cm
Perimeter = 18 cm
Perimeter = 18 cm

Answers for the above questions (going clockwise from the sides given):
- a) 2 cm, 4 cm
- b) 3 cm, 6 cm, 3 cm
- c) 5 cm, 4 cm, 5 cm

Draw a square and inform your students that the perimeter is 20 cm. What is the length of each side? (Answer: 5 cm.) Repeat with other multiples of four for the perimeter.

If a student is failing to perform an operation correctly, isolate the problematic step. Give them a number of questions that have been worked out to the point where they have trouble and have them practise doing just that one step until they master it.

A full size copy of this lesson can be downloaded from our website.
Sample Problem Solving Lesson

As much as possible, allow your students to extend and discover ideas on their own (without pushing them so far that they become discouraged). It is not hard to develop problem solving lessons (where your students can make discoveries in steps) using the material on the worksheets. Here is a sample problem solving lesson you can try with your students.

1. Warm-up
   Review the notion of perimeter from the worksheets. Draw the following diagram on a grid on the board and ask your students how they would determine the perimeter. Tell students that each edge on the shape represents 1 unit (each edge might, for instance, represent a centimeter).

   ![Diagram of a shape on a grid]

   Allow your students to demonstrate their method (e.g., counting the line segments, or adding the lengths of each side).

2. Develop the Idea
   Draw some additional shapes and ask your students to copy them onto grid paper and to determine the perimeter of each.

   ![Additional shapes on grid]

<table>
<thead>
<tr>
<th>Check</th>
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<th>Try Again?</th>
</tr>
</thead>
<tbody>
<tr>
<td>The perimeters of the shapes above are 10 cm, 10 cm and 12 cm respectively.</td>
<td>Have your students make a picture of a letter from their name on graph paper by colouring in squares. Then ask them to find the perimeter and record their answer in words. Ensure students only use vertical and horizontal edges.</td>
<td>Students may need to use some kind of system to keep them from missing sides. Suggest that your students write the length of the sides on the shape.</td>
</tr>
</tbody>
</table>

3. Go Further
   Draw a simple rectangle on the board and ask students to again find the perimeter.

   ![Rectangle on grid]

   Add a square to the shape and ask students how the perimeter changes.

   ![Rectangle with square added]

   Draw the following polygons on the board and ask students to copy the four polygons on their grid paper.

   ![Four polygons on grid]
Sample Problem Solving Lesson  
(continued)

Ask your students how they would calculate the perimeter of the first polygon. Then instruct them to add an additional square to each polygon and calculate the perimeter again.

<table>
<thead>
<tr>
<th>Check</th>
<th>Bonus</th>
<th>Try Again?</th>
</tr>
</thead>
</table>
| Have your students demonstrate where they added the squares and how they found the perimeter. | • Ask your students to calculate the greatest amount the perimeter can increase by when you add a single square.  
• Ask them to add 2 (or 3) squares to the shape below and examine how the perimeter changes.  
• Ask them to create a T-table where the two columns are labelled “Number of Squares” in the polygon and “Perimeter” of the polygon (see the Patterns section for an introduction to T-tables). Have them add more squares and record how the perimeter continues to change. | Ask students to draw a single square on their grid paper and find the perimeter (4 cm). Then have them add a square and find the perimeter of the resulting rectangle. Have them repeat this exercise a few times and then follow the same procedure with the original (or bonus) questions. |

| ![Image of polygon with squares added] |
| ![Image of polygon with squares added] |

Ask your students to discuss why they think the perimeter remains constant when the square is added in the corner (as in the fourth polygon above).

4. Another Step

Draw the following shape on the board and ask your students, “How can you add a square to the following shape so the perimeter decreases?”

![Image of polygon with square added]  

<table>
<thead>
<tr>
<th>Check</th>
<th>Bonus</th>
<th>Try Again?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discuss with your students why perimeter decreases when the square is added in the middle of the second row. You may want to ask them what kinds of shapes have larger perimeters and which have smaller perimeters.</td>
<td>Ask your students to add two squares to the polygons below and see if they can reduce the perimeter.</td>
<td>Have your students try the exercise above again with six square-shaped pattern blocks. Have them create the polygon as drawn above and find where they need to place the sixth square by guessing and checking (placing the square and finding the perimeter of the resulting polygon).</td>
</tr>
</tbody>
</table>

| ![Image of polygon with square added] |
| ![Image of polygon with square added] |
5. Develop the Idea

Hold up a photograph that you’ve selected and ask your students how you would go about selecting a frame for it. What kinds of measurements would you need to know about the photograph in order to get the right sized frame? You might also want to show your students a CD case and ask them how they would measure the paper to create an insert for a CD/CD-ROM.

Show your students how the perimeter of a rectangle can be solved with an addition statement (e.g., Perimeter = 14 cm is the sum of 3 + 3 + 4 + 4). Explain that the rectangle is made up of two pairs of equal lines and that, because of this, we only need two numbers to find the perimeter of a given rectangle.

\[ \text{Perimeter} = 3 \text{ cm} + 4 \text{ cm} \]

Show your students that there are two ways to find this:

a) Create an addition statement by writing each number twice: \( 3 \text{ cm} + 3 \text{ cm} + 4 \text{ cm} + 4 \text{ cm} = 14 \text{ cm} \)

b) Add the numbers together and multiply the sum by 2: \( 3 \text{ cm} + 4 \text{ cm} = 7 \text{ cm}; 7 \text{ cm} \times 2 = 14 \text{ cm} \)

Ask your students to find the perimeters of the following rectangles (not drawn to scale).

<table>
<thead>
<tr>
<th>Check</th>
<th>Bonus</th>
<th>Try Again?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Take up the questions (the perimeters of the rectangles above, from left to right, are 8 cm, 16 cm and 22 cm).</td>
<td>Continue creating questions in this format for your students and gradually increase the size of the numbers.</td>
<td>Have students draw a copy of the rectangle in a notebook and copy the measurements onto all four sides. Have them create an addition statement by copying one number at a time and then crossing out the measurement:</td>
</tr>
</tbody>
</table>

6. Go Further

Demonstrate on grid paper that two different rectangles can both have a perimeter of 10 cm.
Ask your students to draw all the rectangles they can with a perimeter of 12 cm.

<table>
<thead>
<tr>
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<th>Try Again?</th>
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<tbody>
<tr>
<td>After your students have finished, ask them whether they were able to find one rectangle, then two rectangles, then three rectangles.</td>
<td>Ask students to find (and draw) all the rectangles with a perimeter of 18 cm. After they have completed this, they can repeat the same exercise for rectangles of 24 cm or 36 cm.</td>
<td>If students find only one (or zero) rectangles, they should be shown a systematic method of finding the answer and then given the chance to practise the original question. On grid paper, have students draw a pair of lines with lengths of 1 and 2 cm each.</td>
</tr>
</tbody>
</table>

7. **Raise the Bar**

   Draw the following rectangle and measurements on paper:

   ![](image)

   Perimeter = 6 cm

   Ask students how they would calculate the length of the missing sides. After they have given some input, explain to them how the side opposite the one measured will always have the same measurement. Demonstrate how the given length can be subtracted twice (or multiplied by two and then subtracted) from the perimeter. The remainder, divided by two, will be the length of each of the two remaining sides.

   Draw a second rectangle and ask students to find the lengths of the missing sides using the methods just discussed.

   ![](image)

   Perimeter = 14 cm
**8. Assessment**

Draw the following diagrams of rectangles and perimeter statements, and ask students to complete the missing measurements on each rectangle.

<table>
<thead>
<tr>
<th></th>
<th>a)</th>
<th>b)</th>
<th>c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 cm</td>
<td>4 cm</td>
<td>6 cm</td>
<td></td>
</tr>
<tr>
<td></td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Perimeter = 12 cm</td>
<td>Perimeter = 18 cm</td>
<td>Perimeter = 18 cm</td>
<td></td>
</tr>
</tbody>
</table>

**Check**

Answers for the above questions (going clockwise from the sides given):

a) 2 cm, 4 cm
b) 3 cm, 6 cm, 3 cm
c) 5 cm, 4 cm, 5 cm

**Bonus**

Draw a square and inform your students that the perimeter is 20 cm. What is the length of each side? (Answer: 5 cm.) Repeat with other multiples of four for the perimeter.
Mental Math
Addition and Subtraction

Teacher
If any of your students don’t know their addition and subtraction facts, teach them to add and subtract using their fingers by the methods taught below. You should also reinforce basic facts using drills, games and flash cards. There are mental math strategies that make addition and subtraction easier: some effective strategies are taught in the next section. (Until your students know all their facts, allow them to add and subtract on their fingers when necessary.)

To ADD 4 + 8, Grace says the greater number (8) with her fist closed. She counts up from 8, raising one finger at a time. She stops when she has raised the number of fingers equal to the lesser number (4):

She said “12” when she raised her 4th finger, so: 4 + 8 = 12

1. Add:
   a) 5 + 2 =       b) 3 + 2 =       c) 6 + 2 =       d) 9 + 2 =       
   e) 2 + 4 =       f) 2 + 7 =       g) 5 + 3 =       h) 6 + 3 =       
   i) 11 + 4 =      j) 3 + 9 =      k) 7 + 3 =      l) 14 + 4 =      
   m) 21 + 5 =      n) 32 + 3 =      o) 4 + 56 =      p) 39 + 4 =      

To SUBTRACT 9 – 5, Grace says the lesser number (5) with her fist closed. She counts up from 5 raising one finger at a time. She stops when she says the greater number (9):

She has raised 4 fingers when she stopped, so: 9 – 5 = 4

2. Subtract:
   a) 7 – 5 =       b) 8 – 6 =       c) 5 – 3 =       d) 5 – 2 =       
   e) 9 – 6 =       f) 10 – 5 =      g) 11 – 7 =      h) 17 – 14 =      
   i) 33 – 31 =     j) 27 – 24 =    k) 43 – 39 =    l) 62 – 58 =      

Teacher
To prepare for the next section, teach your students to add 1 to any number mentally (by counting forward by 1 in their head) and to subtract 1 from any number (by counting backward by 1).
Teacher

Students who don’t know how to add, subtract or estimate readily are at a great disadvantage in mathematics. Students who have trouble memorizing addition and subtraction facts can still learn to mentally add and subtract numbers in a short time if they are given daily practice in a few basic skills.

SKILL 1: Adding 2 to an Even Number

This skill has been broken down into a number of sub-skills. After teaching each sub-skill, you should give your students a short diagnostic quiz to verify that they have learned the skill. I have included sample quizzes for Skills 1 to 4.

i) Naming the next one-digit even number:

Numbers that have ones digit 0, 2, 4, 6 or 8 are called the even numbers. Using drills or games, teach your students to say the sequence of one-digit even numbers without hesitation. Ask students to imagine the sequence going on in a circle so that the next number after 8 is 0 (0, 2, 4, 6, 8, 0, 2, 4, 6, 8… ) Then play the following game: name a number in the sequence and ask your students to give the next number. Don’t move on until all of your students have mastered the game.

ii) Naming the next greatest two-digit even number:

CASE 1: Numbers that end in 0, 2, 4 or 6

Write an even two-digit number that ends in 0, 2, 4 or 6 on the board. Ask your students to name the next greatest even number. Students should recognize that if a number ends in 0, then the next even number ends in 2; if it ends in 4 then the next even number ends in 6, etc. For instance, the number 54 has ones digit 4: so the next greatest even number will have ones digit 6.

CASE 2: Numbers that end in 8

Write the number 58 on the board. Ask students to name the next greatest even number. Remind your students that even numbers must end in 0, 2, 4, 6, or 8. But 50, 52, 54 and 56 are all less than 58 so the next greatest even number is 60. Your students should see that an even number ending in 8 is always followed by an even number ending in 0 (with a tens digit that is one higher).

iii) Adding 2 to an even number:

Point out to your students that adding 2 to any even number is equivalent to finding the next even number: EXAMPLE: $46 + 2 = 48$, $48 + 2 = 50$, etc. Knowing this, your students can easily add 2 to any even number.
SKILL 2: Subtracting 2 from an Even Number

i) Finding the preceding one-digit even number:
Name a one-digit even number and ask your students to give the preceding number in the sequence. For instance, the number that comes before 4 is 2 and the number that comes before 0 is 8. (REMEMBER: the sequence is circular.)

ii) Finding the preceding two-digit number:

CASE 1: Numbers that end in 2, 4, 6 or 8
Write a two-digit number that ends in 2, 4, 6 or 8 on the board. Ask students to name the preceding even number. Students should recognize that if a number ends in 2, then the preceding even number ends in 0; if it ends in 4 then the preceding even number ends in 2, etc. For instance, the number 78 has ones digit 8 so the preceding even number has ones digit 6.

CASE 2: Numbers that end in 0
Write the number 80 on the board and ask your students to name the preceding even number. Students should recognize that if an even number ends in 0 then the preceding even number ends in 8 (but the ones digit is one less). So the even number that comes before 80 is 78.

ii) Subtracting 2 from an even number:
Point out to your students that subtracting 2 from an even number is equivalent to finding the preceding even number: EXAMPLE: 48 – 2 = 46, 46 – 2 = 44, etc.

QUIZ
Add:
a) 26 + 2 = _____  b) 82 + 2 = _____  c) 40 + 2 = _____  d) 58 + 2 = _____  e) 34 + 2 = _____

QUIZ
Name the preceding even number:
a) 48 : _______  b) 26 : _______  c) 34 : _______  d) 62 : _______  e) 78 : _______

QUIZ
Name the preceding even number:
a) 40 : _______  b) 60 : _______  c) 80 : _______  d) 50 : _______  e) 30 : _______

QUIZ
Subtract:
a) 58 – 2 = _____  b) 24 – 2 = _____  c) 36 – 2 = _____  d) 42 – 2 = _____  e) 60 – 2 = _____
SKILL 3: Adding 2 to an Odd Number

i) Naming the next one-digit odd number:

Numbers that have ones digit 1, 3, 5, 7, and 9 are called the odd numbers. Using drills or games, teach your students to say the sequence of one-digit odd numbers without hesitation. Ask students to imagine the sequence going on in a circle so that the next number after 9 is 1 (1, 3, 5, 7, 9, 1, 3, 5, 7, 9…). Then play the following game: name a number in the sequence and ask you students to give the next number. Don’t move on until all of your students have mastered the game.

ii) Naming the next greatest two-digit odd number:

CASE 1: Numbers that end in 1, 3, 5 or 7

Write an odd two-digit number that ends in 1, 3, 5, or 7 on the board. Ask you students to name the next greatest odd number. Students should recognize that if a number ends in 1, then the next even number ends in 3; if it ends in 3 then the next even number ends in 5, etc. For instance, the number 35 has ones digit 5: so the next greatest even number will have ones digit 7.

CASE 2: Numbers that end in 9

Write the number 59 on the board. Ask students to name the next greatest number. Remind your students that odd numbers must end in 1, 3, 5, 7, or 9. But 51, 53, 55, and 57 are all less than 59. The next greatest odd number is 61. Your students should see that an odd number ending in 9 is always followed by an odd number ending in 1 (with a tens digit that is one higher).

 iii) Adding 2 to an odd number:

Point out to your students that adding 2 to any odd number is equivalent to finding the next odd number: EXAMPLE: 47 + 2 = 49, 49 + 2 = 51, etc. Knowing this, your students can easily add 2 to any odd number.

Add:

a) 27 + 2 = _____ b) 83 + 2 = _____ c) 41 + 2 = _____ d) 59 + 2 = _____ e) 35 + 2 = _____
SKILL 4: Subtracting 2 from an Odd Number

i) Finding the preceding one-digit odd number:
Name a one-digit even number and ask your students to give the preceding number in the sequence. For instance, the number that comes before 3 is 1 and the number that comes before 1 is 9. (REMEMBER: the sequence is circular.)

ii) Finding the preceding odd two-digit number:
CASE 1: Numbers that end in 3, 5, 7 or 9
Write a two-digit number that ends in 3, 5, 7 or 9 on the board. Ask students to name the preceding even number. Students should recognize that if a number ends in 3, then the preceding odd number ends in 1; if it ends in 5 then the preceding odd number ends in 3, etc. For instance, the number 79 has ones digit 9, so the preceding even number has ones digit 7.

CASE 2: Numbers that end in 1
Write the number 81 on the board and ask your students to name the preceding odd number. Students should recognize that if an odd number ends in 1 then the preceding odd number ends in 9 (but the ones digit is one less). So the odd number that comes before 81 is 79.

iii) Subtracting 2 from an odd number:
Point out to your students that subtracting 2 from an odd number is equivalent to finding the preceding even number: EXAMPLE: 49 – 2 = 47, 47 – 2 = 45, etc.

SKILLS 5 and 6
Once your students can add and subtract the numbers 1 and 2, then they can easily add and subtract the number 3: Add 3 to a number by first adding 2, then 1 (EXAMPLE: 35 + 3 = 35 + 2 + 1). Subtract 3 from a number by subtracting 2, then subtracting 1 (EXAMPLE: 35 – 3 = 35 – 2 – 1).
NOTE: All of the addition and subtraction tricks you teach your students should be reinforced with drills, flashcards and tests. Eventually students should memorize their addition and subtraction facts and shouldn’t have to rely on the mental math tricks. One of the greatest gifts you can give your students is to teach them their number facts.

SKILLS 7 and 8
Add 4 to a number by adding 2 twice (EXAMPLE: $51 + 4 = 51 + 2 + 2$). Subtract 4 from a number by subtracting 2 twice (EXAMPLE: $51 – 4 = 51 – 2 – 2$).

SKILLS 9 and 10
Add 5 to a number by adding 4 then 1. Subtract 5 by subtracting 4 then 1.

SKILL 11
Students can add pairs of identical numbers by doubling (EXAMPLE: $6 + 6 = 2 \times 6$). Students should either memorize the 2 times table or they should double numbers by counting on their fingers by 2s.

Add a pair of numbers that differ by 1 by rewriting the larger number as 1 plus the smaller number (then use doubling to find the sum): EXAMPLE: $6 + 7 = 6 + 6 + 1 = 12 + 1 = 13$; $7 + 8 = 7 + 7 + 1 = 14 + 1 = 15$, etc.

SKILLS 12, 13 and 14
Add a one-digit number to 10 by simply replacing the zero in 10 by the one-digit number: EXAMPLE: $10 + 7 = 17$.

Add 10 to any two-digit number by simply increasing the tens digit of the two-digit number by 1: EXAMPLE: $53 + 10 = 63$.

Add a pair of two-digit numbers (with no carrying) by adding the ones digits of the numbers and then the tens digits: EXAMPLE: $23 + 64 = 87$.

SKILLS 15 and 16
To add 9 to a one-digit number, subtract 1 from the number and then add 10: EXAMPLE: $9 + 6 = 10 + 5 = 15$; $9 + 7 = 10 + 6 = 16$, etc. (Essentially, the student simply has to subtract 1 from the number and then stick a 1 in front of the result.)

To add 8 to a one-digit number, subtract 2 from the number and add 10: EXAMPLE: $8 + 6 = 10 + 4 = 14$; $8 + 7 = 10 + 5 = 15$, etc.

SKILLS 17 and 18
To subtract a pair of multiples of ten, simply subtract the tens digits and add a zero for the ones digit: EXAMPLE: $70 – 50 = 20$.

To subtract a pair of two-digit numbers (without carrying or regrouping), subtract the ones digit from the ones digit and the tens digit from the tens digit: EXAMPLE: $57 – 34 = 23$. 
Further Mental Math Strategies

1. Your students should be able to explain how to use the strategies of “rounding the subtrahend (EXAMPLE: the number you are subtracting) up to the nearest multiple of ten.”

**EXAMPLES:**

<table>
<thead>
<tr>
<th>Subtrahend</th>
<th>Subtrahend rounded to the nearest tens</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 37 – 19 = 37 – 20 + 1</td>
<td></td>
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<tr>
<td>b) 64 – 28 = 64 – 30 + 2</td>
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<tr>
<td>c) 65 – 46 = 65 – 50 + 4</td>
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</tr>
</tbody>
</table>

**PRACTICE QUESTIONS:**

a) 27 – 17 = 27 – ______ + ______

b) 52 – 36 = 52 – ______ + ______

c) 76 – 49 = 76 – ______ + ______

d) 84 – 57 = 84 – ______ + ______

e) 61 – 29 = 61 – ______ + ______

**NOTE:** This strategy works well with numbers that end in 6, 7, 8 or 9.

2. Your students should be able to explain how to subtract by thinking of adding.

**EXAMPLES:**

a) 62 – 45 = 5 + 12 = 17

b) 46 – 23 = 3 + 20 = 23

c) 73 – 17 = 6 + 50 = 56

**PRACTICE QUESTIONS:**

a) 88 – 36 = ______ + ______ = ______

b) 58 – 21 = ______ + ______ = ______

3. Your students should be able to explain how to “use doubles.”

**EXAMPLES:**

<table>
<thead>
<tr>
<th>Minuend</th>
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</thead>
<tbody>
<tr>
<td>a) 12 – 6 = 6</td>
</tr>
<tr>
<td>b) 8 – 4 = 4</td>
</tr>
</tbody>
</table>

**PRACTICE QUESTIONS:**

a) 6 – 3 = ______

b) 10 – 5 = ______

c) 14 – 7 = ______

d) 18 – 9 = ______

e) 16 – 8 = ______

f) 20 – 10 = ______
NOTE TO TEACHER: Teaching the material on these worksheets may take several lessons. Students will need more practice than is provided on these pages. These pages are intended as a test to be given when you are certain your students have learned the materials fully.

Teacher

Teach SKILLS 1, 2, 3 AND 4 as outlined on pages 2 through 5 before you allow your students to answer Questions 1 through 12:

1. Name the even number that comes after the number. Answer in the blank provided:
   a) 32 _______  b) 46 _______  c) 14 _______  d) 92 _______  e) 56 _______
   f) 30 _______  g) 84 _______  h) 60 _______  i) 72 _______  j) 24 _______

2. Name the even number that comes after the number:
   a) 28 _______  b) 18 _______  c) 78 _______  d) 38 _______  e) 68 _______

3. Add. REMEMBER: adding 2 to an even number is the same as finding the next even number:
   a) 42 + 2 = _______  b) 76 + 2 = _______  c) 28 + 2 = _______  d) 16 + 2 = _______
   e) 68 + 2 = _______  f) 12 + 2 = _______  g) 36 + 2 = _______  h) 90 + 2 = _______
   i) 70 + 2 = _______  j) 24 + 2 = _______  k) 66 + 2 = _______  l) 52 + 2 = _______

4. Name the even number that comes before the number:
   a) 38 _______  b) 42 _______  c) 56 _______  d) 72 _______  e) 98 _______
   f) 48 _______  g) 16 _______  h) 22 _______  i) 66 _______  j) 14 _______

5. Name the even number that comes before the number:
   a) 30 _______  b) 70 _______  c) 60 _______  d) 10 _______  e) 80 _______

6. Subtract. REMEMBER: subtracting 2 from an even number is the same as finding the preceding even number:
   a) 46 – 2 = _______  b) 86 – 2 = _______  c) 90 – 2 = _______  d) 14 – 2 = _______
   e) 54 – 2 = _______  f) 72 – 2 = _______  g) 12 – 2 = _______  h) 56 – 2 = _______
   i) 32 – 2 = _______  j) 40 – 2 = _______  k) 60 – 2 = _______  l) 26 – 2 = _______

7. Name the odd number that comes after the number:
   a) 37 _______  b) 51 _______  c) 63 _______  d) 75 _______  e) 17 _______
   f) 61 _______  g) 43 _______  h) 81 _______  i) 23 _______  j) 95 _______

8. Name the odd number that comes after the number:
   a) 69 _______  b) 29 _______  c) 9 _______  d) 79 _______  e) 59 _______
9. Add. **REMEMBER:** Adding 2 to an odd number is the same as finding the next odd number:
   a) 25 + 2 = ________  b) 31 + 2 = ________  c) 47 + 2 = ________  d) 33 + 2 = ________
   e) 39 + 2 = ________  f) 91 + 2 = ________  g) 5 + 2 = ________  h) 89 + 2 = ________
   i) 11 + 2 = ________  j) 65 + 2 = ________  k) 29 + 2 = ________  l) 17 + 2 = ________

10. Name the odd number that comes before the number:
    a) 39 ________  b) 43 ________  c) 57 ________  d) 17 ________  e) 99 ________
    f) 13 ________  g) 85 ________  h) 79 ________  i) 65 ________  j) 77 ________

11. Name the odd number that comes before the number:
    a) 21 ________  b) 41 ________  c) 11 ________  d) 91 ________  e) 51 ________

12. Subtract. **REMEMBER:** Subtracting 2 from an odd number is the same as finding the preceding odd number.
    a) 47 – 2 = ________  b) 85 – 2 = ________  c) 91 – 2 = ________  d) 15 – 2 = ________
    e) 51 – 2 = ________  f) 73 – 2 = ________  g) 11 – 2 = ________  h) 59 – 2 = ________
    i) 31 – 2 = ________  j) 43 – 2 = ________  k) 7 – 2 = ________  l) 25 – 2 = ________

**Teacher**

Teach **SKILLS 5 AND 6** as outlined on pages 5 and 6 before you allow your students to answer Questions 13 and 14:

13. Add 3 to the number by adding 2, then adding 1 (**EXAMPLE:** 35 + 3 = 35 + 2 + 1):
    a) 23 + 3 = ________  b) 36 + 3 = ________  c) 29 + 3 = ________  d) 16 + 3 = ________
    e) 67 + 3 = ________  f) 12 + 3 = ________  g) 35 + 3 = ________  h) 90 + 3 = ________
    i) 78 + 3 = ________  j) 24 + 3 = ________  k) 6 + 3 = ________  l) 59 + 3 = ________

14. Subtract 3 from the number by subtracting 2, then subtracting 1 (**EXAMPLE:** 35 – 3 = 35 – 2 – 1):
    a) 46 – 3 = ________  b) 87 – 3 = ________  c) 99 – 3 = ________  d) 14 – 3 = ________
    e) 8 – 3 = ________  f) 72 – 3 = ________  g) 12 – 3 = ________  h) 57 – 3 = ________
    i) 32 – 3 = ________  j) 40 – 3 = ________  k) 60 – 3 = ________  l) 28 – 3 = ________

15. Fred has 49 stamps. He gives 2 stamps away. How many stamps does he have left?

16. There are 25 minnows in a tank. Alice adds 3 more to the tank. How many minnows are now in the tank?
Exercises

Teacher

Teach SKILLS 7 AND 8 as outlined on page 6.

17. Add 4 to the number by adding 2 twice (EXAMPLE: 51 + 4 = 51 + 2 + 2):
   a) 42 + 4 = _______  b) 76 + 4 = _______  c) 27 + 4 = _______  d) 17 + 4 = _______
   e) 68 + 4 = _______  f) 11 + 4 = _______  g) 35 + 4 = _______  h) 8 + 4 = _______
   i) 72 + 4 = _______  j) 23 + 4 = _______  k) 60 + 4 = _______  l) 59 + 4 = _______

18. Subtract 4 from the number by subtracting 2 twice (EXAMPLE: 26 – 4 = 26 – 2 – 2):
   a) 46 – 4 = _______  b) 86 – 4 = _______  c) 91 – 4 = _______  d) 15 – 4 = _______
   e) 53 – 4 = _______  f) 9 – 4 = _______  g) 13 – 4 = _______  h) 57 – 4 = _______
   i) 40 – 4 = _______  j) 88 – 4 = _______  k) 69 – 4 = _______  l) 31 – 4 = _______

Teacher

Teach SKILLS 9 AND 10 as outlined on page 6.

19. Add 5 to the number by adding 4, then adding 1 (or add 2 twice, then add 1):
   a) 84 + 5 = _______  b) 27 + 5 = _______  c) 31 + 5 = _______  d) 44 + 5 = _______
   e) 63 + 5 = _______  f) 92 + 5 = _______  g) 14 + 5 = _______  h) 16 + 5 = _______
   i) 9 + 5 = _______  j) 81 + 5 = _______  k) 51 + 5 = _______  l) 28 + 5 = _______

20. Subtract 5 from the number by subtracting 4, then subtracting 1 (or subtract 2 twice, then subtract 1):
   a) 48 – 5 = _______  b) 86 – 5 = _______  c) 55 – 5 = _______  d) 69 – 5 = _______
   e) 30 – 5 = _______  f) 13 – 5 = _______  g) 92 – 5 = _______  h) 77 – 5 = _______
   i) 45 – 5 = _______  j) 24 – 5 = _______  k) 91 – 5 = _______  l) 8 – 5 = _______

Teacher

Teach SKILLS 11 as outlined on page 6.

21. Add:
   a) 6 + 6 = _______  b) 7 + 7 = _______  c) 8 + 8 = _______
   d) 5 + 5 = _______  e) 4 + 4 = _______  f) 9 + 9 = _______

22. Add by thinking of the larger number as a sum of two smaller numbers:
   a) 6 + 7 = 6 + 6 + 1  b) 7 + 8 = _______  c) 6 + 8 = _______
   d) 4 + 5 = _______  e) 5 + 7 = _______  f) 8 + 9 = _______
Teacher

Teach SKILLS 12, 13 AND 14 as outlined on page 6.

23. a) 10 + 3 = __________ b) 10 + 7 = __________ c) 5 + 10 = __________ d) 10 + 1 = __________
   e) 9 + 10 = __________ f) 10 + 4 = __________ g) 10 + 8 = __________ h) 10 + 2 = __________
24. a) 10 + 20 = __________ b) 40 + 10 = __________ c) 10 + 80 = __________ d) 10 + 50 = __________
   e) 30 + 10 = __________ f) 10 + 60 = __________ g) 10 + 10 = __________ h) 70 + 10 = __________
25. a) 10 + 25 = __________ b) 10 + 67 = __________ c) 10 + 31 = __________ d) 10 + 82 = __________
   e) 10 + 43 = __________ f) 10 + 51 = __________ g) 10 + 68 = __________ h) 10 + 21 = __________
   i) 10 + 11 = __________ j) 10 + 19 = __________ k) 10 + 44 = __________ l) 10 + 88 = __________
26. a) 20 + 30 = __________ b) 40 + 20 = __________ c) 30 + 30 = __________ d) 50 + 30 = __________
   e) 20 + 50 = __________ f) 40 + 40 = __________ g) 50 + 40 = __________ h) 40 + 30 = __________
   i) 60 + 30 = __________ j) 20 + 60 = __________ k) 20 + 70 = __________ l) 60 + 40 = __________
27. a) 20 + 23 = __________ b) 32 + 24 = __________ c) 51 + 12 = __________ d) 12 + 67 = __________
   e) 83 + 14 = __________ f) 65 + 24 = __________ g) 41 + 43 = __________ h) 70 + 27 = __________
   i) 31 + 61 = __________ j) 54 + 33 = __________ k) 28 + 31 = __________ l) 42 + 55 = __________

Teacher

Teach SKILLS 15 AND 16 as outlined on page 6.

28. a) 9 + 3 = __________ b) 9 + 7 = __________ c) 6 + 9 = __________ d) 4 + 9 = __________
   e) 9 + 9 = __________ f) 5 + 9 = __________ g) 9 + 2 = __________ h) 9 + 8 = __________
29. a) 8 + 2 = __________ b) 8 + 6 = __________ c) 8 + 7 = __________ d) 4 + 8 = __________
   e) 5 + 8 = __________ f) 8 + 3 = __________ g) 9 + 8 = __________ h) 8 + 8 = __________

Teacher

Teach SKILLS 17 AND 18 as outlined on page 6.

30. a) 40 – 10 = __________ b) 50 – 10 = __________ c) 70 – 10 = __________ d) 20 – 10 = __________
   e) 40 – 20 = __________ f) 60 – 30 = __________ g) 40 – 30 = __________ h) 60 – 50 = __________
31. a) 57 – 34 = __________ b) 43 – 12 = __________ c) 62 – 21 = __________ d) 59 – 36 = __________
   e) 87 – 63 = __________ f) 95 – 62 = __________ g) 35 – 10 = __________ h) 17 – 8 = __________
Mental Math Advanced

Multiples of Ten

STUDENT: In the exercises below, you will learn several ways to use multiples of ten in mental addition or subtraction.

1. Warm up:
   a) $536 + 100 = \Box$
   b) $816 + 10 = \Box$
   c) $124 + 5 = \Box$
   d) $540 + 200 = \Box$
   e) $234 + 30 = \Box$
   f) $345 + 300 = \Box$
   g) $236 – 30 = \Box$
   h) $442 – 20 = \Box$
   i) $970 – 70 = \Box$
   j) $542 – 400 = \Box$
   k) $160 + 50 = \Box$
   l) $756 + 40 = \Box$

2. Write the second number in expanded form and add or subtract one digit at a time.
The first one is done for you:
   a) $564 + 215 = \Box$
   b) $445 + 343 = \Box$
   c) $234 + 214 = \Box$

3. Add or subtract mentally (one digit at a time):
   a) $547 + 312 = \Box$
   b) $578 – 314 = \Box$
   c) $845 – 454 = \Box$

4. Use the tricks you’ve just learned:
   a) $845 + 91 = \Box$
   b) $456 + 298 = \Box$
   c) $100 – 84 = \Box$
   d) $1000 – 846 = \Box$

EXAMPLE 1

Sometimes you will need to carry:

$545 + 172 = 545 + 100 + 70 + 2 = 645 + 70 + 2 = 715 + 2 = 717$

EXAMPLE 2

If one of the numbers you are adding or subtracting is close to a number with a multiple of ten, add the multiple of ten and then add or subtract an adjustment factor:

$645 + 99 = 645 + 100 – 1 = 745 – 1 = 744$
$856 + 42 = 856 + 40 + 2 = 896 + 2 = 898$

EXAMPLE 3

Sometimes in subtraction, it helps to think of a multiple of ten as a sum of 1 and a number consisting entirely of 9s (EXAMPLE: $100 = 1 + 99; 1000 = 1 + 999$). You never have to borrow or exchange when you are subtracting from a number consisting entirely of 9s.

$100 – 43 = 1 + 99 – 43 = 1 + 56 = 57$ [Do the subtraction, using 99 instead of 100, and then add 1 to your answer.]
$1000 – 543 = 1 + 999 – 543 = 1 + 456 = 457$

4. Use the tricks you’ve just learned:
   a) $845 + 91 = \Box$
   b) $456 + 298 = \Box$
   c) $100 – 84 = \Box$
   d) $1000 – 846 = \Box$
Mental Math

Game: Modified Go Fish

Purpose

If students know the pairs of one-digit numbers that add up to particular target numbers, they will be able to mentally break sums into easier sums.

**EXAMPLE:** As it is easy to add any one-digit number to 10, you can add a sum more readily if you can decompose numbers in the sum into pairs that add to ten.

\[
7 + 5 = 7 + 3 + 2 = 10 + 2 = 12
\]

*These numbers add to 10.*

To help students remember pairs of numbers that add up to a given target number I developed a variation of “Go Fish” that I have found very effective.

The Game

Pick any target number and remove all the cards with value greater than or equal to the target number out of the deck. In what follows, I will assume that the target number is 10, so you would take all the tens and face cards out of the deck (Aces count as one).

The dealer gives each player 6 cards. If a player has any pairs of cards that add to 10 they are allowed to place these pairs on the table before play begins.

Player 1 selects one of the cards in his or her hand and asks the Player 2 for a card that adds to 10 with the chosen card. For instance, if Player 1’s card is a 3, they may ask the Player 2 for a 7.

If Player 2 has the requested card, the first player takes it and lays it down along with the card from their hand. The first player may then ask for another card. If the Player 2 doesn’t have the requested card they say: “Go fish,” and the Player 1 must pick up a card from the top of the deck. (If this card adds to 10 with a card in the player’s hand they may lay down the pair right away). It is then Player 2’s turn to ask for a card.

Play ends when one player lays down all of their cards. Players receive 4 points for laying down all of their cards first and 1 point for each pair they have laid down.

**NOTE:** With weaker students I would recommend you start with pairs of numbers that add to 5. Take all cards with value greater than 4 out of the deck. Each player should be dealt only 4 cards to start with.

I have worked with several students who have had a great deal of trouble sorting their cards and finding pairs that add to a target number. I’ve found the following exercise helps:

Give your student only three cards; two of which add to the target number. Ask the student to find the pair that add to the target number. After the student has mastered this step with 3 cards repeat the exercise with 4 cards, then 5 cards, and so on.

**NOTE:** You can also give your student a list of pairs that add to the target number. As the student gets used to the game, gradually remove pairs from the list so that the student learns the pairs by memory.
## Mental Math Checklist #1

<table>
<thead>
<tr>
<th>Student Name</th>
<th>Can Add 1 to Any Number</th>
<th>Can Subtract 1 from Any Number</th>
<th>Can Add 2 to Any Number</th>
<th>Can Subtract 2 from Any Number</th>
<th>Knows All Pairs that Add to 5</th>
<th>Can Double 1-Digit Numbers</th>
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<tbody>
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# Mental Math Checklist #2

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<thead>
<tr>
<th>Student Name</th>
<th>Can Add Near Doubles. EXAMPLE: $6 + 7 = 6 + 6 + 1$</th>
<th>Can Add a 1-Digit Number to Any Multiple of 10. EXAMPLE: $30 + 6 = 36$</th>
<th>Can Add Any 1-Digit Number to a Number Ending in 9. EXAMPLE: $29 + 7 = 30 + 6 = 36$</th>
<th>Can Add 1-Digit Numbers by “Breaking” them Apart into Pairs that Add to 10. EXAMPLE: $7 + 5 = 7 + 3 + 2 = 10 + 2$</th>
<th>Can Subtract Any Multiple of 10 from 100. EXAMPLE: $100 – 40 = 60$</th>
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<tr>
<th>Student Name</th>
<th>Can Mentally Make Change from a Dollar. SEE: Workbook Sheets on Money.</th>
<th>Can Mentally Add Any Pair of 1-Digit Numbers.</th>
<th>Can Mentally Subtract Any Pair of 1-Digit Numbers.</th>
<th>Student Can Multiply and Count by:</th>
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Teacher

Trying to do math without knowing your times tables is like trying to play the piano without knowing the location of the notes on the keyboard. Your students will have difficulty seeing patterns in sequences and charts, solving proportions, finding equivalent fractions, decimals and percents, solving problems etc. if they don’t know their tables.

Using the method below, you can teach your students their tables in a week or so. If you set aside five or ten minutes a day to work with students who need extra help, the pay-off will be enormous. There really is no reason for your students not to know their tables!

DAY 1: Counting by 2s, 3s, 4s and 5s

If you have completed the Fractions Unit you should already know how to count and multiply by 2s, 3s, 4s and 5s. If you don’t know how to count by these numbers you should memorize the hands below:

If you know how to count by 2s, 3s, 4s and 5s, then you can multiply by any combination of these numbers. For instance, to find the product 3 × 2, count by 2s until you have raised 3 fingers.

DAY 2: The Nine Times Table

The numbers you say when you count by 9s are called the MULTIPLES of 9 (zero is also a multiple of 9). The first ten multiples of 9 (after zero) are: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90. What happens when you add the digits of any of these multiples of 9 (EXAMPLE: 1 + 8 or 6 + 3)? The sum is always 9!

Here is another useful fact about the nine times table: Multiply 9 by any number between 1 and 10 and look at the tens digit of the product. The tens digit is always one less than the number you multiplied by:

You can find the product of 9 and any number by using the two facts given above. For instance, to find 9 × 7, follow these steps:

STEP 1: Subtract 1 from the number you are multiplying by 9: 7 – 1 = 6

Now you know the tens digit of the product.
Teacher

1. Make sure your students know how to subtract (by counting on their fingers if necessary) before you teach them the trick for the nine times table.

2. Give a test on STEP 1 (above) before you move on.

\[
\begin{align*}
\text{STEP 2:} & & 9 \times 7 = \\
& & \underline{__} \quad \underline{__} \\
\text{These two digits add to 9.} & & \text{So the missing digit is } 9 - 6 = 3
\end{align*}
\]

(You can do the subtraction on your fingers if necessary).

Practise these two steps for all of the products of 9: \(9 \times 2, 9 \times 3, 9 \times 4, \text{ etc.}\)

**DAY 3: The Eight Times Table**

There are two patterns in the digits of the 8 times table. Knowing these patterns will help you remember how to count by 8s.

**STEP 1:** You can find the ones digit of the first five multiples of 8, by starting at 8 and counting backwards by 2s.

\[
\begin{align*}
8 \\
6 \\
4 \\
2 \\
0
\end{align*}
\]

**STEP 2:** You can find the tens digit of the first five multiples of 8, by starting at 0 and count up by 1s.

\[
\begin{align*}
08 \\
16 \\
24 \\
32 \\
40
\end{align*}
\]

(Of course you don’t need to write the 0 in front of the 8 for the product \(1 \times 8\).)

**STEP 3:** You can find the ones digit of the next five multiples of 8 by repeating step 1:

\[
\begin{align*}
8 \\
6 \\
4 \\
2 \\
0
\end{align*}
\]

**STEP 4:** You can find the remaining tens digits by starting at 4 and count up by 1s.

\[
\begin{align*}
48 \\
56 \\
64 \\
72 \\
80
\end{align*}
\]

Practise writing the multiples of 8 (up to 80) until you have memorized the complete list. Knowing the patterns in the digits of the multiples of 8 will help you memorize the list very quickly. Then you will know how to multiply by 8:

\[
8 \times 6 = \underline{48}
\]

Count by eight until you have 6 fingers up: 8, 16, 24, 32, 40, 48.
DAY 4: The Six Times Table

If you have learned the eight and nine times tables, then you already know $6 \times 9$ and $6 \times 8$.

And if you know how to multiply by 5 up to $5 \times 5$, then you also know how to multiply by 6 up to $6 \times 5$! That’s because you can always calculate 6 times a number by calculating 5 times the number and then adding the number itself to the result. The pictures below show why this works for $6 \times 4$:

\[
\begin{array}{c}
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\end{array}
\quad 6 \times 4 = 5 \times 4 + 4 = 20 + 4 = 24
\]

Similarly:

\[
\begin{array}{c}
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\end{array}
\quad 6 \times 2 = 5 \times 2 + 2; \quad 6 \times 3 = 5 \times 3 + 3; \quad 6 \times 5 = 5 \times 5 + 5.
\]

Knowing this, you only need to memorize 2 facts:

**ONE:** $6 \times 6 = 36$  
**TWO:** $6 \times 7 = 42$

Or, if you know $6 \times 5$, you can find $6 \times 6$ by calculating $6 \times 5 + 6$.

DAY 5: The Seven Times Table

If you have learned the six, eight and nine times tables, then you already know:

\[
6 \times 7, \quad 8 \times 7 \quad \text{and} \quad 9 \times 7.
\]

And since you also already know $1 \times 7 = 7$, you only need to memorize 5 facts:

\[
\begin{array}{c}
1. \quad 2 \times 7 = 14 \\
2. \quad 3 \times 7 = 21 \\
3. \quad 4 \times 7 = 28 \\
4. \quad 5 \times 7 = 35 \\
5. \quad 7 \times 7 = 49
\end{array}
\]

If you are able to memorize your own phone number, then you can easily memorize these 5 facts!

**NOTE:** You can use doubling to help you learn the facts above. 4 is double 2, so $4 \times 7 (= 28)$ is double $2 \times 7 (= 14)$. 6 is double 3, so $6 \times 7 (= 42)$ is double $3 \times 7 (= 21)$.

Try this test every day until you have learned your times tables:

\[
\begin{array}{cccc}
1. \quad 3 \times 5 = & 2. \quad 8 \times 4 = & 3. \quad 9 \times 3 = & 4. \quad 4 \times 5 = \\
5. \quad 2 \times 3 = & 6. \quad 4 \times 2 = & 7. \quad 8 \times 1 = & 8. \quad 6 \times 6 = \\
9. \quad 9 \times 7 = & 10. \quad 7 \times 7 = & 11. \quad 5 \times 8 = & 12. \quad 2 \times 6 = \\
13. \quad 6 \times 4 = & 14. \quad 7 \times 3 = & 15. \quad 4 \times 9 = & 16. \quad 2 \times 9 = \\
17. \quad 9 \times 9 = & 18. \quad 3 \times 4 = & 19. \quad 6 \times 8 = & 20. \quad 7 \times 5 = \\
21. \quad 9 \times 5 = & 22. \quad 5 \times 6 = & 23. \quad 6 \times 3 = & 24. \quad 7 \times 1 = \\
25. \quad 8 \times 3 = & 26. \quad 9 \times 6 = & 27. \quad 4 \times 7 = & 28. \quad 3 \times 3 = \\
29. \quad 8 \times 7 = & 30. \quad 1 \times 5 = & 31. \quad 7 \times 6 = & 32. \quad 2 \times 8 = \\
\end{array}
\]
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A Note for Patterns

Before students can create or recognize a pattern in a sequence of numbers, they must be able to tell how far apart the successive terms in a particular sequence are. There is no point in introducing students to sequences if they don’t know how to find the gap between a given pair of numbers, either by applying their knowledge of basic addition and subtraction, or by counting on their fingers as described below.

For weaker students, use the following method for recognizing gaps:

How far apart are 8 and 11?

**STEP 1:**
Say the lower number (“8”) with your fist closed.

![Hand with fist closed](image)

**STEP 2:**
Count up by ones, raising your thumb first, then one finger at a time until you have reached the higher number (11).

![Hand with fist closed](image)

**STEP 3:**
The number of fingers you have up when you reach the final number is the answer (in this case you have three fingers up, so three is the difference between 8 and 11.)

Using the method above, you can teach even the weakest student to find the difference between two numbers in one lesson. (You may have to initially hold your student’s fist closed when they say the first number—some students will want to put their thumb up to start—but otherwise students find the method easy.)

Eventually, you should wean your student off using their fingers to find the gap between a pair of numbers. The exercises in the MENTAL MATH section of this manual will help with this.
To introduce the topic of patterns, tell your students that they will often encounter patterns in sequences of numbers in their daily lives. For instance, the sequence 25, 20, 15, 10... might represent the amount of money in someone’s savings account (each week) or the amount of water remaining in a leaky fish tank (each hour).

Ask your students to give other interpretations of the pattern. You might also ask them to say what the numbers in the pattern mean for a particular interpretation. For instance, if the pattern represents the money in a savings account, the person started with $25 and withdrew $5 each week.

Tell your students that you would like to prepare them to work with patterns and you will start by making sure they are able to find the difference between two numbers. There is nothing wrong with counting on their fingers until they have learned their number facts—they are our built-in calculator. If there are students who know their subtraction facts, tell them they are in perfect shape and can do the calculations mentally.

Show students how to find the difference between two numbers using the method described in the note at the beginning of this unit. (Say the smaller number with your fist closed. Count up to the larger number raising one finger at a time. The number of fingers you have raised when you say the larger number is the difference).

**NOTE:** Before you allow students to try any of the questions on a particular worksheet you should assign sample questions (like the ones on the worksheet) for the whole class to discuss and solve. You should also give a mini quiz or assessment consisting of several questions that students can work on independently (either in a notebook or individual pieces of paper). The quiz or assessment will tell you whether students are ready to do the work on the worksheet.

The bonus questions provided in a lesson plan may either be assigned during the lesson, (to build excitement, by allowing students to show off with harder looking questions) or during the assessments (for students who have finished their work early, so you have time to help slower students).

**Sample Questions**

Find the difference:

- a) 3 \( \Box \) 7  
- b) 23 \( \Box \) 25  
- c) 39 \( \Box \) 42  
- d) 87 \( \Box \) 92

**Bonus**

- a) 273 \( \Box \) 277  
- b) 1768 \( \Box \) 1777  
- c) 1997 \( \Box \) 2003  
- d) 32 496 \( \Box \) 32 502
**Assessment**

Find the difference:

a) 4 \( \phantom{7} \) 7  

b) 21 \( \phantom{25} \) 25  

c) 37 \( \phantom{42} \) 42  

d) 70 \( \phantom{77} \) 77

Show students the difference between two numbers on a number line as follows:

What number added to 6 gives 9?  
\[ 6 + ? = 9 \]

Count 3 spaces between 6 and 9 on a number line, checking with each hop:

- Add 1, get 7, 7 is 1 more than 6.
- Add 2, get 8, 8 is 2 more than 6.
- Add 3, get 9, 9 is 3 more than 6,

**SO:** 6 + \( \phantom{3} \) = 9 AND: 3 is called the **difference** between 9 and 6

Ask students what number they would add to 45 to get 49. They should see that they add the “difference” or “gap” between the numbers. Tell your students that you will now give them problems that are the reverse of the ones they have solved.

Given the difference between two numbers, they can find the greater number by adding the difference to the smaller number.

If students don’t know their number facts, they can always add using the method shown in the box in the middle of the worksheet.

Ask your students to add the number in the circle to the number beside it, and write their answer in the blank:

a) 6 \( \phantom{4} \) b) 17 \( \phantom{2} \) c) 9 \( \phantom{3} \) d) 25 \( \phantom{4} \)

e) 38 \( \phantom{5} \) f) 29 \( \phantom{4} \) g) 56 \( \phantom{8} \) h) 29 \( \phantom{7} \)

i) 80 \( \phantom{9} \) j) 46 \( \phantom{9} \) k) 74 \( \phantom{7} \) l) 93 \( \phantom{5} \)

---

**Assessment**

1. Find the difference:

a) 4 \( \phantom{7} \) b) 6 \( \phantom{11} \) c) 21 \( \phantom{25} \) d) 37 \( \phantom{42} \) e) 70 \( \phantom{77} \) f) 46 \( \phantom{52} \) g) 67 \( \phantom{76} \)

2. Fill in the missing numbers:

a) \( \phantom{\text{is 3 more than 63}} \) b) \( \phantom{\text{is 6 more than 75}} \) c) \( \phantom{\text{is 10 more than 26}} \)

d) \( \phantom{\text{is 4 more than 32}} \) e) \( \phantom{\text{is 7 more than 41}} \) f) \( \phantom{\text{is 8 more than 57}} \)
Bonus

Fill in the missing numbers:

a) 167 is ___ more than 163  
   b) 378 is ___ more than 371  
   c) 983 is ___ more than 978
   
d) 107 + ___ = 111  
   e) 456 + ___ = 459  
   f) 987 + ___ = 992.
   
g) ___ is 4 more than 596  
   h) ___ is 6 more than 756

NOTE: It is extremely important that your students learn their number facts and eventually move beyond using their fingers. We recommend that you use the exercises in the MENTAL MATH section of this Guide to help students learn their number facts.

ACTIVITY

Let your students play the Modified Go Fish Game in pairs (see the MENTAL MATH section for instructions). Start with the number 5 as the target number and proceed gradually to 10. This will help them find larger differences between numbers.
In this lesson, weaker students can use their fingers to extend a pattern as follows:

**EXAMPLE:** Extend the pattern 3, 6, 9... up to six terms.

**STEP 1:** Identify the gap between successive pairs of numbers in the sequence (You may count on your fingers, if necessary). The gap, in this example, is three. Check that the gap between successive terms in the sequence is always the same, otherwise you cannot continue the pattern by adding a fixed number. (Your student should write the gap between each pair of successive terms above the pairs.)

\[
3, 6, 9, \underline{\text{ }}, \underline{\text{ }}, \underline{\text{ }},
\]

**STEP 2:** Say the last number in the sequence with your fist closed. Count by ones until you have raised three fingers (the gap between the numbers). The number you say when you have raised your third finger is the next number in the sequence.

\[
3, 6, 9, 12, \underline{\text{ }}, \underline{\text{ }},
\]

**STEP 3:** Continue adding terms to extend the sequence.

\[
3, 6, 9, 12, 15, 18
\]

To start the lesson write the sequence: 1, 2, 3, 4, 5, ... on the board. Ask your students what the next number will be. Then write the sequence 2, 4, 6, 8, ... Ask what the next number is this time. Ask students how they guessed that the next number is 10 (skip counting by 2). Ask what they did to the previous number to get the next one. Ask what the difference between the numbers is.

Tell your students that the next question is going to be harder, but that you still expect them to find the answer. Write the sequence 2, 5, 8, 11... with room to write circles between the numbers:

\[
2, 5, 8, 11,
\]

Ask students if they can see any pattern in the numbers. If your students need help, fill in the difference between the first two terms in the circle.

\[
2, 5, 8, 11
\]

Add another circle and ask students to find the difference between 5 and 8.

\[
2, 5, \underline{\text{ }}, 8, 11
\]
After students have found the difference between 8 and 11, ask them if they can predict the next term in the sequence.

Explain that sequences like the one you just showed them are common in daily life. Suppose George wants to buy a skateboard. He saves $7 during the first month and, in each of the next months, he saves $4 more.

How much money does he have after two months? Write \(7 \quad \underline{\phantom{0}} \quad 4\) , and ask a volunteer to fill in the next number.

Ask volunteers to continue George’s savings’ sequence. Mention that patterns of this kind are called “increasing sequences” and ask if the students can explain why they are called increasing. Point out that in all of the exercises so far the difference between terms in sequence has been the same. Later your students will study more advanced sequences in which the difference changes.

Extend the patterns:

a) \(6, 9, 12, 15, \underline{\phantom{0}} , \underline{\phantom{0}} \) 
b) \(5, 11, 17, 23, \underline{\phantom{0}} \)

c) \(2, 10, 18, 26, \underline{\phantom{0}} \)

Do not progress to word problems before all your students can extend a given pattern.

**Bonus**

a) 99, 101, 103, … 
b) 654, 657, 660, …

**Sample Word Problems**

1. A newborn Saltwater Crocodile is about 25 cm long. It grows 5 cm in a month during the first 4 months. How long is a 2-month-old Saltie? 4-mth old one? If its length is 40 cm, how old is it?

2. A 14-year-old Tyrannosaurus Rex is 300 cm (3 m) long and grows 14 cm every month. How long is it at the age of 14 years and 1 month? 14 years and 2 months? 14 years and 6 months? 15 years?

**Extensions**

1. A baby blue whale weighs 2700 kg. Each hour it gains 4 kilograms. How much does a two-hour-old baby weigh? A 4-hour old?

   **Bonus**

   How much does a 24-hour old whale weigh?

   **Double Bonus**

   What is the weight of a whale calf that is 1 week old?

2. For each sequence, create a word problem that goes with the sequence.

   a) \(7, 9, 11, 13, \underline{\phantom{0}} , \underline{\phantom{0}} \) 
   b) \(5, 9, 13, 17, \underline{\phantom{0}} , \underline{\phantom{0}} \)

3. **GAME:** Each group needs two dice (say blue and red) and a token for each player. Use the “Number Line Chasing Game” from the BLM. The blue die shows the difference in the pattern.
The red die shows the number of terms in the sequence to calculate. The players start at 1. If he throws 3 with the blue die and 2 with the red die, he will have to calculate the next 2 terms of the sequence: $1, \underline{3}, \underline{3}$ (4 and 7). 7 is a “smiley face”. When the player lands on a “smiley face”, he jumps to the next “smiley face”, in this case 17. His next sequence will start at 17. The first player to reach 100 is the winner.

**PA5-3**

**Counting Backwards**

**GOALS**

Students will find the difference between two numbers counting backwards using fingers and a number line.

**PRIOR KNOWLEDGE REQUIRED**

Count backwards from 100 to 1

**VOCABULARY**

difference

**ASK A RIDDLE:** I am a number between 1 and 10. Subtract me from 8, and get 5. Who am I?

Explain that you can use a number line to find the difference. Draw the number line on the board and show how to find the difference by counting backwards. **WRITE:** “5 is 3 less than 8.”

Show a couple more examples on the board, this time using volunteers.

**Sample Problems**

I am a number between 1 and 10. Who am I?

a) If you subtract me from 46, you get 41.
b) If you subtract me from 52, you get 43.
c) If you subtract me from 54, you get 46.

Ask your students how they could solve the problem when they do not have a number line. Remind them how to find the difference counting backwards on their fingers.

You might need to spend more time with students who need more practice finding the gap between pairs of numbers by subtracting or counting backwards. Your students will not be able to extend or describe patterns if they cannot find the gap between pairs of numbers. You also might use flashcards similar to those you used with the worksheet **PA5-1**.

Draw two number lines on the board:
What number added to 24 is 27? Solve by counting forward on the upper line.

What number is subtracted from 27 to get 24? Solve by counting backward on the lower line. (Draw arrows to indicate the “hops” between the numbers on the number line).

What are the similarities between the methods? What are the differences?

Students should notice that both methods give the difference between 24 and 27 and are merely different ways to subtract. **ASK:** How many times did you hop? What did you measure? (distance, difference, gap) What is this called in mathematics? What other word for the gap did we use? (difference)

When you measured the distance (or difference) between two points on the number line, the first time you went from left to right, and the second time you went from right to left.

What operation are you performing when you go from the smaller number to the larger? (adding)
From the larger number to the smaller one? (subtracting)

You might even draw arrows: from right to left and write “−”, and from left to right and write “+” below and above the respective number lines.

Congratulations your students on discovering an important mathematical fact—addition and subtraction are opposite movements on a number line, and both counting backwards and forwards are used to find the difference.

**Assessment**

**MORE RIDDLES:** I am a number between 1 and 10. Who am I?

a) If you subtract me from 27, you get 21.
b) If you subtract me from 32, you get 24.
c) If you subtract me from 34, you get 27.

**Bonus**
Find the gap between the numbers:

a) 108 105  
b) 279 274  
c) 241 238  
d) 764 759
PA5-4
Decreasing Sequences

Remind your students what an increasing sequence is. Ask them where they encounter increasing sequences in life. **POSSIBLE EXAMPLES:** height, length, weight of plants and animals, growing buildings, etc. Are all sequences that they will encounter in life increasing? Ask students to discuss this point and give examples of sequences that are not increasing.

Give an example, such as: Jenny was given $20 as a monthly allowance. After the first week she still had 16 dollars, after the second week she had 12 dollars, after the third week she had 8 dollars. If this pattern continues, how much money will she have after the fourth week? The amount of money that Jenny has decreases: 20, 16, 12, 8, so this type of sequences is called a “decreasing sequence”. Write the term on the board and later include it in spelling tests together with “increasing sequence”.

Have your students do some warm-up exercises, such as:

1. Subtract the number in the circle from the number beside it. The minus reminds you that you are subtracting. Write your answer in the blank:
   a) 13 \(-\) 2  
   b) 12 \(-\) 6  
   c) 11 \(-\) 8  
   d) 9 \(-\) 5  

2. Fill in the missing numbers:
   a) ____ is 4 less than 37  
   b) ____ is 5 less than 32  

Then write two numbers on the board:

14  
   11

Ask what the difference between the numbers is. Write the difference in the circle with a minus sign then ask students to extend the sequence.

14  
   11  
   8  
   ____  ,  ____

**Assessment**

Extend the decreasing sequences:

a) 21 , 19 , 17 , 15 ,  ,  

b) 34 , 31 , 28 ,  ,  

c) 48 , 41 , 34 , 27 ,  ,  

---

**GOALS**

Students will find the differences between two numbers by subtraction, and extend decreasing sequences.

**PRIOR KNOWLEDGE REQUIRED**

Count to 100  
Find difference between two numbers

**VOCABULARY**

difference  
decreasing sequence  
pattern
**Bonus**

Extend the decreasing sequences:

a) 141, 139, 137, 135

b) 548, 541, 534, 527

c) 234, 221, 208, ,

**A Game for Two**

The students will need two dice. The first player throws both dice (blue and red). The number on the red die determines how many numbers in the sequence the player has to calculate and the number on the red die indicates the difference of the sequence. The first number in this sequence is 100. The next player starts at the number the first player finished at. The player who ends at 0 or below is the winner.

**EXAMPLE:** The first throw yields red 4, blue 3. The first player has to calculate four numbers in the sequence whose difference is 3 and whose first number is 100. The player calculates: 97, 94, 91, 88. The next player starts with 88 and rolls the dice.

**ADVANCED VERSION:** Give 1 point for each correct term in the sequence and 5 points to the first player to reach 0. The player with the most points is the winner.

**ALTERNATIVE:** Use the Number Line Chasing Game from the BLM. Each player uses a moving token, similar to the game in section PA5-2.

**Extension**

If you used the “Number Line Chasing Game” from the BLM or the Activity above, your students might explore the following question:

Two players start at the same place. John throws the dice and receives red 3 and blue 4. Mike gets red 4 and blue 3. Do they end at the same place? Does this always happen? Why?
**GOALS**
Students will distinguish between increasing and decreasing sequences, and extend sequences.

**PRIOR KNOWLEDGE REQUIRED**
Count to 100
Find difference between two numbers
Increasing sequences
Decreasing sequences

**VOCABULARY**
difference
increasing sequence
decreasing sequence
pattern

Remind your students about how they can extend a sequence using the gap provided. Give examples for both increasing and decreasing sequences. For increasing sequences, you might draw an arrow pointing upwards beside the sequence, and for decreasing sequences, an arrow pointing downwards.

Write several sequences on the board and **ASK**: Does the sequence increase or decrease? What sign should you use for the gap? Plus or minus? What is the difference between the numbers?

Let your students practice finding the next term in a sequence using the gap provided:

a) $4, 9, \ldots$  
b) $25, 22, \ldots$  
c) $23, 31, \ldots$  
d) $49, 42, \ldots$

Ask students what they would do if they have the beginning of the sequence but they do not know the gap:

$4, 7, 10, \ldots$

First ask if the sequence goes up or down. Is the gap positive or negative? Then find the difference. When the gaps are filled in, extend the sequence.

**Assessment**
Extend the sequences. First find the difference (gap). **(GRADING TIP: check to see if the students have the correct “gap”):**

a) $21, 18, 15, \ldots$  
b) $34, 36, 38, \ldots$  
c) $47, 53, 59, \ldots$  
d) $51, 47, 43, \ldots$  
e) $84, 76, 68, \ldots$  
f) $60, 65, 70, \ldots$

**Bonus**

a) $128, 134, 140, \ldots$  
b) $435, 448, 461, \ldots$  
c) $961, 911, 861, \ldots$
A Game
Calculating three terms of a sequence each time, one point for each correct term. The students will need two dice of different colors (say red and blue) and a cardboard coin with the signs “+” and “−” written on its sides. The students throw the dice and the coin.

If the coin shows “+”, then the number on the red die is the starting point of the sequence and the number on the blue die is the difference. For example, if the coin shows “+”, the red die shows 5 and the blue die shows 3, the student has to write: 5, 8, 11, 14.

If the coin shows “−”, then the starting point of the sequence will be 10 or 100 times the number on the red die. The number on the blue die is the difference. For example, if the red die shows 5 and the blue die shows 3, the student has to write: 50, 47, 44, 41.

Give a point for each correct term.

Another possibility is to use the “Number Line Chasing Game” at the BLM.

Extension

1. You could make up some fanciful word problems for your students, possibly based on things they are reading:

A newborn dragon (called Herbert by its proud owner) grows 50 cm every day. At hatching, it is 30 cm long. How long will Herbert be in 3 days? In a week?

2. A GAME FOR PAIRS: The first player throws a die so that the second player does not see the result. The die indicates the difference between terms in a sequence. The first player chooses the first term of the sequence. The first player then gives the second player the first and the third terms in the sequence. For example, if the die shows 3, the first player might select 13 as the first term and write: 13, ______, 19. The second player then has to guess the second term and the difference. In this case, the second term is 16. (The trick is to find the number that is exactly in the middle of the two given terms) After students have tried the game, you might ask them to make the T-table of differences:

<table>
<thead>
<tr>
<th>The Die Roll</th>
<th>The Difference Between the First and the Second Terms</th>
<th>The Difference Between the First and the Third Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>...</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

Students will be soon able to see the pattern.

3. ADVANCED VERSION: only for the players who have mastered the previous game:

Students are given a coin with the number 1 on one side and 2 on the other side. The coin gives the number of missing terms and is visible to both players. If the coin shows 1, then only one term is withheld and the game is played in the same way as shown above. If the coin shows 2, and the die shows 3 as before, the first player writes 13, ______, ______, 22.
Extending a Pattern Using a Rule

This section provides a basic introduction to pattern rules. For more advanced work including word problems, applications and communication, see sections PA5-10 to 30.

Write a number sequence on the board: 10, 13, 16, 19, …

Draw the circles for differences.

\[ 10, 13, 16, 19, \]

Then ask them if they could describe the sequence without saying all of the individual terms.

For a challenge you might ask students to work in pairs. One student writes an increasing sequence without showing their partner. The student must get their partner to reproduce their sequence but they are not allowed to give more than one term of the sequence. Students should see that the best way to describe a sequence is to give the first term and the difference between terms. The sequence above could be described by the rule “Start at 10 and add 3 to get the next term.”

For practice, give your students questions such as:

Continue the patterns:

a) (add 3) 14, 17, __, __, __, __

b) (subtract 2) 35, 33, __, __, __, __

c) (add 7) 12, 19, __, __, __, __

d) (subtract 8) 77, 69, __, __, __, __

e) (add 11) 15, 26, __, __, __, __

f) (subtract 9) 97, 88, __, __, __, __

Create a pattern of your own:

__, __, __, __

My rule: ______________________________________

For practice you might assign the game in the activity below.

Tell your students the following story and ask them to help the friends:

Bonnie and Leonie are two Grade 5 friends who are doing their homework together. A sequence in on their homework is: 43, 36, 29, 22, … Bonnie said: “If you count backwards from 43 to 36 you say eight numbers.”

1 2 3 4 5 6 7 8

43, 42, 41, 40, 39, 38, 37, 36

“So the difference between 43 and 36 is 8.”

GOALS

Students will extend patterns using a verbal rule.

PRIOR KNOWLEDGE REQUIRED

Addition

Subtraction

Extension of patterns

Sequence

VOCABULARY

pattern rule
Leonie said “If you count the number of “hops” on a number line between 43 and 36, you see there are 7 hops so the difference is 7.” Who is right?

Assessment
Continue the patterns:

a) (add 8) 12, 20, __, __, __, __

b) (subtract 6) 77, 71, __, __, __, __

Create a pattern of your own:

__ __ __ __ __

My rule: ______________________________________

A Game
The students need a die and a coin. The coin has the signs “+” and “−” on the sides. The player throws the coin and the die and uses the results to write a sequence. The die gives the difference. The coin indicates whether the difference is added or subtracted (according to the sign that faces up). The player chooses the first number of the sequence. The game might also be played in pairs, where one of the players checks the results of the other and supplies the initial term of the sequence. Give one point for each correct term.

Extension
Extend the sequences according to the rules:

a) (add 15) 12, 27, __, __, __

b) (subtract 26) 177, 151, __, __, __, __

c) (add 108) 12, 120, __, __, __, __, __

d) (subtract 38) 276, 238, __, __, __, __, __

e) (multiply by 2) 6, 12, __, __, __, __, __

f) (add two previous numbers) 1, 2, 3, 8, __, __, __, __, __
Identifying Pattern Rules

Tell your students that today you will make the task from the previous lesson harder—you will give them a sequence, but you will not tell them what the rule is; they have to find it themselves. Give them an example—write a sequence on the board: 2, 6, 10, 14… Is it an increasing or a decreasing sequence? What was added? Let students practice with questions like:

Write the rule for each pattern:

a) 69, 64, 59, 54, subtract _____
b) 35, 39, 43, 47, add _____
c) 43, 54, 65, 76, add _____
d) 119, 116, 113, subtract _____
e) 25, 32, 39, 46, add _____
f) 98, 86, 74, subtract _____

Tell another story about pattern fans Bonnie and Leonie: The teacher asked them to write a sequence that is given by the rule “add 2”. Bonnie has written the sequence: “3, 5, 7, 9…”. Leonie has written the sequence: “2, 4, 6, 8…”. They quarrel—whose sequence is the right one?

Ask your students: how can you ensure that the sequence given by a rule is the one that you want? What do you have to add to the rule? Students should see that you need a single number as a starting point. Write several sequences on the board and ask your students to make rules for them.

Explain to your students that mathematicians need a word for the members of the sequences. If your sequence is made of numbers, you might say “number”. But what if the members of the sequence are figures? Show an example of a pattern made of blocks or other figures.

Tell the students that the general word for a member of a sequence is “term”.

Form a line of volunteers. Ask them to be a sequence. Ask each student in the sequence to say in order: “I am the first/second/third… term.” Ask each term in the sequence to do some task, such as: “Term 5, hop 3 times”, or “All even terms, hold up your right arm!” Ask your sequence to decide what simple task each of them will be do, and ask the rest of the class to identify which term has done what. Thank and release your sequence.

MORE PRACTICE:

a) What is the third term of the sequence 2, 4, 6, 8?
b) What is the fourth term of the sequence 17, 14, 11, 8?
c) Extend each sequence and find the sixth term:
   i) 5, 10, 15, 20   ii) 8, 12, 16, 20   iii) 131, 125, 119, 113, 107
Bonus
What operation was performed on each term in the sequence to make the next term:
2, 4, 8, 16… (HINT: neither “add”, nor “subtract”. ANSWER: Multiply the term by 2.)

Extensions
1. Find the mistake in each pattern and correct it.
   a) 2, 5, 7, 11 add 3
   b) 7, 12, 17, 21 add 5
   c) 6, 8, 14, 18 add 4
   d) 29, 27, 26, 23 subtract 2
   e) 40, 34, 30, 22 subtract 6

2. Divide the students into groups or pairs. One player writes a rule, the other has to write a sequence according to the rule, but that sequence must have one mistake in it. The first player has to find the mistake in the sequence. Warn the students that making mistakes on purpose is harder than simply writing a correct answer, because you have to solve the problem correctly first! The easiest version is to make the last term wrong. More challenging is to create a mistake in any other term.

3. Find the missing number in each pattern. Explain the strategy you used to find the number.
   a) 2, 4, ________, 8
   b) 9, 7, ________, 3
   c) 7, 10, ________, 16
   d) 16, ________, 8, 4
   e) 3, ________, 11, 15
   f) 15, 18, ________, 24, ________, 30
   g) 14, ________, ________, 20
   h) 57, ________, ________, 45

4. One of these sequences was not made by a rule. Find the sequence and state the rules for the other two sequences. (Identify the starting number and the number added or subtracted.)
   a) 25, 20, 15, 10
   b) 6, 8, 10, 11
   c) 9, 12, 15, 18

5. Students should know the meaning of “term” and should be able to connect each term in a sequence with its term number. (For instance, in the sequence 4, 10, 16, 22, the first term is 4, the second term is 10, and so on.)

Here are some questions that will give students practice with this skill:
   a) What is the third term of the sequence 2, 4, 6, 8?
   b) What is the fourth term of the sequence 17, 14, 11, 8?
   c) Extend each sequence and find the sixth term.
i) 5, 10, 15, 20  
ii) 8, 12, 16, 20  
iii) 111, 125, 119, 113, 107

d) What operation was performed on each term in the sequence to make the next term:  
2, 4, 8, 16… (Answer: Multiply the term by 2.)

6. From the description of each sequence, find the 4th term of the sequences:

a) Start at 5 and add 3.
b) Start at 40 and subtract 7.

---

**PA5-8**

**Introduction to T-tables**

Draw the following sequence of figures on the board and tell your students that the pictures show several stages in the construction of a castle made of blocks:

![Sequence of figures]

Ask your students to imagine that they want to keep track of the number of blocks used in each stage of the construction of the castle (perhaps because they will soon run out of blocks and will have to buy some more). A simple way to keep track of how many blocks are needed for each stage of the construction is to make a T-table (the central part of the chart resembles a T—hence the name). Draw the following table on the board and ask students to help you fill in the number of blocks used in each stage of construction.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

Ask students to describe how the numbers in the table change—they should notice that the number of blocks in each successive figure increases by 2, or that the difference between successive terms in the right hand column is 2. Write the number 2 (the “gap” between terms) in a circle between each pair of terms.
Ask students if they can state a rule for the pattern in the table (Start at 4 and add 2) and predict how many blocks will be used for the fifth figure. Students should see that they can continue the pattern in the chart (by adding the gap to each new term) to find the answer. Point out that the T-table allows them to calculate the number of blocks needed for a particular structure even before they have built it.

Draw the following sequence of figures on the board and ask students to help you make a T-table and to continue the table up to five terms.

Before you fill in any numbers, ask your students if they can predict the gap between terms in the T-table. They should see—even without subtracting terms in the table—that the gap between terms is 3 because the castle has three towers and you add one block to each tower at every stage of the construction.

Draw the following T-table on the board:

<table>
<thead>
<tr>
<th>Figure 1</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>23</td>
</tr>
</tbody>
</table>

Tell students that the T-table gives the number of blocks used at each stage in the construction of a castle. The castle was built in the same way as the others (towers are separated by a gate with a triangular roof, and one block is added to each of the towers at each stage), but this particular castle has more towers. Ask students if they can guess, from the pattern in the number of blocks, how many towers the castle has. From the fact that the gap is 5, students should see that the castle must have five towers. Ask a student to come to the board to draw a picture of the first stage in the building of the castle. Then ask students to help you extend the T-table to five terms by adding the gap to successive terms.

After completing these exercises, give students a quiz with several questions like Questions 1 a) and 2 a) on the worksheet, or have them work through the actual questions on the worksheet. As a final exercise, ask students to imagine that you have 43 blocks and that you would like to build a castle with five towers each seven blocks high. Do you have enough blocks? (Remember that there are 8 non-tower blocks.) Students should check if there are enough blocks by extending the T-table for the castle with five towers to seven lines (since each line in the T-table corresponds to the growth of each tower by one block).
Ask your students to construct a sequence of shapes (for instance, castles or letters of the alphabet) that grow in a fixed way. You might also use pattern blocks for this activity.

**ACTIVITY 1**

![Figure of blocks](image)

Ask students to describe how their pattern grows and to predict how many blocks they would need to make the 6th figure. (If students have trouble finding the answer in a systematic way, suggest that they use a T-table to organize their calculation.) This activity is also good for assessment.

**ACTIVITY 2**

Give each student a set of blocks and ask them to build a sequence of figures that grows in a regular way (according to some pattern rule) and that could be a model for a given T-table. Here are some sample T-tables you can use for this exercise.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

**Extensions**

1. The following T-table gives the number of blocks used in building a structure. The same number of blocks were added each time. Fill in the missing numbers.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>25</td>
</tr>
</tbody>
</table>

2. A castle was made by adding one block at a time to each of four towers. Towers are separated by a gate with a triangular roof. Altogether 22 blocks were used. How high are the towers? How many blocks are not in the towers?
3. Claude used one kind of block to build a structure. He added the same number of blocks to his structure at each stage of its construction. He made a mistake though in copying down the number of blocks at each stage. Can you find his error and correct it?

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
</tr>
</tbody>
</table>

4. You want to construct a block castle following the steps shown below. You would like each tower to be 5 blocks high. Each block costs five cents and you have 80 cents altogether. Do you have enough money to buy all the blocks you need? (HINT: make a T-table with three columns: Figure, Number of Blocks and Cost).

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**PA5-9**

**T-tables**

**GOALS**
Students will create a T-table for growing patterns and identify rules of number patterns using T-tables.

**PRIOR KNOWLEDGE REQUIRED**
Addition  | Skip counting
Subtraction | Number patterns

**VOCABULARY**
T-table  | repeating pattern
cart chart | growing pattern
term      | line segment

Draw the following figure on the board and ask students to count the number of line segments (or edges) in the figure.

![Figure 1](image)

Suggest that students mark the edges as they count so they don’t miss any.

Raise the bar by drawing a sequence of more complex figures.

![Figure 2](image)

After students have counted the line segments in the two figures shown above, ask them if they can see any pattern in the number of line segments: Can they predict the number of segments in the next figure? Draw the figure so they can test their predictions.

When you are sure that students can count the line segments without missing any, let them answer **QUESTIONS 1 TO 5** on the worksheet.
Bonus
How many line segments will the 5th figure have?

Additional Problems
If the first term of a sequence is the same as the gap in the sequence (as in QUESTION 6 on the worksheet), then it is particularly easy to extend the sequence: you just have to skip count by the first number in the sequence.

Here are some problems using this idea.

Jane grows flowers, reads a book, and saves money from babysitting. Extend the charts.

<table>
<thead>
<tr>
<th>Days</th>
<th>Pages Read</th>
<th>Days</th>
<th>Flowers Opened</th>
<th>Days</th>
<th>Money Saved</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>2</td>
<td>6</td>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>3</td>
<td></td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

In many sequences that students will encounter in life—for instance in a recipe—two quantities will vary in a regular way.

If a recipe for muffins calls for 3 cups of flour for every 2 cups of blueberries, you can keep track of how many cups of each ingredient you need by making a double chart.

<table>
<thead>
<tr>
<th>Number of Muffin Trays</th>
<th>Number of Cups of Flour</th>
<th>Number of Cups of Blueberries</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>6</td>
</tr>
</tbody>
</table>

QUESTION 7 on the worksheet, and several questions below, can be solved using a double chart.
Extensions

1. Tommy’s father bakes cookies for Tommy’s birthday. He needs at least 5 cookies per guest and 20 cookies for the family. For each cookie he needs 10 chocolate chips. He has a bag of 400 chocolate chips. Tommy invited 5 guests. Does he have enough chips? How many guests does he have enough chips for? (Luckily, Tommy does the calculation and goes to buy some more chips—to help Dad.)

If students have trouble with this question you might suggest that they make a double table to keep track of the number of cookies needed for the guests.

<table>
<thead>
<tr>
<th>Guests</th>
<th>Number of Cookies</th>
<th>Number of Chips</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>250</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>300</td>
</tr>
<tr>
<td>3</td>
<td>35</td>
<td>350</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
After they have completed the table for the guests, they can add on the number of chips needed for the family’s cookies.

2. Ask students to write a description of how the patterns in QUESTIONS 3 and 4 in section PA5-9 are made. They should explain precisely how each element differs from the one before it.

**PA5-10**  
Problems Involving Time

Explain that today you are going to use T-tables to solve realistic problems— involving money and time. Students will need to solve problems of this kind often in their lives, so it is important to know how to approach the problem systematically. Make sure all of your students know the days of the week and the months of the year.

Give an EXAMPLE: Jenny started babysitting in the middle of June and earned $10 in June. She continued throughout the summer and earned $15 every month after that. She saved all the money. How much money did she save through the whole summer? Make a chart:

<table>
<thead>
<tr>
<th>Month</th>
<th>Savings ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>June</td>
<td>10</td>
</tr>
<tr>
<td>July</td>
<td>25</td>
</tr>
<tr>
<td>August</td>
<td>40</td>
</tr>
</tbody>
</table>

Write only the months in the left column and 10 in the right column. Ask the students how much money was added in July, then write “+15” in the upper circle. Ask the volunteers to continue the pattern and to finish the problem.

Let your students practice with more problems, like:

1. Lucy’s family spends the holidays hiking the Bruce Trail. They started on Saturday going 25 km. Seeing they will never do the distance they planned at that pace, they hike now 30 km every day. Make a chart to show their progress. How far did they go along the trail at the end of the holiday, next Sunday?

2. Natalie started saving for her Mom’s birthday present in March. She saved $8 in March and $5 every month after that. Her Mom’s birthday is July 1. Will she have enough money to buy a present that costs $22? Make a T-table to show her savings.

Use volunteers to solve another problem:

Ronald received $50 as a birthday present on March 1. Every month he spends $8. How much money will remain by the end of June? When will Ronald spend the last of the money?

Allow your students to practice more decreasing patterns.
Assessment
1. There are 15 crates of strawberries in a market at 4:00 pm. Every hour 4 crates are sold. How many crates are left in the market at 7:00 pm?
2. A conservation centre is breeding a group of critically endangered Florida Panthers. The group now consists of 17 animals. Every year 6 new panthers are born. Make a T-table to show how many panthers will be at the centre in the years 2007-2011.

Bonus
Jeremy’s maple sapling grows 5 cm in June. It grows 6 cm each month after that. Judy’s sapling grows 4 cm in June. It grows 7 cm each month after that. Whose sapling is higher by the end of September? (Both start out the same height.)

ACTIVITY
Divide your students into pairs, give them play money and let them play a selling and buying game: One is a seller, the other is a buyer. Let them record the money they have after each “sell” in a T-chart—one for the buyer and another one for the seller.

Point out that the “buyer’s” money is shrinking—and explain that this analog of decreasing sequence is called a shrinking pattern. What other patterns do they know? Was the “seller’s” money shrinking, growing or repeating?

Extension
Imogen’s candle is 23 cm high when she lights it at 7 pm. It burns down 3 cm every hour. Fiona’s candle is 22 cm high when she lights it at 7 pm. It burns down 4 cm every hour. Whose candle is taller at 11 pm?
Remind your students that the general word for a member of a sequence is “term”.

Ask students to predict whether the 7th term of each sequence below will be greater or less than 100. Then have them check by continuing the sequence:

a) 70, 75, 80, ...

b) 81, 84, 87, ...

c) 49, 56, 63, ...

Ask students to find the missing term in each sequence.

a) 8, 12, ______, 20

b) 11, ______, ______, 26

c) 15, ______, ______, 24, ______

d) 59, ______, ______, ______, 71

Each T-Table was made by adding a number repeatedly. Find and correct any mistakes in the tables.

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>19</td>
</tr>
<tr>
<td>5</td>
<td>21</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>34</td>
</tr>
<tr>
<td>4</td>
<td>37</td>
</tr>
<tr>
<td>5</td>
<td>41</td>
</tr>
</tbody>
</table>

Give students several word problems with larger numbers. For EXAMPLE:

Ernie is saving money for a playstation that costs $300. He has saved $150. He saves $40 every week. How many weeks will it take before he can buy the new playstation?

Suggest to your students that they make a T-table showing the money that Ernie has saved. How can this help them find the answer? What do we actually need to find to solve the problem? (The number of the first term that is larger than 300).

Extension

1. From the description of each sequence, find the 4th term of the sequences:

   a) Start at 5 and add 3.
   b) Start at 40 and subtract 7.
2. A newborn Siberian Tiger cub weighs 1 300 g. It gains 100 g a day. A newborn baby weighs 3 300 g. It gains 200 g every week.
   
a) A cub and a baby are born on the same day. Who weighs more after...
   
i) 2 weeks?  
   ii) 6 weeks?
   
b) After how many weeks would the cub and the baby have the same weight?

---

**GOALS**

Students will find the core of the pattern and continue the repeating pattern.

**PRIOR KNOWLEDGE REQUIRED**

Ability to count  
Attributes

**VOCABULARY**

attribute  
length of core  
core

---

**PA5-12**

**Repeating Patterns**

Explain that a pattern is repeating if it consists of a “core” of terms or figures repeated over and over. Show an EXAMPLE:

```
   O   O   O   O   O   O   O   O
   O   O
```

The core of the pattern is the part that repeats. Circle it. Write several more examples, such as:

```
   O   O   O   O   O   O   O   O
   O   O   O   O   O   O   O   O
   O   O   O   O   O   O   O   O
```

c) R A T R A T R A  
d) S T O P S T O P S  
e) u 2 u 2 u 2 u 2 u

Show the pattern M O M M O M and extend it as follows: M O M M O M M O M. Ask students if they agree with the way you extended the sequence. You may pretend not to see your mistake or even to ask the students to grade your extension. Give more difficult examples of sequences where the core starts and ends with the same symbol, such as:

```
   △  △  △  △  △  △  △  △  △  △  △
```

b) A Y A Y A Y A Y
c) 1 2 3 1 1 2 3 1 1

Warn your students that these patterns are trickier.

Explain that the “length” of the core is the number of terms in the core. For instance, the pattern 3, 2, 7, 3, 2, 7... has a core of length 3. It’s core is “3, 2, 7”. Ask volunteers to find the length of the cores for the examples you’ve drawn.
Assessment
Circle the core of the pattern, and then continue the pattern:

C A T D O G C A T D O G C A T _________________
Core length: ___________

A M A N A A M A N A A M A _________________
Core length: ___________

Extensions
1. Can you always find the core of a pattern that repeats in some way? Do the following sequences have a core? No. Can you still continue the pattern? Yes.

a) __ __ __ __ __ __

b) __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __ __
Remind your students how to recognize the core of a pattern. Draw a sequence of blocks with a simple pattern like the shown one below (where B stands for blue and Y for yellow).

```
BYBBY
```

Then draw a rectangle around a set of blocks in the sequence and follow the steps below.

```
BYBBY
```

**STEP 1:**
Ask students to say how many blocks you have enclosed in the rectangle (in this case, three).

**STEP 2:**
Ask students to check if the pattern (BYB) inside the rectangle recurs exactly in the next three blocks of the sequence, and in each subsequent block of three, until they have reached the end of the sequence (if you had enclosed four blocks in the rectangle, then students would check sets of four, and so on). If the pattern in the rectangle recurs exactly in each set of three boxes, as in the diagram above, then it is the core of the sequence. Otherwise students should erase your rectangle, guess another core and repeat steps 1 and 2. (Students should start by looking for a shorter core than the one you selected.)

You could also have your students do this exercise with coloured blocks, separating the blocks into sets, rather than drawing rectangles around them.

Draw several sequences on the board and, for several of the sequences, draw a box around a group of numbers or symbols that is not the core. Ask your students to grade your work. If they think you have made a mistake, ask them to correct the mistake.

**EXAMPLES:**

```
AABBAAABB
ACACACACA
GYGGYGGY
```

Do a quick assessment to make sure everyone is able to find the core and its length in the pattern:

```
REDDREDD
DADDDAADDAGREGGREG
```

Ask your students if they remember what special word mathematicians use for the figures or numbers in patterns or sequences. Write the word “term” on the board.
Draw a pattern on the board:

Circle the core of the pattern, find its length and continue the pattern. Ask which terms are circles. What will the 20th term be—a circle or a diamond? Write down the sequence of the term numbers for the circles (1, 3, 5, 7, …) and diamonds (2, 4, 6, 8, …), and ask which sequence the number 20 belongs to.

For more advanced work, students could try predicting which elements of a pattern will occur in particular positions. For instance, you might ASK: “If the pattern below were continued, what would the colour of the 23rd block be?”

A method of solving this sort of problem using a hundreds chart is outlined on the worksheet. Students could also use number lines, rather than the hundreds charts to solve the problem as shown below.

**STEP 1:**

Ask a volunteer to find the length of the core of the pattern:

The core is five blocks long.

**STEP 2:**

Let another volunteer mark off every fifth position on a number line and write the colour of the last block in the core above the marked position.

The core ends on the 20th block and starts again on the 21st block. You might even group the core with an arc. Ask at which term the new core starts (21). Let another volunteer write the letters of the new core in order on the number line starting at 21: the 23rd block is yellow.

Eventually you should encourage students to solve problems like the one above by skip counting. The students might reason as follows:

“I know the core ends at every fifth block so I will skip count by 5s until I get close to 23. The core ends at 20, which is close to 23 so I’ll stop there and write out the core above the numbers 21, 22 and 23.”

The 23rd block is yellow.

I know the core ends here. A new core starts again here.
To give practice predicting terms; you might wish to use the activity games at this point.

**Assessment**
What is the 20\textsuperscript{th} term of the pattern: A N A A N A N A N A N A A?
What is the 30\textsuperscript{th} term?

**Bonus**
Find terms 20, 30, 40, 50, \ldots, 100 of the pattern:

\[ R W W R W W R W R W R W W R W W \ldots \]

The students will need pattern blocks or coloured beads and one or two dice for all of the following activities:

1. Player 1 rolls the die so that the partner does not see the result. Then he builds a sequence of blocks or beads with the core of the length given by the die. Player 2 has to guess the length of the core and to continue the sequence.

2. Player 1 throws two dice (unseen by Player 2), and uses the larger number as the core length and the smaller as the number of different blocks or beads to use. For example, if the dice show three and four, the core length is four but only three types of beads might be used. **EXAMPLE:** RGYR GYY.

3. Player 1 throws a die (unseen by Player 2), and uses the number as the core length for his sequence. Player 2 throws the die and multiplies the result by 10. He has to predict the bead for the term he got. For instance, if Player 2 got three on the die, he has to predict the 30\textsuperscript{th} term of the sequence.

**Extensions**

1. Ask students to use the word “term” in their answers to questions 11 and 12; **EXAMPLE:** for **QUESTION 12 a)** they might write: “The 19\textsuperscript{th} term is a penny.” Here are some questions using the word “term”:

   a) What is the 16\textsuperscript{th} term in this sequence?
   \[ R Y Y R Y Y R Y Y \]

   b) What is the 50\textsuperscript{th} term in this sequence?
   \[ \square \bigtriangleup \bigtriangleup \bigtriangleup \square \bigtriangleup \bigtriangleup \bigtriangleup \]

   c) What direction will the 53\textsuperscript{rd} arrow be pointing in? Explain.
   \[ \uparrow \rightarrow \leftarrow \uparrow \rightarrow \leftarrow \uparrow \]

   d) What will the 47\textsuperscript{th} term of the pattern look like?
   \[ \square \bigtriangleup \bigtriangleup \bigtriangleup \]

\[ \square \bigtriangleup \bigtriangleup \bigtriangleup \]
2. Sometimes in a repeating pattern the repetition does not begin straight away. For example, the leaves of many plants have the same shape, which means they make a repeating pattern. But the first two leaves that spring from the seed often have a different shape! The pattern starts repeating after the first two “atypical” terms. **OR:** A B C B C B C B C is a repeating pattern with core B C.

Ask your students to find the core in these patterns:

![Patterns](image.png)

**PA5-14**

**Number Lines**

**GOALS**

Students will solve simple problems involving decreasing sequences using number lines.

**PRIOR KNOWLEDGE REQUIRED**

Decreasing sequences
Number lines

**VOCABULARY**

decreasing sequence
term
number line

---

Write a sequence on the board: 36, 31, 26, 21. Ask your students if they remember what this kind of sequence is called (a decreasing sequence).

Ask what the difference between the terms in the sequence is. Ask another volunteer to continue the sequence and to explain to you how they continued the sequence. They may say that they counted backwards or used their number facts to subtract. Ask the students what they would do if the numbers were large, so that counting backwards or subtracting would be more difficult. In this case they can use a number line to solve the problem. That’s what you will teach them today.

Present the following **EXAMPLE:** Karen is driving from Vancouver to Edmonton, which is 1 200 km away. She can drive 300 km a day. She starts on Tuesday. How far from Edmonton is she on Thursday night?

Draw a number line:

```
Edmonton       Vancouver
```

Draw the arrows showing her progress every day. You could **ASK:**

When will she reach Edmonton? (Friday evening).

Kamloops is 800 km away from Edmonton. Invite a volunteer to mark Kamloops on the number line. When would Karen pass Kamloops?

Jasper National park is 790 km from Vancouver. When will she pass Jasper?

Ask a volunteer to mark the approximate position of Jasper on the number line.
Let your students draw number lines in their notebooks: from 0 to 50 skip counting by 5s, from 0 to 30 skip counting by 2s, from 0 to 45 skip counting by 3s, from 0 to 80 skip counting by 4s. Here are some problems they could practice with:

A magical post-owl flies 15 km in an hour. How long will it take this owl to reach a wizard that lives 50 km away from the sender?

Jonathan has $30. He spends $8 a week for snacks. How much money will he have after three weeks? When will his money be completely spent?

A snail crawls 6 cm in an hour. It is 45 cm away from the end of the branch. How far from the end of the branch will it be after four hours of crawling?

A dragonfly flies 8 m in a second (You can mention that it is one of the fastest insects, because it hunts other insects). It spots a fly on a tree 80 m away. How far is the dragonfly from its prey after seven seconds of flight?

Point out to students that for problems with large numbers, they may want to use a scale with intervals of 100, 1000, or other multiples of 10. Which scale would be best for the next problem: 10, 100, 1000?

A horse runs 30 km per hour. An errand- rider has to go to a fort that is 200 km from the King’s palace. How far is the rider from the fort after three-hour ride?

**Assessment**

Draw a number line from 0 to 15 and solve the problem:

Jane can walk two blocks in a minute. She is 15 blocks away from home. How far from home will she be in six minutes? How long will it take her to reach home?

**Bonus**

Draw a number line from 330 000 to 380 000 skip counting by 5 000s. A rocket travels from the moon toward the earth at the speed of about 5 000 km an hour. The moon is 380 000 km from the earth. How far from the earth will the rocket ship be after five hours of flight? After seven hours?

**Extension**

What number is represented by the X?

| 89 000 | X | 89 500 |

---
PA5-15
Number Lines (Advanced)

If students do not know how to divide by skip counting you might wait until you have covered NUMBER SENSE PART 2 before you do this lesson.

Explain that today you are going to make the task your students have to do harder—not only will they need to solve the problem using a number line, but they will also have to decide which number line to use for a particular problem. Explain that the number they skip count by to mark the number line is called the “scale.”

Start with an easy problem: ask your students to choose a scale for a number line (say 5) and have a volunteer draw a number line (skip counting by 5s as they subdivide the line). Now ask your students to suppose that the distance involved in a problem is 22 km. Will their scale be convenient? Which scale would be more convenient? (A scale of 2, because two divides into 22 evenly.)

Present the following problem: John and Jane are 20 km from home. John walks at a speed of 5 km an hour. Jane rollerblades at 10 km an hour. How long will it take each of them to get home? Draw a number line without any subdivisions and ask students which scale would be convenient. Five will work for both speeds but 10 will not be a convenient scale for the distance John moves in an hour.

Suppose Jane rollerblades at a speed of 10 km an hour, but she has 200 km to go. Ask students if they would use a scale of 5 for a number line showing how far she travels in a given time. They should see that a number line with a scale of 10 would have fewer subdivisions and be easier to draw.

Ask students to draw a number line for Rachel who bikes at a speed of 15 km an hour and has 75 km to go. Give them the choice of numbers for skip counting: 10, 15, or 30. Invite volunteers to draw arrows to show how far Rachel travels each hour. Which scale is the most convenient? (15.) Why was 30 inconvenient? (It is larger than the speed.) Why was 10 inconvenient? (It does not divide the speed, so the first arrow points between the marks.) Students should see that a convenient scale should divide into the speed evenly, but be large enough so that you avoid long number lines.

It is often convenient to use the speed of the person who is travelling as the scale. But sometimes this approach will not work. If a person travels at 10 km per hour but is travelling 25 km, a scale of 10 on the number line won’t work well. Students should see that they will have to use a smaller scale that divides into both 10 and 25 (EXAMPLE: 5) for their scale.

Present another problem: The speed is 60 km in an hour; the distance is 210 km. Which scale could students use? Ask which numbers divide into 60. If a student gives a number less than 10, ask them to approximate the number of subdivisions their number line will have. Students should eventually see that the most convenient scale is 30, as it is the largest number that it divides evenly into 210 and 60.
Summarize

In choosing a scale, start by checking to see if either condition A or B below holds:

A: the speed divides evenly into the distance from home

B: the speed does not divide evenly into the distance but there is a number greater than one that divides into both the speed and the distance from home.

If case A holds, then the student can label each segment on the number line with the speed. For instance, in question a) in the chart below, each segment stands for 3 km, because 3 (the speed) divides 27 (the distance).

| 0  | 3  | 6  | 9  | 12 | 15 | 18 | 21 | 24 | 27 |

After five hours the cyclist will be 12 km from home.

If condition B holds, then the student can let each segment represent the greatest number that divides both the speed and the distance from home evenly. For example, five is the greatest number that divides 10 and 35 evenly. So each segment in the number line for question b) below can represent five km.

NOTE: In question c) below each segment can represent 2 km, and in question d), 25 km. You can use these two questions for assessment as well.

For each question below ask students to determine how far from home the cyclist will be after travelling towards home for the given time.

<table>
<thead>
<tr>
<th>Distance From Home</th>
<th>Speed</th>
<th>Time Spent Traveling</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 27 km</td>
<td>3 km each hour</td>
<td>5 hours</td>
</tr>
<tr>
<td>b) 35 km</td>
<td>10 km each hour</td>
<td>3 hours</td>
</tr>
<tr>
<td>c) 22 km</td>
<td>4 km each hour</td>
<td>4 hours</td>
</tr>
<tr>
<td>d) 250 km</td>
<td>75 km each hour</td>
<td>2 hours</td>
</tr>
</tbody>
</table>

Extensions

1. Yoko’s house is 20 m from the sidewalk. A dog is tied to a tree halfway between the house and the sidewalk. The dog’s leash is 8 m long. How close to the sidewalk can the dog come?

2. Michael’s house is 15 metres from the ocean. He is sitting in a chair 5 metres away from his house. The tide rises 5 metres every hour. How long will it take before his feet get wet?

3. Aaron is training for football. He runs 5 metres forward and 2 metres back every 4 seconds. How far from where he started will he be after 16 seconds?
PA5-16
Lowest Common Multiples

A number A is a multiple of number B if B divides evenly into A (or in other words if there is a third number C such that $C \times B = A$). For example, 15 is a multiple of 3, as there is a number (5) that is multiplied by 3 to give 15. Remind your students that 0 is a multiple of any number. (Why 0 is a multiple of, say, 2? What number is multiplied by 2 to give 0?)

Draw a number line on the board. Ask volunteers to circle all the multiples of 2 (0, 2, 4, 6...) and to underline all the multiples of 5 (0, 5, 10, 15...). What is the least number (greater than 0) that is both circled and underlined? (10) This number is a multiple of both 2 and 5, and it is called the “lowest common multiple of 2 and 5”. Write the term on the board.

Ask your students to find, using number lines, the least common multiples of 2 and 3, 3 and 5, 2 and 6, 4 and 6. To check that your students understand the concept, ask the following questions: “Jasmine says that the lowest common multiple of two numbers is always the product of the numbers. For example, the lowest common multiple of 2 and 3 is $2 \times 3 = 6$, and the lowest common multiple of 2 and 5 is $2 \times 5 = 10$. Is Jasmine correct?” Students should see that the lowest common multiple of two numbers is sometimes less than the product of the numbers. For instance, the lowest common multiple of 4 and 6 is 12, but their product is 24.

Ask your students if they can invent a method to find the lowest common multiple without drawing a number line. Show them how to find the lowest common multiple using skip counting as follows:

To find the lowest common multiple of 4 and 5: count by 4s: 4, 8, 12, 16, 20, 24... After that count by 5s: 5, 10, 15, 20. The number 20 is the first on both lists, so it is the lowest common multiple.

Ask your students to use skip counting to find the lowest common multiples of 2 and 7, 3 and 7, 6 and 10. You may use the activity below for practice.

Tell your students that lowest common multiple are often used in more advanced math, for instance in adding fractions. However, here is a problem that could occur in real life:

Imagine that there are two tasks that a nurse in a hospital has to perform. One has to be done every 3 hours; the other has to be performed every 4 hours. Both tasks are first performed at midnight. The nurse cannot do both things at the same time. When will she need another nurse to help her? (How many hours after midnight?)

(HINT: Make a list of the times when the nurse performs each task. Does this method differs from the method used to find lowest common multiples?)
Assessment
Find the least common multiples of 6 and 9, 5 and 10, 7 and 8.

Jo visits the library every 4th day of the month, starting from the 4th, and swims every 6th day a month, stating from the 6th. When will he swim and go to the library on the same day?

Bonus
How many times in a month will Jo visit the library and the swimming pool on the same day?

ACTIVITY
Each student will need two dice of one colour and two dice of another colour. The player rolls the dice, adds the results of the dice of the same colour and finds the lowest common multiple of the numbers. For EXAMPLE: red: 3 and 6 (3 + 6 = 9), blue: 2 and 4 (2 + 4 = 6). The player has to find the least common multiple of 9 and 6.

Extension
Paul visits the library every 4th day of the month, Herda visits every 6th day, and Nigel every 8th day. On what day of the month will they all visit the library together?
PA5-17
Describing and Creating Patterns

GOALS
Students will identify and describe increasing, decreasing and repeating patterns.

PRIOR KNOWLEDGE REQUIRED
Identify and describe increasing and decreasing sequences
Identify and continue repeating patterns
Find difference between sequence terms

VOCABULARY
increasing sequence
decreasing sequence
repeating pattern
difference

Write a sequence on the board: 3, 5, 7, 9, … Ask your students what this sequence is called. (The numbers grow, or increase, therefore it’s called an “increasing sequence”). Write another sequence: 95, 92, 89, 86, … and ask students what this sequence is called. Write “decreasing sequence” on the board.

Ask your students: Are all the sequences in the world increasing or decreasing? For example, Jennifer saved $80. She receives a $20 allowance every two weeks. She spends money occasionally:

<table>
<thead>
<tr>
<th>Date</th>
<th>Balance ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sep 1</td>
<td>80</td>
</tr>
<tr>
<td>Sep 5</td>
<td>100</td>
</tr>
<tr>
<td>Sep 7</td>
<td>89</td>
</tr>
<tr>
<td>Sep 12</td>
<td>79</td>
</tr>
<tr>
<td>Sep 19</td>
<td>99</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date</th>
<th>Balance ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sep 1</td>
<td>80 +20 Allowance</td>
</tr>
<tr>
<td>Sep 5</td>
<td>100 -11 Book</td>
</tr>
<tr>
<td>Sep 7</td>
<td>89 -10 Movie + Popcorn</td>
</tr>
<tr>
<td>Sep 12</td>
<td>79 +20 Allowance</td>
</tr>
</tbody>
</table>

Jennifer’s money pattern increases between the first two terms, decreases for the next two terms, and then increases again.

Write several sequences on the board and ask volunteers to put a “+” sign in any circle where the sequence increases, and the “-” sign where it decreases. Let your students practice this skill, and after that ask them to mark the sequences: “I” for increasing, “D” for decreasing, and “B” (for “both”) if the sequence sometimes increases and sometimes decreases.

Assessment

h) 1 , 5 , 8 , 3
i) 18 , 13 , 17 , 23
j) 28 , 26 , 19 , 12
k) 17 , 8 , 29 , 25
l) 53 , 44 , 36 , 38

Bonus

a) 257 , 258 , 257 , 260
b) 442 , 444 , 447 , 449
c) 310 , 307 , 304 , 298
d) 982 , 952 , 912 , 972
After your students have finished the assessment, explain that now their task will be harder: they will have find not only if the sequence is increasing or decreasing but also the magnitude of the difference between successive terms. Invite volunteers to find differences by counting forwards and backwards, both using their fingers and number lines. Use simple examples such as:

a) 7, 9, 11, 13  

b) 2, 5, 9, 12  

c) 10, 7, 4, 1  

d) 32, 36, 40, 44  

e) 15, 23, 31, 39  

f) 72, 52, 32, 12  

Invite volunteers to identify the difference as follows:

Put a “+” sign in the circle where the sequence increases and a “–” sign when it decreases. Then write the difference between the terms:

a) 7, 10, 11, 13  

b) 12, 8, 7, 12  

c) 20, 17, 14, 21  

d) 32, 46, 50, 64  

e) 15, 24, 33, 42  

f) 77, 52, 37, 12  

Give several possible descriptions of sequences, such as:

A: Increases by different amounts  

B: Decreases by different amounts  

C: Increases by constant amount  

D: Decreases by constant amount  

Ask your students to match the descriptions with the sequences on the board. When the terms of the sequence increase or decrease by the same amount, you might ask students to write a more precise description. For example, the description of sequence e) could be “Increases by 9 each time.” or “Start at 15 and add 9 each time.”  

Next let your students do the activity below.

**Assessment**  
Match description to the pattern. Each description may fit more than one pattern.

17, 16, 14, 12, 11  

3, 5, 7, 3, 5, 7, 3  

A: Increases by the same amount  

10, 14, 18, 22, 26  

4, 8, 11, 15, 18,  

B: Decreases by the same amount  

54, 47, 40, 33, 26  

A: Increases by different amounts  

4, 8, 12, 8, 6, 4, 6  

11, 19, 27, 35, 43  

D: Decreases by different amounts
74, 69, 64, 59, 54 ... 12, 15, 18, 23, 29 ... E: Repeating pattern
98, 95, 92, 86, 83 ... 67, 71, 75, 79, 83 ... F: Increases and decreases

For patterns that increase or decrease by the same amount, write an exact rule.

**Bonus**

Match description to the pattern.

97, 96, 94, 92, 91 ... 3, 5, 8, 9, 3, 5, 8, 9 ... A: Increases by the same amount
210, 214, 218, 222 ... 444, 448, 451, 456 ... B: Decreases by the same amount
654, 647, 640, 633 ... 741, 751, 731, 721 ... C: Increases by different amounts
D: Decreases by different amounts
E: Repeating pattern
F: Increases and decreases by different amounts

For patterns that increase or decrease by the same amount, write an exact rule.

**A Game for Pairs**

Your students will need the spinner shown. Player 1 spins the spinner so that Player 2 does not see the result. Player 1 has to write a sequence of the type shown by the spinner.

Player 2 has to write the rule for the sequence or describe the pattern. For example, if the spinner shows, "Decreases by the same amount," Player 1 can write "31, 29, 27, 25." Player 2 has to write "Start at 31 and subtract 2 each time."
PA5-18
2–Dimensional Patterns

GOALS
Students will identify and describe patterns in 2-dimensional grids.

PRIOR KNOWLEDGE REQUIRED
Identify increasing and decreasing sequences
Describe increasing and decreasing sequences
Identify repeating patterns

VOCABULARY
increasing sequence
decreasing sequence
repeating pattern
column
row
diagonal

Draw a 2-dimensional grid on the board. Remind your students that rows go vertically and columns go horizontally. Show the diagonals as well.

Draw the chart.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>V</td>
<td>D</td>
</tr>
<tr>
<td>R</td>
<td>E</td>
<td>H</td>
</tr>
<tr>
<td>O</td>
<td>I</td>
<td>O</td>
</tr>
<tr>
<td>W</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>L</td>
<td>L</td>
<td></td>
</tr>
</tbody>
</table>

Explain that in 2-dimensional patterns you generally count columns from left to right but there are two common ways of counting rows. In coordinate systems, which they will learn later, you count from the bottom to the top. But in this lesson, you will count rows from the top to the bottom. Ask a volunteer to number the columns and rows on your grid.

Ask your students to draw a 4 × 4 grid in their notebooks and to fill it in:

<table>
<thead>
<tr>
<th>13</th>
<th>17</th>
<th>21</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>14</td>
<td>18</td>
<td>22</td>
</tr>
<tr>
<td>7</td>
<td>11</td>
<td>15</td>
<td>19</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
</tr>
</tbody>
</table>

Ask them to shade the first column, the first row, the third row, and to circle both diagonals. Let them describe the patterns that they see in the shaded rows and columns and in diagonals.

After that allow your students to do the activities below for more practice.

ACTIVITY 1
Shade in the multiples of nine on a hundreds chart. Describe the position of these numbers.

a) Add the ones digit and the tens digit of each multiple of nine. What do you notice?

b) What pattern do you see in the ones digits of the multiples of nine?

c) What pattern do you see in the tens digits of the multiples of nine?
Ask students to circle the multiples of 3 and 4 on a hundreds chart (They should know the multiples of 5 automatically)

Then give students a set of base ten blocks (ones and tens only). Ask them to build base ten representations of the following numbers.

a) You need three blocks to build me. I am a multiple of five. (ANSWER: 30)
b) You need six blocks to build me. I am a multiple of five. (There are two ANSWERS: 15 and 60).
c) You need five blocks to build me. I am a multiple of four. (ANSWER: 32)

CHALLENGING: Find all the possible answers:
d) You need six blocks to build me. I am a multiple of three. (There are six ANSWERS: 6, 15, 24, 33, 42, and 51)

Use a hundreds chart to solve the following problems.

a) I am a number between 30 and 100. My tens digit is 2 more than my ones.
b) I am an odd number between 35 and 50. The product of my digits is less than the sum.
c) Shade in the multiples of 9 on a hundreds chart. Describe the position of these numbers.

Shade a diagonal on the hundreds chart from left to right. Add the ones digit and the tens digit of each number. What do you notice? Students should notice that the sum of the digits of the numbers in the diagonal increases by 2 as you move down the diagonal. Ask your students to explain why this happens. (Answer: As you move down the diagonal you move down one row, which increases the tens digit by 1 and you move across one column, which increases the ones digit by 1.)

**Extensions**

1. Students will need the hundreds chart BLM. Write the following patterns on the board and ask students to identify in which rows and columns in the hundreds chart the patterns occur:

   |   |   |   |   |
---|---|---|---|---|
| 3 | 38 | 43 | 12 |
| 13| 48 | 54 | 23 |
| 23| 58 | 63 | 34 |
| 33| 68 | 74 | 45 |

After students have had some practice at this, ask them to fill in the missing numbers in the patterns (challenge them to do this without looking at the hundreds chart).

|   |   |   |   |
---|---|---|---|
| 12| 57 | 7 | 45 |
| 22| 77 | 18 | 56 |
| 32| 87 | 38 | 67 |
Make up more of these puzzles for students who finish their work early.

2. Add the digits of some of 2-digit numbers on the hundreds chart. Can you find all the numbers that have digits that add to ten? Describe any pattern you see.

3. Sudoku is an increasingly popular mathematical game that is now a regular feature in many newspapers. In the BLM section of this guide you will find Sudoku suitable for children (with step-by-step instructions to find solutions). Once students master this easier form of Sudoku they can try the real thing.

4. Complete the multiplication chart. Describe the patterns you see in the rows, columns and diagonals of the chart.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**PA5-19**

**Number Pyramids**

**GOALS**
Students will identify and describe patterns they see in two-dimensional pyramids.

**PRIOR KNOWLEDGE REQUIRED**
Addition  
Subtraction

**VOCABULARY**
column  
row  
difference

**More Puzzles**
Draw several number pyramids like the ones below on the board and ask the students to explain what the rule is.

```
   9
  4 5
```
```
  5
2 3
```
```
  6
4 2
```

In these number pyramids, every number above the bottom row is the sum of the two numbers directly below it. Draw an example with a missing number. Ask your students, how they could find the missing number.

```
  2 3
```
```
  5
2 3
```

Ask your students, how they could find the missing number in the next example. This time the missing number can be found using subtraction:

```
  7
3   
```
```
  7
3 4
```
The number pyramids on the worksheets for this section can be solved directly by addition and subtraction (students should start by finding a part of each pyramid ( ), where two squares are already filled).

Here is a bonus question you can assign your students that can be solved by trial and error.

Students should guess the number in the middle square of the bottom row, then fill in the squares in the second row, checking to see if they add up to the number in the top square. A good way to do so is by using a T-table:

<table>
<thead>
<tr>
<th>The Number in the Middle of the Bottom Row</th>
<th>The Numbers in the Middle Row</th>
<th>The Number on the Top</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5, 6</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>6, 7</td>
<td>13</td>
</tr>
</tbody>
</table>

Even with larger numbers the students would soon be able to see that the numbers in the last column grow by two each time. This will allow them to reach the right number quickly.

Students might also notice the following trick: adding the two numbers in the bottom row, subtracting their sum from the number in the top row, then dividing the result by two gives the missing number in the bottom row. To see why this trick works, notice that the missing number in the bottom row appears in the second row twice, as in the following example, where the missing number is represented by the letter A:

As the number in the top row is the sum of the numbers in the middle row, we have:

\[
15 = 4 + A + 5 + A \quad \text{OR:} \quad 15 = 2A + 9
\]

Hence we have the rule stated above: to find the missing number A, subtract the sum of the two numbers in the bottom row \((4 + 5 = 9)\) from the number in the top row (15):

\[
15 - 9 = 2A
\]

Then divide the result by 2:

\[
6 \div 2 = A \quad \text{SO:} \quad A = 3
\]

This exercise can be used to introduce more advanced students to algebra.
Extensions

1. Draw the charts below on the board and ask the students to explain what the rule is. (The sum of the numbers in the top row and in the bottom row is the same. You may ask them to look at the sums in the columns and ask if they see a pattern.)

```
7  8  
3  12 
```

```
9  6  
1  14 
```

```
4  11 
   6  
```

ADVANCED: Adding the numbers in each row and each column gives the same number.

```
4  7  9  
8  9  3  
5  1  5  
```

Find the missing numbers. (HINT: Start with the row or column where only one number is missing.

```
7  2  5  
   3  
4  6  
```

```
7  6  9  
   4  
8  9  
```

2. Find the missing numbers in these pyramids:

```
  35  
 24  
  8  16  
```

```
  29  
 32  
  4  24  
```
PA5-20
Patterns in the Times Tables

Do the activity in question 1 on the worksheet. You might wish to explain where the name “even” comes from. Give your students 20 small objects, like markers or beads or ten base unit blocks to act as counters. Ask them to take 12 blocks and to divide the blocks into two equal groups. Does 12 divide evenly? Ask them to do the same with 13, 14, 15 and any other number of blocks. When the number of blocks is even, they divide evenly. If they do not divide evenly, the number is called “odd”. Write the on the board: “Even numbers: 2, 4, 6, 8, 10, 12, 14… All numbers that end with _____”. Ask volunteers to finish the sentence. After that write: “Odd numbers: 1, 3, 5, 7, 9, 11, 13… All numbers that end with _____ ” and ask another volunteer to finish that sentence. After that you might do the activities.

Assessment
Circle the even numbers. Cross the odd numbers.

23, 34, 45, 56, 789, 236, 98, 107, 3211, 468021.

Let students do the activity in question 2 on the worksheet. Then write several numbers with blanks...

After students have played the game, write several numbers with blanks for the missing digits on the board and ask your students to fill in each blank so that the resulting number is divisible by five. Ask students to list all possible solutions for each number.

2___ 32___ 8___ 56___ 5__5 ___35 8___0

Assessment
1. Circle the multiples of 5.
   75 89 5 134 890 40 234 78 4 99 100 205 301 45675
2. Fill in the blanks so that the numbers become a multiple of 5. In two cases it is impossible to do so. Put an “X” on them.
   3___ 3__5 3__9 ___0 70___ 5__0 ___3

Bonus
Fill in the blanks so that the numbers become multiples of five, (list all possible solutions). In two cases it is impossible to do so. Put an “X” beside these.

344___ 34___25 136___9
7867___0 456770___ 2345___6780
134600___3 154___0000 678345___
Extensions

1. The colouring page in the BLM section of this guide. (ANSWER: the Quebec flag.)

2. Who am I?
   1. I am a number between 31 and 39. I am a multiple of five.
   2. I am an even multiple of five between 43 and 56.
   3. I am an odd multiple of five between 76 and 89.
   4. I am the largest 2-digit multiple of five.
   5. I am the smallest 3-digit multiple of five.
   6. I am the smallest 2-digit odd multiple of 5.
   7. I am a multiple of 5 and a multiple of 3. I am between 40 and 50.
   8. I am the smallest non-zero even multiple of 5 and 3.
   9. I am the largest two-digit multiple of 5 and 3.
   10. I am the largest two-digit multiple of 3, so that the number one less than I am is a multiple of 5.

Give a group of volunteers each a card with a whole number on it.

1. Ask the rest of the class to give the volunteers orders, such as “Even numbers, hop” or “Odd numbers, raise your right hand.”

2. ADVANCED: Ask a volunteer to skip count by 3s, starting with three. These are the multiples of three. Ask your “numbers”: Are you a multiple of three? Repeat skip counting by 5s or other numbers.

   ADVANCED TASKS: All numbers that are multiples of five or even numbers, clap. All numbers that are odd and are multiples of five, stomp.

ACTIVITY 2

A Game for Pairs

Decide which one of the players will be “pro-5” person. The other will be the “anti-5” person. The anti-5 player starts by adding either 3 or 5 to the number 4. The pro-5 player is then allowed to add either 3 or 5 to the result. Each player then takes one more turn. If either player produces a multiple of 5, the pro-5 player gets a point. Otherwise the anti-5 player gets a point. The players exchange roles and start the game again. Students should quickly see that there is always a winning strategy for the pro-5 player.
PA5-21
Patterns in the Eight Times Tables

GOALS
Students will identify even and odd numbers, multiples of 5 and 3.

PRIOR KNOWLEDGE REQUIRED
Skip counting by 2, 3 and 5
Rows, columns and diagonals

VOCABULARY
column
row
diagonal
multiple

Write the first five multiples of eight vertically—students should see that the ones digit of the multiples decreases by 2 as you move down the columns, and the tens digit increases by 1.

Invite students to explore this pattern by writing the next five multiples of eight in a column. Extension 2 explains how to check if a three-digit number is a multiple of 8.

Assessment
Continue the pattern up to five columns.

Extensions
1. “Who am I?”
   a) I am a number between 31 and 39. I am a multiple of eight.
   b) I am the largest 2-digit multiple of eight.
   c) I am the smallest 3-digit multiple of eight.
   d) I am a two digit number larger than 45. I am a multiple of eight and a multiple of five as well.

2. Check if a three-digit number is a multiple of 8. EXAMPLE: 266.

   STEP 1:
   Circle the stem of the three-digit number (the hundreds and the tens digit):
   266

   STEP 2:
   Skip count by 4s, stop just before 26: 4, 8, 12, 16, 20, 24.
   Write out the pattern for multiples of 8:
   24 8
   25 6
   26 4
   27 2
   28 0

   266 is not on the list, so it is not a multiple of 8.
More **EXAMPLES**: 273 and 256. 273 is odd, it cannot be a multiple of 8. For 256, do Steps 1 and 2 as above. 256 is on the list, so it is a multiple of 8. Ask students to figure out why this method works.

**“Eight-Boom” Game**
The players count up from one, each saying one number in turn. When a player has to say a multiple of eight, he says “Boom!” instead: 1, 2, 3, 4, 5, 6, 7, “Boom!”, 9, … If a player makes a mistake, he leaves the circle.

**ADVANCED VERSION**: When a number has “8” as one of its digits, the player says “Bang!” instead. If the number is a multiple of 8 and has 8 as a digit, one says both. **EXAMPLE**: “6, 7, “Boom, Bang!”, 9,” or “15, “Boom!”, 17, “Bang!”, 19”.

Both games are a fun way to learn the 8 times table, and can be used for multiples of any other number you would like to reinforce.

---

**PA5-22**

**Times Tables (Advanced)**

**GOALS**
Students will reinforce their knowledge of multiples of two, five and eight.

**PRIOR KNOWLEDGE REQUIRED**
Multiples of two, five, eight

**VOCABULARY**
multiple

Review Venn diagrams (**SEE**: the sections on Venn diagrams in **PROBABILITY AND DATA MANAGEMENT PART 1**).

Draw a Venn diagram with the properties:

1. Multiples of five
2. Multiples of two

Ask your students to sort the numbers between 20 and 30 in the diagram.

Draw another Venn diagram, this time with multiples of five in one circle and multiples of eight in the other. Ask your students to sort out the numbers between 30 and 41, then between 76 and 87 into this diagram. Ask volunteers to sort some 3-digit numbers in the diagram (for instance, 125 and 140).

Draw a more complicated Venn diagram:

1. Multiples of two
2. Multiples of five
3. Multiples of eight

Ask volunteers to sort into it the numbers:

5, 8, 14, 15, 27, 28, 56, 40, 25, 30, 99

Ask why there are parts that are empty. (There are no numbers that are multiples of eight and not multiples of two.) Ask your students to add two numbers to each part of the diagram that is not empty.
Assessment
Sort the following numbers into the last diagram:

5, 7, 10, 15, 26, 30, 42, 40, 50

Bonus
Add three 3-digit numbers to each non-empty part of the last diagram.

Extension

“Who am I?”

1. I am a number between 31 and 49. I am a multiple of five and a multiple of two.
2. I am an even multiple of five between 63 and 76.
3. I am an odd multiple of five between 96 and 109.
4. I am the largest 2-digit multiple of five and eight.
5. I am the smallest 3-digit multiple of five and two.
6. I am the smallest 2-digit even multiple of five.
7. I am a multiple of five and two. I am not divisible by eight. I am larger than 33 and smaller than 57.

PA5-23
Circle Charts

GOALS
Students discover patterns in the ones digits of various multiples.

PRIOR KNOWLEDGE REQUIRED
Skip counting
Number lines
Lowest common multiple

VOCABULARY
multiple
ones digit
divisible by
lowest common multiple

Draw a circle on the board and divide it into 10 parts as shown below. Ask volunteers to skip count by 4s on the board, to underline the ones digits of the written numbers and to mark with a dot the underlined digits on the circle chart.

0, 4, 8, 12, ...

Ask students to connect the dots with lines as shown. What pattern can they see? What was the last number before the star closed? (20) Remind your students of what a least common multiple is and ask them to find the least common multiple of 4 and 10. (20) How many rays does the star have?

Draw another circle on the board and repeat the exercise with the number 6. How are the stars for 4 and 6 different and how are they the same? Repeat the exercise for 2 and 8. Ask the students: which stars are drawn in a clockwise direction (2 and 4) and which are drawn in a counter clockwise direction (6 and 8)? Why? Ask students to predict the direction for the star for the number 3. Which number will have a similar star: 1, 5, 7, 9?
Draw a table:

<table>
<thead>
<tr>
<th>Number</th>
<th>Lowest Common Multiple with 10</th>
<th>Number of Rays in the Stars</th>
<th>Star direction (CW or CCW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td>CW</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>5</td>
<td>CW</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>CW</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Continue the table for 4, 6, 7, 8 and 9. Invite volunteers to fill in the information that they have already found and to predict the missing information in the chart. To help students predict the number of the rays, ask them to describe, in the previous cases, the relationship between the numbers, the lowest common multiple of the number and 10, and the number of rays drew. After filling the table, ask your student to check the predictions by drawing the actual patterns.

**Extension**

What can you do to the circle pattern or “star” for the number 4 to derive the star for the number 6? (Reflect it) Where is the mirror line in the circle pattern? Why? Are there other pairs of numbers where the stars are reflections of each other? Why?

What does the star for 5 look like? Try to predict its appearance from the table above. Does it also have a symmetry line? (Yes it has 2 lines of symmetry) Draw BOTH lines.
## PA5 Part 1: BLM List

<table>
<thead>
<tr>
<th>Activity</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Colouring Exercise</td>
<td>2</td>
</tr>
<tr>
<td>Hundreds Chart</td>
<td>3</td>
</tr>
<tr>
<td>Mini Sudoku</td>
<td>4</td>
</tr>
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<td>Number Line Chasing Game</td>
<td>6</td>
</tr>
<tr>
<td>Sudoku—The Real Thing</td>
<td>7</td>
</tr>
<tr>
<td>Sudoku—Warm Up</td>
<td>8</td>
</tr>
</tbody>
</table>
Colouring Exercise

Colour the even numbers blue, and leave the odd numbers white. What do you get?
Hundred Charts

1  2  3  4  5  6  7  8  9  10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48 49 50
51 52 53 54 55 56 57 58 59 60
61 62 63 64 65 66 67 68 69 70
71 72 73 74 75 76 77 78 79 80
81 82 83 84 85 86 87 88 89 90
91 92 93 94 95 96 97 98 99 100

1  2  3  4  5  6  7  8  9  10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48 49 50
51 52 53 54 55 56 57 58 59 60
61 62 63 64 65 66 67 68 69 70
71 72 73 74 75 76 77 78 79 80
81 82 83 84 85 86 87 88 89 90
91 92 93 94 95 96 97 98 99 100
Mini Sudoku

In these mini Sudoku problems, the numbers 1, 2, 3 and 4 are used.

Each number must appear in each row, column and $2 \times 2$ box.

When solving Sudoku problems:

1. Start with a row or square that has more than one number.
2. Look along rows, columns and in the $2 \times 2$ boxes to solve.
3. Only put in numbers when you are sure the number belongs there (use a pencil with an eraser in case you make a mistake).

EXAMPLE:

Here’s how you can find the numbers in the second column:

The 2 and 4 are given so we have to decide where to place the 1 and the 3.

There is already a 3 in the third row of the puzzle so we must place a 3 in the first row of the second column and a 1 in the third row.

Continue in this way by placing the numbers 1, 2, 3 and 4 throughout the Sudoku. Before you try the problems below, try the Sudoku warm-up on the following worksheet.

1. a) b) c)

2. a) b) c)
Mini Sudoku (continued)

Try these Sudoku Challenges with numbers from 1 to 6. The same rules and strategies apply!

Bonus

3.

```
 1 4     3 5
 6 4 5 1
 2 3 6
 4 1 6 2 3
 5 1 2
 2 5 3 4 6
```

4.

```
 5 4 2 1
 1 5 6
 3 6 4 5
 6 4 2 3 1
 2 3 6
```

5.

```
 4     3
 6 4 5 2
 6
 5 1
 1 5 2 3
 2
```

6.

```
 2 3
 4 1 2
 6 5 1
 4 3
 6
 3 5 1
```
Sudoku—The Real Thing

Try these Sudoku puzzles in the original format 9 × 9.

You must fill in the numbers from 1 through 9 in each row, column and box. Good luck!

**Bonus**

```
4  2  1  8  3  9
7  1  4   6
6  7  3  2
3  6  9  5
6  2  8  4  5  1  3
8   7  6
9   4  8
1  8  3  6  5  7
5  7  1  4  2
```

**Super Bonus**

```
5  3  6  7  8
2  8  6
4  7  3  9
7  1  3  4
1  4  5  2
4  9  1
2  4  6  8  7
8  3  9  5  1
9   1  4
```

For more Sudoku puzzles, check the puzzle section of your local newspaper!
Sudoku—Warm Up

1. Each row, column or box should contain the numbers 1, 2, 3, and 4. Find the missing number in each set.

   a) 
   
   b) 
   
   c) 
   
   d) 
   
   e) 
   
   f) 

2. Circle the pairs of sets that are missing the same number.

   a) 
   
   b) 
   
   c) 

3. Find the number that should be in each shaded square below. **REMEMBER:** In sudoku puzzles a number can only appear once in each row, column, or box.

   a) 
   
   b) 
   
   c) 

   d) 
   
   e) 
   
   f) 

4. Fill in the shaded number. Remember that each row, column, and box must have the numbers 1, 2, 3, and 4.

   a) 
   
   b) 
   
   c)
Sudoku—Warm Up (continued)

Bonus
Can you find the numbers for other empty squares (besides the shaded ones)?

6. Try to solve the following puzzles using the skills you’ve learned.

7. Find the missing numbers in these puzzles.

Now go back and solve the mini Sudoku puzzles!
NS5-1
Place Value

Photocopy the BLM “Place Value Cards” and cut out the five cards. Write the number 65321 on the board, leaving extra space between all the digits, and hold the “ones” card under the 3.

ASK: Did I put the card in the right place? Is 3 the ones digit? Have a volunteer put the card below the correct digit. Invite volunteers to position the other cards correctly. Cards can be affixed to the board temporarily using tape or sticky tack.

Now erase the 6 and take away the “ten thousands” card. ASK: Are these cards still in the right place? Write the 6 back in, put the “ten thousands” card back beneath the 6, erase the 1, and remove the ones card. ASK: Are these cards still in the right place? Have a volunteer reposition the cards correctly. Repeat this process with 6 5 2 1 (erase the 3) and 6 5 3 1 (erase the 2) and 6 3 2 1 (erase the 5).

Write 23989 on the board and ask students to identify the place value of the underlined digit. (NOTE: If you give each student a copy of the BLM “Place Value Cards,” individuals can hold up their answers. Have students cut out the cards before you begin.) Repeat with several numbers that have an underlined digit. Include whole numbers with anywhere from 1 to 5 digits.

Vary the question slightly by asking students to find the place value of a particular digit without underlining it. (EXAMPLE: Find the place value of the digit 4 in the numbers: 24001, 42305, 34, 432, 32847.) Continue until students can identify place value correctly and confidently. Include examples where you ask for the place value of the digit 0.

Then introduce the place value chart and have students write the digits from the number 231 in the correct column:

<table>
<thead>
<tr>
<th>Ten Thousands</th>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 231</td>
<td></td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Do more examples together. Include numbers with 1, 2, 3, 4, and 5 digits and have volunteers come to the board to write the numbers in the correct columns.

Write 28306 on the board. SAY: The number 28306 is a 5-digit number. What is the place value of the digit 2? (If necessary, point to each digit as you count aloud from the right: ones, tens, hundreds, thousands). SAY: The 2 is in the ten thousands place, so it stands for 20000. What does the digit 8 stand for? (8 000) The 3? (300) The 0? (0 tens or just 0) The 6? (6)

Explain that 28306 is just a short way of writing 20000 + 8000 + 300 + 6. Which digit did I not include in the sum? Why did I leave that one out? The 2 actually has a value of 20000, the 8 has a value of 8000, the 3 has a value of 300, and the 6 has a value of 6. Another way to say this is that the 2 stands for 20000, the 8 stands for 8000, and so on.
**ASK:** What is 537 short for? 480? 2 035? 9 601? 43 520? 60 003? Write out the corresponding addition statements for each number (also known as the expanded form).

**ASK:** What is the value of the 6 in 2608? In 36 504? In 63 504? In 6 504?

**ASK:** In the number 6 831, what does the digit 3 stand for? The digit 6? The 1? The 8?

**ASK:** What is the value of the 0 in 340? In 403? In 8 097? Write out the corresponding addition statements so that students see that 0 always has a value of 0, no matter what position it is in.

**ASK:** In the number 7 856, what is the tens digit? The thousands digit? Ones? Hundreds?

Repeat for 3 050, 5 003, 455, 77 077, 82 028.

Write the following numbers on the board: 2 350, 5 031, 1 435, 44 573, 30 500. Ask students to identify which digit, the 5 or the 3, is worth more in each number. Students should be using the phrases introduced in the lesson—stands for, has a value of, is short for. (**EXAMPLE:** In 2 350, the 5 stands for 50 and the 3 stands for 300, so the digit 3 is worth more.)

Review the meaning of the phrase “times as many.” **ASK:** If I have 2 marbles and you have 3 times as many marbles, how many marbles do you have? What is $2 \times 3$? If I have 4 marbles and you have 40, how many times more marbles do you have? Write on the board, “$4 \times _____ = 40$.” Have students practise this concept by deciding: How many times more is a dime worth than a penny? A loonie than a dime? A dime than a nickel? A quarter than a nickel? A toonie than a loonie? A loonie than a quarter? A toonie than a quarter? **BONUS:** A toonie than a nickel?

**ASK:** What is the value of the first 1 in the number 1 312? What is the value of the second 1? How many times more is the first 1 worth than the second 1? How many tens blocks would you need to make a thousands block? Repeat with more numbers in which the digit 1 is repeated (**EXAMPLES:** 1 341, 11, 101, 21 314).

**ASK:** How much is the first 3 worth: 3 231? The second 3? How many times more is the first 3 worth than the second 3 in 3 231? Repeat with more numbers in which the digit 3 is repeated (**EXAMPLES:** 32 013, 28 331, 24 303).

**Bonus**

Students who finish quickly can be given larger numbers with the two 3s getting further and further apart, ending with a number like:

823 450 917 863 401.

Have students fill in a chart like the following:

<table>
<thead>
<tr>
<th>Number</th>
<th>How many places over is the second 3 from the first 3?</th>
<th>How many times more is the first 3 worth than the second 3?</th>
<th>How many 0s are in the number from the previous column?</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 423</td>
<td>3</td>
<td>1000</td>
<td>3</td>
</tr>
<tr>
<td>2 313</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>383</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3376</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Make up your own number.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
What pattern do you notice? Does the pattern help you solve the BONUS problem? ANSWER: The second 3 is 9 places over from the first 3 in the BONUS, so to write how many times more the first 3 is worth than the second, we write 1 with 9 zeroes: 1 000 000 000.

Extensions

1. Teach students the Egyptian system for writing numerals, to help them appreciate the utility of place value.

Write the following numbers using both the Egyptian and our Arabic systems:

234

848

423

Invite students to study the numbers for a moment, then ASK: What is different about the Egyptian system for writing numbers? (It uses symbols instead of digits. You have to show the number of ones, tens, and so on individually—if you have 7 ones, you have to draw 7 strokes. In our system, a single digit (7) tells you how many ones there are.) Review the ancient Egyptian symbols for 1, 10, and 100, and introduce the symbols for 1 000 and 10 000:

a) 1 = stroke
b) 10 = arch

c) 100 = coiled rope
d) 1 000 = lotus flower

e) 10 000 = finger

Ask students to write a few numbers the Egyptian way and to translate those Egyptian numbers into regular numbers (using Arabic numberals). Emphasize that the order in which you write the symbols doesn’t matter:

234 = stroke

Have students write a number that is really long to write the Egyptian way (EXAMPLE: 798). ASK: How is our system more convenient? Why is it helpful to have a place value system (i.e. the ones, tens, and so on are always in the same place)? Tell them that the Babylonians, who lived at the same time as the ancient Egyptians, were the first people to use place value in their number system. Students might want to invent their own number system using the Egyptian system as a model.

2. Have students identify and write numbers given specific criteria and constraints.

a) Write a number between 30 and 40.
b) Write an even number with a 6 in the tens place.
c) Write a number that ends with a zero.
d) Write a 2-digit number.
e) Write an odd number greater than 70.
f) Write a number with a tens digit one more than its ones digit.

HARDER:

g) Which number has both digits the same: 34, 47, 88, 90?
h) Write a number between 50 and 60 with both digits the same.
i) Write a 3-digit even number with all its digits the same.
j) Find the sum of the digits in each of these numbers: 37, 48, 531, 225, 444, 372.
k) Write a 3-digit number where all digits are the same and the sum of the digits is 15.
l) Which number has a tens digit one less than its ones digit: 47, 88, 34, 90?

m) Write a 2-digit number with a tens digit eight less than its ones digit.

n) Write a 3-digit number where all three digits are odd.

o) Write a 3-digit number where the ones digit is equal to the sum of the hundreds digit and the tens digit.

Make up more such questions, or have students make up their own.

3. Which is worth more in the following numbers, the 3 or the 6? How many times more?

   a) 63 (the 6; 20 times more)
   b) 623
   c) 6 342
   d) 36
   e) 376
   f) 3 006
   g) 6 731
   h) 7 362
   i) 9 603
   j) 3 568
   k) 3 756
   l) 3 765
   m) 6 532

If students need help, ask them how they could turn each problem into one that they already know how to solve.

EXAMPLE: If it was 323 instead of 623, you would know how to solve it (the first 3 is worth 100 times more than the second 3). How is 623 different from 323? (6 is twice as much as 3, so the 6 is worth 200 times more than the 3).

4. How many times more is the 2 worth than the 5 in:

   a) 25
   b) 253
   c) 2534
   d) 25347
   e) 253470
   f) 2534708
   g) 325
   h) 3 425
   i) 3 425 570
   j) 3 425 708
   k) 3 472 508

Students will discover that as long as the numbers are immediately next to each other, the 2 is always worth 4 times more than the 5.

5. How many times more is the 2 worth than the 5 in:

   a) 2345
   b) 23457
   c) 234576
   d) 2345768

Students will discover through playing with the numbers that they can reduce to the case where the 5 is the ones digit even when that’s not the case.

6. How many times more is the 2 worth than the 5 in:

   a) 25
   b) 235
   c) 2465
   d) 27465

Students will see that the answer multiplies by 10 each time so they might start by pretending the numbers are right next to each other and then adding a zero for each time they have to move over a place.

7. Have students find how many times more the 2 is worth than the 5 for a variety of numbers by using the discoveries they have made. EXAMPLE: In the number, 32 406 756 812 406, we can pretend the 5 is the ones digit and start at the 2, so that the number becomes: 240 675. The 5 is 5 places over from the 2, so we can make a T-chart to help us:

<table>
<thead>
<tr>
<th>Number of places over</th>
<th>How many times more?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>400</td>
</tr>
<tr>
<td>4</td>
<td>4 000</td>
</tr>
<tr>
<td>5</td>
<td>40 000</td>
</tr>
</tbody>
</table>

From the chart, we see that the 2 is worth 40 000 times more than the 5.
NS5-2
Writing Numbers

GOALS
Students will learn to write number words for numbers with up to 5 digits.

PRIOR KNOWLEDGE REQUIRED
Place value
Reading and writing number words to twenty

Write the number words for the 2-digit multiples of 10, from 20 to 90, as follows:

10 × two = twenty
10 × three = thirty
10 × four = forty
10 × five = fifty
10 × six = sixty
10 × seven = seventy
10 × eight = eighty
10 × nine = ninety

ASK: What do the number words for a number and ten times that number have in common? (HINT: Look at the beginnings of the words.) What do the number words for multiples of 10 have in common? (HINT: Look at the ends of the words.)

Erase the board and WRITE:

10 × three = fifty
10 × seven = ninety
10 × four = twenty
10 × five = seventy
10 × nine = thirty
10 × eight = sixty
10 × two = ten
10 × six = fifty
10 × one = sixty

Have volunteers choose and write the correct answer from the right column into the space after each equals sign. They should cross out their answer in the right column so that students know which answers have been used. Ask each volunteer: How many beginning letters does your answer have in common with the number it’s ten times of? Encourage students to look over their answers to make sure they spelled them correctly. Do this even when the word is spelled correctly, to foster the habit of double-checking one’s work.

In most cases, a number and ten times that number both start with the same two, three, or more letters. Have students identify the exception. ASK: Why is that one easy to remember anyway?

Erase everything except for the number words the children wrote. Then write the following numbers on the board and have students write the corresponding number words at their desks: 30, 80, 40, 70, 60, 20, 10, 50, 90. While kids are busy working, correct any spelling mistakes on the board.

Write 72 on the board and ask volunteers how they would say that number. Then have them say it aloud with you. ASK: What two words does that sound
like put together? (seventy and two) Have a volunteer try writing the number word for 72. Point out that there is always a hyphen (-) between the two words—72 is written as “seventy-two.” (You may have to add the hyphen if the student has left it out.) Encourage students to write number words for other 2-digit numbers (EXAMPLES: 36, 48, 75, 41, 29, 83, 77, 64) in their notebooks. Circulate to monitor what students are writing, and model correcting any of the errors you see on the board with the whole class, or with individuals, as needed.

Invite students to find any mistakes in the way the following number words are written and to correct them (some are correct):

- forty-zero
- forty-three
- twenty-eight
- thirty-nine
- eight-five
- seventy-six

Summarize the process for writing numbers between 20 and 99: You can write the 2-digit number by writing the word for the first digit times ten, a hyphen, and then the word for the second digit, as long as it isn’t zero. If the second digit is zero, you write only the word for the first digit times ten. EXAMPLE: 35 = (3 \times 10) + 5 and is written as thirty-five, but 30 is written as thirty, not thirty-zero.

ASK: How is writing the number words for 11 to 19 different? (They don’t follow the same pattern.) Write the number words for 11 through 19 on the board and invite students to look for patterns and exceptions (eleven and twelve are unique; the other numbers have the ending “teen”).

Once students have mastered writing numbers up to 99, tell them that writing hundreds is even easier. There’s no special word for three hundreds like there is for three tens:

- 30 = 10 + 10 + 10 = thirty but
- 300 = 100 + 100 + 100 = three hundred (not three hundreds)

SAY: You just write what you see: three hundred. There’s no special word to remember.

Have students write the number words for the 3-digit multiples of 100: 200, 300, 400, and so on. Remind them not to include a final “s” even when there is more than one hundred.

Tell students that they can write out 3-digit numbers like 532 by breaking them down. Say the number out loud and invite students to help you write what they hear: five hundred thirty-two. Point out that there is no dash between “five” and “hundred.”

Ask students how they would write out the number 3000. SAY: Just as with hundreds, there’s no special word, you just write what you hear: three thousand. On the board, WRITE:

\[
1 \ 342 = \text{one thousand three hundred forty-two}
\]

Have volunteers come up and write the number words for more 4-digit numbers (EXAMPLE: 5 653, 4 887, 1 320)

Write on the board: 32 000. ASK: How many thousands are there? Show students how to write “thirty-two thousand”. Have them write number words for various multiples of a thousand. EXAMPLES: 17 000, 20 000, 89 000, 38 000.

Now write on the board: 17 541. ASK: How many thousands are there? If students answer “7”, emphasize that while 7 is the thousands digit, this number is more than ten thousand, so 7 can’t be right. Then allow students to try again. Write on the board: “seventeen thousand”. Cover up the digits 17 and ask students how they write the rest of the number. Then finish writing the number word so that it reads, “seventeen thousand five hundred forty-one”.

Have students practice writing the number words for other 5-digit numbers. EXAMPLES: 23 802, 54 006, 30 109.
Write the following number words on the board and ask the whole class which number words are incorrectly written and why. When the number words have been corrected, have students write the correct numerals individually in their notebooks.

- One thousand zero hundred twenty (shouldn’t say zero)
- Five thousand thirty-two (correct)
- Zero thousand four hundred sixty-four (should start with “four hundred”)
- Eight-thousand three-hundred seventy (should be no dashes)
- Two thousand nine hundred ninety-zero (shouldn’t write “zero”)
- Seven thousand four hundred seventy three (missing a dash for 73)
- Twenty-eight thousand ninety-three (correct)

Write some typical text from signs and have students replace any number words with numerals and vice versa.

a) Max. Height 3 m
b) Montreal 181 km
c) Seventy-Four Queen Street
d) Speed Limit—110 km/h
e) Saskatoon next four exits
f) Highway 407
g) Bulk Sale! Buy ten for the price of five!
h) Top-Quality Racing Broom for Witches and Wizards—Only $89 999.00!
i) Bridge Toll $3.50

Extensions

1. Tell students that sometimes a number in the thousands is expressed in terms of hundreds.

SAY: I want to know how many hundreds there are in 1 900. How many hundreds are in 1 000? How many are there in 900? So how many hundreds are in 1 900? When they say 19, tell them 1 900 can be written out as “one thousand nine hundred” or “nineteen hundred.” Invite students to help you write 2 400, 3 800, and 7 900 in terms of hundreds. ASK: Which way uses fewer syllables, the old way (using thousands and hundreds) or the new way (using only hundreds)? Challenge students to find a number which will have more syllables the new way (EXAMPLE: twenty hundred instead of two thousand).

ASK: How many hundreds are in a thousand? So what would a thousand be in terms of hundreds? How many syllables are there each way? Are there any other numbers like this? Tell students that we don’t usually write thirty hundred or seventy hundred (when there’s a multiple of ten hundreds) but we do otherwise. Have them practice writing the year they were born both ways. Which way is shorter? What if they write the year their older or younger siblings were born?

Tell students that years are usually written in terms of hundreds instead of thousands, unless the hundreds digit is 0. Usually, the word “hundred” is omitted, so that 1927 is written as nineteen twenty-seven, but 2007 is written as two thousand seven. Have students research the correct year for an event of their choice and to write both the numeral and the number word for homework. Possible topics could be sports, history, their family tree, etc. EXAMPLE:

a) Montreal won the most consecutive Stanley Cups—from 1956 to _____.
b) Toronto last won the Stanley Cup in _____.
c) Canada was founded in _____.
d) My grandmother was born in _____.

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WORKBOOK 5
Part 1
Number Sense
2. Show students a copy of a cheque and explain why it’s important to write the amount using both words and numerals. Show them how easy it is to change a number such as “348.00” to “1 348.00” by adding the digit 1. On the other hand, it would be an obvious forgery if someone then tried to add “one thousand” before “three hundred forty-eight.”

3. Have students find a number word whose spelling has the letters in alphabetical order. **ANSWER:** forty. Students should not do this problem by guessing! Rather, they should eliminate possibilities intelligently.

**HINT:** If the words from one to nine are not alphabetical, can any word containing those words be alphabetical?

**SOLUTION:** First, check all numbers up to twelve individually.

Second, eliminate any number words in the “teens”. Why can you do this?

Third, eliminate any number word that has the 1-digit number words in them, for **EXAMPLE:** twenty-seven cannot be alphabetical since seven is not; one hundred thirty-four cannot be alphabetical because one and four are not. The only numbers this leaves are multiples of 10! Check these individually: twenty, thirty, forty. When you reach forty, you are done!

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**NS5-3**

**Representation with Base Ten Materials**

**As students group and manipulate base ten materials throughout the lesson, monitor the models they create on their desks and/or have students sketch their answers on paper so that you can verify their understanding. Students can work individually or in small groups.**

**GOALS**

Students will practise representing numbers with base ten materials.

**PRIOR KNOWLEDGE REQUIRED**

Place value
Base ten materials

**VOCABULARY**

digit
hundreds block
tens block
ones block

**Hundreds block**

**Tens block**

**Ones block**

Give each student 3 tens blocks and 9 ones blocks. Ask them to make the number 17 and to explain how their model represents that number.

**PROMPTS:** How many tens blocks do you have? (1) How many ones are in a tens block? (10) How many ones are there altogether? (17) Repeat with more 2-digit numbers (**EXAMPLES:** 14, 19, 28, 34, 32, 25). Then give each student 3 hundreds blocks and have them make 3-digit numbers (**EXAMPLES:** 235, 129, 316). Use prompts, if needed, to help students break down their models.

For 235, **ASK:** How many ones are in the hundreds blocks? (200) The tens blocks? (30) The ones blocks? (5) How many ones altogether? (235)

Have students work in pairs to make the following: 12 using exactly 12 blocks, 22 using 13 blocks, 25 using 16 blocks, and 31 using 13 blocks. Some of these models will be non-standard. (**EXAMPLE:** 22 using 13 blocks = 1 tens block and 12 ones blocks, instead of the standard 2 tens blocks and 2 ones blocks) Encourage students to make a standard model first and then to ask themselves whether they need more blocks or fewer blocks. Which blocks can they trade to keep the value the same but increase the number of blocks?
Give your students ones and tens blocks. **ASK:** Which numbers have standard base ten models that can be arranged as rectangles of width at least 2? That is, if tens blocks are arranged horizontally, there are at least 2 rows.

**EXAMPLES:**

- **60**
  
  
  
  
  

- **22**
  
  
  
  
  

- **55**
  
  
  
  
  

- **30 + any multiple of 3 up to 39 (33, 36, 39)**

**NOTE:** This is an open-ended activity: there are many possible answers.

Write a number in expanded form on the board. Can students tell, just by looking at the number, whether its base ten model can be arranged in a rectangle with width at least 2? Ask them to explain how they know.

Numbers in the 60s are especially interesting to consider because 2, 3, and 6 divide evenly into the tens digit (6); if the ones digit is any multiple of these factors, the number can be modeled by a rectangle with width at least 2:

**EXAMPLES:**

- **11**
  
  

- **35**
  
  
  
  
  

Before assigning worksheet **NS5-3**, review with students how to draw the base ten blocks, especially the thousands cube, as shown on the worksheet (page 38).
Extensions

1. Ask students to explain and show with base ten blocks the meaning of each digit in a number with all digits the same (EXAMPLE: 3333).

2. In the number 265 321 which digit is exactly three times the tens digit?

3. Write an odd number whose ten thousands digit is twice its hundreds digit.

4. Have students solve these puzzles using base ten blocks:
   a) I am greater than 20 and less than 30. My ones digit is one more than my tens digit.
   b) I am a 2-digit number: Use 6 blocks to make me. Use twice as many tens blocks as ones blocks.
   c) I am a 3-digit number. My digits are all the same. Use 9 blocks to make me.
   d) I am a 2-digit number. My tens digit is 5 more than my ones digit. Use 7 blocks to make me.
   e) I am a 3-digit number. My tens digit is one more than my hundreds digit and my ones digit is one more than my tens digit: use 6 blocks to make me.
   f) Show 1123 using exactly 16 blocks. (There are 2 answers.)
   g) I am a 4 digit number. My digits are all the same: use 12 blocks to make me.

5. Challenge students to solve these puzzles by only imagining the base ten blocks, but allow them to use base ten blocks for harder questions if necessary. QUESTIONS (a) through (e) have more than one answer—emphasize this by asking students to share their answers.
   a) I have more tens than ones. What number could I be?
   b) I have the same number of ones and tens blocks. What number could I be?
   c) I have twice as many tens blocks as ones blocks. What 2-digit number could I be?
   d) I have six more ones than tens. What number could I be?
   e) You have an equal number of ones, tens, and hundreds and twice as many thousands as hundreds. What might your number be?
   f) You have one set of blocks that make the number 13 and one set of blocks that make the number 22. Can you have the same number of blocks in both sets?
   g) You have one set of blocks that make the number 23 and one set of blocks that make the number 16. Can you have the same number of blocks in both sets?

6. Have students draw what they think a base ten block for 10 000 would look like.

7. Have students make a chart to investigate how many base ten blocks are needed to make different numbers.
Example numbers to put in the chart: 84, 23, 916, 196, 1003, 200, 4321, 9022

When students are done the chart, **ASK:**

a) Do all 3-digit numbers need more blocks than all 2-digit numbers? (no) Are all 3-digit numbers greater than all 2-digit numbers? (yes) Why is this the case?

b) How can you find the number of base ten blocks needed just by looking at the digits? **ANSWER:** Add the digits.

c) What is the greatest number of blocks needed for a 1-digit number? (9) A 2-digit number? (18) A 3-digit number? (27) A 4-digit number? (36) Describe the pattern.

d) Make an organized list to find all 2-digit numbers that need exactly…

... 1 block            ... 2 blocks            ... 3 blocks            ... 15 blocks

e) Make an organized list to find all 3-digit numbers that need exactly…

... 1 block            ... 2 blocks            ... 3 blocks            ... 4 blocks            ... 5 blocks

8. Remind your students of the word “product” before assigning this extension.

Find the 3-digit numbers. One of them is impossible. Can you find it and explain why?

a) The difference between my hundreds and tens digit is twice my ones digit. I have no 0 digit. Make me with 12 blocks.

b) Look at my ones digit. Add my hundreds digit that many times. You will get my tens digit. (The product of my hundreds and ones digits is my tens digit.) All my digits are different. My smallest digit is my ones digit. Make me with 11 blocks.

c) The sum of my ones and tens digits equals my hundreds digit. Make me with 11 blocks.

d) The product of my digits is 8. All my digits are the same.

e) My digits increase from left to right. The product of my digits is 12. Make me with 8 blocks.
ASK: How much is the 4 worth in 43? The 3? WRITE: $43 = 40 + 3$.

Have students write the following 2-digit numbers in expanded form in their notebook:

52, 81, 76, 92, 39, 26, 13, 84.

Tell students that if there is a 0 digit, they don’t need to include it in the expanded form—70 is just 70, not $70 + 0$.

Move on to 3-digit numbers. ASK: How much is the 4 worth in 459? The 5? The 9? Have a volunteer write the expanded form of 459 on the board. Have students write the following numbers in expanded form in their notebooks: 352, 896, 784. Ensure that all students have written at least the first one correctly. Then ask a volunteer to write 350 in expanded form on the board ($350 = 300 + 50$). Have another volunteer write the expanded form of 305. Then have students write the following numbers in expanded form in their notebooks:

207, 270, 702, 403, 304, 430, 340.

Encourage them to refer to the board if necessary. Then proceed to 4- and 5-digit numbers (EXAMPLES: 6 103, 7 064, 8 972, 1 003, 8 000, 29 031, 30 402).

Now write on the board:

32 427 = _______ ten thousands + _______ thousands + _______ hundreds + _______ tens + _______ ones

Have a volunteer fill in the blanks. Repeat with the numbers: 4 589, 38 061, 5 770, 804, 40 300.

**Bonus**

Provide questions where you mix the order of the words and digits. (EXAMPLE: $23 891 = _______ ones + _______ hundreds + _______ tens + _______ ten thousands + _______ thousands).

Then model writing the words “ten thousands,” “thousands,” “hundreds,” “tens,” and “ones” as you write the expanded form of the number 42 503:

42 503 = 4 ten thousands + 2 thousands + 5 hundreds + 0 tens + 3 ones

Have students do the same thing in their notebooks for: 23 587, 54 006, 28 913, 7 403, 39 820.

Invite volunteers to write 2-digit numbers given the expanded form, or sum (EXAMPLE: $30 + 7, 40 + 3, 90 + 9, 80 + 5$). Have students do the following sums in their notebook: $80 + 2, 50 + 4, 60 + 6, 90 + 3, 70 + 2, 20 + 7$. 

**GOALS**

Students will replace a number with its expanded numeral form and vice versa.

**PRIOR KNOWLEDGE REQUIRED**

Place value (ones, tens, hundreds, thousands, ten thousands)

**VOCABULARY**

digit
numeral
Repeat with 3-digit numbers that have no 0 digit (EXAMPLE: 300 + 50 + 7, 400 + 20 + 9, 800 + 60 + 1).

ASK: What is 500 + 7? Is there a 0 in the number? How do you know? What would happen if we didn’t write the 0 digit because we thought the 0 didn’t matter, and we just wrote the 5 and the 7 as “57”? Does 500 + 7 = 57? Emphasize that in expanded form we don’t need to write the 0 because expanded form is an addition statement (or sum) and the 0 doesn’t add anything, but in multi-digit numbers, it means something. It makes sure that each digit’s place value is recorded properly. Mathematicians call 0 a place holder (SEE: glossary) because of this.

Then have students add the following 3-, 4- and 5-digit sums in their notebooks: 300 + 2, 200 + 30, 400 + 5, 500 + 4, 700 + 40, 4 000 + 300 + 70 + 8, 5 000 + 60 + 1, 7 000 + 3, 80 000 + 300, 60 000 + 40 + 9.

Give students examples of 3-, 4- and 5-digit sums where they need to fill in the blank (EXAMPLES: 200 + _______ + 3 = 253, 50 000 + _______ + 20 = 54 020).

Include examples where the order of the addends differs from the order of the digits. EXAMPLE: 4 327 = 300 + 7 + _______ + 4 000.

**Extensions**

1. Ask students to imagine that each thousands block has shrunk to the size of a ones block. Tell them that they will use the ones blocks as thousands blocks. Ask them which base 10 block could be used to represent a million? (ANSWER: The thousands block.) If your students need a hint, suggest that they first figure out which base ten block would represent ten thousand (the tens block), then a hundred thousand (the hundreds block).

2. Some of the following problems have multiple solutions:
   a) In the number 2 735 what is the sum of the tens digit and the thousands digit?
   b) Find a number whose tens digit and thousands digit add to 11.
   c) Write a number whose hundreds digit is twice its ones digit.
   d) In the number 4 923, find a digit that is twice another digit. How much more is the larger digit worth?
   e) Make up a problem like (d) and solve it.

3. Have your students demonstrate expanded form in other ways than base ten.
   a) Ask your students to show different ways to make 100 as sums of 25, 10, and 5 (EXAMPLE: 100 = 25 + 25 + 10 + 10 + 10 + 5 + 5 + 5 + 5 or 100 = 2 twenty-fives + 3 tens + 4 fives)
   b) Show different ways to make 200 as sums of 100, 50, 10.
   c) Show different ways to make 1000 as sums of 500, 250, 100.
4. Instead of the base ten blocks ones, tens, hundreds and thousands, Shelley has the base three blocks ones, threes, nines and twenty-sevens.

She has two of each type of block.

a) Draw the blocks she would need to make:  
i) 4  ii) 12  iii) 7  iv) 35

b) What is the largest number she can make with her blocks?

c) Shelley asked her teacher for another threes block so that she can make the number 19 with a nines block, 3 threes block and a ones block. Her teacher told her that she shouldn’t need the third threes block. Can you see why?

d) Why are Shelley’s blocks called base three blocks instead of base ten blocks?

5. Which of the following are correct representations of 352?

a) $300 + 50 + 2$

b) $1$ hundred $+ 20$ tens $+ 2$ ones

c) $2$ hundreds $+ 15$ tens $+ 2$ ones

d) $34$ tens $+ 12$ ones

Make up more problems of this sort and have students make up problems of their own.

6. Expand a number in as many different ways as you can. EXAMPLE:

$312 = 3$ hundreds $+ 1$ ten $+ 2$ ones or $2$ hundreds $+ 9$ tens $+ 22$ ones, and so on.

Students could use base ten blocks if it helps them. They can exchange blocks for smaller denominations to find different representations.

7. a) Use ones, tens, hundreds, and thousands blocks. Model as many numbers as you can that use exactly 4 blocks. In each case, what is the sum of the digits? Why?

b) A palindrome is a number that looks the same written forwards or backwards (EXAMPLE: 212, 3773). Find as many palindromes as you can where the sum of the digits is 10. (Be sure students understand that the sum of the digits is also the number of blocks.)

8. Journal

Write about zero. Is writing 0 necessary when we write numbers like 702? Would it change the meaning if we didn’t write it? Is 0 necessary when we write addition sentences like $8 + 0 + 9 = 17$? Would it change the meaning if we didn’t write it?
NS5-5
Comparing and Ordering Numbers

Make the numbers 25 and 35 using base ten materials:

\[ \begin{array}{c}
\text{25} \\
\text{35}
\end{array} \]

Have students name the numbers. ASK: Which number is bigger? How can we show which number is bigger using base ten blocks? Explain that 3 tens blocks is more than 2 tens blocks and 5 ones blocks is the same as 5 ones blocks, so 35 is more than 25.

Have students identify the bigger number in the following pairs using base ten materials or models drawn on paper:

a) 352, 452  

b) 405, 401  

c) 398, 358  

d) 3 541, 3 241

e) 4 719, 1 719  

f) 1 001, 2 001  

g) 4 321, 4 326  

h) 2 743, 1 743

Show students that instead of looking at the base ten blocks, you can look at the expanded form of a number and see the same thing. In the following example, 200=200, 20 is less than 30, 5=5, so 225 is less than 235.

\[ \begin{align*}
225 &= 200 + 20 + 5 \\
235 &= 200 + 30 + 5
\end{align*} \]

Write on the board:

\[ \begin{align*}
346 &= 300 + 40 + 6 \\
246 &= 200 + 40 + 6
\end{align*} \]

ASK: Which number is bigger? How do you know? Demonstrate circling the place in the expansion where the numbers differ:

\[ \begin{align*}
346 &= \textcircled{300} + 40 + 6 \\
246 &= \textcircled{200} + 40 + 6
\end{align*} \]

Then ask students to compare the numbers and circle the digit that is different:

\[ \begin{align*}
\textcircled{346} \\
\textcircled{246}
\end{align*} \]

Repeat with several examples of 3- and 4-digit numbers where only 1 digit is different. Include examples with zeroes (EXAMPLE: 302, 312; 4003, 4007) but no examples comparing a 3-digit number to a 4-digit number yet.
Bring out the base ten materials and have students compare 43 to 26.

\[
\begin{align*}
\text{SAY:} & \quad \text{Which has more tens blocks? Which has more ones blocks? Hmmm, 43 has more} \\
\text{tens blocks, but 26 has more ones blocks—how can we know which one is bigger?}
\end{align*}
\]

Challenge students to find a way to change one of the models so that it has both more tens and more ones blocks than the other number. They should not change the value of the model; only trading for blocks of equal value is allowed.

Have students do several examples in their notebook where the number of tens is bigger in one number and the number of ones is bigger in the other number. Invite volunteers to show their work on the board.

Now write 43 and 26 in expanded form and compare them:

\[
\begin{align*}
43 &= 40 + 3 \\
26 &= 20 + 6
\end{align*}
\]

Since 10 is always bigger than any 1-digit number, we can split 43 (the one with more tens) into 30 + 10 + 3:

\[
\begin{align*}
43 &= 30 + 10 + 3 \\
26 &= 20 + 6
\end{align*}
\]

This makes it clear that 43 is more than 26, since it wins on all counts. Compare more 2-digit numbers in this way until it becomes clear that the number with more tens is always bigger.

Then move on to 3-digit numbers. Tell students you want to compare 342 and 257. **ASK:** Which number has more hundreds? More tens? More ones? Which number do you think is bigger, the one with the most hundreds, the most tens or the most ones? Why? Then **WRITE:**

\[
\begin{align*}
542 &= 400 + 100 + 42 \\
257 &= 200 + 57
\end{align*}
\]

Since 100 is bigger than any 2-digit number (it has 10 tens and any 2-digit number has at most 9 tens), it doesn’t matter what the tens and ones are of 257.

Note that there can be more than one way to split the hundreds in the expansion, **EXAMPLE:**

\[
\begin{align*}
500 &= 400 + 100 \\
500 &= 200 + 300
\end{align*}
\]

will both work to show that every term in the expansion of 542 is greater than each term in the expansion of 257.

Ask students to compare more 3-digit numbers with different hundreds digits in this way (**EXAMPLE:** 731 and 550, 403 and 329). Then look at a pair in which the hundreds digit is the same (**EXAMPLE:** 542 and 537).

Draw models on the board for a ones, tens, hundreds and thousands block. Ask a volunteer how they would draw a base ten block for 10 000. **ASK:** How did we draw a tens block from a ones block? How should we draw a ten thousands block from a one thousands block?

Have students compare the following numbers by drawing base ten models:

a) 12 035, 13 035 

b) 31 121, 41 121 

c) 11 111, 11 211
Finally, have students compare numbers that do not have the same number of digits (EXAMPLE: 350 and 1 433). Can they explain why any 4-digit number is always bigger than any 3-digit number?

Review the “greater than” and “less than” symbols. If students have a hard time remembering which is which, draw a face:

![Face drawing]

**SAY:** This hungry person can have only one pile of bananas. Which pile should he take:

34 bananas

27 bananas

34 > 27

*SAY:* 34 is greater than (>) 27, and 27 is less than (<) 34.

Write this statement on the board:

\[
[ ] 2 < [ ] 4
\]

**ASK:** What digits can we put in the boxes to make this statement true? Take more than one answer. Encourage students to look for more possibilities. Record their answers on the board for all to see.

Now write this statement on the board:

\[
[ ] 2 < [ ] 9
\]

**ASK:** Are there any digits that will make this statement true?

**PROMPTS:** How many hundreds are in the second number? Can a number with no hundreds be greater than a number that has hundreds? Remind students that we do not write 0 at the beginning of a number.

**Extensions**

1. Create base ten models of a pair of two digit numbers. Ask students to say how they know which number is greater. You might make one of the numbers in non-standard form as shown for the first number below.

**EXAMPLE:** To compare the numbers students could remodel the first number in standard form by regrouping ones blocks as tens blocks.
2. Ask students to create base ten models of two numbers where one of the numbers...
   a) is 30 more than the other
   b) is 50 less than the other
   c) has hundreds digit equal to 6 and is 310 more than the other
   d) has thousands digit 4 and is 31 000 less than the other

3. Ask students where they tend to see many numbers in increasing order (houses, mailboxes, line-ups when people need to take a number tag, apartment numbers).

4. Have students compare numbers using different units. Which is larger?
   a) 9 tens or 85
   b) 42 tens or 5 hundred
   c) 32 hundred or 4 thousand
   d) 30 thousand or 45 hundred
   e) 21 thousand or 640 hundred
   f) BONUS: 5000 thousand or 3 million

Remind students of the relationship between adding and more, subtracting and less. **ASK:**

- What number is 10 more than 846? How can we write an addition statement to show this? (846 + 10 = 856)
- What number is 100 less than 846? How can we write this as a subtraction statement? (846 – 100 = 746)
- What number is 1 less than 846? How would we express this mathematically? (846 – 1 = 845)
- What number is 100 more than 846? How would we express this mathematically? (846 + 100 = 946)

Have students solve some addition and subtraction statements. **EXAMPLES:**

<table>
<thead>
<tr>
<th></th>
<th>739 + 10</th>
<th>620 + 100</th>
<th>702 + 1</th>
<th>4 350 + 100</th>
<th>35 360 + 10 000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>500 – 100</td>
<td>456 – 10</td>
<td>398 – 1</td>
<td>6 543 – 100</td>
<td>24 568 – 10 000</td>
</tr>
</tbody>
</table>

When students are comfortable with this, have them find what they need to add or subtract to get the answers in new statements (**EXAMPLES:** 427 + _____ = 437; 1 603 – _____ = 1 503). Remember to only provide statements in which multiples of 10, 100, and 1 000 are not crossed (**EXAMPLE:** not 499 + _____ = 509).
Ask students to solve:

\[29 + 1 \quad 89 + 1 \quad 99 + 1 \quad 8149 + 1 \quad 42379 + 1\]

Ask students to solve \(20 - 1\). (PROMPT: What do you say before 20 when counting?) Then ask students to solve the following, prompting as necessary:

\[40 - 1 \quad 30 - 1 \quad 730 - 1 \quad 860 - 1 \quad 9540 - 1 \quad 4500 - 1\]

(Some students may need base ten blocks to help them see why the tens digit decreases by 1 and the ones digit becomes 9.)

To help students see what to do when they have to cross multiples of 100 when counting by 10s, bring out the base ten blocks.

Ask students to use base ten blocks to solve:

a) \(290 + 10\)  b) \(790 + 10\)  c) \(2390 + 10\)  d) \(23590 + 10\)  e) \(27990 + 10\)

ASK: How many tens are in 290? (29) How many tens do you have if you add one ten? (30). What number has 30 tens? (300).

Ask students to use base ten blocks to solve:

a) \(894 + 10\)  b) \(693 + 10\)  c) \(994 + 10\)  d) \(7892 + 10\)  e) \(34997 + 10\)

ASK: How many tens are in 894? (89) Demonstrate covering up the ones digit to show them how they can determine the number of tens from the number. Then ASK: How many tens do you have if you add one ten? (90) Does the ones digit stay the same when you add ten? (yes) What is 894 + 10? (904)

Have students create a number that is a multiple of 100 and then create a number that is 10 less. In order to take away 10, they will need to first trade one of the hundreds blocks for 10 tens blocks. Ask various students to write their answers on the board in terms of a math statement. EXAMPLE:

\[300 = \quad 290 = \quad 300 - 10 = 290\]

ASK: Did you have to change the number of hundreds to answer this question? Do you think you will have to change the number of hundreds to answer 304 – 10? Why? Have students find the answer with and without base ten blocks. PROMPTS: How many tens are in 304? (30) How many tens do you have if you subtract one ten? (29) Does the ones digit stay the same when you subtract ten? (Yes) What is 304 – 10? (294)

Ask students to make a base ten model of 2075 and to show you how the model would change if you took away 100. (Students will see that they have to regroup one of the 2 thousands as 10 hundreds and that there would be 9 hundreds left)

Have students solve similar problems in their notebooks (EXAMPLES: \(402 - 10\), \(4307 - 10\), \(5800 - 10\), \(4600 - 10\), \(8900 - 10\), \(34509 - 10\)). They might draw models of base ten blocks to help them.

When they are done, prompt students to articulate their thinking by asking questions like: How many tens are in 4307? (430) How many tens will there be when we subtract 10? (429) Does the ones digit change when we subtract 10? (No) What is 4307 – 10? (4297)

Teach students how to cross multiples of 100 or 1000 when counting by 1 (EXAMPLES: \(800 - 1\), \(1900 - 1\), \(4500 - 1\), \(700 - 1\), \(1000 - 1\), \(19000 - 1\)). Repeat for multiples of 1000 when counting by 10s (EXAMPLES: \(8000 - 10\), \(7000 - 10\), \(13000 - 10\), \(27000 - 10\), \(9003 - 10\)). Have students start with models of base ten blocks in each case.
Ask students how many tens are in each number and what number has one less 10 than that. **ASK:** Which digit can they cover up so that they can see the number of tens? Demonstrate covering up the ones digit.

Repeat with examples where students will have to cross multiples of 1 000 when counting by 100s, again bringing out the base ten blocks to get them started. **(EXAMPLES):** 5 000 – 100, 4 037 – 100, 2 075 – 100, 5 003 – 100

Ask students how many hundreds are in each number and what number has one less 100 than that. **ASK:** Which digits can they cover up so that they can see the number of hundreds? Demonstrate covering up the tens and ones digits.

Students will need continual practice subtracting 10 and 100 across multiples of 100 and 1 000. Play money might be a useful tool for many students. Some ideas for extra practice include:

1. Give your students 5 loonies and ask them to show you how much money they would have left if they took away a dime. Students will see that they have to exchange one of the loonies for ten dimes, and that there will only be 9 dimes left once one is taken away. (Ask students to translate the result into pennies: 500 pennies becomes 490 pennies.)

2. Ask students to show you in loonies how much money they would have altogether if they had $2.90 and they were given a dime.

Then tell students that instead of adding or subtracting 10, 100, or 1000 from a number, you will give them two numbers that differ by 10, 100, or 1000 and the students’ job is to tell you how much the numbers differ by and which one is more.

| a) 34 076 | BONUS: 70 743 |
| 34 176 | 69 743 |
| 34 076 is 100 less than 34 176 | 70 743 is ______ 69 743 |

**Extensions**

1. Circle the greater number in each pair:

   a) 75 276 or 75 376  
   b) 27 052 or 28 052  
   c) 376 259 or 386 259  
   d) 1 387 531 or 1 377 531

2. What number is 1 010 less than…

   a) 3 182  
   b) 4 275  
   c) 6 052  
   d) 3 900

3. Ask students to create a model of a multiple of 1000 and show a number that is…

   a) 100 less  
   b) 10 less

4. Ask students to show a number…

   a) 100 more than 7,940  
   b) 100 less than 7,056

5. What number is 10 010 less than…

   a) 75 231  
   b) 27 387  
   c) 375 105  
   d) 9331
NS5-7
Comparing Numbers (Advanced)

**GOALS**
Students will order sets of two or more numbers and will find the greatest and smallest number possible when given particular digits.

**PRIOR KNOWLEDGE REQUIRED**
Comparing pairs of numbers

**VOCABULARY**
greater than \( > \)
less than \( < \)

**ASK:** What is the smallest 3-digit number? (What is the first 3-digit number you get to when counting?) What is the largest 3-digit number? (What is the last 3-digit number you say when counting before you get to 4-digit numbers?)

Write on the board: 1, 4, 7. Tell students that you want to make a 3-digit number using each of these digits. Have 2 students volunteer answers. Write their answers on the board. **ASK:** Which one is bigger? Then tell students you want the biggest 3-digit number they can think of that uses these 3 digits. **ASK:** How do you know it’s the biggest? Emphasize that the biggest number has the most hundreds possible, and then the most tens possible.

Ask students to make the smallest 3-digit number possible with those same digits and to explain how they know it’s the smallest. **ASK:** Can you make a smaller number with those digits?

Repeat with several examples of 3-digit numbers and then move on to 4-digit numbers. Note that to make the smallest 3-digit number possible with 0, 3, 9, since we do not start a number with 0, the hundreds digit must be 3, then the tens digit can be 0, and the ones digit needs to be 9.

**SAY:** I want to compare two 2-digit numbers. **ASK:** If the number of tens is different, how can you tell which number is greater? If the number of tens is the same, how can you tell which number is greater?

Have students practice putting groups of three 2-digit numbers in order, from least to greatest. Do examples in the following order:

- The tens digit is always the same:
  - 27, 24, 29; 35, 33, 30; 48, 49, 44; 90, 92, 91
- The ones digit is always the same:
  - 41, 71, 51; 69, 99, 89; 50, 20, 30; 83, 53, 63
- The tens digit is always different, but the ones digit can be the same or different:
  - 49, 53, 29; 57, 43, 60; 43, 50, 29; 30, 25, 63
- Exactly two numbers have the same tens digit:
  - 39, 36, 43; 53, 50, 62; 79, 84, 76; 34, 29, 28

When students are comfortable comparing 2-digit numbers, ask them how they would compare two 3-digit numbers. **ASK:** Which digit should you look at first, the ones digit, the tens digit or the hundreds digit? Why?

Have students practice putting sequences of three 3-digit numbers in order (**EXAMPLES:** 134, 127, 198; 354, 350, 357; 376, 762, 480).

Then ask students to put the following groups of numbers in order from least to greatest:

a) 412, 214, 124, 142, 421, 241
b) 931, 319, 913, 193, 391, 139
Write the digits 3, 5, and 8. Ask students to make all the possible 3-digit numbers from these digits. Tell them to try to do it in an organized way. Take suggestions for how to do that. (Start with the hundreds digit; write all the numbers that have hundreds digit 3 first, then work on the numbers with hundreds digit 5, and then 8.) How many numbers are there altogether? (6—2 starting with each digit). Which of these numbers is largest? Which is smallest? Have students reflect: Could they have found the largest number using these digits without listing all the possible numbers?

**SAY:** I want to find the largest number possible using the digits 1, 3, 5 and 8 each once. Do I need to find all the numbers I can make from these digits to find the largest?

Ask students to create the greatest possible number using the digits (each once):

a) 2 9 7 8 4  
b) 4, 4, 8, 3, 2  
c) 3, 7, 0, 6, 1

Note that a number cannot begin with the digit 0, so the smallest number in c) is 10 367.

Then ask your students to find the least possible number using those same digits, and finally to find a number in between the greatest and least possible numbers.

Have students order these groups of three 4-digit numbers:

a) 3 458, 3 576, 3 479  
b) 4 987, 6 104, 6 087  
c) 4 387, 2 912, 3 006

Finally, combine 3-digit and 4-digit numbers and have students order them (E**X**A**M**P**L**E:** 3 407, 410, 740).

Have students order similar groups of 5-digit numbers.

**Bonus**

Have students order longer lists of numbers.

Finally, combine numbers with different numbers of digits. For example, have students order from least to greatest: 2 354, 798, 54 211.

Have students use the digits 0 to 9 each once so that

___ ___ ___ ___ ___ > ___ ___ ___ ___ ___.

**ASK:** Can you place the digits so that ___ ___ ___ ___ ___ > ___ ___ ___ ___ ___?

Have your students use the digits 0, 1 and 2 to create a number that is:

a) more than 200  
b) between 100 and 200  
c) a multiple of 10

Have your students use the digits 4, 6, 7 and 9 to create a number that is:

a) between 6200 and 6500  
b) larger than 8000  
c) an even number between 9500 and 9700.

Have your students use the digits 0, 3, 4, 7 and 8 to create a number that is:

a) a multiple of 10  
b) between 73 900 and 75 000  
c) more than 40 000 and less than 70 000

Ask students to find the correct digit:

a) 3 2 5 0 < 3 ___ 1 0 < 3 3 4 8  
b) 2 5 3 9 1 < 2 5 ___ 7 0 < 2 5 5 6 3
Extensions

1. Use the digits 5, 6, and 7 to create as many 3-digit numbers as you can (use each digit only once when you create a number). Then write your answers in descending order.

2. List 4 numbers that come between 263 and 527. (NOTE: Some students will show off by being as creative as possible (EXAMPLE: 217, 304, 416, 523) while others will show off that they can do as little work as possible (EXAMPLE: 264, 265, 266, 267). Both of these solutions show excellent mathematical thinking.

3. How many whole numbers are greater than 4000 but less than 4350?

4. What is the greatest 6 digit number you can create so that:
   a) the number is a multiple of 5  
   b) the ten thousands digit is twice the tens digit.

5. What is the greatest 6 digit number you can create so that the sum of the digits is 27?

6. What is the greatest 3 digit number you can create that is a perfect square (i.e. you can multiply a smaller number by itself to get the number)?

7. How many different 3 digit numbers can you create using the digits 7, 8, 9
   a) if you are only allowed to use each digit in a number once?  
   b) If you can use each digit more than once?

8. List 4 numbers that come between 12 263 and 12 527.

9. Name 2 places you might see more than...
   a) 1000 people.  
   b) 10 000 people.
10. Say whether you think there are more or fewer than 1000…
   a) hairs on a dog
   b) fingers and toes in a class
   c) students in the school
   d) grains of sand on a beach
   e) left-handed students in the school

11. Two of the numbers in each increasing sequence are out of order. Circle each pair.
   a) 28, 36, 47, 42, 95, 101
   b) 286, 297, 310, 307, 482
   c) 87, 101, 99, 107, 142, 163
   d) 2725, 2735, 2755, 2745, 2765

12. Create 6 different 4-digit numbers and write them in increasing order (from least to greatest).

13. Fill in the missing numbers with any numbers that fit.
   a) 7, _____, _____, 16
   b) 29, _____, _____, 41
   c) 387, _____, _____, 402

14. Place the numbers in descending order (from greatest to least).
   a) 252, 387, 257
   b) 8752, 3275, 9801
   c) 10325, 10827, 10532
   d) 298, 407, 369, 363, 591, 159
   e) 8790, 7809, 9078, 9807, 7908, 7089
   f) 65789, 13769, 57893, 65340, 13108, 56617, 65792
   g) 127283, 152582, 127381

15. Write the number 98950 on the board and challenge students to find all the numbers that use the same digits and are greater.

16. Write the number 75095 on the board. Have each student find a number that differs from it in only one digit. Have them ask a partner whether their number is greater or lesser than 75095. Whose number is biggest? Have them work in groups of 4, 5, or 6 to order all the numbers they made.
Regrouping

Draw on the board:

\[ \begin{array}{c}
  \bullet \bullet \bullet \bullet \bullet \bullet \\
  \bullet \bullet \bullet \bullet \bullet \bullet \bullet \\
  \bullet \bullet \bullet \bullet \bullet \\
  \end{array} \]

**ASK:** How many ones blocks are there altogether? (34) Ask for strategies for counting the blocks. (Count by 5s and then by 1s, or count by 7s and then by 1s. Students who recall multiplication might say: multiply 7 by 5 then subtract 1, or multiply 6 by 5 then add 4).

**ASK:** Do we have enough to trade for a tens block? How do you know? How many tens blocks can we trade for? How do you know? (Have a volunteer group sets of 10.) Where do you see that number in “34”? (The 3 represents the number of tens.) What does the “4” tell you? (The number of ones left over.)

Draw base ten models with more than ten ones and have students practice trading ten ones blocks for a tens block. They should draw models to record their trades in their notebooks. **EXAMPLE:**

\[ \begin{array}{cccc}
  \begin{array}{c}
  \begin{array}{c}
    \bullet \bullet \\
    \bullet \bullet \\
    \bullet \bullet \\
  \end{array} \\
  \begin{array}{c}
    \bullet \bullet \\
    \bullet \bullet \\
    \bullet \bullet \\
    \bullet \bullet \\
    \bullet \bullet \\
    \bullet \bullet \\
  \end{array} \\
  \begin{array}{c}
    \bullet \bullet \\
    \bullet \bullet \\
    \bullet \bullet \\
    \bullet \bullet \\
    \bullet \bullet \\
    \bullet \bullet \\
    \bullet \bullet \\
  \end{array} \\
  \end{array} \\
  \end{array} \longrightarrow \begin{array}{c}
  \begin{array}{c}
    \bullet \bullet \bullet \\
    \bullet \bullet \bullet \\
  \end{array} \\
  \begin{array}{c}
    \bullet \bullet \bullet \\
    \bullet \bullet \bullet \\
    \bullet \bullet \bullet \\
  \end{array} \\
  \begin{array}{c}
    \bullet \bullet \bullet \\
    \bullet \bullet \bullet \\
  \end{array} \\
  \end{array} \]

\[ 4 \text{ tens} + 19 \text{ ones} = 5 \text{ tens} + 9 \text{ ones} \]

Students should also be comfortable translating these pictures into number sentences:

\[ 40 + 19 = 40 + 10 + 9 = 50 + 9 = 59 \]

**ASK:** What number is 6 tens + 25 ones? How can we regroup the 25 ones to solve this question?

\[ 25 = 2 \text{ tens} + 5 \text{ ones} = 10 + 10 + 5, \text{ so } 6 \text{ tens} + 25 \text{ ones} \]

\[ = (10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 5) \]

\[ = 6 \text{ tens} \quad \text{25 ones} \]
This means 6 tens + 25 ones = 8 tens + 5 ones, as summarized by this chart:

<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>25</td>
</tr>
<tr>
<td>6 + 2 = 8</td>
<td>25 - 20 = 5</td>
</tr>
</tbody>
</table>

Have students practice using such a chart to regroup numbers. Then have them regroup numbers without using the chart:

3 tens + 42 ones = _______ tens + _______ ones

Continue with problems that require:

1) regrouping tens to hundreds (EXAMPLE: 4 hundreds + 22 tens).
2) regrouping ones to tens and tens to hundreds. Include numbers with 9 tens and more than 10 ones, so that it won’t look like they have to regroup tens to hundreds until after they regroup the ones (EXAMPLE: 2 hundreds and 9 tens and 14 ones). Use this to explain the importance of grouping the ones first—you can only tell if there are enough tens to make a hundred after you have grouped all the ones into tens.

3) regrouping hundreds to thousands.

4) regrouping thousands to ten thousands, hundreds to thousands, tens to hundreds, and ones to tens. Again, include examples with 9 hundreds and more than 10 tens, or 9 thousands, 9 hundreds, 9 tens, and more than 10 ones.

EXAMPLES: Regroup:

a) 3 ten thousands + 4 thousands + 12 hundreds + 5 tens + 3 ones
b) 4 ten thousands + 17 thousands + 9 hundreds + 18 tens + 2 ones
c) 5 ten thousands + 12 thousands + 9 hundreds + 3 tens + 38 ones
d) 4 ten thousands + 9 thousands + 13 hundreds + 14 tens + 7 ones
e) 5 ten thousands + 12 thousands + 13 hundreds + 5 tens + 12 ones
f) 9 ten thousands + 8 thousands + 16 hundreds + 34 tens + 11 ones
g) 7 ten thousands + 5 thousands + 9 hundreds + 12 tens + 8 ones
h) 6 ten thousands + 9 thousands + 9 hundreds + 24 tens + 7 ones
i) 8 ten thousands + 9 thousands + 9 hundreds + 9 tens + 26 ones
j) 8 ten thousands + 9 thousands + 13 hundreds + 8 tens + 25 ones

Extensions

1. Ask students to show the regrouping in QUESTIONS 2 and 3 with base ten blocks.
2. If you taught your students Egyptian writing (see Extension for NS5-1) you could ask them to show regrouping using Egyptian writing.

EXAMPLE:

\[\begin{array}{c}
\underline{\text{Hieroglyphic Representation}} \\
\underline{\text{Regrouping}}
\end{array}\]
3. In this exercise, students will learn that regrouping from right to left is more efficient than regrouping from left to right. Introduce standard notation for regrouping, so that 4 ten thousands + 12 thousands + 9 hundreds + 23 tens + 14 ones becomes as shown in the grid below.

Demonstrate regrouping from left to right:

Have students regroup the same numbers, but from right to left (i.e. starting with the ones). Then have students do the regrouping in random order. Did they always get the same answer? Which way takes the fewest steps: from left to right, right to left or in random order? Students should investigate these questions for other examples.

Have students make up an example where they think the left to right algorithm and the right to left algorithm will take the same number of steps and then to check their answer. Then have students make up an example where they think the right to left algorithm will be shorter and then to check their answer. Challenge your students to try to come up with an example where the left to right algorithm will be shorter. Why won’t this work?
**NS5-9**

**Adding with Regrouping**

**GOALS**

Students will add 2-digit numbers with regrouping.

**PRIOR KNOWLEDGE REQUIRED**

Adding 2-digit numbers without regrouping

**VOCABULARY**

regrouping carrying

Tell students you want to add 27 and 15. Begin by drawing base ten models of 27 and 15 on the board:

\[
\begin{array}{c}
27 \\
+15 \\
\end{array}
\]

Then write the addition statement and combine the two models to represent the sum:

\[
\begin{array}{c}
\phantom{27} \\
\phantom{+15} \\
\end{array}
\]

ASK: How many ones do we have in the total? How many tens? Replace 10 ones with 1 tens block. **ASK:** Now how many ones do we have? How many tens? How many do we have altogether?

\[
\begin{array}{c}
27 \\
+15 \\
\end{array}
\]

Use a tens and ones chart to summarize how you regrouped the ones:

<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

After combining the base ten materials

After regrouping 10 ones blocks for 1 tens block

Have students draw the base ten materials and the “tens and ones” charts for:

\[
\begin{array}{c}
36 \\
+45 \\
\end{array}
\quad
\begin{array}{c}
28 \\
+37 \\
\end{array}
\quad
\begin{array}{c}
46 \\
+36 \\
\end{array}
\quad
\begin{array}{c}
19 \\
+28 \\
\end{array}
\]

**Bonus**

\[
\begin{array}{c}
32 \\
+13 \\
\end{array}
\quad
\begin{array}{c}
29 \\
\phantom{+13} \\
\end{array}
\quad
\begin{array}{c}
46 \\
11 \\
\end{array}
\quad
\begin{array}{c}
\phantom{+13} \\
\phantom{+34} \\
\end{array}
\]

Ask them if they really need the base ten materials or if they can add without them.
Draw:

<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>13</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>

SAY: When you use a chart, you can add the tens and ones first and then regroup. When you do the sum directly, you regroup right away: \( 7 + 6 = 13 \), which is 1 ten + 3 ones, so you put the 3 in the ones column and add the 1 to the tens column.

Demonstrate the first step of writing the sum of the ones digits:

\[
\begin{array}{c}
1 \\
2 \quad 7 \\
+ \\
4 \quad 8 \\
\hline \\
5
\end{array}
\]

Ask them how many ones there are when you add the ones digits and how you are regrouping them. Tell them that when we regroup ten ones for a ten, we put the 1 on top of the tens column. Mathematicians call this process “carrying the 1.” Ask students for reasons why this name is appropriate for the notation.

Have students do the first step for several problems, and then move on to problems where students need to do both steps. Some students may need to have the first step done for them at first, so that they can focus only on completing the second step.

Include examples where the numbers add up to more than 100 (EXAMPLES: 85 + 29, 99 + 15).
NS5-10
Adding 3-Digit Numbers and
NS5-11
Adding 4- and 5-Digit Numbers

Have volunteers draw base ten models for 152 and 273 on the board and tell students that you want to add these numbers. **ASK:** How many hundreds, tens, and ones are there altogether? Do we need to regroup? How do you know? How can we regroup? (Since there are 12 tens, we can trade 10 of them for 1 hundred.) After regrouping, how many hundreds, tens, and ones are there? What number is that?

Write out and complete the following statements as you work through the **EXAMPLE:**

\[
\begin{array}{c}
| & \text{hundred} & + & \text{tens} & + & \text{ones} \\
152 & \_ & \_ & \_ \\
+273 & \_ & \_ & \_ \\
\hline
\text{After regrouping:} & \_ & \_ & \_ \\
\end{array}
\]

Have students add more pairs of 3-digit numbers. Provide examples in the following sequence:

- either the ones or the tens need to be regrouped (**EXAMPLES:** 349 + 229, 191 + 440).
- both the ones and the tens need to be regrouped (**EXAMPLES:** 195 + 246, 186 + 593).
- you have to regroup the tens, but you don’t realize it until you regroup the ones (**EXAMPLES:** 159 + 242, 869 + 237).

Now show students the standard algorithm alongside a hundreds, tens, and ones chart for the first example you did together (152 + 273):

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>5</td>
</tr>
<tr>
<td>3 + 1 = 4</td>
<td>12 - 10 = 2</td>
<td>5</td>
</tr>
</tbody>
</table>

Point out that after regrouping the tens, you add the 1 hundred that you carried over from the tens at the same time as the hundreds from the two numbers, so you get 1 + 1 + 2 hundreds.
Have students add more 3-digit numbers, using the chart and the algorithm at first, and then using only the algorithm. Do examples in the same sequence as before: either the ones or tens need to be regrouped; both the ones and tens need to be regrouped; the tens need to be regrouped after the ones have been regrouped.

When students have mastered this, repeat the lesson with 4-digit numbers and then with 5-digit numbers. Do not assume that students will be so familiar with the method that you can skip steps in the process. Use a chart alongside the algorithm to start, and provide examples in which different digits need to be regrouped.

Finally, include both 4- and 5-digit numbers as addends in the same sum. **EXAMPLE:** 32 405 + 9 736.

When students are comfortable adding 3-, 4- and 5-digit numbers, introduce palindromes by writing several numbers on the board and telling the students whether or not they are palindromes: 343 (yes), 144 (no), 23 532 (yes), 2 332 (yes), 4 332 (no), 12 334 321 (no). Then write several more numbers on the board and have students guess whether or not the number is a palindrome. When all students are guessing correctly and confidently, ask a volunteer to articulate what the rule is for determining whether or not a number is a palindrome (a palindrome is a number whose digits are in the same order when written from right to left as when written from left to right).

When students are comfortable with the new terminology, ask them what the 2-digit palindromes are. (11, 22, 33, and so on to 99). Tell them that you’re going to show them how to turn any number into a palindrome by using addition. Write the number 13 on the board and ask your students to find numbers you can add to 13 to make a palindrome. (9, 20, 31, 42, 53, 64, 75, 86). Tell your students that one of these numbers can be obtained from the original 13 in a really easy way. Which number is that? (31) How can we obtain 31 from 13? (use the same digits, but in reverse order) Have your students add these numbers to its reverse: 35, 21, 52. Do they always get a palindrome? (yes) Challenge your students to find a 2-digit number for which you don’t get a palindrome by adding it to its reverse. Tell them to do the same process with their resulting number (for example, if they started with 69, they will see that 69 + 96 = 165 is not a palindrome, so they could repeat the process with 165 (165 + 561 = 726). Ask how many have palindromes now. Have students continue repeating the process until they end up with a palindrome. Starting with 69 (or 96), the sequence of numbers they get will be: 69 (or 96), 165, 726, 1 353, 4 884. Have them repeat the process starting with various 2-digit numbers (**EXAMPLES:** 54, 74, 37, 38, 56, 28) and then with multiple-digit numbers (**EXAMPLES:** 341, 576, 195, 197, 8 903, 9 658, 18 271). Tell your students that most numbers will eventually become palindromes but that mathematicians have not proven whether all numbers will or not. Over 2 000 000 steps have been tried (using a computer of course) on the number 196 and mathematicians have still not found a palindrome.

---

**ACTIVITY**

Game for two players:

1. Each player makes a copy of this grid:

2. Players take turns rolling a die and writing the number rolled in one of their own grid boxes.

The winner is the player who creates two 3-digit numbers with the greatest sum.

(Continued on next page.)
Extensions

1. If you taught your students Egyptian writing (SEE: Extension for NS5-1: Place Value) you could ask them to show adding and regrouping using Egyptian writing.

   **EXAMPLE:**

   ![Egyptian numerals addition example]

2. Have students add more than 2 numbers at a time:

   a) 427 + 382 + 975 + 211  b) 3 875 + 5 827 + 2 132  
   c) 15 891 + 23 114 + 36 209  d) 17 432 + 946 + 3 814 + 56 117

3. When you follow steps 1 to 3 in exercise 7 with a given number, it will not necessarily become a palindrome in one step. Find a number that will not become a palindrome, even after five or ten steps.

4. Systematically list all palindromes from 100 to 200 (it’s easy to give a rule for these) or from 100 to 1000.

5. Have your students investigate: when you add two palindromes, do you always get a palindrome? When do you get a palindrome and when do you not? Can they change the question slightly to make it have a “yes” answer? (When you add two palindromes whose digits are all less than 5, do you always get another palindrome?)

6. Have your students find numbers for which adding it to its reverse results in the following numbers:

   a) 584  b) 766  c) 1 251  d) 193  Which one is not possible? (d).

7. Introduce the concept of word palindromes (words that are spelled the same backward as forward) by writing the following words on the board and asking students what rule they all follow. **EXAMPLES:**

   mom, dad, did, noon, racecar, level, nun, toot.

When students see the pattern, have students find examples of words that are palindromes and words that are not. Introduce the concept of palindromic sentences (sentences where the letters written in reverse order are the same as in the correct order).
Tell your students that you want to subtract 48 – 32 using base ten materials. Have a volunteer draw a base ten model of 48 on the board. SAY: I want to take away 32. How many tens blocks should I remove? (3) Demonstrate crossing them out. ASK: How many ones blocks should I remove? (2) Cross those out, too. ASK: What do I have left? How many tens? How many ones? What is 48 – 32?

Have student volunteers do other problems with no regrouping on the board (EXAMPLES: 97 – 46, 83 – 21, 75 – 34). Have classmates explain the steps the volunteers are taking. Then have students do similar problems in their notebooks.

Give students examples of base ten models with the subtraction shown and have them complete the tens and ones charts:

<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

Have students subtract more 2-digit numbers (no regrouping) using both the chart and base ten models.

When students have mastered this, have them subtract by writing out the tens and ones (as in QUESTION 2 on the worksheet):

\[
46 - 13 = 4 \text{ tens } + 6 \text{ ones} \\
 1 \text{ ten } + 3 \text{ ones} \\
 3 \text{ tens } + 3 \text{ ones} \\
 33
\]

Then have students separate the tens and ones using only numerals (as in QUESTION 3 on the worksheet):

\[
36 - 24 = 30 + 6 \\
20 + 4 \\
10 + 2 \\
12
\]
Now ask students to subtract:

\[
\begin{array}{c}
54 \\
- 23
\end{array}
\]

**ASK:** Which strategy did you use? Is there a quick way to subtract without using base ten materials, or tens and ones charts, or separating the tens and ones? (Yes—subtract each digit from the one above.) What are you really doing in each case? (Subtracting the ones from the ones and the tens from the tens, and putting the resulting digits in the right places.)

Have students draw a base ten model of 624 and show how to subtract 310. Have them subtract using the standard algorithm (**EXAMPLE:** by lining up the digits) and check to see if they got the same answer both ways. Repeat with 4- and 5-digit numbers that do not require regrouping (**EXAMPLE:** 34 586 – 12 433).

Ask students how they learned to subtract 46 – 21. Have a volunteer demonstrate on the board. Ask the class if you can use the same method to subtract 46 – 28. What goes wrong? Should you be able to subtract 28 from 46? If you have 46 things, does it make sense to take away 28 of them? (Sure it does). Challenge students to think of a way to change the problem to one that looks like a problem they did last time. Tell them that they are allowed to change one of the numbers and then adjust their answers. Have them work in pairs. Possible answers may include:

- find 46 – 26 then subtract 2 more because you didn’t subtract enough
- find 48 – 28 then subtract 2 because you added 2 to get 48
- find 46 – 20 then subtract 8 because you didn’t subtract enough
  (you can find 26 – 8 by counting down).
- find 38 – 28 then find 46 – 38 by counting down

Have volunteers come to the board and show their strategies and how they needed to adjust their answer. Restate, add to, or correct the students’ explanations when necessary. Then have all students solve similar problems in their notebooks (**EXAMPLES:** 36 – 19, 23 – 14, 57 – 39). They should solve each one in at least two ways.

Now change the rules: tell students they are not allowed to adjust their answers at the end. They are only allowed to adjust the number of tens and ones (by trading or regrouping) in each number. Have pairs work on 46 – 28 using these new rules. Give them a few minutes to think about the problem, and then have volunteers share their strategies with the group. **ASK:** Which number required trading—the larger number or the smaller number? Why does it help to have more ones blocks in the number 46? Use base ten materials to illustrate—make a standard model of 46 and then regroup 1 ten as 10 ones:
SAY: Now there are 16 ones, so we can take away 8 of them. Since we didn’t change the value of 46—we just traded blocks—there is no need to adjust the answer. Removing the shaded blocks (2 tens and 8 ones), we are left with 1 ten and 8 ones. SO: 46 – 28 = 18.

Show students how to note their trades or regrouping:

3 16
– 28

Have students show this notation alongside a base ten model for several problems (EXAMPLES: 53 – 37, 66 – 39, 50 – 27). Tell them that it is the beginning of the standard algorithm. Then have them actually do the subtraction for problems where you have already done the regrouping for them. ASK: How is this subtraction more like the problems we did last class? (Now we can just subtract the ones from the ones and tens from the tens.) What did we have to do to make the problem more like the problems we did last class? (We had to make sure the larger number had more ones than the smaller number and to get it that way, we had to trade a ten for ten ones.) Tell them that some people call this borrowing because we are borrowing a ten from the tens digit.

Give students several subtraction problems and ask them to determine whether regrouping is required. Ask how they know. Emphasize that if they don’t have as many ones in the larger number as in the smaller number, they will need to regroup. Then provide several problems where students need to regroup only when necessary.

Repeat the lesson with more (3, 4 or 5) digits. Provide examples in the following sequence:

• Only one digit needs to be regrouped. (EXAMPLES: 562 – 417, 62 187 – 41 354)
• Two non-consecutive digits need to be regrouped. (EXAMPLE: 54 137 – 28 052)
• Two consecutive digits need to be regrouped. (EXAMPLE: 9 319 – 6 450)
• Three or four consecutive digits need to be regrouped. (EXAMPLES: 3 695 – 1 697, 1 000 – 854, 10 000 – 4 356)
• Consecutive digits need to be regrouped, but this is not obvious until after you regroup the first digit (EXAMPLES: 3 685 – 1 487, 33 116 – 13 435)

Students should be encouraged to check their regrouping. For example, Show students how a regrouping was done:

34 019
– 8 342 as – 3 4 2
2 6 6 7

Show them how to check the regrouping by adding all the regrouped numbers:

1 20 000 2 ten thousands
14 000 14 thousands
900 9 hundreds
110 11 tens
1 9 ones
35 019

Ask students why this is not the same as the original 34 019. Where was the regrouping done incorrectly? How should the regrouping have been done? Emphasize that when borrowing from 0 to make 9, they must first regroup the 0 to a 10 by borrowing from the digit to the left. This step was left out.
Give students several examples where they could check their regrouping using this process. Ensure that students understand that when borrowing is done, they should never change the value of the number.

Discuss what happens when they borrow when they don’t need to. For **EXAMPLE:**

\[
\begin{array}{c}
6 & 16 \\
\hline
\text{\ } & \text{\ } \\
\text{\ } & \text{\ }
\end{array}
\]

\[
- 2 \ 4
\]

\[
4 \ 1 \ 2
\]

**ASK:** Is this the right answer? Tell them that if we read this as 412, it is incorrect, but if we read the answer as 4 tens and 12 ones, we can regroup to obtain 5 tens and 2 ones. Since we never changed any values by any of the regrouping, this will still be the correct answer. The only problem with this method is that it requires regrouping twice (and you must remember to regroup back at the end) when no regrouping was required in the first place!

---

**ACTIVITY 1**

Ask students to show the regrouping in **QUESTIONS 1 A)** to **D)** with actual base ten blocks.

---

**ACTIVITY 2**

Give students a set of cards with the numbers 1 to 8 on them.

a) Ask students to arrange the cards in two rows as shown so that the difference between the numbers is as small as possible.

\[
\begin{array}{c}
\text{\ } \\
\text{\ } \\
\text{\ } \\
\text{\ } \\
\text{\ } \\
\text{\ } \\
\text{\ } \\
\text{\ } \\
\end{array}
\]

\[
\text{Solution: } 5 \ 1 \ 2 \ 3 \ 4 \ 8 \ 7 \ 6
\]

b) Ask students to place the eight cards so that the three subtraction statements below are correct.

\[
\begin{array}{c}
4 \ 5 \\
\hline
3 \ 2
\end{array}
\]

\[
\begin{array}{c}
2 \ 4 \\
\hline
1 \ 7
\end{array}
\]

**TEACHER:** As a warm up for **Activity 2 a)** above, ask students to place the cards from 1 to 4 in the array to make the smallest difference. **MORE CHALLENGING:** Place the cards 1 to 6 in the array to make the smallest difference.

\[
\begin{array}{c}
\text{\ } \\
\text{\ } \\
\text{\ } \\
\text{\ } \\
\text{\ } \\
\end{array}
\]

\[
\text{Solution: } 3 \ 1 \ 2 \ 4
\]

\[
\begin{array}{c}
\text{\ } \\
\text{\ } \\
\end{array}
\]

\[
\text{Solution: } 4 \ 1 \ 2 \ 3 \ 6 \ 5
\]

---

**ACTIVITY 3**

Give students base ten materials and an individual number with sum of digits at least 5 (81, 63, 52, 23, 41, 87) Ask students to make base ten models for their number and to find as many numbers as they can by taking away exactly 3 blocks. They should make a poster of their results by drawing models of base ten materials and showing how they organized their answers. This can be done over 2 days, giving 2-digit numbers the first day and 3-digit numbers the second day. **EXAMPLES:**

Start with 63 and subtract i) 3 tens (33), ii) 2 tens and 1 one (42), iii) 1 ten and 2 ones (51), iv) 3 ones (60); Start with 311 and subtract i) 3 hundreds (11), ii) 2 hundreds and 1 ten (101), iii) 2 hundreds and 1 one (110), iv) 1 hundred and 1 ten and 1 one (200).
Extensions

1. Teach your students the following fast method for subtracting from 100, 1 000, 10 000 (it helps them avoid regrouping).

   You can subtract any 2-digit number from 100 by taking the number away from 99 and then adding 1 to the result. EXAMPLE:

   \[
   \begin{align*}
   100 &= 99 + 1 \\
   -42 &= -42 \\
   &= 57 + 1 = 58
   \end{align*}
   \]

   You can subtract any 3-digit number from 1 000 by taking the number away from 999 and adding 1 to the result. EXAMPLE:

   \[
   \begin{align*}
   1000 &= 999 + 1 \\
   -423 &= -423 \\
   &= 576 + 1 = 577
   \end{align*}
   \]

2. Make a 2-digit number using consecutive digits (EXAMPLE: 23). Reverse the digits of your number to create a different number, and subtract the smaller number from the larger one (EXAMPLE: 32 – 23). Repeat this several times. What do you notice? Create a 3-digit number using consecutive digits (EXAMPLE: 456). Reverse the digits of your number to create a different number, and subtract the smaller number from the larger one. Repeat this several times. What do you notice? (The result is always 9 for 2-digit numbers and 198 for 3-digit numbers). Have them find the result for 4-digit numbers (the result is always 3 087) and predict whether there will be a constant answer for 5-digit numbers. Some students may wish to investigate what happens when we don’t use consecutive digits (EXAMPLE: 42 – 24 = 18, 63 – 36 = 27, 82 – 28 = 54; the result in this case is always from the 9 times tables).

3. Have students determine which season is the longest if the seasons start as follows (in the northern hemisphere):

   Spring: March 21
   Summer: June 21
   Fall: September 23
   Winter: December 22

   Assume non-leap years only.

   Students will need to be organized and add several numbers together as well as use some subtraction. For example, the number of days in spring is the total of the number of spring days in each month. March: 31 – 20 (since the first 20 days are not part of spring) April: 30 May: 31 June: 20.

   **TOTAL:** Students may either add: 11 + 30 + 31 + 20 or notice that they are adding and subtracting 20 and simply add 31 + 30 + 31 = 92 days. Summer has 10 + 31 + 31 + 22 = 94 days, fall has 8 + 31 + 30 + 21 = 90 days and winter has 10 + 31 + 28 + 20 = 89 days, so in order from longest to shortest: summer, spring, fall, winter. Encourage the students to check their answers by totalling the seasons to get 365 days.

   Students should be encouraged to compare this ordering with temperature. It is premature to discuss scientific reasons for this; just noticing the pattern is enough.

   Students may wish to examine this question for their own region and the current year. The beginning dates of each season may differ from those listed above.
NS5-13
Parts and Totals

Write on the board:

**Red apples**

[ ] [ ] [ ] [ ] [ ]

**Green apples**

[ ] [ ]

Tell students that each square represents one apple and **ASK:** How many red apples are there? How many green apples? Are there more red apples or green apples? How many more? (Another way to prompt students to find the difference is to **ASK:** If we pair up red apples with green apples, how many apples are left over?) How many apples are there altogether?

Label the total and the difference on the diagram, using words and numerals:

- **Difference:** 4 apples
- **Total:** 8 apples

Do more examples using apples or other objects. You could also have students count and compare the number of males and females in the class. (If yours is a single-gender class, you can divide the class by age instead of by gender). **ASK:** How many girls are in the class? How many boys? Are there more boys or girls? How many more? What subtraction sentence could you write to express the difference? How many children are there altogether in the class? What addition sentence could you write to express the total?

Then draw the following chart and have volunteers fill in the blanks:

<table>
<thead>
<tr>
<th>Red Apples</th>
<th>Green Apples</th>
<th>Total Number of Apples</th>
<th>How many more of one colour of apple?</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td></td>
<td>6 more green than red</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td>3 more red than green</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td></td>
<td>2 more green than red</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td></td>
<td>1 more red than green</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>11</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

Then ask students to fill in the columns of the chart from this information:

- a) 3 red apples, 5 green apples
- b) 4 more red apples than green apples, 5 green apples
- c) 4 more red apples than green apples, 5 red apples
- d) 11 apples in total, 8 green apples
- e) 12 apples in total, 5 red apples
**NS5-14**

## Parts and Totals (Advanced)

### GOALS

Students will recognize what they are given and what they need to find in a word problem.

### PRIOR KNOWLEDGE REQUIRED

- Equivalent addition and subtraction statements
- Subtraction as take away
- Subtraction as comparison (how much more than, how much shorter than, and so on)
- Addition as how much altogether, or how many altogether, or how long altogether

### VOCABULARY

- equation
- fact family

---

**Draw on the board:**

![Diagram](image)

Ask your students to suggest number sentences (addition and subtraction) represented by this model. Ensure that either you or the students state what the numbers in each number sentence represent. Continue until all possible answers have been identified and explained: $3 + 4 = 7$ because 3 white circles and 4 dark circles make 7 altogether; $4 + 3 = 7$ because 4 dark circles and 3 white circles make 7 altogether; $7 - 4 = 3$ because removing the 4 dark circles leaves 3 white circles; $7 - 3 = 4$ because removing the 3 white circles leaves 4 dark circles. Tell them that these number sentences all belong to the same fact family because they come from the same picture and involve the same numbers. Tell them that these number sentences are all examples of equations because they express equality of different numbers, for example $3 + 4$ and $7$ or $7 - 4$ and $3$. Write on the board:

- a) $3 + 5 = 8$
- b) $43 - 20 > 7$
- c) $9 - 4 < 20 - 8$
- d) $5 + 12 + 3 = 11 + 9 = 31 - 11$

Ask which number sentences are equations (parts a and d only).

Then have students list all the possible equations in the fact family for different pictures. Include pictures in which more than one attribute varies.

**EXAMPLE:** (dark and white, big and small)

![Diagram](image)

**NOTE:** The fact families for $4 + 5 = 9$ and $6 + 3 = 9$ can both be obtained from this picture, but they are not themselves in the same fact family.

When students are comfortable finding the fact family of equations that correspond to a picture, give them a number sentence without the picture and have them write all the other number sentences in the same fact family. Use progressively larger numbers, sometimes starting with an addition sentence and sometimes with a subtraction sentence (**EXAMPLES:** $10 + 5 = 15$, $24 - 3 = 21$, $46 + 21 = 67$, $103 - 11 = 92$).

Then put the following chart on the board. It is similar to the chart used in the last lesson, but it uses grapes in place of apples and includes a column for the fact family related to the total number of grapes.
### Green Grapes | Purple Grapes | Total Number of Grapes | Fact Family for the Total Number of Grapes | How many more of one type of grape? |
|-------------|-------------|------------------------|-------------------------------------------|----------------------------------|
| 7 | 2 | 9 | $\begin{align*} 7 + 2 &= 9 \\
9 - 7 &= 2 \end{align*}$ | 5 more green than purple |
| 5 | 3 | | | |
| 7 | | | | 3 more purple than green |
| 7 | | | | 3 more green than purple |
| 5 | 12 | | | |
| | 6 | 10 | | |

Look at the completed first row together, and review what the numbers in the fact family represent. Demonstrate replacing the numbers in each number sentence with words:

- $7 + 2 = 9$ becomes: green grapes + purple grapes = total number of grapes
- $2 + 7 = 9$ becomes: purple grapes + green grapes = total number of grapes
- $9 - 7 = 2$ becomes: total number of grapes – green grapes = purple grapes
- $9 - 2 = 7$ becomes: total number of grapes – purple grapes = green grapes

Have students complete the chart in their notebooks, and ask them to write out at least one of the fact families using words.

Look at the last column together and write the addition sentence that corresponds to the first entry, using both numbers and words:

- $5 + 2 = 7$: how many more green than purple + purple grapes = green grapes

Have students write individually in their notebooks the other equations in the fact family, using both numbers and words:

- $2 + 5 = 7$: purple grapes + how many more green than purple = green grapes
- $7 - 5 = 2$: green grapes – how many more green than purple = purple grapes
- $7 - 2 = 5$: green grapes – purple grapes = how many more green than purple

Make cards labelled:
- # of green grapes
- # of purple grapes
- Total number of grapes
- How many more purple than green
- How many more green than purple

Stick these cards to the board and write “=” in various places.

**EXAMPLE:** # green grapes [ ] # purple grapes = how many more green than purple

Have students identify the missing operation and write in the correct symbol (+ or –). For variety, you might put the equal sign on the left of the equation instead of on the right. Be sure to create only valid equations.
FOR EXAMPLE: How many more purple than green grapes = # green grapes is invalid, since neither addition nor subtraction can make this equation true.

Teach students to identify which information is given and what they need to find out. They should underline the information they are given and circle the information they need to find out.

FOR EXAMPLE:
Cari has 15 marbles. 6 of them are red. How many are not red?

Teach students to write an equation for what they have to find out in terms of what they are given.

EXAMPLE:
not red = marbles − red marbles

not red = 15 − 6 = 9

They then only need to write the appropriate numbers and add or subtract.

Finally, do a simple word problem together. Prompt students to identify what is given, what they are being asked to find, and which operation they have to use. Encourage students to draw a picture similar to the bars drawn in EXERCISE 1 of worksheet NS4-21: Parts and Totals.

EXAMPLE:
Sera has 11 pencils and Thomas has 3 pencils. How many more pencils does Sera have?

I know: # of pencils Sera has (11), # of pencils Thomas has (3)

I need to find out: How many more pencils Sera has than Thomas

How many more Sera has = # of pencils Sera has − # of pencils Thomas has

= 11 − 3

= 8
NS5-15
Addition and Subtraction

**GOALS**
Students will solve word problems involving addition and subtraction.

**PRIOR KNOWLEDGE REQUIRED**
Subtraction as take away
Subtraction as comparison (EXAMPLE: how much more than, how much shorter than, etc.)
Addition as total (EXAMPLE: how much altogether, how many altogether, how long altogether, etc.)
Recognizing what students need to know in a word problem
Equivalent addition and subtraction sentences

**VOCABULARY**
sum
difference

Tell your students that you asked two people to tell you how much longer the second stick is than the first stick. One person answered 3 m and the other person answered 23 m. How did they get their answers? Which person is right? How do you know?

Emphasize that something measuring only 13 m long cannot be 23 m longer than something else.

Ask if anyone has a younger sister or brother. Ask the student their age, then ask how old their sister or brother is. Ask them how much older they are than their sister or brother. How did they get their answer? Did they add the ages or subtract? Why? Emphasize that when they’re asked how much older or how much longer that they’re being asked for the difference between two numbers so they should subtract.

Write:

<table>
<thead>
<tr>
<th>Smaller</th>
<th>Larger</th>
<th>How much more?</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 m</td>
<td>13 m</td>
<td>3 m longer</td>
</tr>
<tr>
<td>5 years</td>
<td>9 years</td>
<td>4 years older</td>
</tr>
</tbody>
</table>

Give several word problems like this and continue to fill in the chart. Try to wait until most hands are raised before allowing a volunteer to come to the board.

**EXAMPLE:**

1. Katie has 13 jelly beans. Rani has 7 jelly beans. Who has more jelly beans? How many more?

2. Sara weighs 29 kg. Ron weighs 34 kg. Who weighs more? How much more?

Anna’s pet cat weighs 15 kg and her new kitten weighs 7 kg. How much more does the cat weigh than the kitten? Be sure that all students understand a kitten is a baby cat, then ask if they needed to know that a kitten was a baby cat to answer the question. Could they have answered the question without the information? Then tell them that someone from Mars came to visit you yesterday and told you that a klug weighs 13 norks and a prew weighs...
17 norks. Which one weighs more? How much more? Do they need to know anything about klugs or prews or norks to answer the question? How would they know if they could lift any of these objects?

If students enjoy this, have them come up with several science fiction math sentences for their classmates to solve.

Continue to use the chart, but this time have students insert a box (□) when they aren’t given the quantity.

**EXAMPLES:**

1. Sally’s yard has an area of 21 m². Tony’s yard is 13 m² larger than Sally’s. How big is Tony’s yard?

   Are you given the larger number or the smaller number? How do you know?

2. Teresa read 14 books over the summer. That’s 3 more than Randi. How many books did Randi read? Are you given the larger number or the smaller number? How do you know?

3. Mark ran 5 100 m on Tuesday. On Thursday, he ran 400 m less. How far did he run on Thursday? Are you given the larger number or the smaller number? How do you know?

For the above questions, have students write subtraction sentences with the question mark in the right place (larger number – smaller number = how much more).

1. □ – 21 = 13
2. 14 – □ = 3
3. 5 100 – □ = 400

Have students change the subtraction sentence so that the box is equal to the sum or the difference. First, make sure they can write all of the math sentences in the same family as 10 – 6 = 4 (10 – 4 = 6, 4 + 6 = 10, 6 + 4 = 10). Give students lots of practice before they go on to changing sentences with boxes.

First repeat this lesson with addition problems that ask how many there are altogether, and then with problems where students are given how many there are altogether but they need to find one of the addends (SEE: glossary).

**EXAMPLE:** Sally and Teresa sold apples together. Altogether they sold 28 baskets of apples. Sally kept track and noted that she sold 13 baskets of apples. How many baskets did Teresa sell?

Students should be aware that phrases like “in all” or “altogether” in a word problem usually mean that they should combine quantities by adding.

Similarly, phrases like “how many more”, “how many are left” and “how many less” indicate that they should find the difference between quantities by subtracting.

However, the wording of a question won’t always tell students which operation to use to solve the question. The two questions below have identical grammatical structure, but in the first question they must add to find the answer and in the second they must subtract.

1. Paul has 3 more marbles than Ted. Ted has 14 marbles. How many marbles does Paul have?
2. Paul has 3 more marbles than Ted. Paul has 14 marbles. How many marbles does Ted have?

In solving a problem it helps to make a model, draw a simple diagram, or make a mental model of the situation. Students should start by getting a sense of which quantity in a question is greater and which quantity is smaller. In the first question, they should recognize that the 14 is the smaller quantity, so they have □ – 14 = 3, which they can change to 14 + 3 = □. In the second question,
they should recognize that 14 is the larger number, so \(14 - \square = 3\), or \(14 - 3 = \square\).

When students have mastered this, combine the problems and use a chart similar to the following:

<table>
<thead>
<tr>
<th>First number</th>
<th>Second number</th>
<th>How many altogether?</th>
<th>How much more than the smaller number is the larger number?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Have students put the given numbers and the box for the unknown in the right place, and then put an X in the part that doesn’t apply for each question. Then have them solve the problem by expressing the \(\square\) as the sum or difference, then adding or subtracting.

**Extensions**

1. Here are some two-step problems your students can try:
   a) Alan had 352 stickers. He gave 140 to his brother and 190 to his sister. How many did he keep?
   b) Ed is 32 cm shorter than Ryan. Ryan is 15 cm taller than Mark. If Ed is 153 cm tall, then how tall is Mark?
   c) Mark ran 3 900 m on Tuesday. He ran 600 m further on Thursday. How far did he run altogether?
   d) Sally had 34 baskets of apples. She sold 11 on Monday. By the end of Tuesday, she had 5 baskets left. How many did she sell on Tuesday?

2. Ask your students to say what information is missing from each problem. Then ask them to make up an amount for the missing information and solve the problem.
   a) Howard bought a lamp for $17. He then sold it to a friend. How much money did he lose?
   b) Michelle rode her bike 5 km to school. She then rode to the community center. How many km did she travel?
   c) Kelly borrowed 3 craft books and some novels from the library. How many books did she borrow altogether?

Make up more problems of this sort.

3. Students should fill in the blanks and solve the problems.
   a) Carl sold _____ apples. Rebecca sold _____ apples. How many more apples did Carl sell than Rebecca.
   b) Sami weighs _____ kg. His father weighs _____ kg. How much heavier is Sami’s father?
   c) Ursula ran _____ km in gym class. Yannick ran _____ km. How much further did Ursula run?
   d) Jordan read a book _____ pages long. Digby read a book _____ pages long. How many more pages were in Jordan’s book?
NS5-16
Larger Numbers

Review **NS5-1** by asking volunteers to identify the place value of each digit in these numbers:

- a) 7
- b) 37
- c) 237
- d) 4237
- e) 42376

Write on the board: 8 943 206. **ASK:** How is this number different from most of the numbers we have worked with in this unit? (It has 7 digits.) Underline the 9 and tell students that its place value is “hundred thousands.” Underline the 8 and ask if anyone knows its place value (millions). Then have a volunteer identify the place value of each digit in f) 5 864 237.

Write all the place value words on the board for reference: ones, tens, hundreds, thousands, ten thousands, hundred thousands, and millions. Then write many 6- and 7-digit numbers, underline one of the digits in each number, and have students identify and write the place value of the underlined digits. Be sure to include the digit 0 in some of the numbers.

Have students write the numbers in a) to f) in expanded form using numerals (e.g. 237 = 200 + 30 + 7). Then have them write more 6- and 7-digit numbers in expanded form. **BONUS:** Assign numbers with more digits.

Read these numbers aloud with students and write the number words for each one:

- 34 000
- 78 000
- 654 000
- 981 000
- 8 000 000
- 7 000 000

Then write on the board: 654 502 = 654 000 + 502. Tell students that to read the number 654 502, they read both parts separately: six hundred fifty-four thousand five hundred two. **ASK:** How would you read 8 654 502?

Have students read several numbers aloud with you **(EXAMPLES):** 356, 2 356, 402 356, 9 402 356 Then have them write corresponding addition sentences for each number **(EXAMPLE):** 9 402 356 = 9 000 000 + 402 000 + 356 and the number words **(EXAMPLE):** nine million four hundred two thousand three hundred fifty-six). Have students write the number words for more 6- and 7-digit numbers. Include the digit 0 in some of the numbers.

Review comparing and ordering numbers with up to 5 digits, then write on the board:

- a) 657 834
- b) 826 789
- c) 2 790 504

- 951 963
- 6 154 802
- 2 789 504
**ASK:** What is the largest place value in which these pairs of numbers differ? Why do we care only about the largest place value that differs? **SAY:** 2 790 504 has 1 more in the ten thousands place, but 2 789 504 has 9 more in the thousands place. Which number is greater? How do you know?

Do a few addition and subtraction questions together, to remind students how to add and subtract using the standard algorithm (**EXAMPLES:** 354 + 42, 25 052 + 13 119, 4 010 – 559) Then teach students to extend the algorithm to larger numbers by adding 6- and 7-digit numbers together.

**Extensions**

1. **What is the number halfway between 99 420 and 100 000?**

2. **Write out the place value words for numbers with up to 12 digits:**

   - ones
   - thousands
   - millions
   - billions
   - tens
   - ten thousands
   - ten millions
   - ten billions
   - hundreds
   - hundred thousands
   - hundred millions
   - hundred billions

Point out that after the thousands, there is a new word every 3 place values. This is why we put spaces between every 3 digits in our numbers—so that we can see when a new word will be used. This helps us to identify and read large numbers quickly. Demonstrate this using the number 3 456 720 603, which is read as three **billion** four hundred fifty-six **million** seven hundred twenty **thousand** six hundred three. Then write another large number on the board—42 783 089 320—and **ASK:** How many billions are in this number? (42) How many more millions? (783) How many more thousands? (89) Then read the whole number together.

Have students practise reading more large numbers, then write a large number without any spaces and **ASK:** What makes this number hard to read? Emphasize that when the digits are not grouped in 3s, you can’t see at a glance how many hundreds, thousands, millions, or billions there are. Instead, you have to count the digits to identify the place value of the left-most digit. **ASK:** How can you figure out where to put the spaces in this number? Should you start counting from the left or the right? (From the right, otherwise you have the same problem—you don’t know what the left-most place value is, so you don’t know where to put the spaces. You always know the right-most place value is the ones place, so start counting from the right.)

Write more large numbers without any spaces and have students re-write them with the correct spacing and then read them (**EXAMPLE:** 87301984387 becomes 87 301 984 387).
If students are engaged in the lesson and time permits, tell them about the Japanese system of naming numbers. In Japanese, there is a new word every 4 places. The place value names are something like:

<table>
<thead>
<tr>
<th>ones</th>
<th>tens</th>
<th>hundreds</th>
<th>thousands</th>
</tr>
</thead>
<tbody>
<tr>
<td>mans</td>
<td>ten mans</td>
<td>hundred mans</td>
<td>thousand mans</td>
</tr>
<tr>
<td>okus</td>
<td>ten okus</td>
<td>hundred okus</td>
<td>thousand okus</td>
</tr>
<tr>
<td>chos</td>
<td>ten chos</td>
<td>hundred chos</td>
<td>thousand chos</td>
</tr>
</tbody>
</table>

In the Japanese system, 3456720603 would be read as thirty-four okus five thousand six hundred seventy-two mans six hundred three. **ASK:** How could we write the number with spaces so that it is easier to read using the Japanese system? (**ANSWER:** 34 5672 0603)

Have students practise writing and reading other numbers using the Japanese system.

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**NS5-17**

**Concepts in Number Sense**

Students will review the number sense concepts learned so far.

Worksheet **NS5-17** is a review of number sense concepts. It can be used as an assessment.

Some practice exercises your students can do include:

1. Find the greatest 3-digit number that you can subtract from 7315 without regrouping.
2. Find the greatest number you can add to 124 429 without regrouping.
3. Find the least number you can add to 5372 so you will have to regroup the tens, hundreds and thousands.
4. Create a 6-digit number so that…
   a) the ones digit is 1 more than the tens digit.
   b) all but one of the digits is odd.
   c) the number is the greatest you can create with these digits.
5. Choose a number less than 10 and greater than 0. **EXAMPLE:** 9
   If the number is even, halve it and add one. If the number is odd, double it. **EXAMPLE:** 9 \( \rightarrow \) 18
   Again, if the number is even, halve it and add one. If the number is odd, double it. **EXAMPLE:** 9 \( \rightarrow \) 18 \( \rightarrow \) 10
   a) Continue the number snake for the example. What happens?
   b) Investigate which number makes the longest snake before repeating.
   c) Try starting a number snake with a 2-digit number. What happens?

**Extension**

Provide the 2 BLMS “Define a Number” and “Define a Number (Advanced).”
Arrays

Your students should know that multiplication is a short form, or abbreviation, for addition. For instance, \(3 \times 5 = 5 + 5 + 5\). The statement \(3 \times 5\) can also be interpreted as \(3 + 3 + 3 + 3 + 3\), but you can explain this when your students are more familiar with the concept of multiplication. In the beginning, always have your students write the addition statement using the second number in the given product.

Have your students write various multiplication statements as addition statements. For instance:

\[
3 \times 2 = 2 + 2 + 2 \\
4 \times 3 = ? \\
5 \times 2 = ?
\]

Your student should also know how to represent a multiplication statement with an array of dots. For instance, \(3 \times 5\) can be represented as 3 rows of 5 dots:

\[
\begin{array}{ccc}
\bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet \\
\end{array}
\]

Draw several arrays on the board and ASK: How many rows? How many dots in each row? How many dots altogether? What multiplication sentence could you write? Emphasize that the number of rows is written first and the number in each row is written second. Include arrays that have only 1 row and arrays that have only 1 dot in each row.

Have your students use tiles or counters to create arrays for the following statements (make sure the number of rows matches the first number in the statement, and the number of dots in each row matches the second number).

a) \(2 \times 5\)  b) \(3 \times 3\)  c) \(4 \times 6\)  d) \(5 \times 3\)

Have your students use tiles or counters to create arrays for:

a) \(3 \times 3\)  b) \(3 \times 2\)  c) \(3 \times 1\)  d) \(3 \times 0\)

Have your students draw arrays in their notebooks for the following products:

a) \(2 \times 6\)  b) \(4 \times 5\)  c) \(6 \times 2\)  d) \(4 \times 4\)

Point out that, in each array, each row contains the same number of dots. Show students that there are 3 ways to arrange 4 dots so that each row contains the same number of dots:

\[
\begin{array}{ccc}
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \\
\bullet & \\
\end{array}
\]

Then give students 6 counters each and have them create as many different arrays as they can. They should draw their arrays in their notebooks and write the corresponding multiplication sentences (or “products”). As students
Part 1
Number Sense

Finish with this first problem, give them 2 more counters for a total of 8, then 4 more for a total of 12, then 3 more for 15, 1 more for 16, and finally 8 more for 24 counters. Some students will work more quickly than others; not all will get to 24 counters. Allow enough time for every student to make and record arrays using 6, 8, and 12 counters. Then ask students to share their answers. For example, some students may have 12 as 4 rows of 3 \((4 \times 3)\) or 3 rows of 4 \((3 \times 4)\), others as 6 rows of 2 or 2 rows of 6, and some as 12 rows of 1 or 1 row of 12. Accept one answer from each student so that everyone has a chance to contribute.

Tell your students that you want to find all the arrays that can be made with 12 counters, not just some of them. Have students tell you the arrays they found and ask them how they can be sure we have all of them. How should we organize our list? (one suggestion could be to organize by the number of rows; another possibility is to organize by the number in each row) Organizing by the number of rows, we can list all the possible arrays:

\[
\begin{array}{cccccccc}
1 \times 12 & 2 \times 6 & 3 \times 4 & 4 \times 3 & 5 \times \_ & 6 \times 2 \\
7 \times \_ & 8 \times \_ & 9 \times \_ & 10 \times \_ & 11 \times \_ & 12 \times 1
\end{array}
\]

We know there are at most 12 rows because we only have 12 counters. Since we’ve listed all possible values for the number of rows, and have found the number in each row whenever possible, we have all possible arrays \((3 \times 4, 4 \times 3, 2 \times 6, 6 \times 2, 1 \times 12, 12 \times 1)\). Tell your students that since you found all the possible arrays, they have found all the factors of 12. Tell your students that the factors of a number are the numbers that multiply to give that particular number, so 1, 2, 3, 4, 6 and 12 are the factors of 12. **Ask:** Is 7 a factor of 12? (no) How do you know? (if there were 7 rows of dots, we could have 7 dots or 14 dots or 21 dots and so on, but never 12). Tell them to look at the arrays they made in their notebooks for the numbers 6, 8, 15, 16 and 24, and then **Ask:** What are the factors of 6? Of 8? Of 15? Of 16? Of 18? Of 24? Challenge them to find all the factors of these numbers, not just some of them.

Ask students to draw arrays for the numbers 2, 3, 5, 7 and 11. What do you notice? (There are only 2 arrays for each number.) Numbers that have only 2 distinct factors (1 and the number itself) are called prime numbers: (i.e. \(7 = 1 \times 7 \text{ and } 7 = 7 \times 1\) are the only factorizations of 7, and 1 and 7 are the only factors of 7, so 7 is prime.) Numbers that have more than 2 distinct factors are called composites. 1 is neither prime nor composite because it only has 1 factor.

**Bonus**

Ask students if they can find the first 10 prime numbers. Then ask them to find the longest string of consecutive composite numbers less than 30.

Now give students multiplication sentences and have them draw the corresponding arrays (**Example:** \(2 \times 4 = 8; 5 \times 2 = 10, 3 \times 3 = 9\)). Remember that the first number is the number of rows and the second number is the number in each row.

Finally, give students multiplication word problems that refer directly to rows and the number in each row. Mix it up as to which is mentioned first: **Examples:**

- There are 3 rows of desks. There are 5 chairs in each row.
- There are 4 desks in each row. There are 2 rows of chairs.
- How many desks are there altogether? How many chairs are there altogether? \((3 \times 4 = 12)\) \((2 \times 5 = 10)\)

Have students draw corresponding arrays in their notebooks (they can use actual counters first if it helps them) to solve each problem. Ensure that students understand what the dots in each problem represent. (In the examples above, dots represent desks and chairs.)
Demonstrate the commutativity of multiplication (the fact that order doesn’t matter—$3 \times 4 = 4 \times 3$). Revisit the first array and multiplication sentence you looked at in this lesson ($4 \times 5 = 20$):

\[ \begin{array}{cccc}
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\end{array} \]

\[ 5 + 5 + 5 + 5 = 4 \times 5 = 20 \]

Then have students count the number of columns and the number of dots in each column and write a multiplication sentence according to those ($5 \times 4 = 20$):

\[ \begin{array}{cccc}
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\end{array} \]

\[ 4 + 4 + 4 + 4 + 4 = 5 \times 4 = 20 \]

**ASK:** Does counting the columns instead of the rows change the number of dots in the array? Are $4 \times 5$ and $5 \times 4$ the same number? What symbol do mathematicians use to show that two numbers or statements are the same? Write on the board: $4 \times 5 \quad 5 \times 4$ and have a volunteer write the correct sign in between ($=$).

Point out to your students that they can easily see the same thing by rotating an array (i.e. you get the same answer regardless of the order in which you multiply a pair of numbers).

Have students write similar multiplication sentences for the arrays they drew in their notebooks earlier. Then have them fill in the blanks in multiplication sentences that include large numbers.

**EXAMPLES:**

\[ 4 \times 7 = 7 \times \_ \_ \_ \]

\[ 19 \times 27 = \_ \_ \_ \times 19 \]

\[ \_ \_ \_ \times 5 = 5 \times 102 \]

\[ 387 \times \_ \_ \_ = 2 501 \times 387 \]

Have students create story problems they could solve by making:

a) 3 rows of 4 counters  
b) 2 rows of 5 counters  
c) 4 rows of 6 counters

Then have them give their story problems to a partner to solve.
Put 10 counters on the table in 2 rows of 5. Have a pair of students come up and look at the array from different directions, so that one student sees 2 rows of 5 and the other sees 5 rows of 2. Have pairs of students view different arrays from different directions at their own desks:

\[ \text{I see 2 rows of 5.} \]

\[ \text{I see 5 rows of 2.} \]

**ACTIVITY 1**

In groups of 3, have students write an addition sentence and a multiplication sentence for the number of shoes people in their group are wearing \((2 + 2 + 2 = 6\) and \(3 \times 2 = 6\)). Then have them write sentences for the number of left and right shoes in the group \((3 + 3 = 6\) and \(2 \times 3 = 6\)). Have them repeat the exercise for larger and larger groups of students.

Instead of left shoes and right shoes, students could group and count circles and triangles on paper:

They can count this as \(2 + 2 + 2 = 6\) \((3 \times 2 = 6)\) or they can count 3 circles + 3 triangles = 6 shapes \((2 \times 3 = 6)\).

**Extensions**

1. Have students add strings of numbers that are “almost” arrays using multiplication and adjusting their answers as needed. **EXAMPLE:** \(3 + 3 + 3 + 2\) is not quite \(3 + 3 + 3 + 3\), but is one less, so you can solve it by multiplying \(4 \times 3\) (12) and then subtracting 1 (11).

Other examples include:

\[ 4 + 4 + 4 + 4 + 5 = 4 \times 6 + 1 = 24 + 1 = 25 \]
\[ 2 + 2 + 2 + 2 + 3 = 5 \times 2 + 1 = 11 \]
\[ 6 + 6 + 6 + 5 = 5 \times 6 - 1 = 30 - 1 = 29 \]

Students could then work on addition sentences that can be turned into multiplication sentences by both adding 1 and subtracting 1, which is the same as moving 1 from one set to another. You can use counters to model such a sentence and the changes made. **EXAMPLE:** \(3 + 4 + 3 + 3 + 2\) is the same as \(3 + 3 + 3 + 3 + 3\), since we can move one counter from the set with 4 to the set with 2 to make 5 groups of 3.

Then have students change 2 numbers to create a multiplication sentence:

a) \(2 + 5 + 8\) \((= 5 + 5 + 5 = 3 \times 5)\)   b) \(4 + 7 + 10\)   c) \(3 + 4 + 4 + 5\)

Finally, have students change as many numbers as they need to in order to create a multiplication sentence:

a) \(5 + 7 + 6 + 6 + 6 + 4 + 8\)   b) \(8 + 6 + 7 + 3 + 10\)   c) \(9 + 11 + 5 + 15 + 4 + 16\)
2. Tell students that writing $4 \times 5 = 5 \times 4$ is a short way of saying that $4 \times 5$ and $5 \times 4$ are both the same number without saying which number that is (20). Introduce the following analogy with language: Instead of saying “The table is brown” and “Her shirt is brown” we can say “The table is the same colour as her shirt” or “The table matches her shirt.”

3. Teach multiplication of 3 numbers using 3-dimensional arrays. Give students blocks and ask them to build a “box” that is 4 blocks wide, 2 blocks deep, and 3 blocks high.

When they look at their box head-on, what do they see? (3 rows, 4 blocks in each row, a second layer behind the first) Write the math sentence that corresponds to their box when viewed this way: $3 \times 4 \times 2$

Have students carefully pick up their box and turn it around, or have them look at their box from a different perspective: from above or from the side. Now what number sentence do they see? Take various answers so that students see the different possibilities. **ASK:** When you multiply 3 numbers, does it matter which number goes first? Would you get the same answer in every case? (Yes, because the total number of blocks doesn’t change.)

4. You could also teach multiplication of 3 numbers by groups of groups:

This picture shows 2 groups of 4 groups of 3. **ASK:** How would we write 4 groups of 3 in math language? ($4 \times 3$) How could we write 2 groups of 4 groups of 3 in math language? ($2 \times 4 \times 3$). If I want to write this as a product of just 2 numbers, how can I do so? Think about how many groups of 3 hearts I have in total. I have $2 \times 4 = 8$ groups of 3 hearts in total, so the total number of hearts is $8 \times 3$.

**SO:** $2 \times 4 \times 3 = 8 \times 3$.

Invite students to look for a different way of grouping the hearts. **ASK:** How many hearts are in each of the 2 groups? (There are $4 \times 3 = 12$ hearts in each of the 2 groups, so $2 \times 4 \times 3 = 2 \times 12$.) Do I get the same answer no matter how I multiply the numbers? (Yes; $8 \times 3 = 2 \times 12$).

Put up this math sentence: $2 \times 4 \times 3 = 2 \times ____$. **ASK:** What number is missing? (12) How do you know? (because $4 \times 3 = 12$). Is 12 also equal to $3 \times 4$? Can I write $2 \times 4 \times 3 = 2 \times 3 \times 4$? Remind students that $2 \times 4 \times 3$ is the same as both $8 \times 3$ and $2 \times 12$. Have students explore different orderings of the factors. Have them try this with $2 \times 6 \times 5$.

Ask if there is any way to order the factors that make it particularly easy to find the product. What is the answer? ($2 \times 6 \times 5 = 2 \times 5 \times 6 = 10 \times 6 = 60$). Then challenge them to find more products of 3 numbers:

a) $25 \times 7 \times 4$

b) $15 \times 7 \times 2$

c) $250 \times 9 \times 4$

d) $50 \times 93 \times 2$
Students who finish quickly may want to try multiplying 4 or even 5 numbers. Challenge them to find: $5 \times 5 \times 4 \times 2$, $25 \times 2 \times 4 \times 5$, $25 \times 5 \times 4 \times 7 \times 6$. They might want to find a way to model $5 \times 5 \times 4 \times 2$ on paper or with cubes. This could be done by drawing 5 groups of 5 groups of 4 groups of 2.

It could also be done by building 5 cubes that are each $5 \times 4 \times 2$.

**NS5-19**

**Multiplying by Skip Counting and Adding On**

Show a number line on the board with arrows, as follows:

```
0 1 2 3 4 5 6 7 8 9 10
```

Ask students if they see an addition sentence in your number line. **ASK:** What number is being added repeatedly? How many times is that number added? Do you see a multiplication sentence in this number line? Which number is written first—the number that’s repeated or the number of times it is repeated? (The number of times it is repeated.) Then write “$5 \times 2 = \square$” and **ASK:** What does the 5 represent? What does the 2 represent? What do I put after the equal sign? How do you know?

Have volunteers show several products on number lines (**EXAMPLES:** $6 \times 2$, $3 \times 4$, $2 \times 5$, $5 \times 1$, $1 \times 5$).

Then have a volunteer draw arrows on an extended number line to show skip counting by 3s:

```
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
```
Have students find $5 \times 3$, $2 \times 3$, $6 \times 3$, $4 \times 3$, and other multiples of 3 by counting arrows.

Then draw 2 hands on the board and skip count with students by 3 up to 30 (you can use your own hands or the pictures):

```
<table>
<thead>
<tr>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>21</td>
<td>24</td>
<td>27</td>
<td>30</td>
</tr>
</tbody>
</table>
```

Have students skip count on their hands to find $5 \times 3$, $2 \times 3$, $6 \times 3$, $4 \times 3$, and other multiples of 3. **ASK:** Which is easier to use to multiply by 3, number lines or your hands? Why? (Since they probably don’t need to count fingers to tell when 6 fingers are held up but they do need to count 6 arrows, students are likely to find it easier to read answers on their hands.)

Finally, tell students that knowing their multiplication facts (or “times tables”) will help them solve problems which would otherwise require a good deal of work adding up numbers. For instance, they can now solve the following problems by multiplying.

a) There are 4 pencils in a box. How many pencils are there in 5 boxes?
b) A stool has 3 legs. How many legs are on 6 stools?
c) A boat can hold 2 people. How many people can 7 boats hold?

---

**NS5-20**

**Multiplying by Adding On**

**GOALS**

Students will turn products into smaller products and sums.

**PRIOR KNOWLEDGE REQUIRED**

Arrays

Multiplication

Draw the following pictures on the board:

```

```

Have students write a multiplication sentence for each picture. Then have them draw their own pictures and invite partners to write multiplication sentences for each other’s pictures. Demonstrate the various ways of representing multiplication sentences by having volunteers share their pictures.

Then draw 2 rows of 4 dots on the board:

```

```

**ASK:** What multiplication sentence do you see? ($2 \times 4 = 8$) (Prompt as needed: How many rows are there? How many in each row? How many altogether?) What happens when I add a row? Which numbers change?
Which number stays the same? (The multiplication sentence becomes $3 \times 4 = 12$; we added a row, so there are 3 rows.) How many dots did we add? Invite a volunteer to write the math sentence that shows how $2 \times 4$ becomes $3 \times 4$ when you add 4. $(2 \times 4 + 4 = 3 \times 4)$.

Look back at the number line you drew above:

```
0  1  2  3  4  5  6  7  8  9  10
```

**ASK:** What multiplication sentence do you see? What addition sentence do you see? (Prompt as needed: Which number is repeated? How many times is it repeated?) Ensure all students see $5 \times 2 = 10$ from this picture. Then draw another arrow:

```
0  1  2  3  4  5  6  7  8  9  10  11  12
```

**ASK:** Now what multiplication sentence do you see? What are we adding to $5 \times 2$ to get $6 \times 2$? Have a volunteer show how to write this as a math sentence: $5 \times 2 + 2 = 6 \times 2$.

Repeat using the 4 sets of 3 hearts and adding another set of 3 hearts.

Have students use arrays to practise representing products as smaller products and sums. Begin by providing an array with blanks (as in **QUESTION 1** on the worksheet) and have volunteers come up and fill in the blanks, as has been done in the following **EXAMPLE**:

![Example Array]

In their notebooks, have students draw an array (or use counters) to show that:

- a) $3 \times 6 = 2 \times 6 + 6$
- b) $5 \times 3 = 4 \times 3 + 3$
- c) $3 \times 8 = 2 \times 8 + 8$

Have students do the following questions without using arrays:

- a) If $10 \times 2 = 20$, what is $11 \times 2$?
- b) If $5 \times 4 = 20$, what is $6 \times 4$?
- c) If $11 \times 5 = 55$, what is $12 \times 5$?
- d) If $8 \times 4 = 32$, what is $9 \times 4$?
- e) If $6 \times 3 = 18$, what is $7 \times 3$?
- f) If $8 \times 2 = 16$, what is $9 \times 2$?
- g) If $2 \times 7 = 14$, what is $3 \times 7$?

Ask students to circle the correct answer: $2 \times 5 + 5 = 2 \times 6$ **OR:** $3 \times 5$. Repeat with various examples.

Finally have students turn products into a smaller product and a sum without using arrays. Begin by giving students statements with blanks to fill in:

- a) $5 \times 8 = 4 \times 8 + ____$
- b) $9 \times 4 = ____ \times 4 + ____$
- c) $7 \times 4 = ____ \times ____ + ____$
- d) $8 \times 5 = ________
NS5-21
Multiples of 10

ASK: What product does each picture represent?

(3 × 2)

(3 × 20)

Have a volunteer draw a picture for 3 × 200. Repeat for 3 × 2 000.

Ask students if they see a pattern in the multiplication sentences that correspond to these pictures:

3 × 2 = 6  3 × 20 = 60  3 × 200 = 600  3 × 2 000 = 6 000

ASK: If you have 2 hundreds 3 times, how many hundreds do you have? (6) If you have 2 tens 3 times, how many tens do you have? (6) If you have 2 baskets of apples 3 times, how many baskets of apples do you have? (6)

ASK: If I have 3 tens 4 times, what number am I repeating 4 times? (30) How do I write that as a multiplication sentence? (4 × 30) How many tens do I have? (12) What number is that? (120) Have a volunteer model this on the board with base ten materials.

Repeat with several examples and then have students do the following questions in their notebooks (they should draw base ten models to illustrate each one):

5 × 20 = 5 × _____ tens = _____ tens = _________
3 × 50 = 3 × _____ tens = _____ tens = _________
4 × 200 = 4 × _____ hundreds = _____ hundreds = _________
3 × 2 000 = 3 × _____ thousands = _____ thousands = _________
2 × 4 000 = 2 × _____ thousands = _____ thousands = _________

When students have mastered this, ASK: What is 5 × 3? How can you use this to find 5 × 300? (I know 5 × 3 = 15, SO: 5 × 300 is 15 hundreds or 1500.) SAY: I know that 3 × 8 = 24. What is 3 × 80? 3 × 800? 3 × 8 000?

Change the order of the factors so that the multiple of 10, 100 or 1000 is the first factor:

a) 400 × 2  b) 30 × 8  c) 6 000 × 9  d) 700 × 8  e) 900 × 5
**Extension**

Teach students to multiply products where both factors are multiples of 10, 100 or 1000.

a) \(30 \times 50 = 30 \times \_ \text{ tens} = \_ \text{ tens} = \_
\)

b) \(300 \times 50 = 300 \times \_ \text{ tens} = \_ \text{ tens} = \_
\)

c) \(20 \times 800 = 20 \times \_ \text{ hundreds} = \_ \text{ hundreds} = \_
\)

d) \(300 \times 700 = 300 \times \_ \text{ hundreds} = \_ \text{ hundreds} = \_
\)

e) \(400 \times 3000 = 400 \times \_ \text{ thousands} = \_ \text{ thousands} = \_
\)

f) \(500 \times 40\)

g) \(60 \times 5000\)

h) \(700 \times 9000\)

i) \(8000 \times 400\)

j) \(80000 \times 400000\)

k) \(60000 \times 500000\)

---

**NS5-22**

**Advanced Arrays**

Ask students what multiplication statement (number of rows \(\times\) number in each row) they see in each picture:

![Array Diagram]

Then do the same thing for each part of this diagram:

![Array Diagram]

Now ask students to identify the multiplication statement for the whole diagram (\(3 \times 13\) since there are 3 rows and 13 in each row). **ASK:** How can we get \(3 \times 13\) from \(3 \times 10\) and \(3 \times 3\)? What operation do you have to do? How do you know? (You have to add because you want the total number of squares.) Then write on the board: \(3 \times 10 + 3 \times 3 = 3 \times 13\). Repeat with several examples, allowing students to write the final math sentence combining multiplication and addition. Include examples where the white part of the array is 20 squares long instead of 10.

Students can make base ten models to show how to break a product into the sum of two smaller products. **QUESTION 2 a)** on worksheet NS5-22 shows
that $3 \times 24 = 3 \times 20 + 3 \times 4$. Ask students to make a similar model to show that $4 \times 25 = 4 \times 20 + 4 \times 5$.

**STEP 1:** Make a model of the number $25$ using different coloured base ten blocks (one colour for the tens blocks and one colour for the ones):

![](image1.png)

**STEP 2:** Extend the model to show $4 \times 25$

![](image2.png)

**STEP 3:** Break the array into two separate arrays to show $4 \times 25 = 4 \times 20 + 4 \times 5$

![](image3.png)

Show students how to draw and label an array to represent the sums of products. For example, $3 \times 5 + 2 \times 5$ may be represented by:

![](image4.png)

Students should see from the above array that: $3 \times 5 + 2 \times 5 = (3 + 2) \times 5 = 5 \times 5$

This is the **distributive property** of multiplication—multiplying $5$ by two numbers that add up to $5$ (in this case, $3$ and $2$) and adding the products.

To practice, have students draw and label arrays for:

a) $2 \times 2 + 3 \times 2$

b) $3 \times 4 + 2 \times 4$

c) $4 \times 5 + 2 \times 5$

**Extension**

Order of operations: Give students the following problem: $10 \times 2 + 4 \times 2 = ____$. Have a volunteer show their answer on the board and how they got it (for example, $20 + 8 = 28$).

**ASK:** What would happen if we did the operations in the order they appear? Do this together, to demonstrate how the order in which the operations are done can drastically affect the answer: $10 \times 2 + 4 \times 2 = 20 + 4 \times 2 = 24 \times 2 = 48$. Show students more “wrong” answers, such as doing the addition before the multiplication: $10 \times 6 \times 2 = 60 \times 2 = 120$. **ASK:** Why is it important for there to be an agreed upon order of operations? Tell students that we always do multiplication before addition. Give students several problems to practice with: a) $7 + 2 \times 5 + 4$

b) $9 + 2 \times 3 + 8$

c) $4 \times 8 + 9 \times 3$. **BONUS:** $3 \times 4 + 7 + 5 \times 6 + 2 \times 3 + 9$. 

**GOALS**

Students will multiply large numbers by breaking them into smaller numbers.

**PRIOR KNOWLEDGE REQUIRED**

Multiplication

Arrays

Distributive property

Commutative property of multiplication

---

**ASK:** How can we use an array to show $3 \times 12$? Have a volunteer draw it on the board. Then ask if anyone sees a way to split the array into two smaller rectangular arrays, as in the last lesson. Which number should be split—the 3 or the 12? **SAY:** Let’s split 12 because 3 is already small enough. Let’s write: $12 = \_\_\_ + \_\_\_$. What’s a nice round number that is easy to multiply by 3? (10) Fill in the blanks ($12 = 10 + 2$) then have a volunteer split the array. **ASK:** What is $3 \times 10$? What is $3 \times 2$? What is $3 \times 12$? How did you get $3 \times 12$ from $3 \times 10$ and $3 \times 2$? Have a volunteer write the math sentence that shows this on the board: $3 \times 12 = 3 \times 10 + 3 \times 2$.

**ASK:** Did we need to draw the arrays to know how to split the number 12 apart? Ask students to split the following products in their notebooks without drawing arrays (they should split the larger number into two smaller numbers, one of which is a multiple of 10):

- a) $2 \times 24$
- b) $3 \times 13$
- c) $4 \times 12$
- d) $6 \times 21$
- e) $9 \times 31$
- f) $4 \times 22$

When students are comfortable splitting these products into the sum of two smaller products, have them solve each problem.

Give early finishers these slightly more difficult problems:

- a) $2 \times 27$
- b) $3 \times 14$
- c) $7 \times 15$
- d) $6 \times 33$
- e) $8 \times 16$

To solve these products, students will have to do more than just copy digits when adding the numbers in the last step (**EXAMPLE:** $2 \times 27 = 2 \times 20 + 2 \times 7 = 40 + 14 = 54$).

**ASK:** To find $2 \times 324$, how can we split the 324 into smaller numbers that are easy to multiply by 2? How would we split 24 if we wanted $2 \times 24$? We split 24 into 2 tens and 4 ones. What should we split 324 into?

$324 = 300 + 20 + 4$, **SO:**

$$2 \times 324 = 2 \times 300 + 2 \times 20 + 2 \times 4$$

Have students multiply more 3-digit numbers by 1-digit numbers by expanding the larger number and applying the distributive property. (**EXAMPLES:** $4 \times 231, 8 \times 126, 5 \times 301$)
Students should record their answers and the corresponding base ten models in their notebooks. Then have them solve additional problems without drawing models. Finally, have them solve problems in their heads.

**Bonus**

Students who finish quickly can do problems with 4- and 5-digit numbers.

**Extensions**

1. Have students draw models for $2 \times 24$ and $24 \times 2$. How would they split the two rectangles? Emphasize that just as $2 \times 24 = 2 \times 20 + 2 \times 4$, we can split $24 \times 2$ as follows: $24 \times 2 = 20 \times 2 + 4 \times 2$. Have students multiply 2-digit numbers by 1-digit numbers, as they did during the lesson, but with the larger number first (**EXAMPLES**: $13 \times 3, 12 \times 4, 21 \times 6$).

2. Have students combine what they learned in this lesson with what they learned in lesson **NS5-21**. Ask them to multiply 3-digit numbers by multiples of 1000 or 10 000.
   
   a) $342 \times 2000$  
   b) $320 \times 6000$  
   c) $324 \times 2000$  
   d) $623 \times 20000$

3. Have students calculate the total number of days in a non-leap year:
   
   a) By adding the sequence of 12 numbers:
      
      $31 + 28 + 31 + \ldots + 30 + 31 = \square$
   
   b) By changing numbers to make a multiplication and addition statement:
      
      $31 + 28 + 31 + 30 + 31 + 30 + 31 + \ldots + 31$
      
      $30 + 1 \quad 30 + 1 \quad 30 + 1 \quad 30 + 1 \quad 30 + 1 \quad 30 + 1$
      
      $= 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30$
      
      $+ 1 + 1 + 1 + 1 + 1 + 1 + 1 - 2$
      
      $= 30 \times 12 + 5 = 360 + 5 = 365$.

   Guide the students by asking what a good round number is that is close to 28 and 31 that they can change all 12 numbers to. Then have them think about how to adjust for the changes they made. If they pretend all the numbers are 30, how many times would they have to add 1 to get the actual number of days? (7) How many times would they have to subtract? (only once for February) What are they subtracting by? (2) What is the overall effect? (adding 5)
**GOALS**

Students will multiply 2-digit numbers by 1-digit numbers using the standard algorithm (without regrouping ones).

**PRIOR KNOWLEDGE REQUIRED**

Breaking 2-digit numbers into a multiple of 10 and a 1-digit number

**VOCABULARY**

algorithm

---

**The Standard Method for Multiplication**

**ASK:** How can I find $42 \times 3$? How can I split the 42 to make it easier? ($42 = 40 + 2$) Why does this make it easier? (Because I know $4 \times 3$ is 12; to find $40 \times 3$ I just add a 0 because $40 \times 3$ is 4 tens $\times 3$ which is 12 tens.)

Tell your students that this strategy is something that mathematicians use often; they break a problem into two easier problems and then they solve the easier problems. The trade-off is that they now have two problems to solve ($40 \times 3$ and $2 \times 3$) instead of only one, but they are both easier to do than the original problem. Put the following base ten model on the board for reference, then show students this modification of the standard method for multiplying:

\begin{align*}
42 \\
\times 3 \\
\underline{2 \times 3:} & \quad 6 \\
\underline{40 \times 3:} & \quad 120 \\
& \quad 126
\end{align*}

**ASK:** How are the two ways of writing and solving $42 \times 3$ the same and how are they different? Put up the following incorrect notation and ask student to identify the error (the 6 resulting from $3 \times 2$ is aligned with the tens, not the ones).

\begin{align*}
42 \\
\times 3 \\
\underline{2 \times 3:} & \quad 6 \\
\underline{40 \times 3:} & \quad 120 \\
& \quad 186
\end{align*}

**ASK:** Why is it important to line up the ones digit with the ones digit and the tens digit with the tens digit? Emphasize that the incorrect alignment adds 60 and 120 but $2 \times 3$ is only 6, not 60.

Have students practise multiplying more numbers using the modified standard method. They can use grid paper to help them line the numbers up, if necessary.

**EXAMPLES:**

\begin{align*}
12 & \times 4 \\
32 & \times 3 \\
21 & \times 8 \\
43 & \times 2 \\
2 \times 4: & \\
10 \times 4: & \text{______}
\end{align*}
Have volunteers write their answers on the board. Then circle the 0 in the second row of each problem, where the tens are being multiplied. **ASK:** Will there always be a 0 here? How do you know? If there is always going to be a 0 here, is there a way we could save time and space when we multiply? Suggest looking at the ones digit of the answer. Where else do students see that digit? What about the other digits? Where else do they see those? Show students how they can skip the intermediate step used above and go straight to the last line, as follows:

\[
\begin{array}{c}
42 \\
\times 3 \\
\hline
126
\end{array}
\]

\[
2 \times 3 \text{ ones} = 6 \text{ ones}
\]

\[
4 \times 3 \text{ tens} = 12 \text{ tens}
\]

\[
12 \text{ tens} + 6 \text{ ones} = 120 + 6 = 126
\]

Have students use this standard method, or algorithm, to multiply more 2-digit numbers in their notebooks. They should indicate with arrows the number of ones and tens, as is done above. Early finishers can solve problems with more digits, but still without regrouping. **(EXAMPLES:** 321 × 4, 342 × 2, 4221 × 3)

**NS5-25**

**Regrouping**

**GOALS**

Students will multiply any 2-digit number by any 1-digit number using the standard algorithm.

**PRIOR KNOWLEDGE REQUIRED**

The standard algorithm for multiplication

Remind students how they used the standard algorithm (modified and unmodified) to multiply numbers in the last lesson:

**Method 1:**

\[
\begin{array}{c}
42 \\
\times 3 \\
\hline
126
\end{array}
\]

\[
2 \times 3: \quad 6
\]

\[
40 \times 3: \quad 120
\]

**Method 2:**

\[
\begin{array}{c}
42 \\
\times 3 \\
\hline
126
\end{array}
\]

Then have them do the following questions using Method 1:

\[
\begin{array}{c}
42 \\
\times 7 \\
\hline
294
\end{array}
\]

\[
35 \\
\times 9 \\
\hline
315
\]

\[
28 \\
\times 4 \\
\hline
112
\]

Have volunteers write their answers on the board. **ASK:** How are these problems different from the problems we did before? Draw a base ten model for the first problem and use it to illustrate the regrouping of the ones and tens when you multiply 42 × 7:

\[
4 \text{ tens} \times 7 = 28 \text{ tens}
\]

\[
2 \text{ ones} \times 7 = 14 \text{ ones}
\]

\[
42 \times 7 = 28 \text{ tens} + 1 \text{ ten} + 4 \text{ ones}
\]

\[
= 29 \text{ tens} + 4 \text{ ones}
\]

\[
= 2 \text{ hundreds} + 9 \text{ tens} + 4 \text{ ones}
\]
Have students solve more multiplication questions by writing out and regrouping the tens and ones in their notebooks, as in the following EXAMPLE:

\[
\begin{array}{c}
36 \\
\times 7
\end{array}
\]

\[
\begin{array}{c c c c c c}
36 &=& 3 \text{ tens} &+& 6 \text{ ones} \\
6 \text{ ones} \times 7 &=& ____ \text{ tens} &+& ____ \text{ ones} \\
3 \text{ tens} \times 7 &=& ____ \text{ tens} &=& ____ \text{ hundreds} &+& ____ \text{ tens} \\
36 \times 7 &=& ____ \text{ hundreds} &+& ____ \text{ tens} &+& ____ \text{ ones}
\end{array}
\]

When students are comfortable with this, show them how to record the ten that comes from regrouping the ones using the standard algorithm:

\[
\begin{array}{c}
1 \\
42 \\
\times 7
\end{array}
\]

\[
\begin{array}{c}
2 \times 7 = 14 \\
2 \text{ ones} \times 7 = 1 \text{ ten} + 4 \text{ ones} \\
\end{array}
\]

Carry the 1 to the tens column

\[
\begin{array}{c}
4 \text{ tens} \times 7 = 28 \text{ tens} \\
28 \text{ tens} + 1 \text{ ten} = 29 \text{ tens} = 2 \text{ hundreds} + 9 \text{ tens}
\end{array}
\]

Have a volunteer multiply 34 \times 8 on the board using the standard algorithm, but invite other students to help him or her. ASK: What should our volunteer do first? What next? Make sure students understand and can explain what the 1 added above the tens column represents.

Put the following problems on the board and ask students to identify which one is wrong and to explain why.

Method A:  Method B:  Method C:

\[
\begin{array}{cc c c c}
42 \\
\times 7
\end{array}
\]

\[
\begin{array}{c c c c}
2 \times 7: & 14 \\
40 \times 7: & 280
\end{array}
\]

\[
\begin{array}{c c c c c c}
1 \\
42 \\
\times 7
\end{array}
\]

\[
\begin{array}{c c c c c c}
2 \times 7 = 14 \\
294
\end{array}
\]

Have students practise using the standard algorithm to multiply. (Be sure to give students lots of practice at this skill.) EXAMPLES:

\[
\begin{array}{c c c c}
13 \\
\times 4
\end{array}
\]

\[
\begin{array}{c c c c}
35 \\
\times 3
\end{array}
\]

\[
\begin{array}{c c c c}
23 \\
\times 8
\end{array}
\]

\[
\begin{array}{c c c c}
43 \\
\times 6
\end{array}
\]

**Extension**

Which is larger: 28 \times 6 or 26 \times 8? How can you tell without actually multiplying the numbers together? Encourage students to actually do the calculations and then to reflect back on how they could have known which was bigger before doing the calculations. Challenge them to decide first which will be larger between the following pairs and then to do the actual calculations to check their predictions:  a) 34 \times 5 or 35 \times 4  b) 27 \times 9 or 29 \times 7  c) 42 \times 3 or 43 \times 2  d) 89 \times 7 or 87 \times 9

Students with a feel for numbers (a “number sense”) should predict that multiplying by the larger single-digit number will be larger since the other two are so close together. For example, adding 35 four times is not much more than adding 34 four times, but adding 34 five times is substantially more than adding 34 four times. Another way to think about it is that both products have the same product of their ones digits (in part a, this is 4 \times 5 or 5 \times 4), but the product of the tens digit with the single-digit number, in part a, is either 3 \times 5 or 3 \times 4.
NS5-26
Multiplying – 3-Digit by 1-Digit

Review with students 3 ways of multiplying $34 \times 2$:

1. Using the expanded form:
   \[ 30 + 4 \times 2 = 60 + 8 \]

2. With base ten materials:

3. Using the standard algorithm:
   \[ 34 \times 2 = 68 \]

**NOTE:** Students have multiplied using the expanded form in previous lessons, but may not have written the problem out exactly like this. Explain what you’re doing aloud as you solve the problem (EXAMPLE: I split 34 up into 30 and 4, then I multiply each number by 2 and add up the results.)

Then tell students that you would like to multiply $213 \times 2$. **ASK:** How can I write 213 in expanded form?

Have a volunteer write it on the board. Then add “× 2”:

\[ 200 + 10 + 3 \times 2 \]

Have another volunteer multiply each number by 2 and add all the numbers to find the answer.

\[ 200 + 10 + 3 \times 2 = 400 + 20 + 6 = 426 \]

Invite other volunteers to solve the problem using base ten materials and the standard algorithm. Do a few more problems that do not require any regrouping (EXAMPLES: $324 \times 2, 133 \times 3, 431 \times 2$). Have volunteers use whichever method they prefer. Finally, solve $39 \times 3$ using the standard algorithm and review the notation for regrouping the tens.

Now solve $213 \times 4$ using the modified standard algorithm and the expanded form, as shown:

\[
\begin{align*}
213 & \quad 200 + 10 + 3 \\
\times 4 & \quad \times 4 \\
\hline
3 \times 4 \text{ ones} = 12 \text{ ones} & \quad 800 + 40 + 12 = 852 \\
1 \times 4 \text{ tens} = 4 \text{ tens} & \\
2 \times 4 \text{ hundreds} = 8 \text{ hundreds} & \\
\hline & 852 
\end{align*}
\]
Make sure students understand why the tens and hundreds digits are positioned as they are in the first solution. **ASK:** How are the two ways of solving the problem similar and how are they different? (You multiply the ones, tens, and hundreds separately in both; you line the digits up in the first method.) Then have a volunteer solve $213 \times 4$ using base ten materials and have them explain how their method relates to the two methods shown above. Finally, have a volunteer do the problem using the standard algorithm:

```
  1
213
× 4
---
  852
```

Together, solve:

- problems that require regrouping ones to tens (**EXAMPLES:** $219 \times 3, 312 \times 8, 827 \times 2$)
- problems that require regrouping tens to hundreds (**EXAMPLES:** $391 \times 4, 282 \times 4, 172 \times 3$)
- problems that require regrouping both ones and tens (**EXAMPLES:** $479 \times 2, 164 \times 5, 129 \times 4$)

Remind students that they are using the distributive property when they multiply using the expanded form:

$$8 \times 365 = 8 \times (300 + 60 + 5) = 8 \times 300 + 8 \times 60 + 8 \times 5$$

Students should be able to articulate the advantages of using the distributive property. (**EXAMPLE:** “I know that $9 \times 52 = 9 \times 50 + 9 \times 2$, and this is easier to calculate in my head because I get $450 + 18 = 468$.”)

Have students do additional problems in their notebooks. Students should solve each problem using base ten materials, the expanded form, and the standard algorithm.

**Extension**

Explain why a 3-digit number multiplied by a 1-digit number must have at most 4 digits. Students may wish to investigate, by using their calculators, the maximum number of digits when multiplying:

- a) 2-digit by 2-digit numbers
- b) 1-digit by 4-digit numbers
- c) 2-digit by 3-digit numbers
- d) 3-digit by 4-digit numbers
- e) 3-digit by 5-digit numbers

Challenge your students to predict the maximum number of digits when multiplying a 37-digit number with an 8-digit number.
Draw 3 rows of 8 and ASK: How many dots do I have here? What multiplication sentence does this show? (3 × 8 = 24) I have 3 rows of 8. If I want to double that, how many rows of 8 should I make? Why? (I should make 6 rows of 8 because 6 is the double of 3.) Add the additional rows and SAY: Now I have twice as many dots as I started with. How many dots did I start with? (24) How many dots do I have now—what is the double of 24? Ask students to give you the multiplication sentence for the new array: 6 × 8 = 48.

Ask students to look for another way to double 3 rows of 8. ASK: If we kept the same number of rows, what would we have to double instead? (the number of dots in each row) So how many dots would I have to put in each row? (16, because 16 is the double of 8) What is 3 × 16? (It is the double of 3 × 8, so it must be 48.) How can we verify this by splitting the 16 into 10 and 6? Have someone show this on the board: 3 × 16 = 3 × 10 + 3 × 6 = 30 + 18 = 48.

Have students fill in the blanks:

a) 4 × 7 is double of __________, so 4 × 7 = __________.

b) 5 × 8 is double of __________, so 5 × 8 = __________.

c) 6 × 9 is double of __________, so 6 × 9 = __________.

d) 3 × 8 is double of __________, so 3 × 8 = __________.

e) 4 × 8 is double of __________, so 4 × 8 = __________.

Encourage students to find two possible answers for the first blank in part e). (2 × 8 or 4 × 4 are both possible)

Ask volunteers to solve in sequence:

2 × 3; 4 × 3; 8 × 3; 16 × 3; What is double of 16? What is 32 × 3? What is double of 32? What 64 × 3? Different volunteers could answer each question.

Repeat with the sequences beginning as follows. Take each sequence as far as your students are willing to go with it.

a) 3 × 5, 6 × 5,

b) 2 × 6, 4 × 6,

c) 3 × 4, 6 × 4,

d) 2 × 7, 4 × 7

e) 2 × 9, 4 × 9

f) 3 × 9, 6 × 9

Ask students to find two possible answers for the first blank in the following:

20 × 32 is double of __________

(ANSWERS: 10 × 32 or 20 × 16)

ASK: Which multiplication is easier to do: 10 × 32 or 20 × 16? What is 10 × 32? What is 20 × 32?
Show the following array on the board:

```
20
32
10
10
```

Write on the board: $20 \times 32 = 2 \times 10 \times 32$

$$= 2 \times 320 = 640$$

Ask a volunteer to explain how this picture shows why this works (a $20 \times 32$ array is the same as two $10 \times 32$ arrays put together.)

**ASK:** How many $10 \times 32$ arrays are in a $30 \times 32$ array? Have students write a multiplication statement to show this. ($30 \times 32 = 3 \times 10 \times 32$). **ASK:** What is $10 \times 32$? What is $30 \times 32$?

Put the following sizes of arrays on the board:

- a) $20 \times 17$
- b) $30 \times 25$
- c) $20 \times 24$
- d) $40 \times 21$
- e) $30 \times 81$

Have students write the first factor as a multiple of ten and then to write the product as a product of three numbers. (**EXAMPLE:** $20 = 2 \times 10$ so $20 \times 17 = 2 \times 10 \times 17$)

Have students finish the multiplication by multiplying the last two factors (one of which is 10) and then multiplying by the first factor). (**EXAMPLE:** $20 \times 17 = 2 \times 10 \times 17 = 2 \times 170 = 340$)

Have volunteers fill in the blanks:

- a) $20 \times 34 = 2 \times _____$
- b) $30 \times 16 = 3 \times _____$
- c) $20 \times 28 = 2 \times _____$
- d) $50 \times 17 = 5 \times _____$
- e) $70 \times 23 = 7 \times _____$
- f) $120 \times 34 = 12 \times _____$
- g) $70 \times 15 = _____ \times 150$
- h) $60 \times 37 = _____ \times 370$
- i) $280 \times 46 = _____ \times 460$
- j) $90 \times 27 = _____ \times _____$
- k) $80 \times 35 = _____ \times _____$
- l) $20 \times 95 = _____ \times _____$

Challenge students to find several products mentally by following the steps:

**STEP 1:** Multiply the second number by 10.

**STEP 2:** Multiply the result by the tens digit of the first number.

(**EXAMPLES:** $40 \times 33, 70 \times 21, 40 \times 31$)

Tell your students that sometimes the multiple of 10 is the second number instead of the first number, as in: $36 \times 50$. **ASK:** How would steps 1 and 2 change for this problem? Have students solve several problems with the multiple of 10 in any position. (**EXAMPLES:** $60 \times 41, 55 \times 30, 80 \times 40$) Challenge your students to write steps 1 and 2 so that it takes into account both types of questions.

Write down the following steps on the board:
STEP 1: Multiply the 2-digit number by the tens digit of the multiple of ten

STEP 2: Add a zero to the result of the last step (this step corresponds to multiplying by ten)

ASK: How are these steps the same and how are they different from the other two steps? (they are in the opposite order; they replace “multiplying by 10” by “adding a 0”; they are more general in that it doesn’t matter which factor, the first or the second, is the multiple of 10; they still give the same answer).

Make sure your students know that the method also works for larger multiples of 10, for instance, to find $2000 \times 32$, multiply $32 \times 2 = 64$ and then add 3 zeroes to get 64 000. (Here is the longer way to represent this: $2000 \times 32 = 2 \times 1000 \times 32 = 2 \times 32000 = 64000$.)

Finally, review rounding with your students, then teach your students to estimate products of 2-digit numbers by rounding each factor to the tens digit. **EXAMPLE:** $25 \times 42 = 1200$.

**Bonus**

a) Estimate products of three 2-digit numbers. **EXAMPLE:** $27 \times 35 \times 41 = 30 \times 40 \times 40 = 48000$.

b) Estimate products of two 3-digit numbers by rounding to the nearest multiple of a hundred. **EXAMPLE:** $271 \times 320 = 300 \times 300 = 900$

To review rounding, remind your students that if the ones digit is 4 or less, they round down and if the ones digit is 5 or more they round up: $34 \rightarrow 30; 35 \rightarrow 40$.

There are several ways to help students remember how to round.

a) If the number is closest to a unique multiple of 10, round it to that multiple of 10;

b) If the ones digit is 5, then we round up. Several ways to remember this fact include:

- The numbers in the thirties (30, 31, …, 39) that round to 30 because they are closer to 30 are 30, 31, 32, 33, 34. The numbers that are closest to 40 are 36, 37, 38, 39. To make the same amount of numbers round to 30 as to 40, where should we put 35?

- When rounding to the nearest hundreds instead of tens, the tens digit usually tells you which multiple of 100 the number is closest to; the only exceptions are numbers such as 50, 150, 250 and so on. Every other number with tens digit 5 would round up, so if we want to round a number by only looking at the tens digit, we should round 350 up to 400 as well as 351, 352 and so on.

- Use the following analogy: You have started to cross a street that is 10 steps long:

```
|   |   |   |   |   |   |   |   |   |   |
```

0 1 2 3 4 5 6 7 8 9 10

You see a truck coming and need to decide between going back to the beginning or finishing crossing the street. **ASK:** Where would you go if you had taken a) 3 steps b) 7 steps c) 6 steps d) 2 steps e) 5 steps. There are two reasons why someone having taken 5 steps might choose to finish crossing the street instead of turning back—they are already facing in that direction and they were trying to cross the street anyway. Thus, solving this problem will obtain the same results as rounding a single-digit number to 0 or 10. The same ideas work for rounding 2-digit numbers to the closest multiple of 10.
Multiplying 2-Digit Numbers by Multiples of Ten

GOALS

Students will multiply 2-digit numbers by 2-digit multiples of ten (10, 20, 30 and so on to 90) by using the standard algorithm.

PRIOR KNOWLEDGE REQUIRED

Multiplication as the number of dots in an array with given number of rows and given number of squares in each row
Multiplying 2-digit numbers by 1-digit numbers by using the standard algorithm
Multiplying 2-digit numbers by 2-digit multiples of ten using mental math

Review 2-digit by single-digit products using the standard algorithm. Have students solve in their notebooks: 37 × 2, 45 × 3, 38 × 7, 32 × 5.

Have students solve, using mental math: 37 × 20, 45 × 30, 38 × 70, 32 × 50.

Write the following products on the board:

\[
\begin{array}{c}
1 \\
\times 2 \\
37 \\
74 \\
\end{array} 
\]

\[
\begin{array}{c}
1 \\
\times 0 \\
37 \\
740 \\
\end{array} 
\]

Discuss the similarities and differences between the two algorithms. Why can we do the multiplication 37 × 20 as though it is 37 × 2 and then just add a 0? Why did we carry the 1 to the tens column in the first multiplication but to the hundreds column in the second multiplication?

Write on the board:

\[
37 \times 20 = 7 \times 20 + 30 \times 20 \]

and show the following picture:

Write on the board:

\[
7 \times 20 = 7 \times 2 \text{ tens} = ___ \text{ tens} = ___ \text{ hundred} + ____ \text{ tens} = _______
\]

\[
30 \times 20 = 30 \times 2 \text{ tens} = ____ \text{ tens} = ____ \text{ hundreds} + ____ \text{ tens} = _______
\]

So 37 × 20 = ____ hundreds + ____ tens = _______

Emphasize that adding together the number of tens in 7 × 20 and 30 × 20 is the same as adding together the number of ones in 7 × 2 and 30 × 2; they are both 14 + 60 = 74, so while 37 × 2 is 74, 37 × 20 is 74 tens or 740. Then have students solve several problems this way. (EXAMPLE: 24 × 30 = 4 × 30 + 20 × 30 = 12 tens + 60 tens = 72 tens = 720.)
Have students compare this method to the standard algorithm. Ensure students understand that the standard algorithm is following the same steps, but is taking short cuts:

- First, multiply $7 \times 20$ by multiplying $7 \times 2$ and adding a 0. Record the 1 in the hundreds column, the 4 in the tens column and the 0 in the ones column.

- Next, multiply $30 \times 20$ by multiplying $3 \times 2$ and put your answer in the hundreds column (don’t forget to add the 1 hundred from $7 \times 20$). This works because $30 \times 20$ is really 600 instead of 6.

Have students practice the first step in each example:

\[
\begin{array}{cccc}
3 & 29 & b) & 34 \\
\times 40 & \times 70 & c) & 29 \\
60 & & \times 70 \\
\end{array}
\]

Students should be able to explain how this step records, for example, the fact that $9 \times 40 = 360$, and why the 3 is in the hundreds column, the 6 in the tens column and the 0 in the ones column.

When students have mastered this, remind your students that we want to finish solving $29 \times 40$ and so far, we have only found $9 \times 40$. \textbf{ASK:} What else do we need to do? (We need to find $20 \times 40$ and then add the result to $9 \times 40$ to find $29 \times 40$.)

Show this using a modified version of the standard algorithm:

\[
\begin{array}{cc}
29 \\
\times 40 \\
360 \\
+800 \\
1160 \\
\end{array}
\]

Have your students use this modified version of the standard algorithm to find the products from questions b, c and d above. Then show the actual standard algorithm for part a alongside the modified algorithm.

\[
\begin{array}{cc}
3 & 3 \\
29 & 29 \\
\times 40 & \times 40 \\
360 & 1160 \\
+800 & 4 \times 2 + 3 = 11 \\
1160 & \\
\end{array}
\]
NS5-29

Multiplying 2 Digits by 2 Digits

GOALS
Students will multiply 2-digit numbers by 2-digit numbers.

Prior Knowledge Required
Multiplying a 2-digit by a 1-digit number
Multiplying a 2-digit number by a 2-digit multiple of 10

Write on the board: \(28 \times 36\). **Ask:** How is this multiplication different from any we have done so far? Challenge them to think of a way to split the problem into two easier problems, both of which we know how to do. Ask students to list the types of multiplication problems that they already know how to do—Can we break this problem into two problems that we already know how to do? Suggest that they change one of the two numbers, either 28 or 36. The possibilities include:

\[
28 \times 36 = 20 \times 36 + 8 \times 36 \quad \text{OR} \quad 28 \times 36 = 28 \times 30 + 28 \times 6.
\]

Ask students to multiply \(28 \times 6\) and \(28 \times 30\) using the methods they have learned so far:

\[
\begin{array}{c}
4 \\
28
\end{array} \times \begin{array}{c}
28 \\
6
\end{array} = \begin{array}{c}
168 \\
840
\end{array}
\]

**Ask:** How can we find \(28 \times 36\) from \(28 \times 30\) and \(28 \times 6\)? (add them together) Have students multiply several pairs of 2-digit numbers by using this method. **Examples:** \(43 \times 27, 54 \times 45, 36 \times 44\).

Then show your students how to use the modified algorithm:

\[
\begin{array}{c}
28 \\
\times 36
\end{array} = \begin{array}{c}
168 \\
+ 840
\end{array} = \begin{array}{c}
1008
\end{array}
\]

and then the standard algorithm:

\[
\begin{array}{c}
4 \\
28
\end{array} \times \begin{array}{c}
24 \\
28
\end{array} \times \begin{array}{c}
24 \\
28
\end{array} = \begin{array}{c}
168 \\
+ 840
\end{array} = \begin{array}{c}
1008
\end{array}
\]
Have your students practise the first step of the standard algorithm (that is, multiplying the first number by the ones digit of the second number) for several problems. (EXAMPLES: $23 \times 17$, $46 \times 19$, $25 \times 32$, $42 \times 26$). Ensure that students leave enough space in their notebook to solve the entire problem, not just the first step.

EXAMPLE:

```
  23  
× 17  
-----
 161  = 23 × 7
```

Ensure that carrying at this stage is put in the tens column and that students can explain why this makes sense. When your students are finished the first step, have them practise the second step of the standard algorithm (that is, multiplying the first number by the tens digit in the second number) for the same problems they have just completed the first step for.

EXAMPLE:

```
  23  
× 17  
-----
 161  = 23 × 7
 230  = 23 × 10
```

Ensure that carrying at this stage is put in the hundreds column and that students can explain why this makes sense. When students are finished, have them practise the final step of the standard algorithm (adding the two results to find the total answer).

EXAMPLE:

```
  23  
× 17  
-----
 161  = 23 × 7
 230  = 23 × 10
 391  = 23 × 17
```

NOTE: To explain why the multiplication algorithm taught on the worksheet works, point out to your students that a multiplication statement like $32 \times 23$ is merely a short form for a sum of four separate multiplication statements, namely:

```
32 × 23 = 30 × 20 + 2 × 20 + 30 × 3 + 2 × 3
```

You can demonstrate this fact by drawing an array of 32 by 23 squares:
From the pictures above you can see that the total number of squares in the array is

\[32 \times 23 \quad \text{OR} \quad 30 \times 20 + 2 \times 20 + 30 \times 3 + 2 \times 3\]

The same fact follows from the “distributivity law”, which you can explain to your student as follows. Suppose you add a pair of numbers and then multiply the result by a third number. For instance, \((3 + 2) \times 4\). You get the same result if you multiply each number in the pair separately by the third number and then add the result:

\[(3 + 2) \times 4 = 3 \times 4 + 2 \times 4\]

This fundamental rule for combining multiplication and addition is called the distributivity law (see section NS5-22 for an explanation of why the law works). The distributivity law is given by the following formula:

\[a \times (b + c) = a \times b + a \times c \quad \text{or} \quad (b + c) \times a = b \times a + c \times a\]

Using the distributive law, we can derive the result given above.

\[
32 \times 23 = 32 \times (20 + 3) \quad \text{(write the second number [23] as a sum)}
= 32 \times 20 + 32 \times 3 \quad \text{(use the distributivity law)}
= (30 + 2) \times 20 + (30 + 2) \times 3 \quad \text{(write the first number [32] as a sum)}
= 30 \times 20 + 2 \times 20 + 30 \times 3 + 2 \times 3 \quad \text{(use the distributivity law twice to create four products)}
\]

The four products that appear in the last line above are hidden in the steps of the standard multiplication algorithm:

\[
\begin{array}{c}
32 \\
\times 23
\end{array}
\begin{array}{c}
\underline{\times 23} \\
6 (= 2 \times 3) \\
90 (= 30 \times 3)
\end{array}
\begin{array}{c}
32 \\
\times 30
\end{array}
\begin{array}{c}
\underline{\times 23} \\
96
\end{array}
\]

\[
\begin{array}{c}
32 \\
\times 23
\end{array}
\begin{array}{c}
\underline{\times 23} \\
90
\end{array}
\begin{array}{c}
40 (= 20 \times 2) \\
\underline{+ 600 (= 30 \times 20)}
\end{array}
\]

This is the standard way of showing the multiplication

\[
\begin{array}{c}
32 \\
\times 23
\end{array}
\begin{array}{c}
\underline{\times 23} \\
90
\end{array}
\begin{array}{c}
40 (= 20 \times 2) \\
\underline{+ 600 (= 30 \times 20)}
\end{array}
\]

This is the standard way of showing the multiplication

You might assign your students a few questions and then ask them to try multiplying in the non-standard way shown above (i.e. writing each of the four products down separately).
Extension

Have students investigate: Which is larger in each pair:

a) $35 \times 27$ or $37 \times 25$
b) $36 \times 27$ or $37 \times 26$
c) $46 \times 25$ or $45 \times 26$
d) $49 \times 25$ or $45 \times 29$

Students should investigate by actually making the calculations and then try to reflect back on how they could solve the problem using the least amount of effort possible.

NS5-30
Mental Math: Rearranging Products

**Review:**

- doubling single-digit numbers. Ensure that students understand that they can find the double of any number from 1 to 10 by skip counting by 2 on their fingers.

- the relationship between the doubles of two numbers that differ by 5 (e.g. add ten to the double of 4 to get the double of 9).

- doubling 2-digit numbers by separating the tens and ones (31 = 30 + 1 so its double is 60 + 2 = 62; 79 = 70 + 9, so its double is 140 + 18 = 158).

- doubling as a strategy to multiply two numbers (8 × 12 is the double of $8 \times 6 = 48 = 40 + 8$, so $8 \times 12 = 80 + 16 = 96$).

Tell your students that they will be learning new strategies for multiplying numbers. This time, instead of pairs of numbers, they will work with 3 numbers at a time.

Give students blocks and ask them to build a “box” that is 4 blocks wide, 2 blocks deep, and 3 blocks high.

When they look at their box head-on, what do they see? (3 rows, 4 blocks in each row, a second layer behind the first) Write the math sentence that corresponds to their box when viewed this way: $3 \times 4 \times 2$

Have students carefully pick up their box and turn it around, or have them look at their box from a different perspective: from above or from the side. Now what number sentence do they see? Take various answers so that...
students see the different possibilities. **ASK:** When you multiply 3 numbers, does it matter which number goes first? Would you get the same answer in every case? (Yes, because the total number of blocks doesn’t change.)

Students should also look at multiplication of 3 numbers as groups of groups:

![Hearts](image)

This picture shows 2 groups of 4 groups of 3. **ASK:** How would we write 4 groups of 3 in math language? (4 \times 3) How could we write 2 groups of 4 groups of 3 in math language? (2 \times 4 \times 3). If I want to write this as a product of just 2 numbers, how can I do so? Think about how many groups of 3 hearts I have in total. I have 2 \times 4 = 8 groups of 3 hearts in total, so the total number of hearts is 8 \times 3. So 2 \times 4 \times 3 = 8 \times 3.

Invite students to look for a different way of grouping the hearts. **ASK:** How many hearts are in each of the 2 groups? (There are 4 \times 3 = 12 hearts in each of the 2 groups, so 2 \times 4 \times 3 = 2 \times 12.) Do I get the same answer no matter how I multiply the numbers? (Yes; 8 \times 3 = 2 \times 12).

Write the following math sentence on the board: 2 \times 4 \times 3 = 2 \times __. **ASK:** What number is missing? (12) How do you know? (because 4 \times 3 = 12). Is 12 also equal to 3 \times 4? Can I write 2 \times 4 \times 3 = 2 \times 3 \times 4? Remind students that 2 \times 4 \times 3 is the same as both 8 \times 3 and 2 \times 12.

Have students explore different orderings of the factors. Remind them that just like we use the term addend in sums, we use the term factor in products. Have them try this with 2 \times 6 \times 5. Ask if there is any way to order the factors that make it particularly easy to find the product. What is the answer? (2 \times 6 \times 5 = 2 \times 5 \times 6 = 10 \times 6 = 60). Then challenge them to find more products of 3 factors:

- a) 25 \times 7 \times 4
- b) 15 \times 7 \times 2
- c) 250 \times 9 \times 4
- d) 50 \times 93 \times 2

**Bonus**

- a) 125 \times 7 \times 8
- b) 25 \times 40 \times 8
- c) 6 \times 50 \times 20

Challenge your students to find: 5 \times 5 \times 4 \times 2, 25 \times 2 \times 4 \times 5, 25 \times 5 \times 4 \times 7 \times 2. Encourage them to model 5 \times 5 \times 4 \times 2 on paper or with cubes. This could be done by drawing 5 groups of 5 groups of 4 groups of 2.
It could also be done by building 5 cubes that are each $5 \times 4 \times 2$.

When students are comfortable recognizing pairs of numbers that multiply to 10 or 100 ($2 \times 5$, $25 \times 4$, $50 \times 2$, $20 \times 5$), teach them to break numbers apart to make a multiple of 10. (EXAMPLE: $2 \times 35 = 2 \times 5 \times 7 = 10 \times 7 = 70$). Students should recognize that any even number, when multiplied by a multiple of 5, will result in a multiple of 10. (EXAMPLE: $6 \times 45 = 3 \times 2 \times 5 \times 9 = 3 \times 10 \times 9 = 270$) Have them find the following products using this method:

a) $4 \times 45$  b) $8 \times 75$  c) $6 \times 35$  d) $12 \times 15$  e) $18 \times 25$  f) $22 \times 45$

Progress to products of 3 or 4 numbers that require this strategy:

a) $6 \times 7 \times 35$  b) $4 \times 55 \times 3$  c) $14 \times 15 \times 4$  d) $22 \times 3 \times 15$  e) $2 \times 6 \times 15 \times 35$

Give students two copies of an array of dots with 4 rows and 6 dots in each row or have them use two arrays on grid paper with 4 rows of 6 squares each.

Challenge your students to move some of the dots in one of them so that there are 8 rows of 3 instead of 4 rows of 6. Students might find it helpful to actually cut and glue the dots. **ASK:** Did you change the number of dots by moving them around? Write on the board: $4 \times 6 = 8 \times 3$. Challenge students to find a different product that is also equal to $4 \times 6$ by moving the dots in a different way. ($4 \times 6 = 2 \times 12$)

Draw an array with 5 rows of dots and 6 dots in each row. **ASK:** If I move some dots so that there are 10 rows, how many dots will there be in each row? If I double the number of rows and I want to keep the number of dots the same, how do I have to change the number of dots in each row? (the number of dots in each row should become half of what it was)

Have students double the first number and halve the second number to find an equal product:

a) $5 \times 4$  b) $11 \times 6$  c) $12 \times 4$  d) $13 \times 8$  e) $9 \times 12$  f) $3 \times 12$

Have students find each product by doubling one of the factors and halving the other:

a) $32 \times 5 = 16 \times 10 = 160$  b) $5 \times 42$  c) $24 \times 5$  d) $5 \times 84$  e) $15 \times 22$

**Extension**

1. **ASK:** Which is easier—to find the product of two 2-digit numbers or the product of a 1-digit number and a 2-digit number? Why? How much more work is involved in finding the product of two 2-digit numbers? Have students change the following products into a product of a 2-digit and 1-digit number by repeatedly doubling one factor and halving the other. Then find the product. Emphasize that changing a harder problem (in this case, multiplying two 2-digit numbers) into an easier problem (in this case, multiplying a 2-digit number by a 1-digit number) is a common problem-solving strategy.

a) $14 \times 17$  b) $32 \times 13$  c) $36 \times 15$  d) $24 \times 24$

**Bonus**
28 × 26 (this one will become a 3-digit by a 1-digit number)
2. Challenge students to write each number as a product of 3 (not necessarily all different) numbers all greater than 1.
   a) 8 (2 × 2 × 2)
   b) 30 (2 × 3 × 5)
   c) 20 (2 × 2 × 5)
   d) 18 (2 × 3 × 3)
   e) 42 (2 × 3 × 7)
   f) 27 (3 × 3 × 3)

   One approach to this problem is to start with small factors, divide out, and then find the factors of a smaller number. (EXAMPLE: 2 goes into 30 15 times, so we just have to find an array of 15 dots. 5 rows of 3 will do.) Students may wish to use link-it cubes to build boxes with these numbers of cubes to show their answers.

3. Challenge students to write each number as a product of 4 (not necessarily all different) numbers all greater than 1. Students may wish to use link-it cubes again, but here they will need to build a series of boxes of the same dimensions, since we do not have 4 dimensions in space.
   a) 24 (2 × 2 × 2 × 3) Students might build two 2 × 2 × 3 boxes or three 2 × 2 × 2 boxes.
   b) 16 (2 × 2 × 2 × 2)
   c) 60 (2 × 2 × 3 × 5)
   d) 36 (2 × 2 × 3 × 3)
   e) 100 (2 × 2 × 5 × 5)

4. The addition statement 3 + 3 becomes 2 + 2 + 2 when the array model is rotated:

   \[
   \begin{array}{c}
   \text{3 + 3} \\
   \end{array}
   \quad \rightarrow \quad
   \begin{array}{c}
   \text{2 + 2 + 2} \\
   \end{array}
   \]

   Rotate the model and then re-write the addition statement.

   a) 2 + 1 + 1
   b) 3 + 2 + 2 + 1 + 1
   c) 4 + 3 + 3 + 2 + 2 + 1 + 1

   **BONUS:** Without using a model, find another addition statement for 5 + 4 + 4 + 3 + 3 + 2 + 2 + 1 + 1.
These sheets are mostly a review of the multiplication concepts learned so far.

Remind students that 32 753 can be written as 30 000 + 2 000 + 700 + 50 + 3. Have students write the following numbers in a similar way:

a) 41 805  b) 356 970  c) 4 003 990  d) 780 031

Then ask: What is the place value of the 4 in 41 805? When students answer 10 000, write: 40 000 = ____ × 10 000. Tell students that we can write 41 805 as: 4 × 10 000 + 1 × 1 000 + 8 × 100 + 5. Have students write b), c) and d) above in this same manner.

To familiarize your students with deciding whether statements are always, sometimes or never true, have them decide whether these statements are always, sometimes, or never true:

a) My birthday is in the spring. (some students will say always, many will say never, and one or two might say sometimes, e.g. if the student’s birthday is March 20.)

b) Chocolate is brown. (sometimes)

c) The sun is hot. (always)

d) Snow is wet. (sometimes)

e) Maple leaves are green. (sometimes)

f) Squares are round. (never)

Then introduce mathematical sentences where students need to decide between always, sometimes and never.

a) A number less than 1 000 will have 3 digits. (sometimes)

b) A number more than 1 000 will have exactly 3 digits. (never)

c) The sum of two 2-digit numbers is at least 20. (always)

d) The sum of three 2-digit numbers is 25. (never)

e) The sum of two odd numbers is even. (always)

f) The sum of three odd numbers is even. (never)

g) Five times a number is even. (sometimes)

h) Five times a number is odd. (sometimes)
i) Five times a number is a multiple of 5. (always)

j) Five times a number is 0. (sometimes)

Introduce the word “factor.” Explain that the factors of a whole number are the whole numbers that multiply to give the number. For example, $2 \times 4 = 8$, so 2 and 4 are both factors of 8. 3 is not a factor of 8 because 3 does not divide evenly into 8: $3 \times 1 = 3$, $3 \times 2 = 6$, and $3 \times 3 = 9$.

Demonstrate how students can draw arrays to find all the factors of a number. For example, to find the factors of 8, draw:

- 8 rows of 1
- 4 rows of 2
- Can’t draw full rows of 3
- 2 rows of 4
- Can’t draw full rows of 5, 6 or 7
- 1 row of 8.

We can’t have more than 8 in a row because we have only a total of 8.

Now ask students to decide whether these statements are always, sometimes or never true.

a) A factor of a number is greater than the number. (never)

b) 1 is a factor of a number. (always)

c) A factor of a number is less than the number. (sometimes)

d) The product of two factors is less than the number. (sometimes)

Be sure that students explain their answers.

Extension

1. Fill in the missing digits.

   a) $2 \square \times 4 \quad 9 \ 6$

   b) $\square \times 3 \quad 1 \ 6 \ 5$

   c) $\square \times 4 \quad 1 \ 9 \square$

   d) $2 \square \times 6 \quad 1 \ 4 \ 0$

   $\square \square \ 0$

   $1 \ 3 \ 4 \ 4$
**GOALS**

Students will identify the objects to be divided into sets, the number of sets and the number of objects in each set.

**PRIOR KNOWLEDGE REQUIRED**

Sharing equally

**VOCABULARY**

groups or sets
divided or shared

---

**NS5-33**

**Sets**

Draw 12 glasses divided equally on 4 trays. Ask your students to identify the objects to be shared or divided into sets, the number of sets and the number of objects in each set. Repeat with 15 apples divided into 3 baskets and 12 plates placed on 3 tables. Have students complete the following examples in their notebook.

a) Draw 9 people divided into 3 teams (Team A, Team B, Team C)
b) Draw 15 flowers divided into 5 flowerpots.
c) Draw 10 fish divided into 2 fishbowls.

The objects to be divided are put into sets. What are the sets? [**HINT**: the noun immediately preceding “each” is the object, the noun immediately following “each” is the set.]

a) 5 boxes, 4 pencils in each box (pencils are being divided, boxes are sets)
b) 3 classrooms, 20 students in each classroom (students are being divided, classrooms are sets)
c) 4 teams, 5 people on each team (people are being divided, teams are the sets)
d) 5 trees, 30 apples on each tree (apples are being divided, trees are the sets)
e) 3 friends, 6 stickers for each friend (stickers are being divided, friends are the sets)

Then have students use circles to represent sets and dots to represent objects to be divided into sets.

a) 5 sets, 2 dots in each set
b) 7 groups, 2 dots in each group
c) 3 sets, 4 dots in each set
d) 4 groups, 6 dots in each group
e) 5 children, 2 toys for each child
f) 6 friends, 3 pencils for each friend
g) 3 fishbowls, 4 fish in each bowl, 12 fish altogether
h) 20 oranges, 5 boxes, 4 oranges in each box
i) 4 boxes, 12 pens, 3 pens in each box
j) 10 dollars for each hour of work, 4 hours of work, 40 dollars

**Bonus**

k) 12 objects altogether, 4 sets
l) 8 objects altogether, 2 objects in each set
m) 5 fish in each fishbowl, 3 fishbowls
n) 6 legs on each spider, 4 spiders
o) 3 sides on each triangle, 6 triangles
p) 3 sides on each triangle, 6 sides
q) 6 boxes, 2 oranges in each box
r) 6 oranges, 2 oranges in each box
s) 6 fish in each bowl, 2 fishbowls
t) 6 fish, 2 fishbowls
u) 8 boxes, 4 pencils in each box
v) 8 pencils, 4 pencils in each box

NS5-34
Two Ways of Sharing

GOALS
Students will recognize which information is provided and which is absent among: how many sets, how many in each set and how many altogether.

PRIOR KNOWLEDGE REQUIRED
Dividing equally
Word problems

VOCABULARY
set
divide
equally

Divide 12 volunteers into 4 teams, numbered 1-4, by assigning each of them a number in the following order: 1, 1, 2, 1, 1, 3, 1, 1, 4, 1, 1, 1. Separate the teams by their numbers, and then ask your students what they thought of the way you divided the teams. Was it fair? How can you reassign each of the volunteers a number and ensure that an equal amount of volunteers are on each team? An organized way of doing this is to assign the numbers in order: 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4. Demonstrate this method and then separate the teams by their numbers again. Does each team have the same amount of volunteers? Are the teams fair now?

Present your students with 4 see-through containers and ask them to pretend that the containers represent the 4 teams. Label the containers 1, 2, 3 and 4. To evenly divide 12 players (represented by a counter of some sort) into 4 teams, one counter is placed into one container until all 12 counters are placed into the 4 containers. This is like assigning each player a number. Should we randomly distribute the 12 counters and hope that each container is assigned an equal amount? Is there a better method? How is this similar to assigning each volunteer a number?

Now, suppose you want to share 12 cookies between yourself and 3 friends. How many people are sharing the cookies? [4.] How many containers are needed? [4.] How many counters are needed? [12.] What do the counters represent? [Cookies.] What do the containers represent? [People.] Instruct your students to draw circles for the containers and dots inside the circles for the counters. How many circles will you need to draw? [4.] How many dots will you need to draw inside the circles? [12.]

Draw 4 circles.

Counting the dots out loud as you place them in the circles, have your students yell “Stop” when you reach 12. Ask them how many dots are in each circle? If 4 people share 12 cookies, how many cookies does each person
get? If 12 people are divided among 4 teams, how many people are on each team? Now what do the circles represent? Now what do the dots represent? If 12 people ride in 4 cars, how many people are in each car? What do the circles and dots represent? Have students suggest additional representations for circles and dots.

Assign your students practise exercises, drawing circles and the correct amount of dots. If it helps, allow them to first use counters and count them ahead of time so they know automatically when to stop.

- a) 12 cookies, 3 people
- b) 15 cookies, 5 people
- c) 10 cookies, 2 people

So far, questions have told students how many piles (or sets) to make. Now questions will tell students how many to put in each pile (or in each set) and they will need to determine the number of sets.

Tell students that Saud has 30 apples. Count out 30 counters and set them aside. Saud wants to share his apples so that each friend gets 5. He wants to figure out how many people he can share his apples with. ASK: What can we use to represent Saud’s friends? [Containers.] Do you know how many containers we need? [No, because we are not told how many friends Saud has.] How many counters are to be placed in each container? [5.]

Put 5 counters in one container. Are more containers needed for the counters? [Yes.] How many counters are to be placed in a second container? How do they know? Will another container be needed? Continue until all the counters are evenly distributed. The 30 counters have been distributed and each friend received 5 apples. How many friends does Saud have? How do they know? [6, because 6 containers were needed to evenly distribute the counters.]

Now draw dots and circles as before. What will the circles represent? [Friends.] How many friends does Saud have? How many dots are to be placed in each circle? [5.] Place 5 dots in each circle and keep track of how many you use.

```
   ● ● ● ● ●   ● ● ● ● ●   ● ● ● ● ●   ● ● ● ● ●   ● ● ● ● ●
   5          10         15          20         25         30
```

Six circles had to be drawn to evenly distribute 30 apples, meaning 6 friends can share the 30 apples. Emphasize the difference between this problem and the previous problems. [The previous problems identified the number of sets, but not the number of objects in each set.]

Saud has 15 apples and wants to give each friend 3 apples. How many friends can he give apples to? How can we model this problem? What will the dots represent? What will the circles represent? Does the problem tell us how many dots or circles to draw? Or how many dots to draw in each circle? Draw 15 dots on the board and demonstrate distributing 3 dots in each circle:

```
   ● ● ●       ● ● ●       ● ● ●       ● ● ●       ● ● ●
```

There are 5 sets, so Saud can give apples to 5 people.
Have your students distribute the dots into sets of 3, then ask them to count the number of sets.

a) ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● 

b) ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● 

c) ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● 

Repeat with sets of 2 dots:

a) ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● 

b) ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● 

Have students draw the dots to determine the correct number of sets.

a) 15 dots, 5 dots in each set  
b) 12 dots, 4 dots in each set  
c) 16 dots, 2 dots in each set  

**Bonus**

24 dots and

a) 3 dots in each set  
b) 2 dots in each set  
c) 4 dots in each set  
d) 6 dots in each set  
e) 8 dots in each set  
f) 12 dots in each set  

In the following questions, have your students draw dots to represent the objects being divided into sets, and draw circles to represent the sets.

a) 10 apples, 2 apples in each box. How many boxes?  
b) 12 apples, 3 apples in each box. How many boxes?  
c) 18 apples, 6 apples in each box. How many boxes?  

Then assign word problems with full sentences and have your students underline the important words. Demonstrate with the first problem.

a) Sally has **10 apples**. She puts 5 apples in each box. How many boxes has she used?  
b) Sally has 12 apples. She gives 4 apples to each of her siblings. How many siblings does she have?  
c) Sally has 10 apples. She puts 2 apples on each plate. How many plates has she used?  
d) Sally has 8 stamps. She puts 2 stamps on each envelope. How many envelopes does she have?  

**Bonus**

e) Sally has 18 apples. She shares them among herself and some friends. Each person gets 6 apples. How many friends has she shared with?  

Sometimes a problem provides the number of sets, and sometimes it provides the number of objects in each set. Have your students tell you if they are provided with the number of sets or the number of objects in each set for the following problems.

a) There are 15 children. There are 5 children in each canoe.  
b) There are 15 children in 5 canoes.  
c) Aza has 40 stickers. She gives 8 stickers to each of her friends.  

Have your students draw and complete the following table for questions a)–i) below. Insert
a box for information that is not provided in the problem.

<table>
<thead>
<tr>
<th>EXAMPLE</th>
<th>What has Been Shared or Divided into Sets?</th>
<th>How Many Sets?</th>
<th>How Many in Each Set?</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>18 strings</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>b)</td>
<td>24 strings</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

EXAMPLE: There are 6 strings on each guitar. There are 18 strings.

a) There are 24 strings on 4 guitars.
b) There are 3 hands on each clock. There are 15 hands altogether.
c) There are 18 holes in 6 sheets of paper.
d) There are 15 rings on 5 binders.
e) 15 people sit on 5 couches.
f) There are 15 people. 5 people fit on each couch.
g) There are 4 cans of tennis balls. There are 3 tennis balls in each can.
h) There are 20 chairs in 4 rows.
i) There are 3 rows of chairs. There are 9 chairs altogether.

Ask your students to draw circles for the sets and dots for the objects being divided into sets.

a) 10 dots, 2 sets. How many in each set?
b) 10 dots, 2 in each set. How many sets are there?
c) 12 dots, 2 sets. How many in each set?
d) 12 dots, 3 in each set. How many sets are there?
e) There are 12 children in 2 boats. How many children are in each boat?
f) There are 15 children. 3 children can fit into each car. How many cars are needed?
g) Luybava has 12 pencils. She puts 4 pencils in each box. How many boxes does she have?
h) There are 12 strings on 2 guitars. How many strings are on each guitar?
i) There are 24 strings. There are 6 strings on each guitar. How many guitars are there?
j) Tina has 30 stickers. She wants to divide them among herself and 4 friends. How many stickers will each person receive?
k) Tina has 30 stickers. She wants each person to receive 3 stickers. How many people will receive stickers?
l) There are 24 toothpicks. Sally wants to make triangles with the toothpicks. How many triangles can Sally make?
m) There are 24 toothpicks. Sally wants to make squares with the toothpicks. How many squares can Sally make?
NS5-35
Dividing by Skip Counting

Distribute 15 counters to each of your students and then ask them to divide the counters into sets of 3. How many sets do they have? Explain that 15 objects divided into sets of 3 equals 5 sets. This is written as \(15 \div 3 = 5\), and read as “fifteen divided by 3 equals 5.”

The following website provides good worksheets for those having trouble with this basic definition of division:


With symbols similar to those below, have your students solve \(12 \div 2\), \(12 \div 3\), \(12 \div 4\), and \(12 \div 6\).

Have your students illustrate each of the following division statements.

\[
6 \div 3 = 2 \quad 12 \div 3 = 4 \quad 9 \div 3 = 3 \quad 8 \div 2 = 4 \quad 8 \div 4 = 2
\]

Then have your students write the division statement for each of the following illustrations.

WRITE: \(15 \div 3 = 5\) (15 divided into sets 3 equals 5 sets).

\[
3 + 3 + 3 + 3 + 3 = 15
\]
Explain that every division statement implies an addition statement. Ask your students to write the addition statements implied by each of the following division statements. Allow them to illustrate the statement first, if it helps.

\[
\begin{align*}
15 \div 5 & = 3 \\
12 \div 2 & = 6 \\
10 \div 2 & = 5 \\
6 \div 3 & = 2 \\
6 \div 2 & = 3
\end{align*}
\]

Write: \(15 \div 3 = 5\)

Ask your students to write the following division statements as addition statements, without illustrating the statement this time.

\[
\begin{align*}
12 \div 4 & = 3 \\
12 \div 3 & = 4 \\
18 \div 6 & = 3 \\
18 \div 3 & = 6 \\
18 \div 2 & = 9 \\
18 \div 9 & = 2 \\
25 \div 5 & = 5
\end{align*}
\]

Bonuses:

\[
\begin{align*}
132 \div 43 & = 3 \\
1700 \div 425 & = 4 \\
90 \div 30 & = 3 \\
1325 \div 265 & = 5
\end{align*}
\]

Then have your students illustrate and write a division statement for each of the following addition statements.

\[
\begin{align*}
4 + 4 + 4 & = 12 \\
2 + 2 + 2 + 2 + 2 & = 10 \\
6 + 6 + 6 + 6 & = 24 \\
3 + 3 + 3 & = 9 \\
3 + 3 + 3 + 3 + 3 + 3 + 3 & = 21 \\
5 + 5 + 5 + 5 + 5 + 5 + 5 & = 30
\end{align*}
\]

Then have your students write division statements for each of the following addition statements, without illustrating the statement.

\[
\begin{align*}
17 + 17 + 17 + 17 + 17 & = 85 \\
21 + 21 + 21 & = 63 \\
101 + 101 + 101 + 101 & = 404 \\
2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 & = 36 \\
1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 & = 21
\end{align*}
\]

Draw:

What division statement does this illustrate? What addition statement does this illustrate? Is the addition statement similar to skip counting? Which number could be used to skip count the statement?

Explain that the division statement \(18 \div 3 = ?\) can be solved by skip counting on a number line.

How many skips of 3 does it take to reach 18? (6.) So what does \(18 \div 3\) equal? How does the number line illustrate this? (Count the number of arrows.)
Explain that the division statement expresses a solution to $18 \div 3$ by skip counting by 3 to 18 and then counting the arrows.

Ask volunteers to find, using the number line: $12 \div 2; 12 \div 3; 12 \div 4; 12 \div 6$.

SAY: If I want to find $10 \div 2$, how could I use this number line?

What do I skip count by? How do I know when to stop? Have a volunteer demonstrate the solution.

Ask your students to solve the following division statements with a number line to 18.

$8 \div 2, 12 \div 3, 15 \div 3, 14 \div 5, 14 \div 2, 16 \div 4, 16 \div 2, 18 \div 3, 18 \div 2$

Then have your students solve the following division problems with number lines to 20 (see the BLM “Number Lines to Twenty”). Have them use the top and bottom of each number line so that each one number line can be used to solve two problems. For example, the solutions for $8 \div 2$ and $8 \div 4$ might look like this:

$8 \div 2 \quad 8 \div 4 \quad 6 \div 3 \quad 14 \div 2 \quad 9 \div 3 \quad 15 \div 3$

$10 \div 5 \quad 20 \div 4 \quad 20 \div 5 \quad 18 \div 2 \quad 20 \div 2 \quad 16 \div 4$

Then provide numerous solutions of division problems and have your students express the division statement and the addition statement. For example, students should answer with “$20 \div 4 = 5$” and “$4 + 4 + 4 + 4 + 4 = 20$” for the following number line.

Draw number lines for $16 \div 8, 10 \div 2, 12 \div 4$, and $12 \div 3$.

Explain that skip counting can be performed with fingers, in addition to on a number line. If your students need practice skip counting without a number line, see the MENTAL MATH section of this teacher’s guide. They might also enjoy the following interactive website:

http://members.learningplanet.com/act/count/free.asp

Begin with skip counting by 2. Have your students keep track of the count on their fingers.
To solve $15 \div 3$, skip count by 3 to 15. The number of fingers it requires to count to 15 is the answer. [5.]

Perform several examples of this together as a class, ensuring that students know when to stop counting (and in turn, what the answer is) for any given division problem. Have your students complete the following problems by skip counting with their fingers.

- a) $12 \div 3$
- b) $6 \div 2$
- c) $10 \div 2$
- d) $10 \div 5$
- e) $12 \div 4$
- f) $9 \div 3$
- g) $16 \div 4$
- h) $50 \div 10$
- i) $25 \div 5$
- j) $15 \div 3$
- k) $15 \div 5$
- l) $30 \div 10$

Two hands will be needed to keep track of the count for the following questions.

- a) $12 \div 2$
- b) $30 \div 5$
- c) $18 \div 2$
- d) $28 \div 4$
- e) $30 \div 3$
- f) $40 \div 5$

**Bonus**

- a) $22 \div 2$
- b) $48 \div 4$
- c) $65 \div 5$
- d) $120 \div 10$
- e) $26 \div 2$

Then have your students express the division statement for each of the following word problems and determine the answers by skip counting. Ask questions like: What is to be divided into sets? How many sets are there and what are they?

- a) 5 friends share 30 tickets to a sports game. How many tickets does each friend receive?
- b) 20 friends sit in 2 rows at the movie theatre. How many friends sit in each row?
- c) $50$ is divided among 10 friends. How much money does each friend receive?

Have your students illustrate each of the following division statements and skip count to determine the answers.

- $3 \div 3$
- $5 \div 5$
- $8 \div 8$
- $11 \div 11$

Without illustrating or skip counting, have your students predict the answers for the following division statements.

- $23 \div 23$
- $180 \div 180$
- $244 \div 244$
- $1896 \div 1896$

Then have your students illustrate each of the following division statements and skip count to determine the answers.

- $1 \div 1$
- $2 \div 1$
- $5 \div 1$
- $12 \div 1$

Then without illustrating, have your students predict the answers for the following division statements.

- $18 \div 1$
- $27 \div 1$
- $803 \div 1$
- $6692 \div 1$
NS5-36
Division and Multiplication

GOALS
Students will understand the relationship between division and multiplication.

PRIOR KNOWLEDGE REQUIRED
- Relationship between multiplication and skip counting
- Relationship between division and skip counting

VOCABULARY
divided by

Write “10 divided into sets of 2 results in 5 sets.” Have one volunteer read it out loud, another illustrate it, and another write its addition statement. Does the addition statement remind your students of multiplication? What is the multiplication statement?

\[ 10 \div 2 = 5 \]

\[ 2 + 2 + 2 + 2 + 2 = 10 \]

\[ 5 \times 2 = 10 \]

Another way to express “10 divided into sets of 2 results in 5 sets” is to write “5 sets of 2 equals 10.”

Have volunteers illustrate the following division statements, write the division statements, and then rewrite them as multiplication statements.

a) 12 divided into sets of 4 results in 3 sets. \((12 \div 4 = 3\) and \(3 \times 4 = 12\))

b) 10 divided into sets of 5 results in 2 sets.

c) 9 divided into sets of 3 results in 3 sets.

d) 15 divided into 5 sets results in sets of 3.

e) 18 divided into 9 sets results in sets of 2.

f) 6 people divided into teams of 3 results in 2 teams.

g) 8 fish divided so that each fishbowl has 4 fish results in 2 fishbowls.

h) 12 people divided into 4 teams results in 3 people on each team.

i) 6 fish divided into 3 fishbowls results in 2 fish in each fishbowl.

Assign the remaining questions to all students.

d) 15 divided into 5 sets results in sets of 3.

e) 18 divided into 9 sets results in sets of 2.

f) 6 people divided into teams of 3 results in 2 teams.

g) 8 fish divided so that each fishbowl has 4 fish results in 2 fishbowls.

h) 12 people divided into 4 teams results in 3 people on each team.

i) 6 fish divided into 3 fishbowls results in 2 fish in each fishbowl.

Then ask students if there is another multiplication statement that can be obtained from the same picture as \(3 \times 4 = 12\)? How can the dots be grouped to express that 3 sets of 4 is equivalent to 4 sets of 3?

\[ \begin{array}{c}
\bullet \bullet \bullet \\
\bullet \bullet \bullet \\
\bullet \bullet \bullet \\
\end{array} \]

\[ \begin{array}{c}
\bullet \bullet \bullet \\
\bullet \bullet \bullet \\
\bullet \bullet \bullet \\
\end{array} \]

3 sets of 4 is equivalent to 4 sets of 3.

ANSWER: The second array of dots should be circled in columns of 3.

Then ask them what other division statement can be obtained from the same picture as \(12 \div 4 = 3\). Can the 12 dots be divided into 3 sets of 4 instead of 4 sets of 3? (Place one dot of each colour into each circle.)
What division statement does this result in? \[12 \div 3 = 4\]

Draw similar illustrations and have your students write two multiplication statements and two division statements for each. Have a volunteer demonstrate the exercise for the class with the first illustration.

Demonstrate how multiplication can be used to help with division. For example, the division statement \(20 \div 4 = \) can be written as the multiplication statement \(4 \times \) \(= 20\). To solve the problem, skip count by 4 to 20 and count the number of fingers it requires. Demonstrate this solution on a number line, as well.

Assign students the following problems.

a) \(9 \times 3 = 27\), \(\text{SO: } 27 \div 9 = \) 

b) \(2 \times 6 = 12\), \(\text{SO: } 12 \div 2 = \) 

c) \(8 \times \) \(= 40\), \(\text{SO: } 40 \div 8 = \) 

d) \(10 \times \) \(= 30\), \(\text{SO: } 30 \div 10 = \) 

e) \(5 \times \) \(= 30\), \(\text{SO: } 30 \div 5 = \) 

f) \(4 \times \) \(= 28\), \(\text{SO: } 28 \div 4 = \)
NS5-37
Knowing When to Multiply or Divide

Have your students fill in the blanks.

\[
\begin{array}{ccccccc}
\text{I} & \text{I} & \text{I} & \text{I} & \text{I} & \text{I} & \text{I} \\
\end{array}
\rightarrow
\begin{array}{ccccccc}
\text{I} & \text{I} & \text{I} & \text{I} & \text{I} & \text{I} \\
\end{array}
\]

\[
\begin{array}{cccc}
\text{___ sets} & \text{___ sets} \\
\text{___ objects per set} & \text{___ objects per set} \\
\text{___ objects altogether} & \text{___ objects altogether} \\
\end{array}
\]

When you are confident that your students are completely familiar with the terms “set,” “group,” “for every,” and “in each,” and you’re certain that they understand the difference between the phrases “objects in each set” and “objects” (or “objects altogether,” or “objects in total”), have them write descriptions of the diagrams.

\[
\begin{array}{ccccccc}
\text{I} & \text{I} & \text{I} & \text{I} & \text{I} & \text{I} & \text{I} \\
\end{array}
\rightarrow
\begin{array}{ccccccc}
\text{I} & \text{I} & \text{I} & \text{I} & \text{I} & \text{I} \\
\end{array}
\]

\[
\begin{array}{cccc}
\text{___ groups} \\
\text{___ objects in each group} \\
\text{___ objects} \\
\end{array}
\]

Explain that a set or group expresses three pieces of information: the number of sets, the number of objects in each set, and the number of objects altogether. For the problems, have your students explain which piece of information isn’t expressed and what the values are for the information that is expressed.

a) There are 8 pencils in each box. There are 5 boxes. How many pencils are there altogether?

b) Each dog has 4 legs. There are 3 dogs. How many legs are there altogether?

c) Each cat has 2 eyes. There are 10 eyes. How many cats are there?

d) Each boat can fit 4 people. There are 20 people. How many boats are needed?

e) 30 people fit into 10 cars. How many people fit into each car?

f) Each apple costs 20¢. How many apples can be bought for 80¢?

g) There are 8 triangles divided into 2 sets. How many triangles are there in each set?

h) 4 polygons have a total of 12 sides. How many sides are on each polygon?
Introduce the following three problem types.

**TYPE 1:** Knowing the number of sets and the number of objects in each set, but not the total number of objects.

**ASK:** If you know the number of objects in each set and the number of sets, how can you find the total number of objects? What operation should you use—multiplication or division?

Write on the board:

\[
\text{Number of sets} \times \text{Number of objects in each set} = \text{Total number of objects}
\]

If there are 3 sets and 2 objects in each set, how many objects are there in total? \[3 \times 2 = 6\]
If there are 5 sets and 4 objects in each set, how many objects are there in total?
If there are 3 objects in each set and 4 sets, how many objects are there in total?
If there are 2 objects in each set and 7 sets, how many objects are there in total?
If there are 100 objects in each set and 3 sets, how many objects are there in total?
If there are 100 sets and 3 objects in each set, how many objects are there in total?

**TYPE 2:** Knowing the total number of objects and the number of objects in each set, but not the number of sets.

\[
\text{Number of sets} \times \text{Number of objects in each set} = \text{Total number of objects}
\]

If there are 4 objects in each set and 12 objects in total, how many sets are there?
\[
4 \times \_ \_ \_ = 12, \text{ SO: } 12 \div 4 = 3. \text{ There are 3 sets.}
\]
Elaborate on this problem type with several examples, as above.

**TYPE 3:** Knowing the total number of objects and the number of sets, but not the number of objects in each set.

\[
\text{Number of sets} \times \text{Number of objects in each set} = \text{Total number of objects}
\]

If there are 6 sets and 12 objects in total, how many objects are in each set?
\[
6 \times \_ \_ \_ = 12, \text{ SO: } 12 \div 6 = 2. \text{ There are 2 objects in each set.}
\]
Elaborate on this problem type with several examples, as above.

If there are 5 sets and 40 objects altogether, how many objects are in each set? Vary the wording, as in: If there are 28 objects in 4 sets, how many objects are in each set? Have students write multiplication statements for the following problems with the blank in the correct place.

a) 2 objects in each set.
   6 objects in total.
   How many sets? \[ \_ \_ \_ \times 2 = 6 \]
   Which of these problems are division problems? [Multiplication is used to find the total number of objects, and division is used if the total number of objects is known.]
Assign several types of problems.

a) 5 sets, 4 objects in each set. How many objects altogether?

b) 8 objects in total, 2 sets. How many objects in each set?

c) 3 sets, 6 objects in total. How many objects in each set?

d) 3 sets, 6 objects in each set. How many objects in total?

e) 3 objects in each set, 9 objects altogether. How many sets?

When your students are comfortable with this lesson’s goal, introduce alternative contexts for objects and sets. Start with point-form problems (EXAMPLE: 5 tennis courts, 3 tennis balls on each court. How many tennis balls altogether?), and then move to complete sentence problems (EXAMPLE: If there are 5 tennis courts, and 3 balls on each court, how many tennis balls are on the tennis courts altogether?).

Emphasize that understanding the context of objects and sets within the word problem is unnecessary for answering the problem. Tell your students that you met someone from Mars last weekend, and they told you that there are 3 dulgs on each flut. If you count 15 dulgs, how many fluts are there? Explain the problem-solving strategy of replacing unknown words with words that are commonly used. For example, replace the object in the problem (dulgs) with students, and replace the set in the problem (flut) with bench. So, if there are 3 students on each bench and you count 15 students, how many benches are there? It wouldn’t make sense to replace the object (dulgs) with benches and the set (flut) with students, would it? [If there are 3 benches on each student and you count 15 benches, how many students are there?] A good strategy for replacing words is to replace the object and the set and then invert the replacement with the same two words. Only one of the two versions of the problem with the replacement words should make sense.

Students may wish to create their own science fiction word problems for their classmates. Encourage them to use words from another language, if they speak another language.

Extension

Have your students circle the given information in the following word problems, and then underline the information that isn’t given and needs to be determined. They should start by looking for the question mark; reading that sentence will tell them what needs to be determined. Then they should find the given information that they need to find the answer and circle it. It will be especially important to start by determining what the question is asking when irrelevant information is included. The two examples below contrast the situations where very little except the relevant information is given and where irrelevant information such as “are being used for tournaments” is included.

If there are 5 tennis courts and 3 balls on each court, how many tennis balls altogether?

5 tennis courts are being used for tournaments. There are 3 balls on each court. How many tennis balls are there altogether?

Start with problems that have very few extraneous words.

a) If there are 4 cans of tennis balls and 12 tennis balls altogether, how many tennis balls are in each can?

b) If there are 7 chairs in each row, and 3 rows of chairs, how many chairs are there altogether?

c) If 20 students ride in 4 vans, how many students ride in each van?
Constantly identify the information that is needed in a word problem in order to solve the word problem. For example, we need to know the number of students and vans to determine how many students will ride in each van.

For the following set of problems, write the interrogative clause first (for EXAMPLE: “How many hockey cards do they each have?”) and ask your students to identify the information that is needed to solve the problem. Then write the problem in its entirety and ask them to solve it.

d) Three friends share 15 hockey cards. How many hockey cards do they each have?

e) If 15 students are going to the zoo, and 5 students can ride in each van, how many vans are needed?

f) Sally has 3 white binders with pretty flowers on the cover. Each binder holds 200 pages. How many pages will the binders hold altogether?

When your students are comfortable with circling the proper information, have them rewrite the following word problems in point-form with only the essential information (EXAMPLE: 3 binders. Each binder holds 200 pages. How many pages will the binders hold?).

g) Sally has 3 white binders that she uses for different subjects. Each binder has 7 purple flowers on it. How many purple flowers are on her binders altogether?

h) Bilal lives in a haunted house. On each window there are 6 spiders. Beside each creaky door there are 5 frogs. If there are 5 windows, how many spiders are there altogether?

i) Sally has 3 blue binders. She uses each binder for 4 different subjects. Each binder has a picture of 5 basketballs on it. How many basketballs are pictured on her binders altogether?

Have your students create a word problem with extraneous information. Then have a partner rewrite the word problem in point-form with only the essential information.
NS5-38
Remainders and
NS5-39
Dividing with Remainders

Draw:

6 ÷ 3 = 2

7 ÷ 3 = 2 Remainder 1

8 ÷ 3 = 2 Remainder 2

9 ÷ 3 = 3

10 ÷ 3 = 3 Remainder 1

Ask your students if they know what the word “remainder” means. Instead of responding with a definition, encourage them to only say the answers for the following problems. This will allow those students who don’t immediately see it a chance to detect the pattern.

7 ÷ 2 = 3  Remainder _____

11 ÷ 3 = 3  Remainder _____

12 ÷ 5 = 2  Remainder _____

14 ÷ 5 = 2  Remainder _____

Challenge volunteers to find the remainder by drawing a picture on the board. This way, students who do not yet see the pattern can see more and more examples of the rule being applied.

SAMPLE PROBLEMS:

9 ÷ 2  7 ÷ 3  11 ÷ 3  15 ÷ 4

15 ÷ 6  12 ÷ 4  11 ÷ 2  18 ÷ 5

What does “remainder” mean? Why are some dots left over? Why aren’t they included in the circles? What rule is being followed in the illustrations? [The same number of dots is placed in each circle, the remaining dots are left uncircled]. If there are fewer uncircled dots than circles then we can’t put
one more in each circle and still have the same number in each circle, so we have to leave them uncircled. If there are no dots left over, what does the remainder equal? [Zero.]

Introduce your students to the word “quotient”: Remind your students that when subtracting two numbers, the answer is called the difference. **ASK:** When you add two numbers, what is the answer called? In $7 + 4 = 11$, what is $11$ called? (The sum). When you multiply two numbers, what is the answer called? In $2 \times 5 = 10$, what is $10$ called? (The product). When you divide two numbers, does anyone know what the answer is called? There is a special word for it. If no-one suggests it, tell them that when you write $10 \div 2 = 5$, the $5$ is called the quotient.

Have your students determine the quotient and the remainder for the following statements.

a) $17 \div 3 = \underline{\phantom{0}}$ Remainder $\underline{\phantom{0}}$

b) $23 \div 4 = \underline{\phantom{0}}$ Remainder $\underline{\phantom{0}}$

c) $11 \div 3 = \underline{\phantom{0}}$ Remainder $\underline{\phantom{0}}$

Write “2 friends want to share 7 apples.” What are the sets? [Friends.] What are the objects being divided? [Apples.] How many circles need to be drawn to model this problem? How many dots need to be drawn?

Draw 2 circles and 7 dots.

To divide 7 apples between 2 friends, place 1 dot (apple) in each circle.

Can another dot be placed in each circle? Are there more than 2 dots left over? So is there enough to put one more in each circle? Repeat this line of instruction until the diagram looks like this:

How many apples will each friend receive? Explain. [There are 3 dots in each circle.] How many apples will be left over? Explain. [Placing 1 more dot in either of the circles will make the compared amount of dots in both circles unequal.]

Repeat this exercise with “5 friends want to share 18 apples.” Emphasize that the process of division and placing apples (dots) into sets (circles) continues as long as there are at least 5 apples left to share. Count the number of apples remaining after each round of division to ensure that at least 5 apples remain.
Have your students illustrate each of the following division statements with a picture, and then determine the quotients and remainders.

\[
\begin{align*}
\text{a) } 11 \div 5 &= ____ \text{ Remainder } ____ \\
\text{b) } 18 \div 4 &= ____ \text{ Remainder } ____ \\
\text{c) } 20 \div 3 &= ____ \text{ Remainder } ____ \\
\text{d) } 22 \div 5 &= ____ \text{ Remainder } ____ \\
\text{e) } 11 \div 2 &= ____ \text{ Remainder } ____ \\
\text{f) } 8 \div 5 &= ____ \text{ Remainder } ____ \\
\text{g) } 19 \div 4 &= ____ \text{ Remainder } ____
\end{align*}
\]

Then have your students explain what the following three models illustrate.

\[
\begin{align*}
\text{Have them explain how these models are the same and how are they different? Have your students complete several division exercises using number lines, and then have them draw number lines for several division statements.}
\end{align*}
\]

Can skip counting show that \(14 \div 3 = 4 \text{ Remainder } 2\)?

\[
\begin{align*}
\text{Why does the count stop at 12? [Continuing the count will lead to numbers greater than 14.]} \\
\text{How can the remainder be determined? [Subtract 12 from 14. 14 – 12 = 2.]} \\
\text{Have your students complete several division exercises by skip counting. Instruct them to now write “R” as the abbreviation in equations for remainder. EXAMPLE: 17 \div 5 = 3 \text{ R } 2.}
\end{align*}
\]

Assign the following exercise to students who have difficulties learning when to stop counting, when skip counting to solve a division statement.

Using a number line from 0 to 25, ask your student to skip count out loud by five and to stop counting before reaching 17. Have them point to the respective number on the number line as they count it. This should enable your student to see that their finger will next point to 20 if they don’t stop counting at 15, passing the target number of 17. You may need to put your finger on 17 to stop some students from counting further. Repeat this exercise with target numbers less than 25. After completing this exercise, most students will know when to stop counting before they reach a given
target number, even if they are counting by numbers other than 5. With a few students, you will have to repeat the exercise with counting by 2s, 3s, etc.

Extensions

1. Which number is greater, the divisor (the number by which another is to be divided) or the remainder? Will this always be true? Have your students examine their illustrations to help explain. Emphasize that the divisor is equal to the number of circles (sets), and the remainder is equal to the number of dots left over. We stop putting dots in circles only when the number left over is smaller than the number of circles; otherwise, we would continue putting the dots in the circles. See the journal section below.

Which of the following division statements is correctly illustrated? Can one more dot be placed into each circle or not? Correct the two wrong statements.

\[
15 \div 3 = 4 \text{ Remainder } 3 \quad 17 \div 4 = 3 \text{ Remainder } 5 \quad 19 \div 4 = 4 \text{ Remainder } 3
\]

14 dots are divided into 3 groups with 2 dots in each group. 5 dots are left over.

Without illustration, identify the incorrect division statements and correct them.

a) 16 \div 5 = 2 \text{ Remainder } 6 \quad b) 11 \div 2 = 4 \text{ Remainder } 3 \quad c) 19 \div 6 = 3 \text{ Remainder } 1

2. Explain how a diagram can illustrate a division statement with a remainder and a multiplication statement with addition.

\[
14 \div 3 = 4 \text{ Remainder } 2 \quad 3 \times 4 + 2 = 14
\]

Ask students to write a division statement with a remainder and a multiplication statement with addition for each of the following illustrations.

3. Compare mathematical division to normal sharing. Often if we share 5 things (say, marbles) among 2 people as equally as possible, we give 3 to one person and 2 to the other person. But in mathematics, if we divide 5 objects between 2 sets, 2 objects are placed in each set and the leftover object is designated as a remainder. Teach them that we can still use division to solve this type of problem; we just have to be careful in how we interpret the remainder.

Have students compare the answers to the real-life problem and to the mathematical problem:

a) 2 people share 5 marbles (groups of 2 and 3; \(5 \div 2 = 2 \text{ R } 1\))

b) 2 people share 7 marbles (groups of 3 and 4; \(7 \div 2 = 3 \text{ R } 1\))

c) 2 people share 9 marbles (groups of 4 and 5; \(9 \div 2 = 4 \text{ R } 1\))
ASK: If $19 \div 2 = 9 \text{ R } 1$, how many marbles would each person get if 2 people shared 19 marbles? Emphasize that we can use the mathematical definition of sharing as equally as possible even when the answer isn’t exactly what we’re looking for. We just have to know exactly how to adapt it to what we need.

4. Find the mystery number. I am between 22 and 38. I am a multiple of 5. When I am divided by 7 the remainder is 2.

5. Have your students demonstrate two different ways of dividing…
   a) 7 counters so that the remainder equals 1.
   b) 17 counters so that the remainder equals 1.

6. As a guided class activity or assignment for very motivated students, have your students investigate the following division statements.
   
   $17 \div 3 = 5 \text{ Remainder 2}$, and $17 \div 5 = \_\_\_\_\_ \text{ Remainder } \_\_\_\_\_ ?$
   
   $22 \div 3 = 7 \text{ Remainder 1}$, and $22 \div 7 = \_\_\_\_\_ \text{ Remainder } \_\_\_\_\_ ?$
   
   $29 \div 4 = 7 \text{ Remainder 1}$, and $29 \div 7 = \_\_\_\_\_ \text{ Remainder } \_\_\_\_\_ ?$
   
   $23 \div 4 = 5 \text{ Remainder 3}$, and $23 \div 5 = \_\_\_\_\_ \text{ Remainder } \_\_\_\_\_ ?$
   
   $27 \div 11 = 2 \text{ Remainder 5}$, and $27 \div 2 = \_\_\_\_\_ \text{ Remainder } \_\_\_\_\_ ?$

   What seems to be true in the first four statements but not the fifth?
   
   $27 \div 2 = 13 \text{ Remainder 1}$, and $27 \div 13 = \_\_\_\_\_ \text{ Remainder } \_\_\_\_\_ ?$
   
   $40 \div 12 = 3 \text{ Remainder 4}$, and $40 \div 3 = \_\_\_\_\_ \text{ Remainder } \_\_\_\_\_ ?$

   Challenge your students to determine the conditions for switching the quotient with the divisor and having the remainder stay the same for both statements. Have students create and chart more of these problems to help them find a pattern. You could start a class chart where students write new problems that they have discovered belongs to one or the other category. As you get more belonging to one of the categories challenge them to find more examples that belong to the other category. Be sure that everyone has a chance to contribute.

   ANSWER: If the remainder is smaller than the quotient, the quotient can be switched with the divisor and the remainder will stay the same for both statements. Note that $23 \div 4 = 5$ Remainder 3 is equivalent to $4 \times 5 + 3 = 23$. But this is equivalent to $5 \times 4 + 3 = 23$, which is equivalent to $23 \div 5 = 4$ Remainder 3. Note, however, that while $8 \times 6 + 7 = 55$ is equivalent to $55 \div 8 = 6$ Remainder 7, it is not true that $6 \times 8 + 7 = 55$ is equivalent to $55 \div 6 = 8$ Remainder 7, because in fact $55 \div 6 = 9$ Remainder 1.

   Journal
   The remainder is always smaller than the divisor because…
NS5-40
Long Division — 2-Digit by 1-Digit

Write

$$\begin{align*}
3 & \longdiv{6} \\
5 & \longdiv{10} \\
4 & \longdiv{12} \\
5 & \longdiv{20} \\
6 & \longdiv{18} \\
9 & \longdiv{18}
\end{align*}$$

Ask your students if they recognize this symbol. If they know what the symbol means, have them solve the problems. If they don’t recognize the symbol, have them guess its meaning from the other students’ answers to the problems. If none of your students recognize the symbol, solve the problems for them. Write more problems to increase the chances of students being able to predict the answers.

$$\begin{align*}
2 & \longdiv{8} \\
2 & \longdiv{6} \\
4 & \longdiv{16} \\
5 & \longdiv{15} \\
4 & \longdiv{8} \\
6 & \longdiv{12}
\end{align*}$$

Explain that $2 \div 6$ is another way of expressing $6 \div 2 = 3$, and that $2 \div 7$ is another way of expressing $7 \div 2 = 3$ Remainder 1.

Ask your students to express the following statements using the new notation learned above.

a) $14 \div 3 = 4$ Remainder 2
b) $26 \div 7 = 3$ Remainder 5
c) $819 \div 4 = 204$ Remainder 3

To ensure that they understand the long division symbol, ask them to solve and illustrate the following problems.

$$\begin{align*}
2 & \longdiv{11} \\
4 & \longdiv{18} \\
5 & \longdiv{17} \\
4 & \longdiv{21} \\
3 & \longdiv{16}
\end{align*}$$

**Bonus**

$$\begin{align*}
6 & \longdiv{45} \\
4 & \longdiv{37} \\
4 & \longdiv{43}
\end{align*}$$

Then demonstrate division using base ten materials.

$$3 \longdiv{63}$$

Have students solve the following problems using base ten materials.

$$\begin{align*}
2 & \longdiv{84} \\
2 & \longdiv{48} \\
3 & \longdiv{96} \\
4 & \longdiv{88}
\end{align*}$$
Then challenge them to solve $3 \div 72$, again using base ten materials, but allow students to trade tens blocks for ones blocks as long as the value of the dividend (72) remains the same.

**ASK:** Can 7 tens blocks be equally placed into 3 circles? Can 6 of the 7 tens blocks be equally placed into 3 circles? What should be done with the leftover tens block? How many ones blocks can it be traded for? How many ones blocks will we then have altogether? Can 12 ones blocks be equally placed into 3 circles? Now, what is the total value of blocks in each circle? [24.] What is 72 divided by 3? Where do we write the answer?

Explain that the answer is always written above the dividend, with the tens digit above the tens digit and the ones digit above the ones digit.

$$
\begin{array}{c}
3 \\ \\
\rightleftharpoons 24 \\
\hline \\
72
\end{array}
$$

Have your students solve several problems using base ten materials.

$$
\begin{array}{c}
4 \\ \\
\rightleftharpoons 92 \\
\hline \\
64
\end{array} \\
\begin{array}{c}
4 \\ \\
\rightleftharpoons 72 \\
\hline \\
45
\end{array} \\
\begin{array}{c}
3 \\ \\
\rightleftharpoons 78
\end{array}
$$

The following problems will have remainders.

$$
\begin{array}{c}
4 \\ \\
\rightleftharpoons 65 \\
\hline \\
82
\end{array} \\
\begin{array}{c}
3 \\ \\
\rightleftharpoons 82 \\
\hline \\
94
\end{array} \\
\begin{array}{c}
2 \\ \\
\rightleftharpoons 35 \\
\hline \\
35
\end{array} \\
\begin{array}{c}
4 \\ \\
\rightleftharpoons 71
\end{array}
$$

Tell your students that they are going to learn to solve division problems without using base ten materials. The solutions to the following problems have been started using base ten materials. Can they determine how the solution is written?

$$
\begin{array}{c}
3 \\ \\
\rightleftharpoons 63 \\
\hline \\
6
\end{array} \\
\begin{array}{c}
2 \\ \\
\rightleftharpoons 64 \\
\hline \\
6
\end{array} \\
\begin{array}{c}
4 \\ \\
\rightleftharpoons 92 \\
\hline \\
8
\end{array}
$$

As in the previous set, illustrate the following problems. Have your students determine how to write the solution for the last problem.

$$
\begin{array}{c}
5 \\ \\
\rightleftharpoons 75 \\
\hline \\
5
\end{array} \\
\begin{array}{c}
2 \\ \\
\rightleftharpoons 91 \\
\hline \\
8
\end{array} \\
\begin{array}{c}
3 \\ \\
\rightleftharpoons 95 \\
\hline \\
9
\end{array} \\
\begin{array}{c}
2 \\ \\
\rightleftharpoons 87 \\
\hline \\
8
\end{array} \\
\begin{array}{c}
2 \\ \\
\rightleftharpoons 78
\end{array}
$$

Challenge students to illustrate several more problems and to write the solution. When all students are writing the solution correctly, ask them how they determined where each number was written. Which number is written above the dividend? [The number of tens equally placed into each circle.] Which number is written below the dividend? [The number of tens placed altogether.]

Then, using the illustrations from the two previous sets of problems, ask your students what the circled numbers below express.
ASK: What does the number express in relation to its illustration? [The number of tens not equally placed into circles.] Why does the subtraction make sense? [The total number of tens minus the number of tens equally placed into circles results in the number of tens blocks left over.]

Teach your students to write algorithms without using base ten materials. Remind them that the number above the dividend's tens digit is the number of tens placed in each circle. For example, if there are 4 circles and 9 tens, as in 4 \[94\], the number 2 is written above the dividend to express that 2 tens are equally placed in each of the 4 circles. Explain that the number of tens placed altogether can be calculated by multiplying the number of tens in each circle (2) by the number of circles (4); the number of tens placed altogether is \(2 \times 4 = 8\). Ask your students to explain if the following algorithms have been started correctly or not. Encourage them to illustrate the problems with base ten materials, if it helps.

Explain that the remaining number of tens blocks should always be less than the number of circles, otherwise more tens blocks need to be placed in each circle. The largest number of tens blocks possible should be equally placed in each circle.

Display the multiplication facts for 2 times 1 through 5 (i.e. \(2 \times 1 = 2\), \(2 \times 2 = 4\), etc.), so that students can refer to it for the following set of problems. Then write

\[2 \div 75\]

ASK: How many circles should be used? If 1 tens block is placed in each circle, how many tens blocks will be placed altogether? [\(2 \times 1 = 2\)] What if 2 tens blocks are placed in each circle? [\(2 \times 2 = 4\)] What if 3 tens blocks are placed in each circle? [\(2 \times 3 = 6\)] And finally, what if 4 tens blocks are placed in each circle? How many tens blocks need to be placed? [Seven.] Can 4 tens blocks be placed in each circle? [No, that will require 8 tens blocks.] Then explain that the greatest multiple of 2 not exceeding the number of tens is required. Have them perform these steps for the following problems.
Then display the multiplication facts for 3 times 1 through to 3 times 5 and repeat the exercise. Demonstrate the steps for the first problem.

\[
\begin{array}{c}
3 \quad \overline{75} \\
\underline{15} \\
3 \times 2 = 6 \text{ tens place} \\
\end{array}
\quad \begin{array}{c}
3 \quad \overline{65} \\
\underline{15} \\
1 \text{ tens block left over} \\
\end{array}
\quad \begin{array}{c}
3 \quad \overline{38} \\
\underline{26} \\
3 \text{ tens in each circle} \\
\end{array}
\quad \begin{array}{c}
3 \quad \overline{81} \\
\underline{27} \\
3 \text{ tens in each circle} \\
\end{array}
\quad \begin{array}{c}
3 \quad \overline{59} \\
\underline{17} \\
1 \text{ tens block left over} \\
\end{array}
\]

Emphasize that the number above the dividend’s tens digit is the greatest multiple of 3 not exceeding the number of tens.

Then, using the illustrations already drawn to express leftover tens blocks (see the second page of this section), explain the next step in the algorithm. **ASK:** Now what do the circled numbers express?

\[
\begin{array}{c}
2 \quad \overline{63} \\
\underline{15} \\
3 \text{ tens to be placed} \\
\end{array}
\quad \begin{array}{c}
2 \quad \overline{64} \\
\underline{16} \\
2 \text{ tens to be placed} \\
\end{array}
\quad \begin{array}{c}
4 \quad \overline{92} \\
\underline{20} \\
4 \text{ tens to be placed} \\
\end{array}
\quad \begin{array}{c}
4 \quad \overline{91} \\
\underline{20} \\
2 \text{ tens to be placed} \\
\end{array}
\quad \begin{array}{c}
3 \quad \overline{95} \\
\underline{21} \\
3 \text{ tens to be placed} \\
\end{array}
\quad \begin{array}{c}
2 \quad \overline{87} \\
\underline{15} \\
2 \text{ tens to be placed} \\
\end{array}
\quad \begin{array}{c}
2 \quad \overline{78} \\
\underline{15} \\
2 \text{ tens to be placed} \\
\end{array}
\]

The circled number expresses the amount represented by the base ten materials not placed in the circles.

Using base ten materials, challenge students to start the process of long division for \(85 \div 3\) and to record the process (the algorithm) up to the point discussed so far. Then ask students to trade the remaining tens blocks for ones blocks, and to circle the step in the algorithm that expresses the total value of ones blocks.

Ensure that students understand the algorithm up to the step where the ones blocks are totalled with the remaining (if any) tens blocks.

\[
\begin{array}{c}
2 \quad \overline{75} \\
\underline{15} \\
15 \text{ ones to be placed} \\
\end{array}
\quad \begin{array}{c}
3 \quad \overline{65} \\
\underline{19} \\
\end{array}
\quad \begin{array}{c}
3 \quad \overline{38} \\
\underline{11} \\
\end{array}
\quad \begin{array}{c}
3 \quad \overline{81} \\
\underline{21} \\
\end{array}
\quad \begin{array}{c}
3 \quad \overline{59} \\
\underline{19} \\
\end{array}
\]

Illustrate all of the placed tens and ones blocks and the finished algorithm, and then ask your students to explain the remaining steps in the algorithm. Perform this for the examples already started (\(63 \div 3\), \(64 \div 2\), \(92 \div 4\), etc.). For example,

\[
\begin{array}{c}
4 \quad \overline{94} \\
\underline{32} \\
14 \text{ Remainder} \\
\end{array}
\quad \begin{array}{c}
4 \quad \overline{94} \\
\underline{32} \\
14 \text{ Remainder} \\
\end{array}
\quad \begin{array}{c}
4 \quad \overline{94} \\
\underline{32} \\
14 \text{ Remainder} \\
\end{array}
\quad \begin{array}{c}
4 \quad \overline{94} \\
\underline{32} \\
14 \text{ Remainder} \\
\end{array}
\quad \begin{array}{c}
4 \quad \overline{94} \\
\underline{32} \\
14 \text{ Remainder} \\
\end{array}
\quad \begin{array}{c}
4 \quad \overline{94} \\
\underline{32} \\
14 \text{ Remainder} \\
\end{array}
\]

So, \(94 \div 4 = 23\) Remainder 2.
Ask your students to explain how the circled numbers are derived. How is the 3 derived? The 12? [Dividing the 14 ones blocks into 4 circles results in 3 blocks in each circle, for a total of 12.] 14 – 12 results in a remainder of 2.

Then challenge students to write the entire algorithm. Ask them why the second subtraction makes sense. [The total number of ones blocks subtracted by the number of ones blocks placed into circles equals the number of ones blocks left over.]

Using base ten materials, have students complete several problems and write the entire algorithms.

\[
\begin{align*}
3 & \longdiv{75} \quad 5 \text{ ones in each circle} \\
& \longdiv{6} \\
& 15 \quad \text{ones to be placed} \\
& \longdiv{15} \quad 5 \times 3 \text{ ones placed} \\
& 0 \quad \text{Remainder (no ones left over)}
\end{align*}
\]

Some students will need all previous steps done so that they can focus on this one.

If you prefer, you may use an example for a problem that has leftover ones. Have students finish the examples they have already started and then complete several more problems from the beginning and use base ten materials only to verify their answers.

\[
\begin{align*}
2 & \longdiv{39} \\
3 & \longdiv{39} \\
2 & \longdiv{57} \\
3 & \longdiv{57} \\
2 & \longdiv{85} \\
3 & \longdiv{94} \\
2 & \longdiv{94}
\end{align*}
\]

With practice, students will learn to estimate the largest multiples that can be used to write the algorithms. When they are comfortable with moving forward in the lesson, introduce larger divisors.

\[
\begin{align*}
4 & \longdiv{69} \\
5 & \longdiv{79} \\
6 & \longdiv{87} \\
4 & \longdiv{57} \\
6 & \longdiv{85} \\
7 & \longdiv{94} \\
8 & \longdiv{94}
\end{align*}
\]

Note that at this point in the lesson, the dividend’s tens digit is always greater than the divisor.

**Extension**

Teach students to check their answers with multiplication. For example:

\[
\begin{align*}
4 & \longdiv{69} \\
& \times 4 \\
& 29 \\
& 28 \\
& 1 \text{R} \quad 68 + 1 = 69
\end{align*}
\]
GOALS
Students will use the standard algorithm for long division to divide 3- and 4-digit numbers by 1-digit numbers.

PRIOR KNOWLEDGE REQUIRED
The standard algorithm for dividing 2-digit numbers by 1-digit numbers
Remainders
Division as finding the number in each group
Tens and ones blocks

VOCABULARY
standard algorithm
remainder
quotient

Teach your students to use long division to divide three-digit numbers by one-digit numbers. Using base ten materials, explain why the standard algorithm for long division works.

EXAMPLE: Divide 726 into 3 equal groups.

STEP 1. Make a model of 726 units.

STEP 2. Divide the hundreds blocks into 3 equal groups.

Keep track of the number of units in each of the 3 groups, and the number remaining, by slightly modifying the long division algorithm.

200 ← 2 hundred blocks, or 200 units, have been divided into each group
600 ← 600 units (200 × 3) have been divided
126 ← 126 units still need to be divided

NOTE: Step 2 is equivalent to the following steps in the standard long division algorithm.
Students should practise Steps 1 and 2 from both the modified and the standard algorithms on the following problems.

\[
\begin{align*}
2 & \longdiv{512} \\
3 & \longdiv{822} \\
2 & \longdiv{726} \\
4 & \longdiv{912}
\end{align*}
\]

Students should show their work using actual base ten materials or a model drawn on paper.

**STEP 3.** Divide the remaining hundreds block and the 2 remaining tens blocks among the 3 groups equally.

There are 120 units, so 40 units can be added to each group from Step 2.

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Base ten materials" /></td>
<td><img src="image2" alt="Base ten materials" /></td>
<td><img src="image3" alt="Base ten materials" /></td>
</tr>
</tbody>
</table>

Keep track of this as follows:

- 40 new units have been divided into each group
- 200
- 3 \(726\) 
- 600
- 126
- 120
- 6 units still need to be divided

**NOTE:** Step 3 is equivalent to the following steps in the standard long division algorithm.

\[
\begin{align*}
2 & \longdiv{726} \\
3 & \longdiv{726} \\
2 & \longdiv{726} \\
3 & \longdiv{726}
\end{align*}
\]

Students should carry out Step 3 using both the modified and standard algorithms on the problems they started above. Then give students new problems and have them do all the steps up to this point. Students should show their work using either base ten materials or a model drawn on paper.
STEP 4. Divide the 6 remaining blocks among the 3 groups equally.

Group 1    Group 2   Group 3

There are now 242 units in each group; hence $726 \div 3 = 242$.

2                            2 new units have been divided into each group
40
200
3    726
−  600
126
−  120
6
−  6                             6 (2 × 3) new units have been divided
0                             There are no units left to divide

NOTE: Step 4 is equivalent to the following steps in the standard long division algorithm.

```
  242
3 726
  6
− 12
  0
```

Students should be encouraged to check their answer by multiplying $242 \times 3$.

Students should finish the problems they started. Then give students new problems to solve using all the steps of the standard algorithm. Give problems where the number of hundreds in the dividend is greater than the divisor. (EXAMPLES: $842 \div 2$, $952 \div 4$) Students should show their work (using either base ten materials or a model drawn on paper) and check their answers using multiplication.

When students are comfortable dividing 3-digit numbers by 1-digit numbers, introduce the case where the divisor is greater than the dividend’s hundreds digit.

Begin by dividing a 2-digit number by a 1-digit number where the divisor is greater than the dividend’s tens digit. (i.e. there are fewer tens blocks available than the number of circles):
ASK: How many tens blocks are in 27? Into how many circles do they need to be divided? Are there enough tens blocks to place one in each circle? How is this different from the problems you just did? Illustrate that there are no tens by writing a zero above the dividend’s tens digit. Then ASK: What is 5 × 0? Write:

\[
\begin{array}{c}
0 \\
5 \longdiv{27} \\
0 \\
27 \quad \text{Number of ones blocks (traded from tens blocks) to be placed}
\end{array}
\]

Have a volunteer finish this problem, and then ask if the zero needs to be written at all. Explain that the algorithm can be started on the assumption that the tens blocks have already been traded for ones blocks.

\[
\begin{array}{c}
5 \\
5 \longdiv{27} \\
25 \quad \text{Number of ones blocks placed} \\
2 \quad \text{Number of ones blocks left over}
\end{array}
\]

Emphasize that the answer is written above the dividend’s ones digit because it is the answer’s ones digit.

Have students complete several similar problems.

\[
\begin{array}{c}
4 \longdiv{37} \\
5 \longdiv{39} \\
8 \longdiv{63} \\
8 \longdiv{71}
\end{array}
\]

Then move to 3-digit by 1-digit long division where the divisor is more than the dividend’s hundreds digit. (EXAMPLES: 324 ÷ 5; 214 ÷ 4; 133 ÷ 2) Again, follow the standard algorithm (writing 0 where required) and then introduce the shortcut (omit the 0).

When students are comfortable with all cases of 3-digit by 1-digit long division, progress to 4-digit by 1-digit long division. Start by using base ten materials and going through the steps of the recording process as before. Some students may need to practise 1 step at a time, as with 3-digit by 1-digit long division.

ASK: How long is each side if an octagon has a perimeter of...

a) 952 cm b) 568 cm c) 8104 d) 3344

Extension

1. By multiplying the divisors by ten and checking if the products are greater or less than their respective dividends, students can determine if the answers to the following problems will have one or two digits.

\[
\begin{array}{c}
3 \longdiv{72} \\
4 \longdiv{38} \\
9 \longdiv{74} \\
6 \longdiv{82} \\
6 \longdiv{34}
\end{array}
\]

For example, multiplying the divisor in 72 ÷ 3 by ten results in a product less than 72 (3 × 10 = 30), meaning the quotient for 72 ÷ 3 is greater than 10 and has two digits. On the other hand, multiplying the divisor in 38 ÷ 4 by ten results in a product greater than 38 (4 × 10 = 40), meaning the quotient for 38 ÷ 4 is less than 10 and has one digit.
Then have students decide how many digits (1, 2, 3 or 4) each of these quotients will have:

- $564 \div 9$
- $723 \div 6$
- $543 \div 9$
- $7653 \div 8$
- $7653 \div 4$
- $563 \div 92$

**BONUS:** $76\,589\,436 \div 9$ (Answer: $9 \times 1\,000\,000 = 9\,000\,000$ is less than the dividend and $9 \times 10\,000\,000 = 90\,000\,000$ is more than the dividend, so the quotient is at least $1\,000\,000$ but is less than $10\,000\,000$, and so has 7 digits.)

---

**NS5-42**

**Topics in Division (Advanced)**

**GOALS**
Students will use division to solve word problems.

**PRIOR KNOWLEDGE REQUIRED**
Long division of 2-, 3-, and 4-digit numbers by 1-digit numbers
Remainders
Word problems

**VOCABULARY**
divisor
quotient
remainder
divisible by

Explain that division can be used to solve word problems. **SAY:** Let’s start with a word problem that doesn’t require long division. A canoe can hold three kids. Four kids are going canoeing together. How many canoes are needed? (2) **ASK:** What happens when long division is used to solve this question? Does long division give us the same answer?

```
  1
3 ) 4
  3
  1 Remainder
```

Long division suggests that three kids will fit in one canoe and one kid will be left behind. But one kid can’t be left behind because four kids are going canoeing together. So even though the quotient is one canoe, two canoes are actually needed.

How many canoes are needed if 8 kids are going canoeing together? 11 kids? 12 kids? 14 kids? Students should do these questions by using long division and by using common sense.

**BONUS:** How many canoes are needed for 26 kids? 31 kids? 85 kids?

How does long division help when we have larger numbers?

If Anna reads two pages from her book every day, and she has 13 pages left to read, how many days will it take Anna to finish her book?

If Rita reads three pages every day, how long will it take her to read …

- a) 15 pages?
- b) 17 pages?
- c) 92 pages?
- d) 84 pages?
- e) 67 pages?

If Anna reads eight pages every day, how long will it take her to read …

- a) 952 pages?
- b) 295 pages?
- c) 874 pages?
- d) 1,142 pages?
- e) 4,567 pages?
(Adapted from the Atlantic Curriculum Grade 4) Students should understand that a remainder is always interpreted within the context of its respective word problem. Students should understand when a remainder ...

- needs to be divided further. For example, when 3 children share 7 licorice pieces, each child receives 2 pieces, and the remaining piece is further divided into thirds. So each child receives 2 and a third licorice pieces.

- needs to be ignored. For example, $3.25 will buy four 75¢ notebooks. Since there is not enough money to buy five notebooks, the 25¢ is ignored.

- needs to be rounded up. For example, five 4-passenger cars are needed to transport 17 children because none of the children can be left behind.

- needs to be divided unequally among the groups. For example, if 91 students are to be transported in 3 buses, 30 students will ride in two buses, and 31 students will ride in the other.

To guide students through question 2 on the worksheet, have them solve the following problems by long division: 72 ÷ 3, 88 ÷ 3, 943 ÷ 3, 1011 ÷ 3, 8846 ÷ 3. Then ASK: Which of these numbers are divisible by 3: 72, 88, 943, 1011, 8846? How can you tell? (Look at the remainder—if the remainder when dividing by 3 is 0, then the number is divisible by 3.)

To guide students through question 3 on the worksheet, have them find the pattern in the remainders when dividing by 4. Use a T-chart:

<table>
<thead>
<tr>
<th>Number</th>
<th>Answer when divided by 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 R 1</td>
</tr>
<tr>
<td>2</td>
<td>0 R 2</td>
</tr>
<tr>
<td>3</td>
<td>0 R 3</td>
</tr>
<tr>
<td>4</td>
<td>1 R 0</td>
</tr>
<tr>
<td>5</td>
<td>1 R 1</td>
</tr>
<tr>
<td>6</td>
<td>1 R 2</td>
</tr>
<tr>
<td>7</td>
<td>1 R 3</td>
</tr>
<tr>
<td>8</td>
<td>2 R 0</td>
</tr>
</tbody>
</table>

ASK: What is the pattern in the remainders? (1, 2, 3, 0 then repeat) What is the next number that will have remainder 3 when divided by 4? (11) And the next number after that? (15) How can we continue to find all the numbers that have remainder 3 when divided by 4? (Skip count by 4 starting at 3 to get: 3, 7, 11, 15, and so on).

Have students circle all the numbers on a hundreds chart that have remainder 3 when divided by 4. Repeat with numbers that have remainder 2 when divided by 3, but have them use a coloured pencil to circle the numbers on the same hundreds chart as before. ASK: What is the first number that has a remainder of 3 when divided by 4 and a remainder of 2 when divided by 3?
To guide students through question 4 on the worksheet, begin with dividing only 1 type of snack evenly into packets. Start with 12 raisins. **ASK:** How many ways can we divide 12 raisins evenly into more than one packet (6 each into 2 packets; 4 each into 3; 3 each into 4; or 1 each into 12). If we want to divide 18 pretzels evenly into more than one packet, how many ways can we do so? (9 each into 2; 6 each into 3; 3 each into 6; 2 each into 9; or 1 each into 18) How can we divide 18 pretzels and 12 raisins evenly into more than one packet? (9 pretzels and 6 raisins into 2 packets; or 6 pretzels and 4 raisins into 3 packets) What is the greatest number of packages we can make if we want to divide 18 pretzels and 12 raisins evenly? (3)

What is the greatest number of packages we can make if we want to divide evenly into packages...

- a) 32 raisins and 24 pretzels
- b) 55 raisins and 77 pretzels
- c) 54 raisins and 36 pretzels
- d) 28 raisins, 32 pretzels, and 44 dried cranberries
- e) 52 peanuts, 40 almonds, and 28 cashews

**ACTIVITY**

Distribute a set of base ten blocks and three containers large enough to hold the blocks to each student. Ask students to make a model of 74. Point out that each of the tens blocks used to make the model is made up of ten ones blocks. Then ask them to divide the 74 ones blocks into the three containers as evenly as possible.

Ensure that they understand leftover blocks are to be left out of the containers. Their understanding of “as evenly as possible” could mean that they add the leftover blocks to some of the containers.

Play this game with different numbers, in teams, and score points for correct answers. Track the steps your students use to divide the blocks by writing the respective steps from the algorithm. Ask your students to explain how the steps they used match the steps from the algorithm.
NS5-43
Concepts in Multiplication and Division

GOALS
Students will consolidate their knowledge of multiplication and division.

PRIOR KNOWLEDGE REQUIRED
Multiplication and division
Fact family of equations
Using multiplication to determine higher rates
Using division to determine unit rates
Word problems
Time (hours, minutes)

VOCABULARY
multiple of
divisible by
remainder
perimeter
rate
unit rate
fact family
equation
odd
even

Review the prior knowledge required and vocabulary as needed.

EXAMPLE: Numbers (greater than 0) that are multiples of, or divisible by, four are the numbers we note when skip counting by four: 4, 8, 12, 16, and so on.
NS5-44
Rounding on a Number Line

GOALS
Students will round to the closest ten, hundred or thousand, except when the number is exactly half-way between a multiple of ten, hundred or a thousand.

PRIOR KNOWLEDGE REQUIRED
Number lines
Concept of Closer

Show a number line from 0 to 10 on the board:

```
0 1 2 3 4 5 6 7 8 9 10
```

Circle the 2 and ask if the 2 is closer to the 0 or to the 10. When students answer 0, draw an arrow from the 2 to the 0 to show the distance. Repeat with several examples and then ask: Which numbers are closer to 0? Which numbers are closer to 10? Which number is a special case? Why is it a special case?

Then draw a number line from 10 to 30, with 10, 20 and 30 a different colour than the other numbers.

```
10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
```

Circle various numbers (not 15 or 25) and ask volunteers to draw an arrow showing which number they would round to if they had to round to the nearest ten.

Repeat with a number line from 50 to 70, again writing the multiples of 10 in a different colour. Then repeat with number lines from 230 to 250 or 370 to 390, etc.

Ask students for a general rule to tell which ten a number is closest to. What digit should they look at? How can they tell from the ones digit which multiple of ten a number is closest to?

Then give several examples where the number line is not given to them, but always giving them the two choices. EXAMPLE: Is 24 closer to 20 or 30? Is 276 closer to 270 or 280?

Repeat the lesson with a number line from 0 to 100, that shows only the multiples of 10.

```
0 10 20 30 40 50 60 70 80 90 100
```

At first, only ask them whether numbers that are multiples of 10 (30, 70, 60 and so on) are closer to 0 or 100. EXAMPLE: Is 40 closer to 0 or 100? Draw an arrow to show this. Repeat with several examples, then ask: Which multiples of 10 are closer to 0 and which multiples of 10 are closer to 100? Which number is a special case? Why is it a special case?

Then ask them about numbers that are not multiples of 10. First ask them where they would place the number 33 on the number line. Have a volunteer
show this. Then ask the rest of the class if 33 is closer to 0 or to 100. Repeat with several numbers. Then repeat with a number line from 100 to 200 and another number line from 700 to 800.

Ask your students for a general rule to tell which multiple of a hundred a number is closest to. What digit should they look at? How can they tell from the tens digit which multiple of a hundred a number is closest to? When is there a special case? Emphasize that the number is closer to the higher multiple of 100 if its tens digit is 6, 7, 8 or 9 and it’s closer to the lower multiple of 100 if its tens digit is 1, 2, 3 or 4. If the tens digit is 5, then any ones digit except 0 will make it closer to the higher multiple. Only when the tens digit is 5 and the ones digit is 0 do we have a special case where it is not closer to either.

Attach 11 cards to a rope so that there are 10 cm of rope between each pair of cards. Write the numbers from 30 to 40 on the cards so that you have a rope number line. Make the numbers 30 and 40 more vivid than the rest. Take a ring that can slide free on the rope with the cards and pull the rope through it.

Ask two volunteers to hold taught the number line. Ask a volunteer to find the middle number between 30 and 40. How do you know that this number is in the middle? What do you have to check? (the distance to the ends of the rope—make a volunteer do that). Let a volunteer stand behind the line holding the middle. Explain to your students that the three students with a number line make a rounding machine. The machine will automatically round the number to the nearest ten. Explain that the machine finds the middle number. Put the ring on 32. Ask the volunteer that holds the middle of the line to pull it up, so that the ring slides to 30. Try more numbers. Ask your students to explain why the machine works.
NS5-45
Rounding and
NS5-46
Rounding to Any Decimal Place

Have students round 2-digit numbers to the nearest ten. Do not at first include numbers that have ones digit 5. Ask students how they know which multiples of ten it is between. **ASK:** How many tens are in 37? How many would be one more ten? So 37 is between 3 tens and 4 tens; that means it’s between 30 and 40. Which multiple of ten is it closer to?

How many tens are in 94? How many would be one more ten? What number is ten tens? So 94 is between what two multiples of 10? Which multiple of 10 is it closer to? What about 97? 34? 46? 72? 81? 79? 23? 11?

Then tell your students that when the ones digit is 5, it is not closer to either the smaller or the larger ten, but we always round up. Give them many examples to practice with: 25, 45, 95, 35, 15, 5, 85, 75, 55, 65. If some students find this hard to remember, you could give the following analogy: I am trying to cross the street, but there is a big truck coming, so when I am part way across I have to decide whether to keep going or to turn back. If I am less than half way across, it makes sense to turn back because I am less likely to get hit. If I am more than half way across, it makes sense to keep going because I am again less likely to get hit. But if I am exactly half way across, what should I do? Each choice gives me the same chance of getting hit. Have them discuss what they would do and why. Remind them that they are, after all, trying to cross the street. So actually, it makes sense to keep going rather than to turn back. That will get them where they want to be.

Another trick to help students remember the rounding rule is to look at all the numbers with tens digit 3 (i.e. 30-39) and have them write down all the numbers that we should round to 30 because they’re closer to 30 than to 40. Which numbers should we round to 40 because they’re closer to 40 than to 30? How many are in each list? Where should we put 35 so that it’s fair?

Then move on to 3-digit numbers, still rounding to the nearest tens: 174, 895, 341, 936, etc. Include 4- and 5-digit numbers if students are interested.

Then move on to rounding to the nearest hundreds. **ASK:** Which multiple of 100 is this number closest to? What do we round to? Start with examples that are multiples of 10: 230, 640, 790, 60, 450 (it is not closest to either, but we round up to 500). Then move on to examples that are not multiples of 10. 236, 459, 871, 548, etc.

**SAY:** When rounding to the nearest 100, what digit do we look at? (The tens digit). When do we round down? When do we round up? Look at these numbers: 240, 241, 242, 243, 244, 245, 246, 247, 248, 249. What do these numbers all have in common? (3 digits, hundreds digit 2, tens digit 4).
Are they closer to 200 or 300? How can you tell without even looking at the ones digit?

Then tell your students to look at these numbers: 250, 251, 252, 253, 254, 255, 256, 257, 258, 259.

**ASK:** Which hundred are these numbers closest to? Are they all closest to 300 or is there one that’s different? Why is that one a special case? If you saw that the tens digit was 5, but you didn’t know the ones digit, and you had to guess if the number was closer to 200 or 300, what would your guess be? Would the number ever be closer to 200? Tell your students that when you round a number to the nearest hundred, mathematicians decided to make it easier and say that if the tens digit is a 5, you always round up. You usually do anyway, and it doesn’t make any more sense to round 250 to 200 than to 300, so you might as well round it up to 300 like you do all the other numbers that have tens digit 5.

Then **ASK:** When rounding a number to the nearest hundreds, what digit do we need to look at? (The tens digit.) Then write on the board:

Round to the nearest hundred:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>234</td>
<td>547</td>
<td>651</td>
<td>850</td>
<td>493</td>
</tr>
</tbody>
</table>

Have a volunteer underline the hundreds digit because that is what they are rounding to. Have another volunteer write the two multiples of 100 the number is between, so the board now looks like:

Round to the nearest hundred:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>234</td>
<td>547</td>
<td>651</td>
<td>850</td>
<td>973</td>
</tr>
<tr>
<td>200</td>
<td>500</td>
<td>600</td>
<td>800</td>
<td>900</td>
</tr>
<tr>
<td>300</td>
<td>600</td>
<td>700</td>
<td>900</td>
<td>1000</td>
</tr>
</tbody>
</table>

Then ask another volunteer to point an arrow to the digit they need to look at to decide whether to round up or round down. Ask where is that digit compared to the underlined digit? (It is the next one). **ASK:** How do they know when to round down and when to round up? Have another volunteer decide in each case whether to round up or down and circle the right answer.

Repeat with 4-digit numbers, having them round to the nearest hundred still. Then repeat with 4-digit and 5-digit numbers rounding to the nearest thousand.

Show your students how numbers can be rounded in a grid. Follow the steps shown below.

**EXAMPLE:** Round 12473 to the nearest thousand

**STEP 1:** Underline the digit you are rounding to.

```
 1 2 4 7 3
```

**STEP 2:** Put your pencil on the digit to the right of the one you are rounding to.

```
 1 2 4 7 3
```

---

**WORKBOOK 5:1 PAGE 103-104**
**STEP 3:** Beside the grid write “round up” if the digit under your pencil is 5, 6, 7, 8, or 9 and “round down” if the digit is 0, 1, 2, 3, 4.

```
1 2 4 7 3
Round Down
```

**STEP 4:** Round the underlined digit up or down according to the instruction you have written. (Write your answer in the grid)

```
1 2 4 7 3
2
```

Stop to give weaker students a diagnostic test on the first 4 steps. If a student is struggling with step 4, make up several examples where the first three steps are done for them, so that the student could focus on rounding the underlined digit up or down. Here is an example of a sample diagnostic quiz:

Round the underlined digit up or down as indicated:

```
Round Down Round Up Round Down
1 6 4 7 3
2 0 7 5 2
5 8 2 1 5
```

Once the student has mastered this step, move on to:

**STEP 5:** Change all numbers to the right of the rounded digit to zeroes.

```
Round Down
1 2 4 7 3
2 0 0 0
```

**STEP 6:** Copy all numbers to the left of the rounded digit as they were.

```
Round Down
1 2 4 7 3
1 2 0 0 0
```

When your students have mastered the rounding without regrouping, give them several examples that would demand regrouping as well. Warn them that the digits on the left of the rounded digit might change while rounding. For example, ask a volunteer to round 17 978 to the nearest hundreds, up to Step 5. Have them write which two hundreds the number is between. How many hundreds are in 17 978? (179) Which multiple of a hundred is the number closest to—17 900 or 18 000? (a hundred and seventy-nine hundreds, or a hundred and eighty hundreds).
Round Up.

Complete the rounding by writing 180, not 179, as the number of hundreds.

Another way to do this rounding is to round the 9 up to 10 and then regroup the 10 hundreds as 1000.

Round to the given digit:

<table>
<thead>
<tr>
<th>thousands</th>
<th>hundreds</th>
<th>tens</th>
<th>hundreds</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 9 6 7 3</td>
<td>1 2 9 7 1</td>
<td>1 2 9 9 3</td>
<td>9 9 8 7</td>
</tr>
</tbody>
</table>
NS5-47
Estimating Sums and Differences and
NS5-48
Estimation

GOALS
Students will estimate sums and differences by rounding each addend to the nearest ten, hundred, or thousand.

PRIOR KNOWLEDGE REQUIRED
Rounding to the nearest ten, hundred, or thousand

VOCABULARY
the “approximately equal to” sign (≈)
estimating

Show students how to estimate 52 + 34 by rounding each number to the nearest ten: 50 + 30 = 80. SAY: Since 52 is close to 50 and 34 is close to 30, 52 + 34 will be close to, or approximately, 50 + 30. Mathematicians have invented a sign to mean “approximately equal to.” It’s a squiggly “equal to” sign: ≈. So we can write 52 + 34 ≈ 80. It would not be right to put 52 + 34 = 80 because they are not actually equal; they are just close to, or approximately, equal.

Tell students that when they round up or down before adding, they aren’t finding the exact answer, they are just estimating. They are finding an answer that is close to the exact answer. ASK: When do you think it might be useful to estimate answers?

Have students estimate the sums of 2-digit numbers by rounding each to the nearest ten. Remind them to use the ≈ sign.

EXAMPLES:

41 + 38  52 + 11  73 + 19  84 + 13  92 + 37  83 + 24

Then ASK: How would you estimate 93 – 21? Write the estimated difference on the board with students:

93 – 21 ≈ 90 – 20

= 70

Have students estimate the differences of 2-digit numbers by again rounding each to the nearest ten.

EXAMPLES:


Then have students practise estimating the sums and differences of:

• 3-digit numbers by rounding to the nearest ten (EXAMPLES: 421 + 159, 904 – 219).

• 3- and 4-digit numbers by rounding to the nearest hundred (EXAMPLES: 498 + 123, 4 501 – 1 511).

• 4- and 5-digit numbers by rounding to the nearest thousand (EXAMPLES: 7 980 + 1 278, 13 891 – 11 990, 3 100 + 4 984).

Students who finish quickly may add and subtract larger numbers, rounding to tens, hundreds, thousands, or even ten thousands.
Teach students how they can use rounding to check if sums and differences are reasonable.

**EXAMPLE:**

Daniel added 273 and 385, and got the answer 958. Does this answer seem reasonable?

Students should see that even rounding both numbers up gives a sum less than 900, so the answer can’t be correct. Make up several examples where students can see by estimating that the answer cannot be correct.

**ASK:** If 48 329 is written as 48 300, what digit has it been rounded to? What digit do you think each of these numbers may have been rounded to?

- a) 39 000
- b) 4 320
- c) 6 500
- d) 1 345 600
- e) 1 230 000
- f) 230 010

Tell students that just because a number has a lot of zeroes doesn’t necessarily mean it has been rounded. Ask the class if they think the number was rounded in each situation below:

- a) The last time the Edmonton Oilers won the Stanley Cup was 1990. (not rounded)
- b) The population of Canada is 33 000 000. (rounded)
- c) Each package has 40 candies. (not rounded)
- d) The probability of precipitation is 40%. (rounded)
- e) Jennifer was born in the year 2000. (not rounded)
- f) Jericho has been a city since around 9000 BCE. (rounded)

**Extensions**

1. Give students sums of 3-digit numbers to estimate, and ask them to decide whether their estimate is too high or too low.

**EXAMPLES:**

- a) 148 + 429 = 150 + 430 = 580. Since both estimates are too high (I rounded both numbers up), my estimated sum is too high.

- b) 148 + 364 = 150 + 360 = 510. Since I rounded one number down by more than I rounded the other number up, my estimated sum is too low.

2. Give students more sums to estimate. Have them decide whether their estimate is too high or too low and by how much. They can use the estimate to calculate the actual answer:

**EXAMPLES:**

- \[ \begin{array}{ccc}
236 & 240 & 4 \\
+ 145 & 150 & 5 \\
\hline
390 & 390 & 9 \\
\end{array} \]

So the actual answer is 381.

- \[ \begin{array}{ccc}
433 & 430 & 3 \\
+ 519 & 520 & 1 \\
\hline
950 & 952 & 2 \\
\end{array} \]

So the actual answer is 952.
GOALS

Students will discover that to multiply a whole number by 10, 100, 1 000, or 10 000, you add 1, 2, 3, or 4 zeroes respectively to the right of the number.

PRIOR KNOWLEDGE REQUIRED

Multiplying by 10 and 100
Rounding
Estimating by rounding

VOCABULARY

estimating
rounding to the leading digit

ASK: How would you find $3 \times 4$ by skip counting? What else could you skip count by? How would you find $10 \times 12$ by skip counting? Is it easier to skip count by 10 twelve times or to skip count by 12 ten times? Why?

Have students find the following products by skip counting:

a) $13 \times 10$

b) $100 \times 11$

c) $12 \times 1000$

d) $34 \times 100$

e) $14 \times 10000$

Have volunteers fill in the blanks with the appropriate number:

To multiply by 10, you add ___ zero.

To multiply by 100, you add ___ zeroes.

To multiply by 1 000, you add ___ zeroes.

To multiply by 10 000, you add ___ zeroes.

Have students continue the patterns:

$$10 \times 6 =$$

$$100 \times 6 =$$

$$1 000 \times 6 =$$

$$10 000 \times 6 =$$

BONUS: $100 000 000 \times 6 =$

$$34 \times 10 =$$

$$34 \times 100 =$$

$$34 \times 1 000 =$$

$$34 \times 10 000 =$$

BONUS: $34 \times 10 000 000 =$

Have students find the products:

a) $32 \times 10$

b) $100 \times 65$

c) $53 \times 1000$

d) $753 \times 100$

e) $81 \times 1000$

f) $10 000 \times 55$

g) $400 \times 60$

h) $4 300 \times 100$

i) $7 100 \times 10 000$

j) $1 000 \times 810$

k) $680 \times 10 000$

Teach students that “rounding to the leading digit” means rounding to the nearest ten of a 2-digit number, the nearest hundred of a 3-digit number, and so on. In other words, the leading digit is the leftmost digit.

Have students estimate products by rounding each factor to its leading digit:

$$22 \times 97 (20 \times 100 = 2000)$$

$$34 \times 132$$

$$48 \times 81$$

$$4137 \times 789$$
Have students determine how many digits each product will have without actually finding the product (students should estimate the product instead):

\[
12 \times 1000 \quad 34 \times 10 \quad 76 \times 1000 \quad 87 \times 998 \quad (3 + 6) \times 100 \quad (7 + 86) \times 1000
\]

**BONUS:** \(130\,000 \times 100\,000\,000\)

---

**NS5-50**

**Other Methods of Estimation**

Show students how rounding one number up and another number down affects an estimate. Write the following sum and estimate on the board:

\[
\begin{array}{ccc}
760 & + & 330 \\
800 & + & 200 \\
\hline
1090 & 1100
\end{array}
\]

**SAY:** My first estimate was off by 40 and my second estimate was off by 30. Why is my answer only off by 10? What happened? Discuss.

Emphasize that rounding one number too high and the other too low will work toward cancelling out the error. Have students estimate the following sums by rounding the first number up and the second number down, and have them decide how high or how low each estimated sum is:

- a) \(580 + 130\)
- b) \(560 + 360\)
- c) \(710 + 830\)
- d) \(530 + 240\)
- e) \(490 + 180\)
- f) \(460 + 230\)

Repeat, but now round the first number down and the second number up.

**ASK:** When you decide to round one number up and the other number down, does it matter which number you round up and which number you round down?

Have students compare this way of rounding (one up and one down) to rounding to the leading digit. Which method gives a better estimate (i.e., a result closer to the answer). Finally, compare to rounding both numbers up or both numbers down.

For the following sums, **ASK:** If you round each number to the leading digit, will you be off by more for the number that you round up or the one that you round down?

- a) \(540 + 790\) (you are off by more when you round down—540 to 500 is a difference of 40—than when you round up—790 to 800 is a difference of 10)
- b) \(520 + 670\)
- c) \(430 + 280\)
- d) \(380 + 410\)
- e) \(380 + 440\)

Then **ASK:** If you were off by more when you rounded up than down, will your answer be too high or too low? (too high) Will the estimates for a) to e) above...
be too low or too high? (EXAMPLE: a) $540 + 790$—you are off by more when you round $540$ down, so the estimate of the sum will be too low)

Students should understand that if both numbers are rounded down, the estimate is too low and if both numbers are rounded up, the estimate is too high. If one number is rounded up and the other is rounded down, whether the estimate is too high or too low depends on which number is off by more.

Have students investigate when rounding one number up and one number down is better than rounding each to the nearest hundred by completing the following chart and circling the estimate that is closest to the actual answer. Students should add at least a couple of their own sums to the chart.

<table>
<thead>
<tr>
<th></th>
<th>763</th>
<th>796</th>
<th>648</th>
<th>602</th>
<th>329</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+751</td>
<td>+389</td>
<td>+639</td>
<td>+312</td>
<td>+736</td>
</tr>
<tr>
<td>actual answer</td>
<td>1514</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>round both to the nearest hundred</td>
<td>800 + 800 = 1600</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>round one up and round one down</td>
<td>700 + 800 = 1500</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tell students about front-end estimation. In front-end estimation, we ignore all but the first digit of the number, regardless of which ten, hundred, or thousand the number is closest to. For example, by front-end estimation, $37$ rounds to $30$. **ASK:** Do you think front-end estimation will give a better estimate or a worse estimate than rounding when finding sums. Why do they think that?

Do an example together to check students’ predictions. **ASK:** What is the estimate for $37 + 41$ by rounding to the nearest ten? $(40 + 40 = 80)$ By front-end estimation? $(30 + 40 = 70)$ What is the actual answer? Which estimate is closest to the actual answer? Repeat with several sums where front-end estimation gives a worse estimate than rounding:

$54 + 29$  $47 + 28$  $31 + 49$  $57 + 34$

Challenge students to find a sum of 2-digit numbers where front end estimation gives the same estimate as rounding to the nearest ten. Allow several students to write their sums on the board, so that students may see the variety. **ASK:** Why does front-end estimation give the same answer as rounding in these cases?

Then move on to 3-digit numbers. Challenge students to find examples of 3-digit sums where rounding to the nearest hundred gives a better answer than front-end estimation. Have several students write their sums on the board so that students can see the variety. **ASK:** Why does rounding give a better estimate than front-end estimation in these cases?

Challenge students to find examples where both methods give the same estimate. Again, have several volunteers share their sums and discuss why they give the same estimate in these cases.
Then apply front-end estimation to subtraction. ASK: Which method of estimation is closer to the actual answer: front-end or rounding?

a) 76 – 57   b) 79 – 13   c) 58 – 41   d) 56 – 44   e) 57 – 36   f) 41 – 23

Challenge students to find their own examples of problems where front-end estimation is better than rounding, the same as rounding, or worse than rounding.

NS5-51
Other Methods of Estimation (Advanced)

Review multiplying and dividing by 10, 100, 1 000, or 10 000.

ASK: What is 32 × 100? What is 3200 ÷ 100? When you multiply a number by 100, how do you change the number? (Add zeroes to the right-hand side.) Why is it particularly easy to divide 3200 by 100? When you multiply a number by 100 and then divide the result by 100, do you always get back your original number? How do you “undo” adding 2 zeroes to a number? (Remove 2 zeroes.)

Have students do the following division problems:

a) 6 300 ÷ 100   b) 400 ÷ 100  c) 97 900 ÷ 100

Review multiplying and dividing by 10, 100, 1 000, or 10 000.

d) 183 000 ÷ 100  e) 70 800 ÷ 100  f) 540 000 ÷ 100

g) 1 000 000 ÷ 100  h) 100 200 300 ÷ 100  i) 54 000 ÷ 1 000

j) 780 000 ÷ 1 000  k) 7 080 000 000 ÷ 10 000  l) 50 400 ÷ 10

Tell students that you want to estimate 7 683 ÷ 97. ASK: What is a number close to 97 that is easy to divide by? So far, we have only divided numbers that end with at least 2 zeroes by 100, but 7 683 doesn’t have 2 zeroes at the end. Can we find a number close to 7 683 that does? What digit should we round to? (the hundreds digit) Have a volunteer round 7 683 to the nearest hundred and then write on the board: 7 700 ÷ 100. SAY: 7 683 is close to 7 700 and 97 is close to 100. Do you think that 7 683 ÷ 97 will be close to 7 700 ÷ 100? So what is our estimate for 7 683 ÷ 97? (77) Have several volunteers punch this into their calculators. How close was our estimate to the actual answer?

Have students estimate several quotients using this strategy:

a) 8 742 ÷ 103   b) 90 803 ÷ 99   c) 108 999 ÷ 101

d) 64 932 877 ÷ 95  e) 6 583 ÷ 997  f) 8 004 ÷ 1 003

g) 9 876 ÷ 9  h) 8 907 641 ÷ 11

Students might find it easier to round both numbers to the leading digit. (EXAMPLE: 8 907 641 ÷ 11 is approximately 9 000 000 ÷ 10 = 900 000)

Have a volunteer write the 6 times table on the board, then write the following problem on the board: 47 ÷ 6. ASK: Is 47 close to a number from the 6 times
table? What is $48 \div 6$? Is $47 \div 6$ close to $48 \div 6$? What is a good estimate for $47 \div 6$?

Have students estimate the following quotients using this strategy:

- a) $55 \div 6$
- b) $37 \div 6$
- c) $43 \div 6$
- d) $62 \div 6$
- e) $20 \div 6$
- f) $17 \div 6$

**ASK:** If $55 \div 6$ is close to 9, what is $550 \div 6$ close to? Will this be close to $554 \div 6$? Have students estimate the following quotients by finding a multiple of 6 that is close to the first 2 digits of the dividend:

- a) $356 \div 6$
- b) $417 \div 6$
- c) $302 \div 6$
- d) $291 \div 6$
- e) $354 \div 6$
- f) $492 \div 6$

Challenge students to decide in each case whether their estimate is higher or lower than the actual answer. *(EXAMPLE: 360 ÷ 6 = 60 is higher than 356 ÷ 6, so this estimate is too high.)* Students should calculate the actual answers and compare them with their estimates.

Challenge students to do the same for the following problems—make an estimate, predict whether the estimate will be higher or lower than the actual answer, then check the prediction using long division.

- a) $647 \div 5$
- b) $238 \div 3$
- c) $291 \div 4$
- d) $906 \div 7$
- e) $733 \div 8$
- f) $9000 \div 7$

**NOTE:** If some students are not comfortable with their 7 and 8 times tables, divide only by 2, 3, 4, and 5.

Draw a $4 \times 5$ array on the board and ask students what two multiplication statements this shows. *(4 × 5 = 20 and 5 × 4 = 20)* What two division statements can you get from the same picture? *(20 ÷ 5 = 4 and 20 ÷ 4 = 5)*

Emphasize that if you have 20 objects divided into sets of size 5, you will have 4 sets. **ASK:** If I put twice as many objects in each set, but still keep 4 sets, what happens to the total number of objects? (It doubles as well.) Show this on the board using arrays and write the corresponding multiplication and division statements:

- 4 rows of 5 gives 20 dots
- 4 rows of 10 gives 40 dots
- $4 \times 5 = 20$
- $4 \times 10 = 40$
- $20 \div 5 = 4$
- $40 \div 10 = 4$

**ASK:** If you double both numbers in a division statement such as $20 \div 5$, do you change the answer? Why not? Emphasize that if you have the same number of rows, but twice as many dots in each row, you are just making another copy of the array beside the original one, so you are doubling the total number of dots.

Have students find the quotients successively and explain their answers as they go:

- a) $12 \div 3$
- b) $42 \div 7$
- c) $24 \div 6$
- d) $84 \div 14$
- e) $48 \div 12$
- f) $168 \div 28$
- g) $96 \div 24$
- h) $336 \div 56$

Tell your students that you want to solve $142 \div 5$. **ASK:** What happens when I double both numbers? How does this help me to solve the original problem? Why is 5 a particularly convenient number for using this strategy?
Have students use this doubling strategy for many cases of dividing by 5. (EXAMPLE: \(476 \div 5\))

**BONUS:** \(13421 344 132 \div 5\)

Finally, teach your students an estimating strategy for adding lists of numbers. First, ensure that students can add long lists of numbers, like these:

a) \(2 + 3 + 6 + 7 + 3 + 5 + 8 + 4\)

b) \(20 + 30 + 60 + 80 + 20 + 40 + 20 + 20 + 10\)

c) \(500 + 300 + 600 + 900 + 200 + 800 + 100\)

Teach students to use strategies such as pairing or grouping numbers (to get 10, 100, or 1 000) to get an estimate. For example, in a)

\[
2 + 3 + 6 + 7 + 3 + 5 + 8 + 4 \rightarrow \begin{array}{c}
2 + 3 + 5 \\
2 + 3 + 6 + 4 \\
2 + 3 + 5 + 6 + 4 \end{array}
\]

\[
= 10 + 10 + 10
\]

Write on the board: \(235 + 349 + 262 + 448 + 421 + 237 + 619 + 522\). Have students first estimate this sum by rounding to the nearest hundreds and adding. Then have your students estimate the sum by rounding half the numbers up and half the numbers down.

Finally, circle the tens and ones digit of each number in the sum:

\[
235 + 349 + 262 + 448 + 421 + 237 + 619 + 522
\]

Show students how 35 can be paired with 62 as approximately adding to 100. Write on the board:

\[
200 + 349 + 200 + 448 + 421 + 237 + 619 + 522 + 35 + 62
\]

\[
\approx 100
\]

**ASK:** What can be paired with the 49 (in 349) to make something about 100? (48 from 448) Look at the remaining circled numbers. Can they be grouped to make a number close to 100? (Yes! These numbers are close to 20, 40, 20, and 20 which together add to 100.)

Have volunteers calculate the actual answer using a calculator and then, as a class, compare the three estimation strategies discussed so far—rounding each number to the nearest hundreds; rounding half of the numbers up and the other half down; grouping numbers that add to about 100. Which estimate is closest to the actual answer?

Have students estimate the sums of lists of numbers, beginning with short lists (2, 3, or 4 numbers) then moving to longer lists. (EXAMPLES: \(325 + 478, 230 + 129 + 443, 518 + 683 + 111 + 238 + 254\))

Ask students to estimate:

a) the number of chocolate chips in a bag of cookies
b) the number of students in the school
c) the number of ridges in a bag of ridged potato chips
d) the number of questions in a JUMP Math workbook

For example, if students estimate that there are 5 questions on each page of the workbook, then the number of questions in the whole book should be estimated as 5 times the number of pages.

**ACTIVITY 1**
Bring in a full jar of grapes. Have students estimate how many grapes are in a jar. Take various guesses and write them on the board. Then have a volunteer remove 10 grapes and eat them if they wish. Ask students if they wish to revise their guess. Continue having volunteers remove 10 grapes at a time and asking students to revise their guess.

**ACTIVITY 2**
Bring a transparency with text on it, and ask students to estimate the number of words on the page. Then have a volunteer circle the first 25 words and have students revise their prediction if they wish. Continue in this way until they’ve circled the words on about half the page. How can students now estimate the total number of words on the page?

**ACTIVITY 3**
Ask students how high they think a stack of 10 000 pennies will be? Then have them measure the height of 10 pennies. Now how high do they think 10 000 pennies will be? How high do they think 30 pennies will be? Have students measure the height of 30 pennies compare it to their estimate. As an extension, teach students that their measurement will be more exact when they measure 30 pennies than when they measure 10 pennies and multiply by 3 (because any error they had when measuring the 10 pennies will be multiplied by 3 when they extend the result to 30 pennies).

**Extension**
Instead of doubling both terms before you divide them, you can multiply both terms by 3. If you think of the array created by the problem, you can see that tripling the numbers triples the number in each row and the total number of dots, but the number of rows (the quotient) remains the same. Thus, 700 ÷ 8 is approximately 2 100 ÷ 24, which in turn is approximately 2 100 ÷ 25. This last problem can be further simplified by first multiplying both terms by 4. (8 400 ÷ 100 = 84) Students should see that they can multiply both numbers by any number, as long as it is the same number for both numbers.
GOALS
Students will count any combination of coins up to a dollar.

PRIOR KNOWLEDGE REQUIRED
Skip counting
Count to 100
T-table

VOCABULARY
penny
nickel
do
dime
quarter

Begin by reviewing the names and values of pennies, nickels, dimes and quarters. Try asking the class, “What coin has a maple leaf on it? What is its value?” etc.

Hold up each coin and list its name and value on the board. It’s also a good idea to have pictures of the coins on the classroom walls at all times while studying money.

Explain that different coins can add up to the same amount. Ask students how you could make 5¢ using different coins. (5 pennies, or 1 nickel.) Then ask how you could make 10¢. (10 pennies, 2 nickels, 1 dime and a bonus answer: 1 nickel and 5 pennies.)

Complete the first page of the worksheet together. Review using the finger counting technique to keep track of your counting. It might help to point to a large number line when skip counting with numbers over 100. It’s a good idea to keep this large number line on the wall while studying money. Let your students continue with the worksheets independently. Stop them after QUESTION 5.

Give each student a handful of play money coins. Ask the students to sort them by denomination—putting all the pennies together, all the nickels together, etc. Once they are sorted, demonstrate how to find the value of all the coins by skip counting in different units starting by tens. Do some examples on the board (EXAMPLE: if you had 3 dimes, 2 nickels and 3 pennies you would count 10, 20, 30, 35, 40, 41, 42, 43).

Have students practice counting up the play money they have. They can also trade some coins with other classmates to make different amounts each time.

NOTE: Allow your students to practice the skill in QUESTIONS 6 and 7 of the worksheet with play money. Do not move on until they can add up any combination of coins up to a dollar. Another way to help struggling students is to cross out the counted money on the worksheet.

Assessment
Count the given coins and write the total amount:

a) $25c, $1c, $10c, $5c, $5c, $25c$
Total amount =

b) $10c, $1c, $1c, $25c, $25c, $5c$
Total amount =
Place ten to fifteen play money coins on a table. Ask students to estimate the amount of money, and then to count the value of the coins. (Students could play this game in pairs, taking turns placing the money and counting the money.)

Ask students to estimate the total value of a particular denomination (EXAMPLE: quarters, pennies, etc.) that would be needed to cover their hand or book. Students could use play money to test their predictions. This activity is a good connection with the measurement section.

**Extension**

**How Many Coins Could This Be?** Have the students form pairs and give the pairs a list of values (EXAMPLE: 25¢, 15¢, 10¢). Have students try to find as many ways as possible to make up that value using different coins. This activity might be used to develop systematic search techniques.

Use a T-table to find all the possibilities to make the value.

<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickles</th>
<th>Pennies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>14</td>
</tr>
</tbody>
</table>

Start with the largest coin we can use.

For example, 14¢: We start with the dimes. (Why not quarters?) What is the largest number of dimes we can use? (1) If we have 1 dime, how many cents are left? (4) How can we make 4¢? Fill in the first line of the table. Now suppose we have no dimes. A nickel is the next largest coin we can use. What is the largest number of nickels we can use? (2. Why not 3?) What is the value of two nickels? How much more do we need? (4¢). How do we do that?

Continue to 1 and 0 nickels and fill in the rest of the table.

Students might start with none of the largest denomination, and then list the possible numbers of these coins in ascending order (EXAMPLE: no dimes, then one dime, and so on).

Students might find it helpful to add a “Total” column to their T-table:

<table>
<thead>
<tr>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 + 1 + 1 + 1 + 1 = 14</td>
</tr>
<tr>
<td>5 + 5 + 1 + 1 + 1 + 1 = 14</td>
</tr>
<tr>
<td>5 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 14</td>
</tr>
<tr>
<td>1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 14</td>
</tr>
</tbody>
</table>

If students are having trouble with this exercise, you might limit the number of types of coins. For instance, you might ask them to find all the ways they can make 25¢ using only nickels and dimes.
NS5-53
Counting by Different Denominations

Begin by reviewing how to skip count by 10s and 5s from numbers not divisible by 10 or 5. (i.e. count by 5s from 3. Count by 10s from 28).

Have students practice writing out skip counting sequences in their notebooks. Assign start points and what to count by. Some sample problems:

a) Count up by 10s from 33
b) Count up by 10s from 47
c) Count up by 5s from 7
d) Count up by 5s from 81

Have students notice the number patterns. When counting by 10s, all the numbers will end in the same digit. When counting by 5s, all the numbers will end in one of two digits.

Give your students some play coins (cents only), and let them play the following game—one player takes 3 or 4 coins, counts the money he has and says the total to the partner. Then he hides one coin and gives the rest to the partner. The partner has to guess which coin is hidden.

Next, modify the above game. Give the students additional play coins. One player will now hide several coins (that must be of the same denomination). He then says the total value of all the coins, as well as the denomination of the hidden coins. The other player has to guess the number of hidden coins (of each denomination).

Introduce the loonie and toonie coins. Sketch them on the board and write out their values and names. Give students play money loonies and toonies to add to their play money from the previous activity.

Assessment

Draw the additional money you need to make the total. You may use only 2 coins (or less) for each question. Can you give more than one solution for a)?

a) 25¢, 1¢, 10¢, 5¢, 5¢, 25¢
   Total amount = 81¢

b) 10¢, 1¢, 1¢, 25¢, 25¢, 5¢
   Total amount = 97¢

c) $2, $2, $2, $2, $1
   Total amount = $12

d) $2, $1, 25¢, 10¢
   Total amount = $5.40
Extensions

1. Tuan says he can make $8.76 using only nickels dimes and quarters. Is he correct? Explain.
2. Lynne says she can make $3.80 using only quarters and dimes. Is she correct? Explain.
3. Show the least number of extra coins you would need to buy…
   a) a book for $24.79 if you had $22.65.
   b) a plant for $17.99 if you had $14.25.
4. Each week, Sheila is given an allowance. She is allowed to select four coins to a maximum value of 85¢. What coins should she choose to get the most money?
5. a) Ella paid for a bottle of water with $2 and received 35¢ in change. How much did the water cost?
   b) Jordan paid for some spring rolls with $5 and received 47¢ in change. How much did his meal cost?
   c) Paige paid for a shirt with $10 and received 68¢ in change. How much did the shirt cost?
   d) Geoff paid for a hockey stick with $20 and received 54¢ in change. How much did the hockey stick cost?
NS5-54
Least Number of Coins and Bills

GOALS
Students will make specific amounts of money using the least number of coins.

PRIOR KNOWLEDGE REQUIRED
Creating amounts of money using a variety of coins

VOCABULARY
penny nickel dime quarter loonie toonie cent dollar

Begin with a demonstration. Bring in $2 worth of each type of coin (EXAMPLE: 4 rolls of pennies, 2 loonies, etc.). Let students pick up each $2 amount to compare how heavy they are. Explain that it is important to figure out how to make amounts from the least number of coins, because then you can avoid carrying large amounts of extra coins.

Tell the class that the easiest way to make an amount with the least number of coins is to start with the largest possible denomination and then move to smaller ones.

Ask students how they could make exactly 10¢ with the smallest number of coins. They will probably give the right answer immediately. Next, ask them to make up a more difficult amount, such as 35¢. Will a toonie work? What about a loonie? A quarter could be used, so put one aside (you can use the large pictures of the denominations to demonstrate). We have 25¢, so what else is needed? Check a dime. Now we have 25¢ + 10¢ = 35¢. That’s just right. Then look for another combination. Ask the students how to make 35¢ using dimes, nickels and pennies, but no quarters. Compare the arrangements and the number of coins used in each.

Let your students practice making amounts with the least number of coins.

Try totals such as 15¢, 17¢, 30¢, 34¢, 50¢, 75¢, 80¢, 95¢.

Ask if they noticed what coin is best to start with (the largest possible without going over the total).

When giving larger sums, you might use more than one volunteer—ask the first to lay out only the necessary quarters, the next one—only the dimes, etc.

Show a couple of examples with incorrect numbers of coins, like 30¢ with 3 dimes, 45¢ with 4 dimes and a nickel, 35¢ with a quarter and two nickels. Ask if you laid the least number of coins in the right way. Let them correct you.

Let your students do the worksheets. Trading coins might be done, in pairs, with play money.

Here are the weights of some Canadian coins rounded to the nearest gram:

- Nickel: 4 grams
- Dime: 2 grams
- Quarter: 4 grams
- Loonie: 7 grams

ASK: Which is heavier—a quarter or 2 dimes and a nickel? How can you make 50¢ using the lightest combination of coins possible? The heaviest combination? Ask the same question for $2.
Assessment

Make the following amounts with the least number of coins:

- a) 9¢
- b) 55¢
- c) 63¢
- d) 17¢
- e) 3¢
- f) 92¢

Extension

1. Repeat the weighing activity at the end of the lesson with the actual weights of the coins:
   
   Nickel: 3.95 grams, Dime: 1.75 grams, Quarter: 4.4 grams, Loonie: 7 g

2. For each set of coins, find an equivalent amount of money that i) weighs more, and
   ii) weighs less.
   
   - a) 2 nickels
   - b) 1 quarter and 5 pennies
   - c) 3 dimes
   - d) 2 dimes and 5 pennies

3. Jomar has a 3 loonies, 5 quarters, 8 dimes, 12 nickels, and 17 pennies. When he buys something, he likes to use the heaviest combination of coins possible so that he is left with the lightest possible coins to carry around. Which coins should he use if he wants to buy something worth:
   
   - a) 15¢
   - b) 30¢
   - c) $1
   - d) 62¢
   - e) 94¢
   - f) $1.75
   - g) $2
   - h) $2.63

Literature Connection

Smart by Shel Silverstein, contained in Where the Sidewalk Ends.

Read this poem to your class. With each stanza, stop and ask the students how much money the boy in the story has. Give students play money to use as manipulatives so they can each have the same amount of coins as described in the poem.

This activity effectively demonstrates that having more coins does not necessarily mean having more money.
Dollar and Cent Notation

**GOALS**
Students will express monetary values in dollar and cent notation.

**PRIOR KNOWLEDGE REQUIRED**
Place value to tenths and hundredths
Canadian coins

**VOCABULARY**
penny    loonie
nickel    toonie
dime      cent
quarter    dollar
tenths     notation
hundredths

Draw five quarters on the board. Ask students how much money you have in cents (or pennies). Write “125¢” on the board. Ask students if they know another way to write one hundred twenty-five cents.

Explain that there are two standard ways of representing money. The first is cent notation: you simply write the number of cents or pennies that you have, followed by the ¢ sign. In dollar notation, “one hundred twenty-five cents” is written $1.25. The number to the left of the decimal point tells how many dollars you have, the number to the right of the decimal represents the number of dimes, and the next digit to the right represents the number of pennies.

Write the total amount in dollar and cent notation:

a) 25¢ 10¢ 5¢ 5¢ 1¢ 1¢
   Total amount = _____ ¢   = $ ______

b) 25¢ 10¢ 5¢ 1¢
   Total amount = _____ ¢   = $ ______

c) $2 $2 $1 25¢ 10¢ 10¢ 10¢ 5¢ 1¢ 1¢
   Total amount = _____ ¢   = $ ______

Write on the board a random arrangement of coins; make sure the coins are not in order from highest denomination to lowest denomination. (EXAMPLE: 25¢, 1¢, 10¢, $2, 10¢, 5¢, 25¢) Show students how to tally the number of each particular denomination and then add the totals together.

Changing from dollar to cent notation is easy—all you need to do is to remove the decimal point and dollar sign, and then add the ¢ sign to the right of the number.

Changing from cent notation to dollar notation is a little trickier. Make sure students know that, when an amount in cent notation has no tens digit (or a zero in the tens place), the corresponding amount in dollar notation must have a zero. EXAMPLE: 6¢ is written $.06 or $0.06, not $0.60.

Convert between dollar and cent notations:

$.24 = ____¢   _____=82¢   _____ = 6¢   $12.03 = _____

$120.30 = ______
Make a chart like the one shown below on the board and ask volunteers to help you fill it in.

<table>
<thead>
<tr>
<th>Amount</th>
<th>Amount in Cents</th>
<th>Dollars</th>
<th>Dimes</th>
<th>Pennies</th>
<th>Amount in Dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 quarters, 3 dimes, 2 nickels</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 toonies, 2 loonies</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 quarters, 2 pennies</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Students could also practice skip counting by quarters, toonies and other coins (both in dollar and in cent notation).

- $2.00, $4.00, ____, ____
- $0.25, $0.50, ____, ____, ____, ____
- 200¢, 400¢, ____, ____
- 25¢, 50¢, ____, ____, ____, ____

**Bonus**

What coin is being used for skip counting?

- $1.00, ____, ____, ____, $1.20
- $2.00, ____, ____, ____, $3.00

You may wish to give your students some play money and let them practise in pairs—one student picks several coins and writes the amount in dollar and cent notations, and the other checks the answer.

**Bonus**

1. Change these numbers to dollar notation: 273258¢, 1234567890¢.

2. Change $245.56 to cent notation, and then either write or say the number of cents properly (twenty-four thousand five hundred fifty-six cents). Try these numbers: $76.34, $123.52, $3789.49.

Write two money amounts on the board: 234¢ and $2.15. Ask which represents more money. Ask a volunteer to change the amount in dollar notation into cent notation to compare. Ask why converting to cents is more convenient (no need to deal with decimal points and fractions). Give more practice questions, such as:

- 1234¢ or $7.56
- $987 or 98756¢
- 9¢ or $0.08
- 40¢ or $0.04

Ask how these questions are similar to:

Which is longer: 1234 cm or 7.56 m? 9875 m or 98756 cm?

Present a problem: Jane emptied her piggy bank. She asked her elder brother John to help her...
count the coins. He suggested she stack the coins of the same value in different stacks and count each stack separately. Jane has:

- 19 pennies
- 8 nickels
- 21 dimes
- 7 quarters
- 6 loonies
- 3 toonies

How much money is in each stack? Ask students to write the amount in dollar notation and in cent notation. Give them more practice questions, such as:

- 9 pennies
- 18 nickels
- 7 dimes
- 6 quarters
- 5 loonies
- 9 toonies

**Assessment**

1. Write each amount in dollar notation, and then circle the greater amount of money:
   - Ten dollars thirty-five cents or 1125¢
   - Three dollars fifty cents or 305¢

2. Write each amount in cent notation and in dollar notation:
   - 7 pennies
   - 21 nickels
   - 5 dimes
   - 2 quarters

**Extensions**

1. Let students know that in decimal notation, the digit after the decimal point always stands for tenths and the next digit stands for hundredths. Ask students why this notation would be used for money. What is a dime a tenth of? And what is a penny a hundredth of?

2. Show how you would make these dollar amounts using exactly four coins:
   - a) $4.30
   - b) $3.50
   - c) $5.10
   - d) $1.40
GOALS
Students will discover how to use the least amount of coins by always starting with the highest denomination possible.

PRIOR KNOWLEDGE REQUIRED
Finding the least number of coins required to make any amount less than a dollar

VOCABULARY
value

ASK: What is the largest coin value I can have if I have:

- a) $1.57  
- b) $3.69  
- c) 57¢  
- d) 69¢  
- e) 13¢  
- f) 24¢  
- g) 61¢  
- h) 7¢  
- i) 39¢

ASK: What is the greatest amount I could pay in $20 bills without exceeding the amount:

- a) $47  
- b) $36  
- c) $22  
- d) $43  
- e) $49  
- f) $35

Challenge students to find all the possible ways to make 13¢ using Canadian coins. (13 pennies, 1 nickel and 8 pennies, 2 nickels and 3 pennies, 1 dime and 3 pennies) ASK: Which way uses the fewest coins?

Repeat for 17¢. (17 pennies; a nickel and 12 pennies; 2 nickels and 7 pennies; 3 nickels and 2 pennies; 1 dime and 7 pennies; 1 dime, 1 nickel, and 2 pennies).

Give students 7 pennies, 7 nickels, 3 dimes, 5 quarters, a loonie, and a toonie. Tell students that they should make a sequence of trades where they can give away any number of coins but can only receive one coin back at a time (e.g., 5 pennies for a nickel, 2 dimes and a nickel for a quarter). The goal is to have the fewest number of coins possible at the end. When students are done, ASK: When you traded, did the coins you got back have higher value or less value than the ones you gave? Emphasize that you tend to give away many lower-value coins to get back 1 higher-value coin. So when you are done, you end up with as many high-value coins as possible.

Tell students that when you gave them their coins, you gave them a total of $4.97. ASK: What is the largest coin we can use to make $4.97? (a toonie) How many toonies can we use? (2) Draw 2 toonies on the board. We now have $4.00 and we need to make $4.97, so we still need 97¢. What is the highest-value coin we can use now? (a quarter) How many quarters do we need? (3) Draw 3 quarters beside the 2 toonies. ASK: How much money have we gained by adding 3 quarters? (75¢) We’ve added 75¢ but we need 97¢—how much more money do we need? (22¢) Continue until you have a total of $4.97.

Teach the following steps, one at a time, to help students find the fewest coins that can be used to give a specific amount of money.

STEP 1: Decide the biggest coin value you need to make the given amount of money.

EXAMPLES: To make 79¢, the greatest coin value is a quarter; for 23¢, the greatest coin value is a dime; for $1.28, the greatest coin value is a loonie.

STEP 2: Decide how many of those coins you can use to make the given money amount.
EXAMPLES: To make 79¢, you need 3 quarters; to make 23¢, you need 2 dimes; to make $1.28, you need 1 loonie.

STEP 3: Find out how much money you still need to make the given amount.

EXAMPLES: 79¢ - 75¢ = 4¢, 23¢ - 20¢ = 3¢, $1.28 - $1.00 = $0.28 = 28¢.

STEP 4: Repeat Steps 1-3 until you have the entire given amount of money.

EXAMPLES: Start with 84¢, 61¢, 42¢, $3.94.

Pose a sample problem. You would like to buy a pen which costs 18¢, but you only have a quarter to pay with. How much change should you get back?

Rather than starting with a complicated subtraction question that includes decimals and carrying, show students how to ‘count up’ to make change. Demonstrate that you can first count up by 1s to the amount. In this case you would start at 18 and count up: 19, 20, 21, 22, 23, 24, 25. You would need 7¢ as change. Review the finger counting technique, keeping track of the number counted on their fingers.

Model more examples, getting increasingly more volunteer help from the class as you go along.

Have students practice this skill with play money, if helpful. They should complete several problems of this kind before moving on.

Some sample problems:

a) Price of an orange = 39¢ Amount paid = 50¢
b) Price of a hairband = 69¢ Amount paid = 75¢
c) Price of a sticker = 26¢ Amount paid = 30¢

Pose another problem: You would like to buy a flower that costs $80¢, but you only have a loonie to pay with. How much change should you get back? Before the students start to count by 1s to 100, tell them that you will show them a faster way to do this. Explain that instead of counting up by 1s, you can count by 10s to 100. Starting at 80 you would count up: 90, 100 to $1.

Give students several problems to practice. Some sample problems:

a) Price of a newspaper = 50¢ Amount Paid = $1.00
b) Price of a milk carton = 80¢ Amount Paid = $1.00
c) Price of a paper clip = 10¢ Amount Paid = $1.00

Pose another problem: You would like to buy a postcard that costs 55 cents, but you only have a loonie to pay with. How much change should you get back?
Explain that you can count up by 1s to the nearest 10, then you can count by 10s to $1 (or the amount of money paid). In this case you would start at 55 and count up to 60: 56, 57, 58, 59, 60. Then count up by 10s. Starting at 60 you would count up: 70, 80, 90, 100. At the end, you will have counted five ones and four 10s, so the change is 45¢.

Demonstrate the above problem using the finger counting technique. Explain to the students that this can be clumsy, because it is easy to forget how many 1s or how many 10s you counted by.

How could you use this method to solve the problem without the risk of forgetting?

Model another example. You would like to buy a candy bar that costs 67¢, but you only have a loonie to pay with.

**STEP 1:** Count up by 1s to the nearest multiple of 10 (Count 68, 69, 70 on fingers).

**STEP 2:** Write in the number that you have moved forward and also the number that you are now ‘at’ (Write “counted 3, got to 70¢”).

**STEP 3:** Count up by 10s to $1.00. Write in the number that you have counted up. (Count 80, 90, 100, write 30¢)

**STEP 4:** Add the differences to find out how much change is owing (3¢ + 30¢ = 33¢).

Let a couple of volunteers model this method with sample problems. Give the students more problems to complete in their notebooks, such as:

- a) Price of a bowl of soup = 83¢ Amount Paid = $1.00
- b) Price of a stamp = 52¢ Amount Paid = $1.00
- c) Price of a pop drink = 87¢ Amount Paid = $1.00
- d) Price of an eraser = 45¢ Amount Paid = $1.00
- e) Price of a candy = 9¢ Amount Paid = $1.00

Then have students calculate change for dollar values. Start with dollar values that differ by less than $10.

(EXAMPLE: $33 → $40, $57 → $60)

Then move on to dollar values that differ by more than $10.

**EXAMPLES:** $39 → $50, $57 → $100

Challenge students to do this type of question in their heads. ASK: Can you figure out the change you would get if you paid with $100 for an item that costs:

a) $74 b) $86 c) $63 d) 24 e) $47 f) $9

Finally, include money amounts that involve both dollars and cents.

**EXAMPLE:** $37.23 → $37.30 → $38 → $40 → $100 requires 7¢ + 70¢ + $2 + $60 = $62.77 in change.
Play Shop Keeper
Set up the classroom like a store, with items set out and their prices clearly marked. The prices should be under $1.00.

Tell the students that they will all take turns being the cashiers and the shoppers. Explain that making change (and checking you’ve got the right change!) is one of the most common uses of math that they will encounter in life!

Allow the students to explore the store and select items to ‘buy’. Give the shoppers play money to ‘spend’ and give the cashiers play money to make change with. Ask the shoppers to calculate the change in their heads at the same time as the cashiers when paying for the item. This way, students can double check and help each other out. Reaffirm that everyone needs to work together, and should encourage the success of all of their peers.

Allow the students a good amount of time in the store. Plan at least 30 minutes for this exploration.

Extensions
1. Once students have calculated the change for any question, have them figure out the least number of coins that could be used to make that change.

2. This would be a great lead up to an actual sale event. If your community or school has an initiative that your class wanted to support, you could plan a bake sale, garage sale, etc. that would raise funds for the cause. Students would find the pretend task even more engaging if it was a ‘dress rehearsal’ for a real event.
NS5-58
Adding Money

Remind your students of how to add 2-digit numbers. Demonstrate the steps: line up the numbers correctly, add the digits in each column starting from the right. Start with some examples that would not require regrouping, and then move on to some which do, EXAMPLE: 15 + 23, 62 + 23, 38 + 46.

Use volunteers to pass to 3- and 4-digits addition questions, first without regrouping, then with regrouping. SAMPLE QUESTIONS:

\[
545 + 123, 1234 + 345, \text{then } 1324 + 1259, 2345 + 5567, 5678 + 789, 3456 + 3987.
\]

Model the steps for adding money. The difference is in the lining up—the decimal point is lined up over the decimal point. ASK: Are the ones still lined up over the ones? The tens over the tens? The dimes over the dimes? Tell them that if the decimal point is lined up, all the other digits must be lined up correctly too, since the decimal point is between the ones and the dimes. Students can model regrouping of terms using play money: for instance, in $2.33 + $2.74 they will have to group 10 dimes as a dollar.

Students should complete a number of problems in their workbooks. Some SAMPLE PROBLEMS:

a) $5.08 + $1.51  
b) $3.13 + $2.98  
c) $10.74 + $15.22  
d) $23.95 + $42.28  
e) $12.79 + $2.83

Use volunteers to solve several word problems, such as: Julie spent $14.98 for a T-shirt and $5.78 for a sandwich. How much did she spend in total?

Students should also practice adding coins and writing the amount in dollar notation, such as:

There are 19 pennies, 23 nickels and 7 quarters in Jane’s piggybank. How much money does she have?

Helen has a five-dollar bill, 6 toonies and 9 quarters in her pocket. How much money does she have?

Randy paid 2 twenty-dollar bills, 9 toonies, a loonie, 5 quarters and 7 dimes for a parrot. How much did his parrot cost?

A mango fruit costs 69¢. I have a toonie. How many mangos can I buy? If I add a dime, will it suffice for another one?

Assessment

1. Add:

\[
\begin{align*}
a) \quad & 18.25 + 71.64 \\
b) \quad & 23.89 + 67.23 \\
c) \quad & 45.08 + 8.87 \\
d) \quad & 78.37 + 4.79
\end{align*}
\]
2. Sheila saved 6 toonies, 7 dimes and 8 pennies from babysitting. Her brother Noah saved a five-dollar bill, 2 toonies, a loonie and 6 quarters from mowing a lawn.

   a) Who has saved more money?
   
   b) They want to share money to buy a present for their mother. How much money do they have together?
   
   c) They’ve chosen a teapot for $23.99. Do they have enough money?

Extension

Fill in the missing information in the story problem and then solve the problem.

   a) Betty bought _____ pairs of shoes for _____ each. How much did she spend?
   
   b) Una bought ____ apples for _____ each. How much did she spend?
   
   c) Bertrand bought _____ brooms for _____ each. How much did he spend?
   
   d) Blake bought _____ comic books for ____ _ each. How much did he spend?

ACTIVITY

Bring in fliers from local businesses. Ask students to select gifts to buy for a friend or relative as a birthday gift. They must choose at least two items. They have a $20.00 budget. What is the total cost of their gifts?
NS5-59
Subtracting Money

Begin by explaining that you will be learning how to subtract money today. Start with a review of 2-digit and 3-digit subtraction questions. Model a few on the board, or use volunteers. Start with some examples that would not require regrouping: $45 - 23, 78 - 67, 234 - 123, 678 - 354.$

Show some examples on the board of numbers lined up correctly or incorrectly and have students decide which ones are done correctly.

Demonstrate the steps: line up the numbers correctly, subtract the digits in each column starting from the right. Move onto questions that require regrouping, **EXAMPLE:** $86 - 27, 567 - 38, 782 - 127, 673 - 185, 467 - 369.$

**Sample Problems**
Subtract:

\[
\begin{align*}
\text{a)} & \quad 98.89 - 71.64 \\
\text{b)} & \quad 89.00 - 67.23 \\
\text{c)} & \quad 45.00 - 38.87 \\
\text{d)} & \quad 78.37 - 9.79
\end{align*}
\]

Next, teach your students the following fast way of subtracting from powers of 10 (such as 10, 100, 1 000 and so on) to help them avoid regrouping:

For example, you can subtract any money amount from a dollar by taking the amount away from 99¢ and then adding one cent to the result.

\[
\begin{align*}
1.00 & - .57 \\
.99 & + .01 \\
= .42 & + .01 = .43 = 43¢
\end{align*}
\]

Another example, you can subtract any money amount from $10.00 by taking the amount away from $9.99 and adding one cent to the result.

\[
\begin{align*}
10.00 & - 8.63 \\
9.99 & + .01 \\
= 1.36 & + .01 = 1.37
\end{align*}
\]

**NOTE:** If students know how to subtract any one-digit number from 9, then they can easily perform the subtractions shown above mentally. To reinforce this skill have students play the Modified Go Fish game (in the **MENTAL MATH** section) using 9 as the target number.

**Assessment**
1. Subtract:

\[
\begin{align*}
\text{a)} & \quad 96.85 - 71.64 \\
\text{b)} & \quad 75.00 - 61.23 \\
\text{c)} & \quad 55.00 - 39.88 \\
\text{d)} & \quad 78.37 - 9.88
\end{align*}
\]
2. Alyson went to a grocery store with $15.00. She would buy buns for $1.69, ice cream for $6.99 and tomatoes for $2.50. Does she have enough money? If yes, how much change will she get?

**Literature Connection**

*Alexander, Who Used to Be Rich Last Sunday.* By Judith Viorst.

Last Sunday, Alexander’s grandparents gave him a dollar—and he was rich. There were so many things that he could do with all of that money!

He could buy as much gum as he wanted, or even a walkie-talkie, if he saved enough. But somehow the money began to disappear…

A great activity would be stopping to calculate how much money Alexander is left with every time he ends up spending money.

Students could even write their own stories about Alexander creating a subtraction problem of their own.

**Extension**

Fill in the missing information in the story problem and then solve the problem.

- a) Kyle spent $4.90 for a notebook and pencils. He bought 5 pencils. How much did the notebook cost?
- b) Sally spent $6.50 for a bottle of juice and 3 apples. How much did the juice cost?
- c) Clarke spent $9.70 for 2 novels and a dictionary. How much did the dictionary cost?
- d) Mary spent $16.40 for 2 movie tickets and a small bag of popcorn. How much did her popcorn cost?
NS5-60

Estimating with Money

Begin with a demonstration. Bring in a handful of change. Put it down on a table at the front of the class. Tell your students that you would like to buy a magazine that costs $2.50. Do they think that you have enough money? How could they find out?

Review rounding. Remind the class that rounding involves changing numbers to an amount that is close to the original, but which is easier to use in calculations. As an example, ask what is easier to add: 8 + 9 or 10 + 10.

Explain to the class that they will be doing two kinds of rounding—rounding to the nearest 10¢ and rounding to the nearest dollar. To round to the nearest 10¢ look at the digit in the ones column. If it is less than 5 (ask which numbers does this mean), you round down. If it is 5 or more (which numbers?), up. For example 33¢ would round down to 30¢, but 38¢ would round up to 40¢. Use volunteers to round: 39¢, 56¢, 52¢, 75¢, 60¢, 44¢.

Model rounding to the nearest dollar. In this case, the number to look at is in the tenths place (the dimes place). If an amount has 50¢ or more, round up. If it has less than 50¢, round down. So, $1.54 would round up to $2.00. $1.45 would round down to $1.00. Use volunteers to round: $1.39, $2.56¢, $3.50, $4.75, $0.60, $0.49.

Have students practice estimating by using play money coins. Place ten to fifteen play money coins on a table. Ask students to estimate the amount of money before they count the coins. (Students could play this game in pairs, taking turns placing the money and counting the money.)

Give several problems to show how rounding can be used for estimation:

Make an estimate and then find the exact amount:

a) Dana has $5.27. Tor has $2.38. How much more money does Dana have?

b) Mary has $18.74. Sheryl has $5.33. How much money do they have altogether?

c) Jason has saved $12.95. Does he have enough money to buy a book for $5.96 and a binder for $6.99? Why is rounding not helpful here?

Ask the students to explain why rounding to the nearest dollar isn’t helpful for the following question:

“Millicent has $8.15. Richard has $7.97. About how much more money does Millicent have than Richard?” (Both round to 8. Rounding to the nearest 10 cents helps.)

Make sure students understand that they can use rounding to check the reasonableness of answers. Ask students to explain how they know that $7.52 + $8.95 = $20.47 can’t be correct.
Assessment

1. Make an estimate and then find the exact amount: Molina has $7.89. Vinijaa has $5.79. How much more money does Molina have?

2. Benjamin spent $6.94 on pop, $12.85 on vegetables and dip and $2.15 on bagels. About how much did he spend altogether?

Extension

What is the best way to round when you are adding two numbers: to round both to the nearest dollar, or to round one up and one down?

Explore which method gives the best answer for the following amounts:

\[
\begin{align*}
$12.56 + $3.68 & \quad $34.55 + $54.57 & \quad $76.61 + $21.05 \\
\end{align*}
\]

Students should notice that, when two numbers have cent values that are both close to 50¢ and that are both greater than 50¢ or both less than 50¢, rounding one number up and one down gives a better result than the standard rounding technique. For instance, rounding $7.57 and $7.54 to the nearest dollar gives an estimated sum of $16.00 ($7.57 + $7.54 = $8.00 + $8.00), whereas rounding one number up and the other down gives an estimated sum of $15.00, which is closer to the actual total.

ACTIVITY

Ask students to estimate the total value of a particular denomination (EXAMPLE: dimes, loonies, etc.) that would be needed to cover their desk. Students could use play money to test their predictions.
Define a Number _______________________________________________________ 2
Define a Number (Advanced) _____________________________________________ 3
Number Lines to Twenty _________________________________________________ 4
Place Value Cards _______________________________________________________ 5
## Define a Number

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td><strong>B</strong></td>
<td><strong>C</strong></td>
</tr>
<tr>
<td>The number is even.</td>
<td>The number is odd.</td>
<td>You can count to the number by 2s.</td>
</tr>
<tr>
<td><strong>D</strong></td>
<td><strong>E</strong></td>
<td><strong>F</strong></td>
</tr>
<tr>
<td>You can count to the number by 3s.</td>
<td>You can count to the number by 5s.</td>
<td>You can count to the number by 10s.</td>
</tr>
<tr>
<td><strong>G</strong></td>
<td><strong>H</strong></td>
<td><strong>I</strong></td>
</tr>
<tr>
<td>The number is greater than 15.</td>
<td>The number is less than 25.</td>
<td>The number has 1 digit.</td>
</tr>
<tr>
<td><strong>J</strong></td>
<td><strong>K</strong></td>
<td><strong>L</strong></td>
</tr>
<tr>
<td>The number has 2 digits.</td>
<td>The number has two digits that are the same.</td>
<td>The number has a zero in it.</td>
</tr>
<tr>
<td><strong>M</strong></td>
<td><strong>N</strong></td>
<td><strong>O</strong></td>
</tr>
<tr>
<td>The number is 7.</td>
<td>The number is less than 7.</td>
<td>You can see the number on the face of a clock.</td>
</tr>
<tr>
<td><strong>P</strong></td>
<td><strong>Q</strong></td>
<td><strong>R</strong></td>
</tr>
<tr>
<td>The ones digit is smaller than 6.</td>
<td>The sum of its digits is less than 9.</td>
<td>The number is divisible by 2.</td>
</tr>
</tbody>
</table>

1. Write a number that statement **A** applies to ________
2. Statements **B, D, H,** and **J** all apply to the number 15. Which other statements apply to the number 15 as well? ______________________________________
3. Choose a number between 1 and 25 (besides 15). Find all the statements that apply to that number.
   Your number ___________ Statements that apply to that number ______________________
4. a) Which statements apply to both 3 and 18? _________________
   b) Which statements apply to both 7 and 11? _________________
5. a) Can you find a number that statements **A, E, J, L** and **R** apply to? (NOTE: There may be more than one answer.)

   ______________________________________

   b) Can you find a number that statements **B, D, H** and **Q** apply to?

   ______________________________________

c) Can you find a number that statements **C, K, H** and **P** apply to?

   ______________________________________
Define a Number (Advanced)

**TEACHER:** You should only assign a few questions on this page at a time.

1. a) Is it possible for statements A and B to both apply to a number? Explain.  
   
   _______________________________________________________________
   _______________________________________________________________

   b) Is it possible for statements D and E to both apply to a number? ______________________

   c) Is it possible for statements C and R to both apply to a number? ______________________

2. a) Can you think of a number where statements E and F apply, but not H? _________________

   b) Can you think of a number where statements H and K apply, but not G? _________________

3. a) If statement N applies to a number does statement O always have to apply? _______________

   b) If statement O applies to a number does statement N always have to apply? _______________

4. Fill out the chart below by writing in a number for each statement. Don’t use the same number more than once.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>D</td>
<td>E</td>
<td>F</td>
</tr>
<tr>
<td>G</td>
<td>H</td>
<td>I</td>
</tr>
<tr>
<td>J</td>
<td>K</td>
<td>L</td>
</tr>
<tr>
<td>M</td>
<td>N</td>
<td>O</td>
</tr>
<tr>
<td>P</td>
<td>Q</td>
<td>R</td>
</tr>
</tbody>
</table>
Number Lines to Twenty
Place Value Cards

Ones

Tens

Hundreds

Thousands
Telling Time—Review

Use a play clock to show various times. First, ask a volunteer to identify the minute and the hour hands. Ask another volunteer to set the clock to the position of a round hour (EXAMPLES: 3 o’clock, 8 o’clock). Show several positions of the minute hand (start with those where the minute hand points directly at a number), and ask your students to tell you how many minutes have passed since the round hour. They can use skip counting by fives and then count on by ones. Then put the hour hand in various positions and ask your students to tell you what the hour is.

Next, ask your students to combine the two skills they just practised and to tell the time. They should write the time in both digital form (8:45) and in words (accept both “forty-five minutes after eight” and “fifteen minutes before nine,” as well as “quarter to nine”). Draw a digital clock on the board, and ask volunteers to turn the hands of the analogue clock to the positions of the digital clock and to read the time aloud:

- 08:30
- 12:45
- 10:40
- 4:15
- 5:21
- 2:37
- 11:04

You can also ask your students to practise telling time in pairs—one student draws an analogue clock, the other draws the digital clock showing the same time, and vice versa. In this case, ask students to use only the minute numbers that end in 5 or 0 (e.g., 15, 40).

Assessment

1. What time is it? Write the time in digital and in verbal form:

   - 12:50
   - 2:38
   - 1:22

2. What time is it? Draw the hands on the analogue clock and write the time in verbal form:

   - 12:50
   - 2:38
   - 1:22

Ask your students to record (both in digital and analogue form) the times when they get up, eat breakfast, go to school, and so on. They can draw a timeline of their morning activities:

- Wake up: 7:00
- Get dressed: 7:10
- Brush teeth: 7:20
- Eat breakfast: 7:30
- Go to school: 8:00
- Arrive at school: 8:15
- Lessons start: 8:30

GOALS

Students will tell time in 5-minute and 1-minute intervals using an analogue clock.

PRIOR KNOWLEDGE REQUIRED

Skip counting by 5s
Telling time in 1-minute intervals
Number lines

VOCABULARY

<table>
<thead>
<tr>
<th>term</th>
<th>definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>minute</td>
<td>analogue clock</td>
</tr>
<tr>
<td>hour</td>
<td>digital clock</td>
</tr>
<tr>
<td>quarter</td>
<td>minute hand</td>
</tr>
<tr>
<td>half</td>
<td>hour hand</td>
</tr>
</tbody>
</table>

ACTIVITY

Ask your students to record (both in digital and analogue form) the times when they get up, eat breakfast, go to school, and so on. They can draw a timeline of their morning activities:
ME5-2
Telling Time—The Second Hand

Ask your students what unit of measurement is used for intervals of time shorter than 1 minute. (seconds) How many seconds are in 1 minute? How long is 1 second? One second is about the time it takes you to blink 3-4 times, or the time it takes you to say “twenty two.” Ask students to give several examples of actions or events that take about a second (or you might use a longer interval, such as 4 seconds). You may use a stopwatch to check the exact duration. A good way to check the duration of a very brief event is to use an average. For example, to measure the duration of 1 clap, clap your hands 20 times while measuring the time with a stopwatch, then divide the measured time by 20. Discuss with students why this method gives better results.

Review reading the clock using the second hand. Explain to your students that the second hand is read exactly the same way as the minute hand. Let them practise telling the time with these and other examples:

Draw two analogue clocks on the board, showing 5:15:50 and 5:16:10. 

**ASK:** How much time has passed between the times shown? Has the hour hand moved? There should be no perceptible change in the position of the hour hand, so much less time than an hour has passed. Has the minute hand moved? Yes, very slightly—from 15 to 16. When did it point directly to 16? (at 5:16:00) How much time elapsed from 5:15:50 to 5:16:00? (10 seconds) How much time elapsed from 5:16:00 to 5:16:10? (another 10 seconds) How much time elapsed in total?

Try a more complicated question: How much time has elapsed between 5:15:50 and 5:17:45? Suggest to your students that they count the seconds by 5s or 10s till they get to the round minute (5:16:50 to 5:16:00), then count the minutes till they get to the number of minutes in the final time (5:16:00 to 5:17:00), and then count the additional seconds to the final time (5:17:00 to 5:17:45). The total time elapsed is 10 sec + 1 min + 45 sec = 1 min 55 sec. Students might find it useful to calculate elapsed time using a number line. Give them several questions to practise with (both by counting on and by drawing times on analogue clocks). **EXAMPLES:** 3:25:48 to 3:27:16, 12:47:10 to 12:48:05.
Assessment

1. What time is it?

2. How much time has passed between 12:23:45 and 12:27:15?

Extensions

1. How many...

   a) seconds are in 3 minutes?  b) minutes are in 2 hours?

2. If you wake up at 7:30 a.m. and it takes you

   • 15 minutes and 30 seconds to eat breakfast,
   • 3 minutes and 10 seconds to brush your teeth,
   • 20 minutes to wash and get dressed,
   • 45 seconds to find your things,
   • 20 minutes and 15 seconds to get to school,

   will you be at school by 8:30 a.m.?
ME5-3
Telling Time in Different Ways

ASK: What time is it now? Express the answer in different ways, such as: ten minutes to nine, ten minutes before nine, fifty minutes after eight, eight fifty. Ask your students which forms of the answer they hear often and which forms are rarely heard. Let them explain why some forms are used more often than others.

Draw several analogue clocks on the board. Draw only the minute hand. Ask volunteers to shade the space that is between the minute hand and the 12, so that the shaded part is less than half the circle. Ask your students to tell you the number of shaded and unshaded minutes. (EXAMPLE: When the minute hand is at 40, 20 minutes are shaded and 40 minutes are unshaded.)

ASK: If you know that 15 minutes are shaded, how do you find the number of unshaded minutes? (60 – 15 = 45). Why do you subtract from 60 minutes? Which is shorter to say—forty-five or fifteen?

Next, draw two analogue clocks on the board. In the first, draw the minute hand between the 12 and 6 positions and shade the right side of the clock. In the second, draw the minute hand between the 6 and 12 positions and shade the left side of the clock. Explain to your students that when the minute hand is between 12 and 6 and the right side is shaded, then we read the earlier round hour. If the minute hand is between the 6 and the 12 and the left side is shaded, then we can read either the previous round hour or the next round hour, and you can tell the time in two ways.

Draw more analogue clocks on the board and ask students to read the time in two ways. For example 12:35 can be read “thirty-five minutes past twelve” and “twenty-five minutes to one.” Which one is more convenient? Which one is shorter? Which would they use?

Read several times aloud, such as 20 minutes past 7 or 25 minutes to 12, and ask your students where the hour hand points for each time. Then give students several more times and ask them to draw an analogue clock showing the time and to write the time in digital form. Invite a pair of volunteers to write more times in digital and analogue forms. Include times that can be read using the words “quarter to” and “quarter after,” such as quarter to six, quarter past eleven, and so on.

Assessment
1. Draw an analogue clock and write the time in digital form:
   a) 17 minutes after 3           b) 11 minutes before 5
   c) quarter to 7                d) 10 minutes past 10
2. What time is it? Write the time in two ways.

a) 12:45

b) 8:45 12:20 3:53 5:17

ME5-4

Elapsed Time

GOALS
Students will find elapsed time.

PRIOR KNOWLEDGE REQUIRED
Telling time in 1-minute intervals
Skip counting by 5s
Number lines

VOCABULARY
minute  analogue clock
hour    digital clock
minute hand hour hand

Draw a clock on the board or use a play clock to show 7:15. SAY: This is the time Rita woke up. She ate breakfast at 7:40. How much time passed between the time Rita woke up and the time she had breakfast? One way to answer the question is to count by 5s. Turn the hand slowly and ask a volunteer to count the minutes on the clock by 5s. Give your students more examples to practise with, such as:

- She brushed her teeth at 7:30 and got to school at 8:15. How much time elapsed?
- She arrived at school at 8:15. The math lesson started at 9:05. How much time elapsed?

Let your students solve more problems by counting by 5s. Invite volunteers to present their solutions. Sample problems:

- Rita put the cake in the oven at 7:45. The cake should bake for 35 minutes. When should she take the cake out?
- The third lesson starts at 1:20 and lasts for 55 minutes. When does it end?
- The TV show ended at 8:25. It lasted 40 minutes. When did it start?

Remind students that another convenient way to find elapsed time is to use a number line, or timeline. Draw a timeline for Rita’s morning on the board:
ASK: What is the scale of Rita’s timeline? How do you know? Ask a volunteer to figure out how much time passes between different events in Rita’s morning routine, for example, from the moment Rita gets dressed to her arrival at school, or from the moment she wakes up to the start of lessons. Let your students do the same using their own timelines, if they prepared them in ME5-1.

Explain that for larger time intervals, you need not draw timelines to scale. For example: Rita returns from school at 3:40. She goes to bed at 9:05. How much time passes in between? Draw a timeline to illustrate the question:

3:40 3:50 4:00 5:00 6:00 7:00 8:00 9:00 9:05

Let your students count by 5s or 10s till they get to the round hour. How much time elapsed? Draw an arrow between 3:40 and 4:00 and mark it “20 min.” Ask a volunteer to count by hours from 4:00 to 9:00. Draw an arrow from 4:00 to 9:00 and mark it “5 hours.” Ask another volunteer to draw an arrow from 9:00 to Rita’s bedtime and to mark the arrow “5 min.” Ask a volunteer to add up the times above the arrows to get the total time Rita has in the evening.

Give students more practice calculating elapsed time. Ask them to find how much time has passed:

- from 10:30 to 12:05
- from 11:25 to 3:40
- from 7:55 to 1:45

Ask your students if they could find the elapsed time using subtraction. You may show an example where the standard method for subtraction works well: Lessons start at 8:30 and Rita wakes up at 7:15. How much time elapsed in between?

Write a subtraction sentence on the board:

\[
\begin{align*}
8:30 & - 7:15 \\
1:15 & 
\end{align*}
\]

Students can practise calculating elapsed time using subtraction with more examples:

\[
\begin{align*}
12:55 - 9:15 & = 3:40 \\
10:45 - 6:25 & = 4:20 \\
6:58 - 1:32 & = 5:26
\end{align*}
\]

Ask your students if they think that this method is always convenient. Ask them to find an example when this will be impossible to use. (EXAMPLE: 5:12 – 12:25) What would they do in this case?

**Assessment**

1. Cyril has to catch a school bus at 8:15. He woke up at 7:40. How much time does he have before the bus leaves?

2. Find the time elapsed:
   - From 10:40 to 13:25
   - From 1:25 to 3:42
   - From 10:53 to 2:35
**Bonus**

A young witch is cooking an invisibility potion. Her potion turns lilac at 7:45 p.m. Twenty minutes after it turns lilac, the recipe says to add some shredded snake. When should the witch add the shredded snake? After adding the snake, the recipe says to stir the potion 3 times clockwise, add some frog spawn, let the potion simmer for 77 minutes, then add 3 hairs and stir the potion counterclockwise. When should the witch add the 3 hairs? The potion must now boil for 35 minutes and then it will be ready. When should the witch take the cauldron from the fire? It takes the potion 2 hours and 45 minutes to take effect. The witch needs to be invisible by midnight. If she takes the potion as soon as it’s ready, will she be invisible by midnight?

**Extension**

You can find elapsed time using subtraction with regrouping. **EXAMPLE:** 5:12 – 3:25

You can’t subtract these times as they are because you can’t take 25 away from 12. You need more minutes in the first time. Regroup 1 hour as 60 minutes and add the result to the minutes, so that 5:12 becomes 4:72. Now subtraction is easy:

\[
\begin{align*}
4:72 \\
- 3:25 \\
\hline
1:47
\end{align*}
\]

ME5-5  
The 24-Hour Clock

**ASK:** How many hours are in the day? So why are we using a clock that has only 12 hours on its face? Explain that the 12-hour clock comes from ancient Egypt, where people divided the night and the day into 12 hours each. The ancient Romans started to use midday, or noon, as the point of division, and that created the 12-hour a.m./p.m. clock that we use today. However, a 24-hour clock is sometimes useful and convenient. For example, many airports prefer to use the 24-hour clock for timetables.

You can draw a large clock or use the 24-hour clock if available:

![24-hour clock diagram]

Explain to your students that the 24-hour clock starts at midnight. Midnight is 00:00. After that, the hours go as in the regular a.m. clock: 1:00 a.m. is 01:00, 2:00 a.m. is 02:00, and so on. Ask your students to convert several a.m. times into 24-hour times. (**EXAMPLES:** 5:00 a.m., 7:30 a.m., 2:15 a.m., 9:45 a.m., 11:20 a.m.)

Tell students that it is midday, or noon. **ASK:** Is midday 12:00 a.m. or 12:00 p.m.? It is 12:00 p.m., because a.m. is finished and p.m. has begun. (And it would look strange to see 12:00 a.m. and 12:01 p.m. a minute later.) In the 24-hour clock, midday is just 12:00. In the 24-hour clock, the next hour (1:00 p.m.) will be 13:00. So 2:00 p.m. is 14:00, and so on. What do you do to the p.m. times (excluding midday or midnight) to get the 24-hour time? (Add 12 to the hours.) Ask your students to convert several p.m. times into 24-hour times. (**EXAMPLES:** 5:00 p.m., 7:15 p.m., 2:30 p.m., 10:45 p.m.)

Write several 24-hour times on the board. (**EXAMPLES:** 07:00, 12:15, 13:30, 15:20, 12:00, 17:00, 20:05, 23:40, 24:00) Ask your students whether the times are a.m. or p.m. Then ask students what they usually do at these times. You may also ask them to convert the times into a.m./p.m. notation.

You may wish to make a timeline of a typical day in your classroom using the 24-hour clock. Students could also convert the timelines they produced in **ME5-1** to the 24-hour notation.

Put the following table on the board:

**GOALS**
Students will learn to use a 24-hour clock.

**PRIOR KNOWLEDGE REQUIRED**
Telling time in 5-minute intervals  
Finding elapsed time using number lines  
Times of day

**VOCABULARY**
minute a.m.  
hour p.m.
Part 1
Measurement

<table>
<thead>
<tr>
<th>Destination</th>
<th>Flight Number</th>
<th>Departure Time</th>
<th>12-Hour Clock</th>
<th>24-Hour Clock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vancouver</td>
<td>AC 1175</td>
<td>9:00 a.m.</td>
<td>09:00</td>
<td></td>
</tr>
<tr>
<td>New York</td>
<td>II 903</td>
<td>4:30 a.m.</td>
<td>04:30</td>
<td></td>
</tr>
<tr>
<td>Calgary</td>
<td>WS 665</td>
<td>3:15 p.m.</td>
<td>15:15</td>
<td></td>
</tr>
<tr>
<td>Edmonton</td>
<td>AC 175</td>
<td>10:05 a.m.</td>
<td>10:05</td>
<td></td>
</tr>
<tr>
<td>Fredericton</td>
<td>QK 8958</td>
<td>9:35 p.m.</td>
<td>21:35</td>
<td></td>
</tr>
</tbody>
</table>

Ask your students which departure times are easier to read—those in the 12-hour clock or those in the 24-hour clock? Why might airports prefer to use the 24-hour clock for timetables? Discuss the advantages and disadvantages of each system and when one might be more useful than the other.

Assessment
1. Fill in the missing times:

<table>
<thead>
<tr>
<th>12-Hour Clock</th>
<th>24-Hour Clock</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:15 a.m.</td>
<td>13:45</td>
</tr>
<tr>
<td></td>
<td>10:20</td>
</tr>
<tr>
<td>3:35 p.m.</td>
<td></td>
</tr>
</tbody>
</table>

2. It is midnight. What do the 12-hour clock and the 24-hour clock show?

3. It is midday. What do the 12-hour clock and the 24-hour clock show?

Extensions
1. **PROJECT:** Which countries use the 12-hour clock and which countries use the 24-hour clock? What are the advantages and disadvantages of the two systems? **POSSIBLE SOURCES:**


2. Where does the 12-hour clock come from? How did the Ancient Romans and Egyptians use it?

   The Romans used the term "p.m." the same way we do. However, to them, the term "a.m." meant "before noon." For example, in Roman time 1:00 a.m. would have been one hour before noon—what we would call 11:00 a.m.
The Ancient Egyptians divided the day and the night into 12 hours of equal length. In the summer, when night was shorter than day, the “night hours” were shorter than the “day hours.” The Egyptians used sundials to measure time during the day and water clocks to measure time at night. Your students may want to research ways to construct sundials and water clocks, as well as their origins.

POSSIBLE SOURCES:
http://en.wikipedia.org/wiki/12-hour_clock
http://www.ancientegypt.co.uk/time/story/main.html
http://library.thinkquest.org/J002046F/technology.htm

3. Calculating elapsed time is easy using the 24-hour clock. Use the method of subtraction with regrouping (from the Extension in ME5-4) to find the elapsed time:
From 7:30 to 12:45  From 8:50 to 15:20  From 9:30 to 17:10

ME5-6
Topics in Time

Ask your students which units people use to measure time. Record the answers on the board. Ask students which units they would use to measure:

- the length of the Christmas holidays
- the length of summer holidays
- the age of a person
- the length of a lesson
- the length of a movie
- the age of a baby that is not yet one month old
- the time it takes you to swallow medicine
- the age of a country
- the age of Egyptian pyramids

Explain to your students that longer periods of time are measured not only in years, but also in decades and centuries. Write both terms on the board. What is the plural of “century”? Write that as well. Explain that a decade is 10 years long and a century is 100 years long. Ask your students to give an example of a period of time that is measured in decades or in centuries (EXAMPLES: age of a country, age of a tree, time since the beginning of the new era). List several periods of time and ask your students to write them in centuries and/or decades.

EXAMPLES:
50 years = ___ decades
90 decades = ___ years = ___ centuries
300 years = ___ centuries = ___ decades
210 years = ___ centuries + ___ decades
**ASK:** How many minutes are in an hour? How many seconds are in a minute? Which is greater—2 hours or 140 minutes? Give students more time intervals to compare.

**EXAMPLES:**

- 1 minute 20 seconds or 85 seconds
- 178 seconds or 3 minutes
- 2 hours 30 minutes or 140 minutes
- 7 hours or 400 minutes

Give students the following problem: An ostrich runs at a speed of 72 kilometres per hour. How far will it run in a minute? In a second?

Explain that to solve these questions, you have to convert kilometres to metres and hours to minutes or seconds. Summarize each step of the solution on the board. **ASK:** If the ostrich runs 72 kilometres in an hour, how many metres will the ostrich run in an hour? (72 000 m) There are 60 minutes in an hour so we know that the ostrich runs 72 000 m in 60 min. To figure out the distance an ostrich can run in 1 minute (or the speed in metres per minute), students will have to divide the distance (72 000 m) by the number of minutes in an hour (60). Point out to students that when you divide 2 numbers that end in zero, you can first divide both numbers by 10. This is equivalent to dropping a zero from the end of each number, i.e., 72 000 ÷ 60 = 7 200 ÷ 6.

Explain to your students that when a dividend (72 000) and a divisor (60) are both divided by the same number (in this case 10), the result of the division (the quotient) is unchanged. To find 7 200 ÷ 6 students can either use long division or reason as follows: The number 7200 can be thought of as 72 hundreds. If we divide 72 hundreds into 6 equal groups, each group would contain 12 hundreds (since 72 ÷ 6 = 12). Hence, 72 hundreds divided by 6 is 12 hundreds, or 1200. The ostrich runs 1 200 m in one minute.

To convert the speed to metres per second, divide by 60 again: 1 200 ÷ 60 = 120 ÷ 6 = 20, which means the ostrich can run 20 m in one second.

Let your students practise solving more questions of this type. **EXAMPLE:** A grizzly bear can run 15 metres in a second. An elk can run 81 km in an hour. Who will run farther in a minute?

Ask your students to use a T-table to solve the following problem: A gazelle runs 25 metres in a second. A cheetah can maintain a speed of 30 metres per second for about 20 seconds only. A cheetah in a tree sees a gazelle 25 metres away. They run in the same direction. How long will it take the cheetah to catch the gazelle?

Help your students to draw the T-table:

<table>
<thead>
<tr>
<th>Time</th>
<th>0 s</th>
<th>1 s</th>
<th>2 s</th>
<th>3 s</th>
<th>4 s</th>
<th>5 s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance from the tree</td>
<td>Gazelle</td>
<td>25 m</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cheetah</td>
<td>0 m</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Explain that in making the table it is convenient to assume that the cheetah starts at a position of 0 on a number line, and the gazelle starts at a position of 25. As before, have students solve more such questions for practice.

**Assessment**

1. How much time passed from the battle on the Plains of Abraham (1759) till British Columbia joined the Confederation (1871)? About how many decades is that?
2. Convert the time intervals:

\[
\begin{align*}
70 \text{ years} &= \_ \_ \text{ decades} & 310 \text{ years} &= \_ \_ \text{ decades} \\
50 \text{ decades} &= \_ \_ \text{ years} &= \_ \_ \text{ centuries} & 2 \text{ centuries} &= \_ \_ \text{ decades} &= \_ \_ \text{ years} \\
500 \text{ years} &= \_ \_ \text{ centuries} &= \_ \_ \text{ decades} & 440 \text{ years} &= \_ \_ \text{ centuries} + \_ \_ \text{ decades}
\end{align*}
\]

3. Which unit of time would you use to determine the age of Canada? The age of a baby? The time it takes you to get home from school? The time it takes you to sing ‘Happy birthday’?

**Bonus**

Alex has a nightmare. He is running at a speed of 90 kilometres per hour. He is being chased by a grizzly bear, who runs at a speed of 900 metres per minute, and his favourite teacher, who runs at a speed of 30 metres per second. Alex runs for 2 minutes and stops (this is a nightmare, remember?) and his pursuers start chasing him 1 minute after he starts running. He wakes up 3 minutes after he started running. Make a T-table to see what happens to Alex—will he escape, be eaten by a bear, or caught by his favourite teacher?

**CROSS-CURRICULAR CONNECTION:**

**PROJECT:** Research a person who had an important part in Canada’s history and create the timeline of his/her life.

**Extensions**

1. If a ones block (in base ten materials) represents 1 year, which block would represent…
   a) a decade?   b) a century?

2. How many decades are in:
   a) a century?   b) 2 centuries?
ME5-7
Thermometers

GOALS
Students will read a thermometer and identify whether given temperatures are hot or cold.

PRIOR KNOWLEDGE REQUIRED
Thermometer

VOCABULARY
thermometer
degree Celsius

Hold up a large thermometer and ask students to identify what it is. **ASK:**
What do we measure with a thermometer? (temperature) Where or when do people use thermometers? (to measure the temperature of: a home, a greenhouse, the outdoors, the fridge or freezer, the body, and so on)

Draw a thermometer on the board or use a model thermometer. Explain that temperature is measured in units called degrees Celsius. Draw the symbol (°C) and write the term on the board. The scale of the thermometer is based on the properties of water, so that anyone can build his or her own thermometer. Explain that 0°C is the temperature at which water freezes and 100°C is the temperature at which water boils. So to make a new thermometer, you need to freeze some water to find the 0°C mark and then boil some water to establish the 100°C mark. Then you divide the scale into 100 parts and your thermometer is ready. You can illustrate the process while drawing a thermometer on the board.

**ASK:** Is -5°C hot or cold? Is water at this temperature a liquid or is it ice?
Explain that if you have a piece of ice at -5°C, you have to heat your piece of ice for some time to make it melt. How much do you have to heat it? Enough to raise it’s temperature by 5°C.

Write several temperatures on the board. Ask students to identify each temperature as hot or cold. Ask them to tie the temperature to familiar events or activities, such as:

- 0°C – cold, water freezes
- 5°C – cold, water is nearly frozen, the ice has just melted
- -10°C – cold, snow, skating
- 20°C – the air is warm, water still cold
- 36°C – the air is very hot, near to normal human body temperature
- 50°C – very hot, desert. (You can bake eggs in the sand!)

Add to this list the range of temperatures for each season where you live, to give students the information they need to complete the worksheet.

Have students solve several problems in which they have to add or subtract to find the temperature. **SAMPLE PROBLEMS:**

- The temperature today is 15°C. Yesterday it was 5°C higher. What was the temperature yesterday?

- The average temperature in winter is -10°C. The average temperature in summer is 25°C. What is the difference between the average temperatures in winter and in summer? (Let students use the thermometer to solve this problem.)
Assessment

What was the temperature on Monday?
What was the temperature on Wednesday?
How much warmer was it on Monday than on Wednesday?
A jar of water was left outside. What was in it on Monday—water or ice? And on Wednesday?

<table>
<thead>
<tr>
<th>Monday</th>
<th>Wednesday</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-10</td>
<td>-10</td>
</tr>
<tr>
<td>-20</td>
<td>-20</td>
</tr>
<tr>
<td>-30</td>
<td>-30</td>
</tr>
</tbody>
</table>

ACTIVITY 1

Hang a thermometer on the class window and let volunteers check and record the temperature over a week, a month, or throughout the whole year. On any given day, ask students whether it is cold or hot, whether it is colder or warmer than the day before, and compare the measurement results. The recorded data can be used in the Data Management unit.

ACTIVITY 2

Build Your Own Thermometer

WEB SOURCE:
http://www.weatheroffice.pyr.ec.gc.ca/skywatchers/exGamesAct/equipment_therm_e.html

The site provides instructions for building your own thermometer. You can make the scale card (index card in the Equipment section) wider to allow the students to write their own comments about the temperatures. They can mention activities appropriate to the temperature (“skiing” or “too cold for bathing”, and so on).
Classifying Data

Have students identify attributes that all members of the following groups satisfy:

a) grey, green, grow, group (EXAMPLE: starts with gr, one syllable, words)

b) pie, pizza, peas, pancakes (EXAMPLE: food, starts with p)

c) 39, 279, 9, 69, 889, 909 (EXAMPLE: odd, ones’ digit 9, less than 1000)

d) 5412 9807 7631 9000 9081 (EXAMPLE: numbers, four digits, greater than 5000)

e) Hat, cat, mat, fat, sat, rat (EXAMPLE: three-letter words, rhymes with at)

Have students write two attributes for the following groups:

a) 42, 52, 32, 62, 72  b) lion, leopard, lynx

Which 2 attributes could have been used to sort the items into the groups shown?

Group 1:  Group 2:

- at least one right angle
- 4 sides or more
- quadrilaterals
- no right angles
- 3 sides
- hexagons

Which categories do all shapes in both groups belong to? Which categories do no shapes in either group belong to? What categories do some shapes in group 1 and some shapes in group 2 belong to? What category do all shapes in group 1 and no shapes in group 2 belong to? What category do no shapes in group 1 but all shapes in group 2 belong to? How were these groups categorized?

Label a cardboard box with “toy box,” then ask your students if various items (some items toys and some items not toys) belong inside or outside of the box. Then tell them that you want to classify items as toys without having to put them in a box. Draw a circle, write “toys” in the circle and tell your students that you want all of the toy names written inside the circle, and all of the other items’ names written outside the circle. Ask your students to tell you what to write inside the circle. To save space, have students problem-solve a way to not write the entire word inside the circle. If they suggest you write just the first letter, ask them what will happen if two toys start with the same letter. Explain to them that you will instead write one letter for each word:

A. legos  B. bowl  C. toy car  D. pen
Which of the letters go inside the circle, and which of the letters go outside the circle?

Toys

Draw several shapes and label them with letters.

A.  B.  C.  D.  E.  F.

Draw:

Shapes

Triangles

Ask your students if all the letters belong inside the box and to explain why. Which letters belong inside the circle? Which letters belong outside the circle but still inside the box? Then ask them why the circle is inside the box? Are all of the triangles shapes? Explain that everything inside the box is a shape, but in order to be inside the circle the shape has to be a triangle. Change the word inside the circle and repeat the exercise. [Suggested words to use include: dark, light, quadrilaterals, polygons, circles, light, dark triangles, dark circles, etc.] Ask your students why the circle is empty when it’s relabelled as “dark circles”? Can they think of another property that they could use to label the circle so that the circle would remain empty? Remind them that the word inside the circle should reflect the fact that the entire box consists of shapes, so “rockets” probably isn’t a good word to use, even though the circle would still be empty.

Clearly label a hula hoop as “pens” and another as “blue,” then ask your students to assign several coloured pens (black and red) and pencils (blue, red and yellow) to the proper position—inside either of the hoops or outside both of them. Do not overlap the hoops at this point. Then present your students with a blue pen and explain that it belongs in both hoops. Allow them to problem-solve a way to move the hoops so that the blue pen is circled by both hoops at the same time. Have a student volunteer to show the others how it’s done. But if students move the hoops closer together without overlapping them, so that part of the blue pen is circled by the “blue” hoop and part of the blue pen is circled by the “pens” hoop, be sure to ask them if it makes sense for part of the pen to be outside of the “pens” hoop. Shouldn’t the entire pen be circled by the “pens” hoop?

Draw:

A.  B.  C.  D.  E.  F.
And draw:

**Shapes**

![Venn diagram with two overlapping circles labeled 'Dark' and 'Triangles'.]

Explain to your students that the picture is called a Venn diagram. Ask them to explain why the two circles are overlapping. Have a volunteer shade the overlapping area of the circles and ask the class which letters go in that area. Have a second volunteer shade the area outside the circles and ask the class which letters go in that area. Ask them which shape belongs in the “dark” circle and which shape belongs in the “triangle” circle. Change the words representing both circles and repeat the exercise. [Suggested words to use include: light and quadrilateral, light and dark, polygon and light, circles and light.]

This is a good opportunity to tie in ideas learned in other subjects. For example, have students categorize words by their first or last letters, by their sound, by the number of syllables, etc. (“rhymes with tin” and “2 syllables”: A. begin; B. chin; C. mat; D. silly). Start with four words, then add four more words to be classified into the same categories. Encourage students to suggest words and their place in the diagram. [Cities, provinces and food groups are also good categories.]

**ACTIVITY**

Tape a big circle to the floor and have students that are ten years old stand in the circle. Then write “10 years old” inside the circle.

10 Years Old

Repeat with several examples:

- **Wearing Yellow**
- **Girl**
- **Wearing Blue**
- **Takes the Bus to School**

Have your students stand inside the circle if the property applies to them. Have them suggest properties and race to the appropriate circle before you finish writing the words. They should learn to strategically pick properties that take a long time to write. After the lesson, repeat the activity by overlapping two circles.
Extensions

1. Distribute the BLM “Shapes” (comprising of 16 shapes that are either striped or plain, big or small, triangles or squares) or the Set™ card game between pairs of students. Each sheet has 2 of each card, so that students can work individually.

Ask them to separate the striped triangles from the rest of the shapes, then challenge them to describe the remaining group of shapes (plain or square). Allow them to use words like “or” or “and” in their answers, but do not encourage use of the word “not” at this point since in this case, “not striped” simply means plain. If they need help, offer them choices: plain or triangle, striped and square, plain or square.

2. Using the BLM “Shapes”, challenge students to describe the remaining group of shapes when they separate out the:
   a) small squares
   b) plain squares
   c) big triangles
   d) shapes that are small or triangular
   e) shapes that are striped or triangular
   f) shapes that are striped or big
   g) shapes that are both striped and big
   h) shapes that are both plain and big
   i) shapes that are striped or small

3. Distribute the BLM “Shapes” between pairs of students and have them individually separate the eight shapes into Venn diagrams. Possible categories include:

   a) striped or triangle, big squares

   b) plain or triangle; plain squares
   c) plain triangles; big or plain
   d) small or triangle; striped and big
   e) small or striped; big or square
   f) big squares; small squares

   In addition, you can distribute the BLM “Circles” between groups of four students and have them separate the shapes from both BLMs into Venn diagrams. Possible categories include:

   g) plain or triangle; not a circle
   h) striped, but not a square; circle or big

4. Combine the BLM “Circles” and BLM “Shapes” and have your students continue Extension 2. Allow them to use the word “not” when describing the separated shapes. For example, when you separate the plain triangles, the remaining shapes can be described as: “striped or not a triangle” or “striped or square or circle”. Although “not a plain triangle” is correct, students should be encouraged not to use this phrase.
PDM5-2
Venn Diagrams (Advanced)

GOALS
Students will translate between classification charts and Venn diagrams.

PRIOR KNOWLEDGE REQUIRED
Venn diagrams with two categories
Tables and charts

VOCABULARY
property

Explain to your students that data can be classified with Venn diagrams and with charts, and that they should be able to translate from one form to the other. Have your students choose two properties that can be used to divide the following names into groups, and to copy the chart with their properties and then to complete the chart.

<table>
<thead>
<tr>
<th>Name</th>
<th>Property 1</th>
<th>Property 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Alice</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>Bob</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Cathy</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>Don</td>
<td></td>
</tr>
<tr>
<td>J</td>
<td>Jen</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>Kevin</td>
<td></td>
</tr>
</tbody>
</table>

When all students have at least copied the chart with 2 chosen properties, discuss the properties that they have used to complete their charts. Sample properties: number of letters, number of syllable, gender, ends in “n”, Then have students complete their charts and create Venn diagrams. Encourage them to insert additional names into their charts and Venn diagrams.

![Venn Diagram](image)

**Bonus**
Distribute the BLM “Shapes”. Have your students draw a Venn diagram with three circles (labelled striped, square and small) then problem-solve a way to arrange the circles so that there is an area for the small striped squares. Then have them categorize all of the BLM “Shapes” into the Venn diagram. Have your students select a third category for the Venn diagram exercise completed earlier in class, and complete that Venn diagram, or have them categorize the following words into the categories red, green and yellow:

- rainbow
- strawberry
- raspberry
- ripe banana
- sky
- grass
- clover
- sun
- blood
- rose
- traffic light
- stop sign
Extension

Have your students determine the categories used for the following Venn diagrams, or have them invent their own and challenge their classmates.

a) "rhymes with art" and "starts with c"

<table>
<thead>
<tr>
<th>Category</th>
<th>Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>smart</td>
</tr>
<tr>
<td>B</td>
<td>heart</td>
</tr>
<tr>
<td>C</td>
<td>cone</td>
</tr>
<tr>
<td>D</td>
<td>frog</td>
</tr>
<tr>
<td>E</td>
<td>candy</td>
</tr>
<tr>
<td>F</td>
<td>time</td>
</tr>
<tr>
<td>G</td>
<td>cool</td>
</tr>
<tr>
<td>H</td>
<td>start</td>
</tr>
<tr>
<td>I</td>
<td>cart</td>
</tr>
</tbody>
</table>

b) Category 1: hit, hat, sit, fog, fig, tug (three-letter words)
   Category 2: this, that, make (four-letter words)
   Both: None
   Neither: to, funny, elephant

c) Category 1: Tina, Cathy, Nancy, Sara (girls’ names)
   Category 2: Mark, Tom, Derek, Fred (boys’ names)
   Both: Teagan, Pat, Casey, Jamie
   Neither: None.

d) Category 1: ball, pencil, song, sun, sea (nouns)
   Category 2: write, sing, see, listen (verbs)
   Both: hint, crack, hit, run
   Neither: funny, from, to, sad

ACTIVITY

Distribute the BLM “Spots and Stripes” and have your students draw two Venn diagrams: an adult animal diagram with circles labelled as “has spots” and “has stripes,” and an embryonic animal diagram with circles labelled as “has short and round part” and “has long and thin part.” Have them compare the two Venn diagrams. Explain to them that mathematicians have proven that if a developing animal is long and thin enough it will become striped; if it is short and round enough it will become spotted. Tell them that scientists knew for hundreds of years that some animals have spotted bodies and striped tails, but no animals have striped bodies and spotted tails, but they could only recently explain why. Can your students explain why?
PDM5-3

Choosing a Scale for a Bar Graph

Using a scale of two, draw a bar graph for the following data:

<table>
<thead>
<tr>
<th>Favourite Animal</th>
<th>Horse</th>
<th>Cat</th>
<th>Dog</th>
<th>Hamster</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
<td>8</td>
<td>5</td>
<td>6</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

Allow your students to compare the graph with the data, then ask them how they think the graph shows the data. Explain to them that the scale of the bar graph expresses how much each mark on the grid represents. Ask them what scale was used for the bar graph. How does the bar graph represent two students? How would the bar graph represent one student?

Ask your students how they can determine from the graph: Which animal is the most popular? How many people like dogs? How many more people like horses than hamsters?

Then ask a volunteer to illustrate on the bar graph the addition of four students choosing a cow as their favourite animal.

Instruct students to plot the following data on a bar graph. Remind them to include a title, a scale (suggest a scale of two) and labels.

<table>
<thead>
<tr>
<th></th>
<th>Earnings From Babysitting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pat</td>
<td>$12</td>
</tr>
<tr>
<td>Samir</td>
<td>$8</td>
</tr>
<tr>
<td>Ali</td>
<td>$6</td>
</tr>
</tbody>
</table>

Bonus

Draw a bar graph for the following data: Carmen earned six more dollars babysitting than Ron. Carmen earned four more dollars than Gita.

NOTE: Students will need to decide how much Gita earned.

Draw the following vertical scales for bar graphs (on a grid, if possible):

- 0, 2, 4, 6, 8, 10, 12, 14;
- 0, 5, 10, 15, 20, 25;
- 0, 50, 100, 150, 200, 250, 300

Ask your students what these scales have in common (they all start with zero and increase by a constant amount each time) and how they differ (they all increase by a different amount (2, 5, and 50) and have a different quantity of numbers per scale (8, 6, and 7).
Add the following vertical scales to the board:

0, 2, 4, 6, 8, 10;
0, 2, 4, 6, 8, 10, 12, 14, 16;
0, 2, 4, 6, 8, 10, 12

Ask your students what these scales have in common (they all start at zero and increase by two) and how they differ (they end with different numbers). Explain that these scales are identified by the amount they increase by and by the number they stop at. Now have students describe all of the previous scales.

Draw the following vertical scales for bar graphs (on a grid, if possible).

\[ \begin{align*}
&60 \quad 50 \quad 45 \quad 40 \quad 35 \quad 30 \quad 0 \\
&27 \quad 24 \quad 21 \quad 18 \quad 0
\end{align*} \]

\[ \begin{align*}
&390 \quad 380 \quad 370 \quad 360 \quad 350 \quad 0
\end{align*} \]

**ASK:** What do these scales have in common? (none of them start at 0, they all have a broken vertical axis to indicate that, they all go up by a constant amount) How are they different? (they start and end at different numbers, the amount they go up by is different for each one—5, 3 and 10 respectively).

**ASK:** If I tell you that the first scale goes up by 5 and stops at 60, have I told you everything about it?

Draw a scale that starts at 20, goes up by 5 and stops at 60. **ASK:** Does this scale also go up by 5 and stop at 60? Is it the same scale? What’s different about it? Explain to students that a scale is identified by the number it starts at, how much it goes up by and the number it stops at, so the first scale is “start at 30, go up by 5 and stop at 60.” Have volunteers identify the second and third scales. Draw several more scales on the board for students to identify individually.

Ask your students what their favourite season is and draw a tally.

<table>
<thead>
<tr>
<th>Spring</th>
<th>Summer</th>
<th>Fall</th>
<th>Winter</th>
</tr>
</thead>
</table>

Total the tallies and then **ASK:** How many students are here today? Did everyone vote for one or the other? If not everyone voted, ask the class why someone might not select a favourite season. (Maybe they don’t have a favourite season, or maybe they like all of the seasons equally.)

Ask your students what the greatest number of tallies for a particular season is, then have them decide on a good scale for a bar graph. Will there be too many markings if the scale increases by one? How many markings would there be if a scale of 50 were used? Is that a good idea? Demonstrate the disadvantages of using a scale of 50 by marking the first grid on the vertical axis (50) immediately above the zero.
Use the data from the class to draw an incorrect graph. For example, if four students prefer spring, 15 prefer summer, three prefer fall and 12 prefer winter, draw a vertical axis with markings at 3, 4, 12 and 15. Ask your students what is wrong with the scale. (The numbers do not increase by a constant amount). How were the numbers chosen?

Ask your students to examine the heights of the bars. Do twice as many people really prefer spring over fall? Without examining the scale, that’s exactly what the graph would express. Have students explain why it is important for the scale to increase by a constant amount.

Have students draw a correct graph in their notebooks. Have them choose two different scales (both starting at 0, but going up by different amounts) and draw them side by side. **ASK:** How are your two graphs the same? How are they different? Which graph makes it look as though there are bigger differences between seasons? If you didn’t look too closely at the axis, would one graph make it look as though there were more people in the class?

Show several bar graphs and have students recreate what they think the original tallies for the bar graph would look like.

Tell students that each scale below starts at 0 (if a scale didn’t start at 0, the graph would show it by a broken axis). Have students tell you which scale was used for each bar graph:

Have students determine the values of the other bars on the graphs.

![Bar Graphs](image-url)
ACTIVITY 1

For practice reading bar graphs, some students might enjoy the Internet game found at:
http://www.slidermath.com/integer/Footbar.shtml
Students need to answer basic facts about football teams given a bar graph.

ACTIVITY 2

Some students might enjoy the Internet game found at:
http://pbskids.org/cyberchase/games/bargraphs/bargraphs.html
Students need to be quick with their hands to catch bugs. A bar graph charts the amount of each
colour caught. Gives practice at changing scale.

ACTIVITY 3

Encourage students to look at bar graphs in books, magazines, on the Internet or on television
(such as on The Weather Network). Have them record the number of markings on the scales and
the number of bars. About how many markings do most bar graphs use? About how many bars
do most bar graphs use?

Extensions

1. Draw a bar graph to show each situation.
   a) Anne earned 12 more dollars babysitting than Carmen. Carmen earned 4 more
dollars than Gita. Gita earned $10.
   b) Anne earned 12 more dollars babysitting than Carmen. Carmen earned 4 more
dollars than Gita. Carmen earned $10.

2. Based on Karen’s wildlife sightings from Question 4 of the worksheet, answer the
   following questions.
   a) Which bars were the most difficult to read? What strategy did you use to read them?
   b) Karen saw ______ times as many fish as eagles.
   c) Karen saw half as many ______ as ______.
   e) Bobby saw 9 more mallards than ______.
   f) In total, Bobby saw 7 ______ and ______.

3. Plot the following data on a bar graph:

<table>
<thead>
<tr>
<th></th>
<th>Earnings From Paper Route</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pat</td>
<td>$75</td>
</tr>
<tr>
<td>Samir</td>
<td>$150</td>
</tr>
<tr>
<td>Ali</td>
<td>$50</td>
</tr>
</tbody>
</table>

What scale should you use? How much more money did Samir earn than Pat and Ali combined?
Journal
On a bar graph, a scale should increase by a constant amount because...

PDM5-4
Double Bar Graphs

Ask your students what their favourite type of book is and draw a tally. Have volunteers help you tally.

<table>
<thead>
<tr>
<th>Favourite Type of Book</th>
<th>Novel</th>
<th>Comic</th>
<th>Non-Fiction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Remind your students that they can only vote for one type of book. Have students check the tallies to see that everyone voted and that no one voted twice.

Then have students draw a bar graph of the result. What was the most popular type of book? The least popular? What if we want to know if comic books are more popular with girls or boys? What if we want to compare the types of books that girls like to the types of books that boys like? How should the results be tallied? Demonstrate this new tally approach by asking the boys and girls separately as groups if the novel is their favourite type of book. Have volunteers complete the tallies by asking the boys and girls separately as groups if the comic or non-fiction books are their favourite types of books.

The tally chart should look like something like this, depending on your class:

<table>
<thead>
<tr>
<th>Favourite Type of Book</th>
<th>Novel</th>
<th>Comic</th>
<th>Non-Fiction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Have your students draw two separate bar graphs for the tallies, one for the girls and one for the boys, using the same scale for each.

Explain that it will be easier to graph both sets of data on the same grid. Without distinguishing one bar from the other, draw the bars for the boys on the same grid as the girls. Emphasize that the graph seems hard to read, and then ask your students how to fix it. Suggest the use of different colours for the boys’ and girls’ bars.

Colour in either the boys’ or girls’ bars on the bar graph, then ask what can be done to ensure that everyone can distinguish the boys’ bars from the girls’ bars. Introduce the concept of a key.
Provide the following data in a new tally chart:

<table>
<thead>
<tr>
<th>Favourite Type of Book</th>
<th>Novel</th>
<th>Comic</th>
<th>Non-Fiction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys</td>
<td>9</td>
<td>13</td>
<td>18</td>
</tr>
<tr>
<td>Girls</td>
<td>26</td>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>

Ask your students what scales can be used to graph only the boys’ tallies? Only the girls’ tallies? Which scale should be used to graph the boys’ and girls’ tallies together—the boys’ scale or the girls’ scale? What would happen if the boys’ and girls’ tallies were graphed on the same grid using only the boys’ scale?

Allow students sufficient practice in choosing a scale.

<table>
<thead>
<tr>
<th>Favourite Type of Book</th>
<th>Novel</th>
<th>Comic</th>
<th>Non-Fiction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys</td>
<td>5</td>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>Girls</td>
<td>26</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Start at: ________ Count by: ________ Stop at: ________

<table>
<thead>
<tr>
<th>Favourite Type of Book</th>
<th>Novel</th>
<th>Comic</th>
<th>Non-Fiction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys</td>
<td>3</td>
<td>31</td>
<td>6</td>
</tr>
<tr>
<td>Girls</td>
<td>11</td>
<td>14</td>
<td>15</td>
</tr>
</tbody>
</table>

Start at: ________ Count by: ________ Stop at: ________

**Bonus**

<table>
<thead>
<tr>
<th>Favourite Type of Book</th>
<th>Novel</th>
<th>Comic</th>
<th>Non-Fiction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys</td>
<td>75</td>
<td>130</td>
<td>145</td>
</tr>
<tr>
<td>Girls</td>
<td>216</td>
<td>54</td>
<td>80</td>
</tr>
</tbody>
</table>

Start at: ________ Count by: ________ Stop at: ________

Have students draw double bar graphs for the preceding sets of data. Create similar problems if they require additional practice.

**Cross-Curricular Connection**

Does the key relate to anything the students have seen in social studies? Is there something on a map that is similar to the key on a double bar graph?
PDM5-5
Broken Line Graphs—An Introduction

GOALS
Students will read, interpret and draw broken line graphs.

PRIOR KNOWLEDGE REQUIRED
Bar graphs
Labels
Scales

VOCABULARY
broken line graph
scale
labels

Write the following data on the board:

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Su</th>
<th>M</th>
<th>T</th>
<th>W</th>
<th>Th</th>
<th>F</th>
<th>Sa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
<td>26</td>
<td>32</td>
<td>22</td>
<td>16</td>
<td>13</td>
<td>15</td>
<td>13</td>
</tr>
</tbody>
</table>

Have students draw a bar graph of the data in their notebooks. While they are doing so, draw a broken line graph of the same data on the board. Have students discuss the similarities and differences between the two types of graphs. Tell your students that the graph you drew is called a broken line graph and ask why they think it is called that.

ASK: Which day was the warmest? Which day was the coolest? Which two days are the same temperature? How does the bar graph show this? How does the broken line graph show this? Does the temperature seem to be increasing over the week or decreasing? How does the line graph show this? If the temperature is increasing, what direction does the broken line between the points go in—from bottom left to top right or top left to bottom right? What direction does the broken line between the points go in if the temperature is decreasing? Does the temperature increase more or decrease more over the course of the week? How is the line graph good for showing that?

Have students draw a line graph in their notebooks for the following set of data:

<table>
<thead>
<tr>
<th>Pairs of Ice Skates Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>34</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
</tr>
</tbody>
</table>

ASK: From which month to which month is there the greatest increase in sales? How does the line graph show this? (from October to November, the broken line is steepest) Why are there so many skates sold in November and December? (Christmas shopping)
PDM5-6
Discrete and Continuous Data

Tell students that there are some things it makes sense to take a fraction of and there are some things it doesn’t. Ask your students in which of the following situations does it make sense to speak of a fraction of the unit?

Rita’s height is 130 cm—**ASK:** What is the unit? (cm) Can there be a fraction of a cm? (yes)

Rita’s birthday is in August—**ASK:** What is the unit? (months) Can her birthday be only partly in August? (no)

Rita’s favourite sport is running—**ASK:** What is the unit? (sports) Can her favourite sport be partly running? (no)

The temperature in Rita’s home is 23°C—**ASK:** What is the unit? (°C) Can the temperature be 23.12 °C? 23.4 °C? (yes)

Tell students that if data in between the given data values is possible, we say the data is continuous. If not, we say the data is discrete.

Ask students whether the data on each graph is continuous or discrete, and to explain how they know.

![Graphs](a) Number of Chocolate Bars Sold (b) Number of Chocolate Bars Eaten

Then do not give data, only labels:

![Graphs](c) Temperature vs. Time (d) Daily Temperature High vs. Day

- **Number of letters vs. words**
- **Cups of flour vs. recipes**
张1

Part 1
Probability & Data Management

WORKBOOK 5
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PDM5-7
Continuous Line Graphs

GOALS

Students will use continuous line graphs when data is continuous to predict what happens in between data values.

PRIOR KNOWLEDGE REQUIRED

Reading data from broken line graphs

VOCABULARY

line graphs
broken line graphs
discrete data
continuous data

Review reading broken line graphs with your students. Draw a broken line graph on the board for the following data.

<table>
<thead>
<tr>
<th>Rita’s Distance from Home (km)</th>
<th>0</th>
<th>2</th>
<th>5</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (min)</td>
<td>0</td>
<td>15</td>
<td>30</td>
<td>45</td>
<td>60</td>
</tr>
</tbody>
</table>

ASK: How far from home was Rita after 45 minutes? After 15 minutes? After an hour? After half an hour? Ensure students know how to use a ruler to find the data values. For example, to find the distance from home after 30 minutes, line the ruler up with the point at 30 minutes and see where it meets the vertical axis (the markings on the ruler should be parallel to the vertical axis to guarantee that the ruler goes straight across and not on a diagonal):

Then replace the broken lines with continuous lines and tell the students that this is called a continuous line graph. Tell the students that if they know how to read a broken line graph, then they already know how to read a continuous line graph. Demonstrate this for them by asking the same questions again.

Teach students how to use a ruler to find in-between data values. For example, to find how far from home Rita was after 20 minutes, estimate where 20 minutes would be on the horizontal axis and then use a ruler to find the point on the graph straight up from 20 (the markings on the ruler should be parallel to the “time” axis to guarantee that the ruler is straight up and down) then mark this point and use the ruler to find the distance from home at 20 minutes:
Have students draw a continuous line graph for the following data:

<table>
<thead>
<tr>
<th>Hours Tom Worked</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money Earned ($)</td>
<td>0</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>100</td>
</tr>
</tbody>
</table>

**ASK:** How much would Tom earn if he worked for 3 hours? For 5 hours? 2 hours? How much would he earn if he worked for half an hour? For two and a half hours? For four and a half hours?

Have students draw a continuous line graph for the following data:

<table>
<thead>
<tr>
<th>Number</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number × 20</td>
<td>0</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>100</td>
</tr>
</tbody>
</table>

**ASK:** How are the two continuous line graphs the same? How are they different? **ASK:** Does it make sense to ask how much would Tom earn in two and a half hours? Do they think it makes sense to ask what is two and a half times twenty? (accept all answers)

Tell students that mathematicians have defined multiplication by fractions and they can find $2 \frac{1}{2} \times 20$ by using the graph in the same way they used the graph above to predict how much money Tom would earn in $2 \frac{1}{2}$ hours.

Go through several examples as a class and then assign individual questions.

**ACTIVITY**

Have students record the temperature every day for a week and plot a line graph. Ask students what would happen to the data if they took the temperature at 3 p.m. one day and 10 p.m. the next day? Explain the importance of recording the temperature at the same time every day for the week and then discuss any trends they see in the line graph.
PDM5-8
More Discrete and Continuous Data

GOALS
Students will explain why it makes sense to use broken line graphs when data is discrete and continuous line graphs when data is continuous. Students will distinguish between when to use bar graphs and line graphs.

PRIOR KNOWLEDGE REQUIRED
Continuous and discrete data
Continuous and broken line graphs

VOCABULARY
continuous line
broken line
continuous and broken line graphs
continuous and discrete data
trend

Explain to students that when data is continuous, it makes sense to ask what happens in between data values. By drawing a continuous line instead of a broken line, you are symbolically showing the fact that all the in-between values make sense and at least could have been data values.

Give students various data sets and ask students to decide whether they would draw a continuous line graph or a broken line graph.

Ensure students understand that, when data is discrete, it is best to use a bar graph unless you want to see trends in the data (How is the data changing? Is it increasing, decreasing, staying constant?). If you want to see trends in the data, it makes more sense to use a line graph than a bar graph. For example, if you are comparing how many single cans of tomato juice, 6-packs, 12-packs or 24-packs a company has sold, you might draw a line graph to see the trend for larger packs. But if you are comparing the sales of different types of juice, it doesn’t make sense to look for a trend. To illustrate this point, draw two line graphs on the board for how many single cans, 6-packs, 12-packs and 24-packs were sold, where one graph shows the data in that order and the other graph shows the data out of order: EXAMPLE: plot the points in the following order: 12-packs, single cans, 24-packs, 6-packs.

Have students discuss the difference between the two graphs and why one of them is easier to read than the other. Then, after this discussion, draw two line graphs showing the sales of different types of juice, EXAMPLE: Apple, Orange and Grape (switch the order to, say, Orange, Grape and Apple on the second graph). Have students discuss the differences between the two graphs and whether it makes a difference in this case to plot the data points in a different order. Emphasize that, in this case, there is no natural ordering of the data points, so it doesn’t make sense to ask about trends in the data. As a result, a bar graph is better than a line graph.

Have students draw two line graphs for the data below, but in one graph putting the days of the week in order and in the other writing the temperatures in increasing order:

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>12</th>
<th>7</th>
<th>10</th>
<th>8</th>
<th>6</th>
<th>7</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day of the Week</td>
<td>Su</td>
<td>M</td>
<td>T</td>
<td>W</td>
<td>Th</td>
<td>F</td>
<td>Sa</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>7</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day of the Week</td>
<td>Sa</td>
<td>Th</td>
<td>M</td>
<td>F</td>
<td>W</td>
<td>T</td>
<td>Su</td>
</tr>
</tbody>
</table>

ASK: Which ordering captures the most about the data? Which ordering will be easiest to read when presented on a line graph? How does the second graph make it appear as though the temperature is increasing throughout the week? What trends do you see from the first graph?
Then repeat the exercise for the following data:

<table>
<thead>
<tr>
<th>(min)</th>
<th>Time to Run 5 km</th>
<th>34</th>
<th>28</th>
<th>29</th>
<th>36</th>
<th>41</th>
<th>25</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Participants</td>
<td>Sara</td>
<td>Juli</td>
<td>Tom</td>
<td>Mark</td>
<td>Mary</td>
<td>Ron</td>
<td>Tara</td>
</tr>
<tr>
<td>(min)</td>
<td>Time to Run 5 km</td>
<td>24</td>
<td>25</td>
<td>28</td>
<td>29</td>
<td>34</td>
<td>36</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>Participants</td>
<td>Tara</td>
<td>Ron</td>
<td>Juli</td>
<td>Tom</td>
<td>Sara</td>
<td>Mark</td>
<td>Mary</td>
</tr>
</tbody>
</table>

**ASK:** Is there a natural order to put the names of the participants in like there was for days of the week? How does putting the times in order from fastest to slowest make it appear as though there is some sort of trend when really there wasn’t? Explain to your students that because there is not a natural order to put the names in, it makes more sense to draw a bar graph than a line graph.

Have students decide in which situations it is more appropriate to draw a bar graph and in which situations it is more appropriate to draw a line graph.

a) The number of cupcakes each class sold in a school bake sale.

b) The price of buying a math textbook from each company.

c) The price of buying one math textbook, 10 books, 30 books or 100 books from the same company.

d) The temperature over a given 24-hour time period in New York City.

e) The temperature at a given time in New York City, Toronto, Vancouver and Paris.

f) How many winter boots each company sold in February.

g) How many winter boots a particular company sold over each month of the year.

For graphs that they choose to do line graphs by, have students explain why other orderings of the data would make less sense and also to predict any trends they expect to see from the line graph.

**Extensions**

1. Show students a typical 2-week temperature graph seen on the weather channel. Ask students if the graph shows discrete data or continuous data. (The horizontal axis showing the days has discrete data and the vertical axis showing the temperature has continuous data). **ASK:** Does it make sense to ask what happens in between data values? (No, there is no daily high between the daily high for Monday and the daily high for Tuesday) Tell students that if the graph showed the continual change of temperature instead of just the daily high, the data on both axes would be continuous. Ask students how they can tell that the graph does not show the continual change of temperature. (the graph would have more ups and downs for the days and nights; the corners would be less abrupt) Challenge students to explain why the weather channel has chosen a continuous line graph to show the data (people are interested in seeing the overall trends in temperature and the increase or decrease from one day to the next).
2. The following chart shows the average rainfall each month in different parts of the world:

   a) Which graph best matches the data in each column? Write the letter in the space provided under each column:

<table>
<thead>
<tr>
<th>Month</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>25</td>
<td>10</td>
<td>100</td>
<td>120</td>
</tr>
<tr>
<td>February</td>
<td>20</td>
<td>10</td>
<td>100</td>
<td>120</td>
</tr>
<tr>
<td>March</td>
<td>25</td>
<td>10</td>
<td>100</td>
<td>120</td>
</tr>
<tr>
<td>April</td>
<td>35</td>
<td>10</td>
<td>20</td>
<td>120</td>
</tr>
<tr>
<td>May</td>
<td>45</td>
<td>13</td>
<td>10</td>
<td>120</td>
</tr>
<tr>
<td>June</td>
<td>50</td>
<td>18</td>
<td>5</td>
<td>120</td>
</tr>
<tr>
<td>July</td>
<td>60</td>
<td>18</td>
<td>5</td>
<td>120</td>
</tr>
<tr>
<td>August</td>
<td>55</td>
<td>13</td>
<td>5</td>
<td>120</td>
</tr>
<tr>
<td>September</td>
<td>50</td>
<td>10</td>
<td>10</td>
<td>120</td>
</tr>
<tr>
<td>October</td>
<td>40</td>
<td>10</td>
<td>20</td>
<td>120</td>
</tr>
<tr>
<td>November</td>
<td>40</td>
<td>10</td>
<td>60</td>
<td>120</td>
</tr>
<tr>
<td>December</td>
<td>35</td>
<td>5</td>
<td>100</td>
<td>120</td>
</tr>
</tbody>
</table>

   b) Describe any trends you see in the graphs. How do you account for the trends?

   c) Sketch a graph that you think would represent the average monthly temperature where you live.
PDM5-9
Primary and Secondary Data

GOALS
Students will distinguish between observation, measurement and surveys as means of obtaining primary data. Students will distinguish between primary and secondary data.

PRIOR KNOWLEDGE REQUIRED
Bar graphs

VOCABULARY
primary (first-hand) data
secondary (second-hand) data
survey
measurement
observation

Conduct a survey with your students to determine how many of them were born before noon, and how many of them were born after noon. After very few (or none) of the students raise their hands for either option, explain that conducting a survey or using another way to obtain data is decided on by whether or not the people being surveyed will have answers to the survey questions. Have your students think about which topics from the list would be good survey topics:

• What are people’s favourite colours? (good)
• Are people left-handed or right-handed? (good)
• Can people count to 100 in one minute or less? (not so good)
• How many sit-ups can people do in one minute? (not so good)
• What are people’s resting heart rates? (not so good)
• What are people’s favourite sports? (good)
• How fast can people run 100 m? (not so good)
• How do people get to school? (good)
• How many siblings do people have? (good)

Is a survey needed for the following topics or can the data be obtained just by observation?

• Do students in the class wear eyeglasses?
• Do students in the class wear contact lenses?
• What are adults’ hair colours?
• What are adults’ natural hair colours?

Sometimes a measurement or calculation is needed to obtain data. Have your students decide if observation, a survey or a measurement is needed to obtain the data for the following topics:

• What are people’s heart rates? (measurement)
• What are people’s favourite sports? (survey)
• How do people get to school? (survey)
• How many sit-ups can people do in one minute? (measurement)
• How long are people’s arm spans? (measurement)
• What colour of shirt are people wearing? (observation or survey)
• How tall are people? (survey or measurement)
• What are my classmates’ favourite books? (survey—ask them)
• How does the temperature of a glass of ice water change over time? (measurement—measure the temperature as time passes)
• Are more people born in the fall or in the spring? (survey)
• Are more people born before noon or after noon? (observation—go to a hospital and watch how many babies are born before noon and how many are born after noon)

Explain to your students that sometimes they will be able to collect their own data directly and sometimes they will need to rely on data that someone else has collected. Explain that sometimes, students will not have enough resources to find the data themselves. For example, to find the predicted temperature for next week, students cannot take the required measurements themselves; rather, they will need to rely on other people’s measurements and predictions. Tell students that data they collect themselves is called primary data and data they use that was collected by someone else is called secondary data.

Ask students whether they would use primary or secondary data in each situation and ask them to explain their answer.

a) You want to find the favourite television show of students in your class.
b) You want to find the favourite television show of grade 5 students in Canada.
c) You want to know the amount of rain in your hometown in the past week.
d) You want to know which player won the Stanley Cup the most number of times.
e) You want to know how many push-ups each person in the class can do in a minute.
f) You want to know the world record for the most number of push-ups done in a minute.
g) You want to know the temperature outside.
h) You want to know what the temperature will be during March break.
i) You want to know your classmates’ eyeglasses prescriptions.
j) You want to know how many people in your class wear eyeglasses.

Extension

Tell students to look at the temperatures given on the worksheet. Do they think the temperatures are warm cold, hot or cool? Where in the world are temperatures likely to be in the range given? Would the temperatures be likely to occur in Toronto, Canada? Ask if students know what unit is being used for measurement? Remind students that when we measure distances, there are different units of measurement such as cm, m, km. Ask if they think that everyone in the world learns the same units of measuring distance as they do. Tell them that in the United States, they use a different unit and ask if anyone knows what that is. (inches and feet—there are about $2\frac{1}{2}$ cm in an inch) Then ask if anyone knows the unit of measurement that people in the United States use for measuring temperature (degree Fahrenheit instead of degree Celsius). Ask if anyone knows how much in degrees Celsius the given 55 degrees Fahrenheit would be. Tell your students that 55°F is about 13°C and ask again how they would describe this temperature—warm, cold, cool, hot? Is it a temperature that could occur in Toronto, Canada?
Tell your students that sometimes they will want to know things about really large groups of people but they may not be able to gather data for everyone in the population. For example, if I want to know how many people in Canada have read The Wizard of Oz, it wouldn’t be practical to ask everyone if they’ve read it. If I survey a group of people about whether they have read The Wizard of Oz, the fraction of people who say yes, might not be exactly the same as the actual fraction of people in Canada who have read the book, but if I survey a large enough group, then the fraction may be close enough to the actual value.

Tell your students that the idea is that even if you just ask a large group of people in Canada, you might not have the exact same fraction of people as if you asked everyone, but you would be pretty close.

**ASK:** If I want to know what fraction of the class is girls without asking everyone in the class, do you think I will get a good idea by just asking 1 student their gender? By asking 5 students?

Draw the following chart on the board:

<table>
<thead>
<tr>
<th>The First Child</th>
<th>The First 5 Children</th>
<th>The First 10 Children</th>
<th>The Whole Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Girls</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boys</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction of Girls</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ask each student to write their name on a slip of paper to put in a hat. Select 1 student’s name and record the gender in your chart (either Girls: 1, Boys: 0, Fraction: 1 or Girls: 0, Boys: 1, Fraction: 0). Have a volunteer select the next 4 children and record the result of the first five (including the name already selected). Have another volunteer select the next 5 and fill in the third column. Finally, have a volunteer fill in the remaining column. **ASK:** Was looking at just the first child a good indication of what fraction of the class are girls? Did looking at the first 5 children give a better estimate? Did looking at the first 10 children give a closer estimate than that? Do you think that larger samples will provide a better idea of what the whole class is like?

Tell students that you want to find the average shoe size of everyone in Ontario. Ask if you should survey everyone in Ontario or only a sample—have students explain their answer. Repeat with various situations. **EXAMPLE:** If I want to know which books my classmates read last week, should I survey the whole class or only a sample? If I want to know the average shoe size of people on my block, should I survey everyone on the block or only a sample? If I want to know how many people in Canada watched a particular television show, should I ask everyone in Canada or just a sample?)
Bring in several jars with 20 green marbles and 80 purple marbles. Have students work in groups to make a chart as follows:

<table>
<thead>
<tr>
<th></th>
<th>The First Marble</th>
<th>The First 10 Marbles</th>
<th>The First 20 Marbles</th>
<th>The Whole Jar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green Marbles</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Purple Marbles</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction of</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Green Marbles</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**ASK:** What fraction of the marbles in the whole jar were green? To find out the fraction of marbles that are green, would you need to look at the whole jar? Would looking at the first 20 marbles give a close answer? Would looking just at the first 10 give a close answer? Would looking at just the first marble give a close answer?

**PDM5-11**

**Designing and Analyzing a Survey**

**GOALS**

Students will understand how to design a survey of their own.

**PRIOR KNOWLEDGE REQUIRED**

Surveys

**VOCABULARY**

survey

Explain to your students that a topic needs to be selected before a survey can be conducted on the topic; they need to decide what they want to know. Conduct a survey with your students by asking them what their favourite flavour of ice cream is—do not limit their choices at this point.

Write each answer with a tally, then ask students how many bars will be needed to display the results on a bar graph (see Activity 2 from PDM5-3). How can the question be changed to reduce the number of bars needed to display the results? How can the choices be limited? Should choices be limited to the most popular flavours? Why is it important to offer an “other” choice?

Explain to your students that the most popular choices to a survey question are predicted before a survey is conducted. Why is it important to predict the most popular choices? Could the three most popular flavours of ice cream have been predicted?

Have your students predict the most popular choices for the following survey questions:

- What is your favourite colour?
- What is your favourite vegetable?
- What is your favourite fruit?
- What is your natural hair colour?
- What is your favourite animal?
Students may disagree on the choices. Explain to them that a good way to predict the most popular choices for a survey question is to ask the survey question to a few people before asking everyone.

Emphasize that the question has to be worded so that each person can give only one answer. Which of the following questions are worded so as to receive only one answer?

a) What is your favourite ice cream flavour?
b) What flavours of ice cream do you like?
c) Who will you vote for in the election?
d) Which of the candidates do you like in the election?
e) What is your favourite colour?
f) Which colours do you like?

Have your students think about whether or not an “other” category is needed for the following questions:

What is your favourite food group?
- Vegetables and Fruits
- Meat and Alternatives
- Milk and Alternatives
- Grain Products

What is your favourite food?
- Pizza
- Burgers
- Tacos
- Salad

Then ask how they know when an “other” category is needed. Continue with these EXAMPLES:

- What is your favourite day of the week? (List all seven days)
- What is your favourite day of the week? (List only Friday, Saturday and Sunday)
- What is your favourite animal? (List horse, cow, dog, pig, cat)
- How many siblings do you have? (List 0, 1, 2, 3, 4 or more)
- Who will you vote for in the election? (List all candidates)

Have your students explain what they like and don't like about each of the four following proposals for a survey question.

1. Do you like to read for fun?
   - YES
   - NO

2. How many books have you read in the last year for fun? _________

3. How often do you like to read for fun?
   - Any chance I get
   - Often
   - Sometimes
   - Not very often
   - Never

4. How many books have you read in the last year for fun?
   - 0
   - 1-5
   - 6-10
   - 11 or more

Emphasize the importance of choosing questions that people will know the answer to (EXAMPLE: some people may not know how many books they read last year), and that provide enough variety in the way someone can answer (EXAMPLE: some people might feel uncomfortable answering YES or NO if they only sometimes like to read for fun). Question 3 is probably the best question.
since it allows enough variety and although some people may be unsure between two adjacent categories, they will have at least a good idea as to how to answer the question. Furthermore, the number of books may not be the best indication since someone who likes to read may only read a few long books, while someone who doesn’t read very much might read the same number of short books.

Have students make up good survey questions to find out:

a) What kind of games should a grades 1-8 school have available for an end-of-year party?
b) What kind of snack do students in Ontario like best?
c) How many pets do students in your class have?

If a school can only provide certain kinds of games (EXAMPLE: no computer games will be available), would it make sense to include computer games in the survey? Should the school provide an “other” category or force students to choose from among the games that the school could possibly bring?

Ask your students to decide who they would survey for each situation a) to c) above. Would it makes sense to ask only grade 5 students what kinds of games they like? What problems would this cause?

Tell students that they will be designing their own survey. Have students brainstorm topics for a survey. (EXAMPLE: How do people get to school? How long does it take people to get to school? How big is your family? How long does it take people to get ready for school from the time they get up to the time they arrive—do older children take more or less time than younger children? How tall are people in each grade?) Discuss what type of graph would be appropriate for each question. Is there a trend you are looking for?

**Extension**

Ask your students to represent their survey results in 2 different ways and explain the advantages and short comings of each different representation.
Remind students how to double numbers and how to multiply by 5 quickly (multiply by 10 and then halve the result). Tell students that you want to know if they are faster at doubling numbers or multiplying by 5. Give students the following two tests and have students time themselves individually.

**Doubling Test:**
- \(2 \times 3 =\)
- \(2 \times 5 =\)
- \(2 \times 4 =\)
- \(2 \times 2 =\)
- \(2 \times 6 =\)
- \(2 \times 1 =\)
- \(2 \times 9 =\)
- \(2 \times 8 =\)
- \(2 \times 0 =\)
- \(2 \times 7 =\)

**Multiplying by 5 Test:**
- \(5 \times 0 =\)
- \(5 \times 1 =\)
- \(5 \times 2 =\)
- \(5 \times 3 =\)
- \(5 \times 4 =\)
- \(5 \times 5 =\)
- \(5 \times 6 =\)
- \(5 \times 7 =\)
- \(5 \times 8 =\)
- \(5 \times 9 =\)

Ask students if they think this pair of tests is fair. Then ask: What about this pair:

- \(2 \times 13 =\)
- \(2 \times 9 =\)
- \(2 \times 24 =\)
- \(2 \times 39 =\)
- \(2 \times 46 =\)
- \(2 \times 28 =\)
- \(2 \times 6 =\)
- \(2 \times 34 =\)
- \(2 \times 18 =\)
- \(2 \times 67 =\)
- \(2 \times 88 =\)

Challenge students to make up a pair of tests that they think is fair. Then have the students do their own tests and ask them if they are faster at multiplying by 2 or by 5. Have half the class do the multiplying by 2 test first and the other half do the multiplying by 5 test first. Did the order they did the test affect their results?

Tell them that sometimes the trickiest part of doing an experiment is making sure that they are really testing for what they want to be testing and that nothing else influences their results.

Discuss the experiments mentioned in the workbook and the importance of planning ahead.

Tell your students that you want to answer the question: Do ice cubes made from the same amount of water but different shaped containers melt at the same rate?
Discuss the different types of containers that could be used for this experiment and how they would make sure the same amount of water is put into each container. How would they measure the rate of melting? What other equipment do they need? Why is it important to think ahead of time about what kind of equipment they need? Would the experiment be fair if some ice cubes are put in the sun and others in the shade? How would that affect the results of the experiment? Have students predict the results of the experiment—will the container’s shape affect the rate of melting. What type of graph would they use to display their results? Is there a natural ordering of the containers? (no) Is there a possibility of data in between the data values? (no, not unless they choose their containers in a very structured way, i.e. by increasing the length and decreasing the width, in which case there would be a natural ordering of the containers)

Have volunteers bring in the equipment for you to demonstrate with in front of the class. Demonstrate the entire process, from doing the experiment to displaying the results (on a bar graph, since it doesn’t make sense to speak about trends in the shape of the container).

Brainstorm factors that could influence the results of the experiments A, B, and C in Question 2 on worksheet PDM5-12. For example, in A, the thickness and quality of paper or the environment (EXAMPLE: existence of wind) or the design of the airplanes or the quality of bending (how precise the creases are) could affect the results, so these should be kept the same for all three models, in B, the freshness of the strawberries and how evenly the sugar is spread and the container (metal, plastic, glass, airtight, shape and size) and the surrounding temperature and the type of strawberries and the type of sugar should be kept the same for adding the different amounts of sugar; in C, the size and shape of the ice cube should be kept the same, the type of salt should be kept the same (i.e. sea salt, table salt) and the temperature of the surrounding areas and the colour of the plate (white or black plates will absorb the heat differently), and the source of the water should be kept the same as the amount of salt increases.

Note that, in B and C, it does make sense to look for trends and so a line graph is more appropriate. In A, a bar graph is more appropriate.
PDM5 Part 1: BLM List

Circles ________________________________________________________________ 2
Shapes ........................................................................................................ 3
Spots and Stripes ..................................................................................... 4
Shapes
Spots and Stripes

Adult Animals

Leopard

Zebra

Cheetah

Genet

Giraffe

Tiger
Spots and Stripes (continued)

Shapes of Embryonic Animals

Leopard Embryo

Zebra Embryo

Cheetah Embryo

Genet Embryo

Giraffe Embryo

Tiger Embryo
A Note for Geometry

NOTE: Terms and definitions are included in the worksheets. General notes about geometrical concepts are included in some lessons.

Most students find elementary geometry to be uncomplicated. What they find hard, however, is the terminology. If students can’t spell or define relevant terms, they will have trouble describing or classifying figures. Therefore, JUMP recommends that you teach the spelling of geometric terms during spelling lessons. The most important terms are: side, edge, vertex, vertices, transformation, slide, reflection, rotation, congruent, line of symmetry, right angle, triangle, square, rectangle, rhombus, parallelogram and equilateral. Other important terms are pentagon, hexagon and trapezoid.

G5-1
Sides and Vertices of 2-D Figures

GOALS
Students will identify polygons, sides and vertices, and distinguish polygons according to the number of sides.

PRIOR KNOWLEDGE REQUIRED
Ability to count to ten
Ability to distinguish a straight line

VOCABULARY
<table>
<thead>
<tr>
<th>polygon</th>
<th>quadrilateral</th>
</tr>
</thead>
<tbody>
<tr>
<td>sides</td>
<td>pentagon</td>
</tr>
<tr>
<td>vertex</td>
<td>hexagon</td>
</tr>
<tr>
<td>vertices</td>
<td>triangle</td>
</tr>
</tbody>
</table>

Draw and label a polygon with the words “polygon,” “sides” and “vertex/vertices.” Remind students of what a side and vertex are and explain that a side has to be straight. Show students how to count sides—marking the sides as you count—then have them count the sides and the vertices of several polygons. Ask them if they can see a pattern between the number of vertices and the number of sides. Be sure that all students are marking sides properly and circling the vertices, so they don’t miss any sides or vertices.

Construct a large triangle, quadrilateral, pentagon and hexagon using construction paper or bristol board. Label each figure with its name and stick them to the chalkboard. Explain that “gon” means angle or corner (vertex), “lateral” means sides. You might want to leave these figures on a wall throughout the geometry unit. Later (G5-6 and G5-8), you can add special quadrilaterals and triangles.

Explain that “poly” means many, then ask your students what the word polygon means (many angles or vertices). Explain that a polygon is a shape that only has straight sides. Draw a shape with a curved side and ask if it is a polygon. Label it as “not polygon.”

Draw several shapes on the board and ask students to count the sides and sort the shapes according to the number of sides. Also ask them to draw a triangle, a pentagon, a figure with six sides, a figure with four angles, and a figure that is not a polygon but has vertices.

Bonus
Draw a figure that has:

a) two curved sides and three straight sides
b) two straight sides and three curved sides
Assessment
1. Draw a polygon with seven sides.
2. Draw a quadrilateral. How many vertices does it have?
3. Altogether, how many vertices are there in two pentagons and a triangle?
4. Draw a figure that is not a polygon and explain why it is not a polygon (for instance, it has a curved side, a circle is not a polygon, a rectangle with rounded edges does not have proper vertices).

Extensions
1. Count the sides of a paper polygon. Count the vertices. Cut off one of the vertices. Count the sides and vertices again. Cut off another vertex. Repeat the count. Do you notice a pattern? (The number of sides will increase by one and the number of vertices will increase by one.)
2. “Word Search Puzzle (Shapes)” BLM can also be assigned as homework.
G5-2
Introduction to Angles

Ask your students if they know what a right angle is—a right angle is the corner of a square, but there is no need to define it properly at this stage. Ask them where they can see right angles in real life (EXAMPLE: corners of a sheet of paper, doors, windows, etc.). Draw a right angle and then show students how to mark right angles with a small square. Explain to your students that not all angles are right angles; some are sharper than a right angle, some are less sharp. Tell them to think of corners; the sharper the corner is, the smaller the angle is.

NOTE: You may want to perform ACTIVITY 1 here.

Draw two angles.

Ask your students which angle is smaller. Which corner is sharper? The diagram on the left is larger, but the corner is sharper, and mathematicians say that this angle is smaller. The distance between the ends of the arms is the same, but this does not matter. What matters is the “sharpness”. The sharper the angle is, the narrower the space between the angle’s arms. Explain that the size of an angle is the amount of rotation between the angle’s arms. The smallest angle is closed—with both arms together. Draw the following picture to illustrate what you mean by smaller and larger angles.

With a piece of chalk you can show how much an angle’s arm rotates. Draw a line on the chalkboard then rest the chalk along the line’s length. Fix the chalk to one of the line’s endpoints and rotate the free end around the endpoint to any desired position.

You might also illustrate what the size of an angle means by opening a book to different angles.

Draw some angles and ask your students to order them from smallest to largest.

A                  B                  C                  D                  E
Draw a polygon and explain that the polygon’s angles are inside the figure, not outside the figure. Draw several polygons and ask volunteers to mark the smallest angle in each figure.

![Polygons](image)

Note that the smallest angle in the right most figure is A and not B—B is actually the largest angle, since the angles are inside the polygon. (You might hold up a pattern block or a cut-out of a polygon so students can see clearly which angles are inside the figure.)

Do some of the activities before proceeding to the worksheets.

Draw several angles and ask volunteers to identify and mark the right angles. For a short assessment, you can also draw several shapes and ask your students to point out how many right angles there are. Do not mark the right angles in the diagram.

![Angles](image)

Before students try QUESTION 3 on the worksheet, draw and label obtuse and acute angles with the words “obtuse” and “acute”. Show students how to mark an acute angle with one arc and an obtuse angle with two arcs (see the Answers to Assessment 2 below). You can also introduce the straight angle, which equals two right angles, and ask students why it is called a straight angle. Allow the students to finish the worksheet.

**Bonus**

Draw a polygon with a fixed number of right angles—1, 2, 3, 4, 5. The odd numbers are tricky. (HINT: Use a square and cut off corners!)

**Assessment**

1. Copy the shape on grid paper and identify all the right angles with a square, all angles less than a right angle with an arc, and all angles greater than a right angle with a pair of arcs.
2. Draw a quadrilateral with an acute, obtuse and right angle. Mark each angle as in Assessment 1.

Possible answers:

 ACTIVITY 1
 Make a key from an old postcard. Have students run their fingers over the corners to identify the sharpest corner. The sharper the corner is, the smaller the angle. Ask your students to mark the angles from least to greatest according to sharpness. Point out that every vertex makes an angle, but some angles are not sharp at all—they are greater than the straight angle. Such angles are called reflexive angles. The less room for your finger to get to the corner from the outside, the larger the angle is. The key in the picture has 9 angles.

 ACTIVITY 2
 Ask students to use any object (a piece of paper, an index card) with a square corner to identify various angles in the classroom that are “more than,” “less than” or “equal to” a right angle.

 - corner of a desk
 - window corners
 - angle made by a door and the wall
 - corners of base ten materials

 ACTIVITY 3
 Use a geoboard with elastics to make figures with different numbers of right angles: 0, 1, 2, 3, 4, 5. Possible answers for 3 and 5 are:

 As a simple lead-up to this exercise, ask your students to make a right angle, and acute and obtuse angles.

 ACTIVITY 4
 Have your students compare the size of angles on pattern blocks by superimposing various pattern blocks and arranging the angles in order according to size.

 They may notice that there are two angles in the trapezoid that are greater than the angles in the square, and that there are two angles that are less than the angles in the square.

 (Continued on next page.)
They may also notice that the large angle in the trapezoid is equal in size to the angles of two equilateral triangles. To prevent students from thinking that this is a common feature of all trapezoids, try to incorporate different (labelled) trapezoids.

Ask your students if they know how to identify all the angles in the equilateral triangle as equal. They might display this by superimposing two triangles and rotating one of the triangles.

If pattern blocks are not available, see the appropriate BLM page.

Extensions

1. a) Which shape has no right angles?

b) There are 5 reflexive angles inside the figure on the right. Can you find them?

c) Mark the right angles with \( \square \) and the angles that are half of a right angle with \( \Box \).

2. Draw your own shape on grid paper and mark any right, acute, obtuse and reflexive angles.

3. a) On a geoboard create angles that are:

   i) a right angle  
   ii) half a right angle  
   iii) one and a half times a right angle.

b) Use the angles of the shapes above to fill in the table:

<table>
<thead>
<tr>
<th>Less than half a right angle</th>
<th>Half a right angle</th>
<th>Between half a right angle and a right angle</th>
<th>Right angles</th>
<th>Between a right angle and one and a half times a right angle</th>
<th>One and a half of a right angle</th>
<th>More than one and a half of a right angle</th>
<th>Right angles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Explain that there are angles that cannot be compared by superimposing (the angles of an open door and window, for example). This means that to measure such angles your students will need some benchmark angles. Allow your students to prepare some benchmark angles, like wedges of cardboard or paper (see the Extensions for directions on the preparation of wedges). Divide students into groups so that different groups prepare different sets of wedges, then regroup your students so that each group has several examples of each benchmark angle.

Allow your students to compare the wedges. They should see that \( \frac{1}{4} \) of the right angle is less than \( \frac{1}{3} \), and that two \( \frac{1}{4} \) wedges are greater than \( \frac{1}{3} \), but that \( \frac{2}{3} \) of the right angle is more than half a right angle, etc.

Explain that the problem with wedges is that they are too rough: draw two angles on the board, of 50° and 55°. Try to check with wedges of 45° and of 60°—both angles fell in between the two, but they are still different. Protractors were invented to measure angles with more accuracy. A protractor has 180 marks called degrees. Explain the degree sign (45° means forty-five degrees). One thing makes a protractor difficult to read—it has 2 scales. The way to distinguish between them is to use the right angle. It has 90 degrees in it, and you can use this to judge what scale to use.

Draw a right angle and write “90°” and “90 degrees” beside it. Draw an angle on the board. Ask if the angle is less or more than the right angle. Ask whether the degree measure should be greater or less than 90°. Write that on the board. Draw several angles on the board and ask whether they are more or less than 90°.

Draw a protractor on the board that has only multiples of 30° on it.

Explain that this protractor is like the one they have in the worksheets, in QUESTION 2. Hold up a benchmark for \( \frac{1}{3} \) of a right angle. Is that less or more than 90°? Put it on the protractor and write out both numbers that the arm of the angle passes through. Which one is the answer? Do the same for \( \frac{2}{3} \) of the right angle benchmark. Hold up the benchmark for half a right angle. Ask how many of these make a right angle (2). The right angle is 90°. How many degrees are in half a right angle? (45°)

You might mention that there is a slight difference between measuring the benchmarks and angles on paper—you place a benchmark on the protractor
that you cannot move; now you have to place the protractor on the angle that does not move. Show how to put a protractor on the angle on the board. Point out that the vertex of the angle should be in the middle of the straight side of the protractor. Mark and write out both numbers on the arm. Ask which one of them is the right answer. If students need a hint, ask them to check whether the angle is more or less than the right angle, and then to decide whether the result should be more or less than 90°. Have them do QUESTION 2 of the worksheet.

After that tell the story of the Pirate’s Treasure (see below) and use the rest of the worksheet. We give a sample of the QUESTIONS 3 and 4 in the worksheet. For faster students the clues may be longer. Take into account that “fastness” in this case is the function of motor skills and not a mathematical ability. Model extending the arms with a ruler. You might use actual small treasures hidden in your classroom to make deciphering the clues more fun. The answer for the sample worksheet is “Behind the board”.

**Pirate’s Treasure**

Captain Flint—a very blood-thirsty pirate—had his treasure buried in several parts all over a remote island in the Atlantic Ocean. You can retrieve parts of this treasure by solving angle problems!

Measure the angles on the worksheet. Arrange the angles in QUESTION 3 from least to greatest. Each angle is marked with a letter. Write the letters according to the order of the angles. Do the same for QUESTION 4. Combine both parts into a phrase to get a clue to where the treasure is hidden!

**Assessment**

1. How many degrees are there in a right angle? _______
2. An angle has 30° in it. Is it an acute or an obtuse angle?
3. Measure the following angles using a protractor. Don’t forget your units!

![Diagram of angles]

**ACTIVITY 1**

Draw a shape on grid paper (a trapezoid, for instance) and measure all its angles. Remind the students that the angles of a polygon are the angles inside, not outside, the polygon.

**ACTIVITY 2**

Allow your students to measure the angles of pattern blocks with wedges. They should see that the right angle is equal to three of the \( \frac{1}{3} \) wedges, the angle of the equilateral triangle is equal to two of the \( \frac{1}{3} \) wedges, etc. Then allow them to measure various angles in the classroom, like empty juice boxes and the angle between the door and the wall (EXAMPLE: less than a right angle but greater than half a right angle.) After that ask your students to measure the angles in pattern blocks with protractors.
Extensions

1. How to create wedges of $\frac{1}{3}$ of a right angle (30°):

   First, create a square from a sheet of paper.
   
   a) Fold the square in two (vertically, not diagonally) so that a crease divides the square into two rectangles.

   b) Fold the top-right corner of the square down so that the top-right vertex meets the crease. The vertex should be slightly above the bottom edge.

   c) Trace a line along the folded side of the square with a pencil.

   d) Fold the bottom-left corner of the square over the first fold.

   e) Unfold. The right angle is now divided into three equal parts, and the traced line divides the square angle into two angles—one of 30° and the other of 60°.

2. How to create wedges of $\frac{1}{4}$ of a right angle—“paper protractor”:

   a) Fold a sheet of paper in two (vertically, not diagonally) and then turn the sheet so that the fold is at the bottom.

   b) Fold the top-right corner down to meet the fold. Now you have half a right angle.
c) Turn the paper over.

![Image of a folded triangle]

d) Fold the half right angle down so that the corners of the sheet are visible again.

![Image of a folded triangle with an arc]

e) Draw an arc as shown.

![Image of an arc drawn on a page]

f) Cut along the arc and unfold the page.

![Image of a page with arcs and cuts]

3. Draw three angles of the same measure—one with long arms, one with short arms, one with one arm longer than the other. Ask your students if any of the angles are larger than the others. Check them with wedges. Give your students six toothpicks: three short ones and three long ones. For assessment, ask to build three models of half right angles using two long toothpicks for arms, two short toothpicks for arms, and then two different-sized toothpicks for arms. **NOTE:** This exercise is important; the ability to measure angles with different arms is a specific demand of both Ontario and Atlantic curricula.

4. Ask your students how they might compare angles when the wedges are too rough to do it, or they don’t have wedges at all. Then show them how to use tracing paper.

1. Mark the vertex of an angle with a dot and write “v” beside it.
2. Put a bold dot on each of the arms and write “a” beside it.
3. Place the tracing paper so that it covers the dots. Can you see the dots through the paper?
4. Mark the dots and the letters on the tracing paper.
5. Use a ruler to join the v-dot to the a-dots. Do not join the a-dots!

Now they have a tracing of an angle to measure and compare with any other angle. Allow your students to practise copying angles onto tracing paper and comparing them with other angles.

5. Draw a triangle and measure all of the angles in the triangle. Add up the angles. Repeat with several other triangles. What do you notice? **NOTE:** The result will most certainly differ from 180°—due to inaccuracy in measure.

6. Take the paper protractor from Extension 2. Take the smallest wedge. How many of them make a right angle? (4). This means this angle is 22.5°. Two of these angles make half a right angle, so it is 45°. Three such angles are 67.5°. Continue marking the paper protractor, using increasing sequence with the difference of 22.5°. Estimate the angles in **QUESTION 1** of the worksheet in terms of wedges. For example, the angle in **d**) is between 1 and 2 wedges. Then measure the angles with your paper protractor.

7. Estimate the angles in **QUESTION 1** of the worksheet in terms of degrees. Measure the angles with a real protractor to test your predictions. (The Atlantic Curriculum expectation D7)
G5-4
Measuring and Constructing Angles

Review the previous lesson. Make sure every student knows how to choose the scale on the protractor. Ask several volunteers to draw random angles on the board and use more volunteers to measure the angles.

Model drawing an angle of the given measure on the board. Do that step-by-step, stressing the position of the protractor. Make sure that every student places the protractor so that the base line passes through 0° and the end of the base line is at the centre cross of the protractor.

Write several simple measures (like 30°, 50°, 60°, 85°, 140°, 170°) on the board and ask students to draw the angles of the given measure. You may let the students practice drawing more difficult angles during the activity.

Draw an analogue clock that shows 3:00 on the board. Ask your students what angle the hands create. What is the measure of that angle? So if the time is 1:00, what is the measure of the angle between the hands? Do you need a protractor to tell? Ask volunteers to write the measures for each hour from 1:00 to 6:00. Which number should they skip count by?

Assessment
Measure the angles:

Draw the angles of 30° and 125°.

The students will need a die and a protractor. Draw a starting line on the sheet of paper. The first player rolls the die and draws the angle of the measure given by the die so that the base line of his angle is the starting line and the angle is drawn counter clockwise. He labels his angle with its degree measure. Each next player rolls the die and draws an angle in the counter clockwise direction so that the base line of his angle is the arm drawn by the previous player. The measure of his angle is the sum of the result of the die and the measure of the angle of the previous player. The player whose angle overlaps with the starting line is the winner. For example, if the first three rolls are 4, 5 and 3, the picture will be as shown.
Extensions

1. Angles between the hands of the clocks: 5 min = 30°, 1 min = ___°.

The hour hand moves 1 “minute” for each 12 minutes of the hour. So if the time is 12:12, the hour hand points at one minute and the angle that it makes with the vertical line is 6°. The minute hand points at 12 minutes and the angle that it makes with the vertical line is 12 x 6 = 72°. The angle between the hands is 72 – 6 = 64°.

What is the angle between the hands at 12:24? 13:36? 15:48 (Draw the hands first)?

2. Some scientists think that moths travel at a 30° angle to the sun to leave home at sunrise. What angle do they need to go at to find their way back at sunset? HINT: Where is the sun? Note that the sun is far away, so all the rays it sends to us seem parallel.

3. The moth sees the light from the candle flame and thinks it’s the sun. The candle is very near to us, and the rays it sends to us start at the candle and go to all directions. Where does the moth end up? Draw the moth’s path.
G5-5
Angles in Triangles and Polygons

Remind the students what acute, obtuse and right angles are. What is the degree measure of the right angle? If an acute angle is less than a right angle, is it more or less than 90°? What about the obtuse angle?

Draw on the board an acute-angled triangle. Explain that it is called “acute-angled triangle” and ask what the reason for the name is. Ask your students if they can draw a triangle that has three right angles. Two right angles? Why is that impossible? (Start drawing. Draw a horizontal line and two right angles as the beginning of the “triangle”. The “sides” of your triangle are both vertical, so they are parallel, and never meet. You might use volunteers during the construction and ask questions to lead your students to understand that there cannot be two right angles in a triangle. You can also mention that students will learn another explanation why this cannot be. Do not mention the sum of the angles in the triangle yet.) Discuss the same problem with three or even two obtuse angles. Conclude that the right-angled triangle and the obtuse-angled triangle have only one right or obtuse angle respectively.

Draw several triangles of various kinds and ask your students to identify them. Invite volunteers to measure the angles carefully and to add the angles in each triangle. What do they get? Explain that if the angles are measured correctly, the sum of the angles in a triangle should be 180°. Write that on the board.

Draw a square, a regular pentagon and a regular hexagon on the board. Remind your students that a polygon is called regular if all its sides and all its angles are equal. Ask your students to measure the angles in each of them and to add the angles in each polygon. Divide each polygon into triangles. How many triangles are in the square? In the pentagon? In the hexagon? Make a table and use volunteers to fill it:

<table>
<thead>
<tr>
<th>Shape</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of vertices</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of triangles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum of angles</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**ASK:** Do you see any patterns in the rows? Do you see any relationships between the rows? What should you do to the number of the vertices in the polygon to get the number of the triangles? If the angles in each triangle add up to 180°, what is the sum of the angles in each polygon? Check your measurement of the angles—do the angles add up to the right result?
Assessment

1. Measure the angles A and B. What should the measure of the angle C be?

2. What is the sum of the angles of a regular octagon? How large is each angle?

Use a paper triangle to convince yourself that the sum of the angles in it is $180^\circ$: What is the degree measure of a right angle? A straight angle equals two right angles. What is its degree measure?

Draw a line between the middles of two edges of a paper triangle.

Fold the triangle along the new line, so that the top corner meets the base of the triangle. You will get a trapezoid.

Fold the other two corners of the triangle so that they meet the top corner. The three corners together make a straight angle. How large is their sum?

Extensions

1. Use the pattern for the sum of the angles in the regular shapes to write a formula for the sum of the angles in a polygon. If the number of vertices is $V$, what is the expression for the number of triangles in the shape? $(V-2)$ What do you do with the number of triangles to get the sum of angles? Write that in a formula. $(180^\circ \times (V-2))$ Does this formula apply to regular polygons only or does it apply to any polygon?

2. Which figure below has…

   a) all acute angles?  
   b) all obtuse angles?  
   c) some acute and some obtuse angles?
G5-6
Classifying Triangles

Ask your students which names for triangles they know. Remind that last year they learned to classify triangles according to their sides. Write the terms equilateral, isosceles and scalene on the board. Review the meanings of the parts “equi” (equal, Latin), “lateral” (sides, Latin), “iso” (same, Greek) and “sceles” (legs, Greek). Ask your students which other words contain some of these parts. Ask volunteers to draw an equilateral, an isosceles and a scalene triangle on the board. Invite another group of volunteers to check the drawings by measuring sides. Explain that it is important to check the drawing because of the human tendency to draw isosceles triangles instead of scalene, and equilateral instead of isosceles.

Give your students the BLM “Paper Triangles” and ask them to measure the lengths of the sides of the triangles with a ruler, measure the angles with a protractor and to classify them. After that you might do the first activity.

Draw on the board a Venn diagram with properties:

1. Right-angled
2. Isosceles

Ask the students to sort the triangles from the BLM into that Venn diagram.

Repeat with the properties:

1. Acute-angled
2. Scalene

Ask the students to make their own Venn diagram using two properties—one from List A below, and the other from List B. These Venn diagrams are also useful for assessment.

As a challenge, have students list all pairs of terms they can make by using one term from each list below:

<table>
<thead>
<tr>
<th>List A</th>
<th>List B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acute-angled</td>
<td>Equilateral</td>
</tr>
<tr>
<td>Obtuse-angled</td>
<td>Isosceles</td>
</tr>
<tr>
<td>Right-angled</td>
<td>Scalene</td>
</tr>
</tbody>
</table>

Have students discuss which pairs of properties can both occur in a triangle and which cannot. For instance, it’s easy to see that a right-angled triangle cannot be equilateral. Try drawing a right-angled equilateral triangle: once you’ve drawn the right angle and two of the equal sides, the third side is forced to be longer than the other two.
Cut out some paper triangles (use the BLM “Paper Triangles”). Fold each triangle in half to check if some of the sides of the triangles are equal. If the sides are equal, what can you say about the angles adjacent to these sides?

ACTIVITY 1
On a picture from a newspaper or a magazine find as many examples of different triangles and name them. Use two attributes, like “right-angled isosceles triangle.”

ACTIVITY 2
Give students a ruler, scissors, and a set of straws. Have them measure and cut a set of straws of the following lengths.

- 10 cm
- 9 cm
- 8 cm
- 6 cm
- 5 cm
- 5 cm
- 4 cm
- 4 cm
- 3 cm

**Question**

a) How many (distinct) right angle triangles can you make using the straws?
   **ANSWER:** 2 distinct triangles with sides of length 3, 4, 5 and 6, 8, 10.

b) How many isosceles triangles can you make? **ANSWER:** 9 triangles with sides of length: 3, 4, 4; 3, 5, 5; 4, 5, 5; 5, 5, 5; 5, 4, 4; 6, 4, 4; 6, 5, 5; 8, 5, 5; and 9, 5, 5

   **NOTE:** Ask students why side lengths 8, 4, 4; 9, 4, 4; 10, 4, 4; and 10, 5, 5 won’t work (see answer to Question e)

c) Can you make an equilateral triangle? **ANSWER:** Yes, with side lengths 5, 5, 5.

d) Show an example of an...
   i) obtuse triangle (one example 3, 4, 6) ii) a scalene triangle (one example 8, 9, 10) iii) acute triangle (one example 4, 5, 6)

e) Can you give a rule for determining the sets of straws that will make a triangle and those that won’t?

   **RULE:** A set of 3 straws will only make a triangle if the sum of the lengths of the two shortest straws is greater than the length of the longest (otherwise the two shorter straws will not meet). Here is an example of a set that doesn’t work (since 4 + 3 is less than 10).

- 10 cm
- 4 cm
- 3 cm
Extensions

1. I am a triangle. Sketch me and name me. Use two names for each triangle (isosceles acute-angled triangle, for example).
   a) I have two angles of 45°.
   b) I have two equal sides. My third side is only half of these two.
   c) I have three equal angles.
   d) I have two equal angles. One of my sides is one and a half of the other one.
      (Can you draw two different triangles?)
   e) One of my angles is larger than the sum of the other two.

   CHALLENGING:
   f) I have two equal angles. One of my sides is 3 times longer than another.
   g) One of my angles is 5° larger than the second angle and 5° smaller than the third angle.
   h) One of my angles is 30° larger than the second angle and 30° smaller than the third angle.
   i) One of my angles is 40° larger than the second angle and 40° smaller than the third angle.
   j) What is the largest whole number you could use in the last 3 questions for the difference between the angles?

2. If your students know what mean, median, range and mode are, you might ask the following question: Look at the sets of angles for the triangles g), h), i) in Extension 1. These are sets of 3 numbers each:
   
   Question g: 55, 60, 65;
   Question h: 30, 60, 90;
   Question i: 20, 60, 100.

   These sets have a common (circle all that apply): sum, difference, mean, median, range, mode.

   NOTE: There is no such thing as “difference of a set”. Students that say that there is a common difference might be guessing instead of checking.
Review drawing angles using a protractor and drawing lines of given measure using a ruler. Draw a horizontal line on the board. Explain to your students that sometimes they might need to draw a triangle of given dimensions. Suppose you need to draw a triangle with base of 60 cm and angles of 40° and 50°. Write the numbers on the board beside the ends of the line. Invite volunteers to draw the angles at the ends of the base, and ask another volunteer to extend the arms until they meet. Shade the ready triangle. Ask your students: The triangle has the angles of 40° and 50°. What should be the measure of the third angle? (90°) So which kind of triangle is that? Could we predict the result before drawing the triangle? Ask your students to predict the type of the triangle before drawing it, then draw the triangle and check the prediction.

5 cm, 30°, 60°  
6 cm, 30°, 30°  
4 cm, 70°, 85°  
3 cm, 30°, 110°  
6 cm, 90°, 45°

Remind your students about the folding activity they did last lesson. They checked that triangles that have two equal sides (What are they called?) have also equal angles. Do they think that it also works the other way? Suggest that your students check the lengths of the sides of the triangles that have pairs of equal angles. If your students are familiar with symmetry, ask them if the triangles have a line of symmetry. Ask them to draw the symmetry line and to explain why equal angles and equal sides always come together. (The two sides and the two angles that are symmetric in the isosceles triangle should be the same.)

Ask your students what type of triangle has two angles of 60°. What is the measure of the third angle? Does the length of the side change the type of this triangle?

Tell your students that now the task should be slightly different: You will give them two sides and the angle between the sides. Show your students how to draw the triangle on the board. Let them practice building the triangles like:

6 cm, 60°, 3 cm  
5 cm, 60°, 5 cm  
10 cm, 45°, 7 cm  
10 cm, 25°, 3 cm  
4 cm, 90°, 3 cm

What type of triangle do they get? Ask them to check the results. (The third triangle might look isosceles right-angled. It is not; let your students carefully measure the third side and the angles.) Could they predict the type of the triangle from the lengths of the sides and one angle? In which cases? (The second triangle is equilateral, and fifth triangle is right-angled.)

Let your students practice building triangles with the first activity.
Assessment
1. Draw a triangle with the base of 6 cm and the angles of 120° and 30°. What triangle is this?
2. Write three numbers that could be the degree measures of a right-angled triangle. Is the triangle with these angles equilateral, isosceles or scalene?

A Game in Pairs
The students will need two dice (red and blue), rulers and protractors. They start with a horizontal line of 5 cm. The first player rolls the dice. He multiplies the result of the red die by 10 and draws an angle of this measure using the 5 cm line as the base. The blue die gives the length of the side in cm. The player completes the triangle. The second player rolls the dice and builds the new triangle using the numbers from the dice and one of the sides of the existing triangle as the base. The triangles cannot overlap. The player who manages to build a triangle that exactly closes 360° around any vertex is the winner. For example, the game started from 6 red and 5 blue, which gave Triangle 1. The next player had 4 red and 1 blue, and drew Triangle 2. The picture shows the end of the game. Player 2 wins by drawing Triangle 10.
Create an equilateral triangle by paper folding.

First, create a square from a sheet of paper.

a) Fold the square in two (vertically, not diagonally) so that a crease divides the square into two rectangles.

b) Fold the top-right corner of the square down so that the top-right vertex meets the crease. The vertex should be slightly above the bottom edge.

c) Mark the point where the corner meets the crease and trace a line along the folded side of the square with a pencil.

d) Unfold. The right angle is now divided into three equal parts, and the traced line divides the square angle into two angles—one of 30° and the other of 60°. See Extension 1 in G5-3 for an explanation.

e) Repeat the steps b) – d) with the top-left corner of the square. The corner will meet the crease at the same point, which is the vertex of your triangle.

f) Cut the triangle out along the traced lines. You have a perfect equilateral triangle!

Extensions

1. The angle of a right pentagon is 108°. Use a protractor and a ruler to draw a right pentagon with the sides of 5 cm.

2. What is the sum of the angles of a pentagon? If a pentagon has 3 right angles and the two remaining angles are equal, what is the size of the remaining angles? Draw a pentagon with three sides of 5 cm and three right angles.

3. If you know the length of the three sides of a triangle, can you tell what type it is? If two sides are equal, what triangle is it? If all the three sides are different, take the two smaller sides and multiply each length by itself. (This is called “square the number”, and the number multiplied by itself is called “the square of the number”. For example, 25 is the square of 5, and 4 is 2 squared. Can you guess why?) Add the squares. Now square the largest side. If the sum of the squares of the smaller side is the same as the square of the largest side, the triangle is right-angled. If the sum of the squares of the smaller sides is larger than the square of the largest side, it is an acute angled-triangle. If the sum is smaller than the largest side squared, it is an obtuse-angled triangle. Construct the triangles from straws or toothpicks and check the claim:

   3, 4, 5  
   4, 4, 5  
   3, 4, 6

You will learn the explanation of the trick later, when you learn about Pythagoras Theorem.
G5-8
Parallel Lines

Explain that parallel lines are straight lines that are always the same distance apart. Like railway tracks, they never meet. Ask your students where they can see parallel lines around them. In the classroom, for instance (shelves, table sides or legs, the lines where the walls meet, etc.). Draw several parallel lines on the board. Show how to mark parallel lines with arrows.

Ask your students to draw a line (first horizontal, then vertical, then diagonal) in their notebooks (or on grid paper) and to draw another one parallel to it. Mark the parallel lines with arrows. On the grid on the board draw a right-angled but not isosceles triangle (sides of 4 and 3 squares). Ask your students to copy the triangle and to draw lines parallel to each of the sides. The purpose of this exercise is to draw a line parallel to a diagonal line on the grid so that it does not go at the half-right angle to the horizontal line.

Draw several shapes with some parallel sides on the board (it is important to have parallel lines with different slopes and of different length—use various trapezoids, parallelograms and hexagons and not only parallelograms and rectangles) and ask volunteers to mark pairs of parallel sides. You may also do the opposite task—draw a pair of parallel lines and ask your students to join the ends to make a quadrilateral.

Ask your students to draw a polygon that has:

a) exactly one pair of parallel sides
b) more than one pair of parallel sides
c) more than two pairs of parallel sides
d) three parallel sides

SAMPLE ANSWERS:
On a picture from a magazine or a newspaper find as many parallel lines as you can, marking each set with a different colour.

Ask your students if they can draw a parallelogram using only a ruler and a right-angled triangle.

**SOLUTION:**

**STEP 1:** Draw one side of the parallelogram.

**STEP 2:** Draw two of the parallel sides using the triangle.

**STEP 3:** Use the ruler to complete the figure.

**Extension**

In each face of a hexagonal prism, how many pairs of sides are parallel? How many sides intersect at a vertex of any prism? Explain how you know. (Fulfils the demands of the Western Curriculum.)
G5-9
Properties of Shapes

Draw several shapes on the board or use large paper shapes. Assign a letter to each. Draw a table on the board:

<table>
<thead>
<tr>
<th>Number of sides</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shapes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use volunteers to sort the shapes by the number of sides. After that you may sort the shapes by the number of parallel sides, right angles, etc. You may also wish to distribute the shapes—one for each student—and separate the students into groups using the shape properties like: number of parallel sides, number of sides, number of right angles.

Here is an activity that can be used to separate 18 students into pairs or 36 students into groups of four. The table below presents a sample of shapes, so that only two shapes satisfy each pair of properties as described below. The numbers in brackets are given for your convenience to show which shapes pair. Do not give the numbers to the students!

Give each student one shape.

Ask all the students who have shapes without right angles to stand on the right, those who have shapes with only one right angle to assemble in the middle, those who have shapes with two right angles to go to stand on the left and those with more to remain sitting. Now you can sort each of the groups by the number of pairs of parallel lines in their shapes—no parallel lines, one pair of parallel lines or more than one pair.

GOALS
Students will sort shapes according to the number of sides, parallel sides or angles. They will also learn the vocabulary.

PRIOR KNOWLEDGE REQUIRED
Polygons
Quadrilaterals
Parallel lines
Right angles

VOCABULARY
triangle
quadrilateral
hexagon
equilateral
pentagon
NOTE: The first number in the bracket is the number of right angles in each shape, and the second number is the number of pairs of parallel lines. For example, both shapes marked as (1,>1) have one right angle and more than one pair of parallel sides.

After this sorting activity let the students play a game. Students will need a set of shapes from the 2-D shapes game (see BLM) in addition to those they were given for the sorting activity. Player 1 (or Team 1) lays down 3 shapes that have a common feature (for instance, they might all have the same number of sides or all be equilateral). Player 2 (or Team 2) adds a fourth shape with the same feature, is told if he is right and if so, guesses what the shapes have in common. If Player 2 is not right, Player 1 adds another shape with the same feature, and Player 2 guesses anew. They continue adding shapes until Player 2 guesses correctly what the shapes have in common.

ADVANCED: Player 1 puts down 3 shapes that have a common feature and 1 extra shape that doesn’t have that feature, and Player 2 tries to find the shape that doesn’t belong and explains the choice.

Write the word “equilateral” on the board. Ask the students what little words they see in it. What might the parts mean? Where did they meet these parts? (“Equi” in “equal” and “lateral” in “quadrilateral”.) Ask students to extend the following chart to classify the pattern block pieces or a set of shapes from the shape game.

<table>
<thead>
<tr>
<th>Number of Right Angles</th>
<th>Number of Sides</th>
<th>Equilateral</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Ask students to make the shapes shown below with an elastic band on a geoboard. They should connect two pins with an elastic band to make:

a) 2 triangles

b) A quadrilateral and a triangle

---

**Extension**

**PROJECT:** Find the names of polygons and the sizes of the angles for regular polygons with up to 20 sides. Challenge students to find the name for a loonie—they will need to count the sides, by tracing and tallying. The name is “hendecagon”.

**POSSIBLE SOURCE:** [http://www.mathisfun.com/geometry/polygons.html](http://www.mathisfun.com/geometry/polygons.html).

---

**G5-10**

**Special Quadrilaterals**

Introduce your students to the various quadrilaterals with special names by holding up a large picture of each (or a large cardboard figure that can later be attached to the classroom wall). Ask your students where they have met these shapes in real life (rhombus and parallelograms may be trickier—diamond shape on playing cards, old bus or car windows—the latter may be trapezoids as well). It’s also a good idea to have these shapes with their names posted on the walls the entire time you are studying the geometry unit. Include the names of the special quadrilaterals on your next spelling test. Make columns on the board: rectangle, rhombus, square, parallelogram, other. Ask volunteers to put shapes in the right columns. Use the polygons from the shapes game (in the BLM) with tape on the back side.

**QUESTION 6** on the worksheet may be done with the geoboard.

**Assessment**

Who am I?

1. I am a quadrilateral, all my sides are equal. None of my angles is a right angle.
2. I have 4 sides and all my angles are the same. One of them is a right angle.
3. I have 4 sides and two of them are parallel. The other two are not.
4. I am a quadrilateral and my opposite sides are parallel.
5. I am a rhombus that has equal angles.

6. I am a quadrilateral, my opposite sides are parallel. Two of my angles are 45°. One pair of opposites sides is longer than the other.

7. I am a quadrilateral with two parallel sides. One of the parallel sides is twice as long as the other one. Draw me first!

---

**Extensions**

1. **WORD SEARCH:** acute, equilateral, hexagon, obtuse, parallelogram, pentagon, quadrilateral, rectangle, rhombus, square, trapezoid and triangle.

   M L E R U N L Z L L B E
   A A G L H S O L T N T R
   R R E R G O L G L U R A
   G E I L A N M Q A A A U
   O T E P U B A B A X P Q
   L A R E T A L I U Q E S
   E L G N A T C E R S Z H
   L I G T O A P U U T O M
   L R O A I N N T T R I R
   A D A G R E B L A E D N
   R A I O T O R E A P E R
   A U E N S A A O L A D X
   P Q A E T N U U R Q U P
2. There are three vertices of a square on the grid paper. Can you finish the square?

![Grid paper with three vertices of a square](image)

3. Can you draw a square without vertical sides on blank (non-grid) paper? Use a ruler.

4. Miss Maple is a Grade 4 teacher. She wants to teach her students about Venn diagrams. She decides that to help her students to sort the shapes, she will label all the regions of the diagram, and not only the circles. The first diagram she makes looks like this:

![Venn diagram with labels](image)

Names that both boys and girls can have

Miss Maple starts another Venn diagram, but doesn’t finish it. Place the words “parallelograms, rectangles, rhombuses, and squares” in the right place in an empty Venn diagram.
G5-11
Exploring Congruency

GOALS
Students will identify congruent shapes.

PRIOR KNOWLEDGE REQUIRED
Ability to count

VOCABULARY
congruent shapes

Explain that two shapes are congruent if they are the same size and shape. A pair of congruent two-dimensional figures will coincide exactly when one is placed one on top of the other. Have students actually do this with tangrams or pattern blocks. As it isn’t always possible to check for congruency by superimposing figures, mathematicians have found other tests and criteria for congruency.

A pair of two-dimensional figures may be congruent even if they are oriented differently in space (see, for instance, the figures below):

As a first test of congruency, your students should try to imagine whether a given pair of figures would coincide exactly if one were placed on top of another. Have them copy the shapes onto grid paper. Trace over one of them using tracing paper and try to superpose it. Are they congruent? Have students rotate their tracing paper and draw other congruent shapes. Let them also flip the paper!

You might also mention the origin of the word: “congruere” means “agree” in Latin.

Draw (or use an overhead projector) a grid on the board, and draw several shapes on the grid:

Ask volunteers to draw shapes congruent to those, but oriented differently.
**ACTIVITY 1**

Give each student a set of pattern block shapes and ask them to group the congruent pieces. Make sure students understand that they can always check congruency by superimposing two pieces to see if they are the same size and shape. Ask students how they moved a particular piece onto another one to check congruency. (Did they slide the piece? Did they have to rotate or flip the piece?)

**ACTIVITY 2**

Give students a set of square tiles and ask them to build all the non-congruent shapes they can find using exactly 4 blocks. They might notice that this is like Tetris game. Guide them in being organized. They should start with two blocks, and then proceed to three blocks. For each shape of 3 blocks, they should add a block in all possible positions and check whether the new shape is congruent to one of the previous ones. **BONUS:** 5 blocks.

**ANSWER:**

![Diagrams of non-congruent shapes using 4 blocks.]

**ACTIVITY 3**

Make 2 shapes on a geoboard. Use the pins to help you say why they are not congruent.

**EXAMPLE:**

The two shapes are not congruent: one has a larger base (you need 4 pins to make the base).
Review the previous lesson. Draw the first two shapes below on the board and ask your students to explain why they are congruent. Add the third shape and ask them to explain why this shape is not congruent to the other two. Repeat with general polygons. Start with different polygons (like a square and a pentagon) and then use more complicated pairs of shapes like two non-congruent triangles, with different patterns and differently oriented. Then let your students do the worksheet.

Extensions

1. How many non-congruent shapes can you make by adding a square to the figure?

2. How many non-congruent shapes can you make by removing one square from the $3 \times 3$ array?

   **ANSWER:** 3 shapes

3. How many non-congruent shapes can you make by removing 2 squares from the $3 \times 3$ array?

   **ANSWER:** 8 shapes

Encourage your students to proceed systematically in looking for the answer. For instance they might start by finding all the shapes they can make after they have removed a corner square:
Then they could try removing a middle square on the outside of the figure:

(Notice the last 2 shapes have been crossed out because they are already on the list for the previous figure.)

Finally, they could try removing the middle square (but all of the shapes that can be made after removing the middle square are already listed).
G5-13
Symmetry

Explain that a line of symmetry is a line that divides a figure into parts that have the same size and shape (i.e. into congruent parts), and that are mirror images of each other in the line.

You can check whether a line drawn through a figure is a line of symmetry by folding the figure along the line and verifying that the two halves of the figure coincide.

In the picture below, the dotted line cuts the figure into two parts that have the same size and shape.

But the two halves do not coincide when you fold the figure along the line: they are not mirror images of each other in the line. Hence, according to the second part of the definition, this line is not a line of symmetry:

Give an example using the human body. You may wish to draw a symbolic human figure on the board and mark the line of symmetry.

MORE EXAMPLES:

Challenge your students to find all the possible lines of symmetry for the triangle (3) and the star (5). Remind students of words “horizontal” and “vertical”.

Have the students do the worksheet.

Assessment

Draw a shape with...

a) a horizontal line of symmetry
b) a vertical line of symmetry
c) a slanted line of symmetry
d) two lines of symmetry
e) four lines of symmetry
f) six lines of symmetry
Bonus
Draw figures to fill the Venn diagram:

1. Have vertical line of symmetry
2. Have horizontal line of symmetry
3. Have both lines of symmetry

Draw some figures that can go outside of both circles. Can they still have a line of symmetry?

Extensions
1. How many lines of symmetry does an oval have?

2. What geometrical shape has an infinite number of lines of symmetry? (Answer: The circle)

3. Sudha drew a mirror line on a square. Then she drew and shaded a reflection of the corner of the square in the mirror line:
a) Draw a large square on grid paper. Draw a mirror line on the square and reflect part of the square in the line.

b) Can you place the mirror line so that the two parts of the square on either side of the mirror line make:
   i. A rectangle
   ii. A hexagon (a shape with 6 sides)
   iii. An octagon (a shape with 8 sides)

4. Complete the figure so that it will have a line of symmetry and will have the least area.

5. Take a photo of a family member’s face (such as an old passport photo) and put a mirror along the symmetry line. Look at the face that is half the photo and half the mirror image. Does it look the same as the photo?

6. CROSS-CURRICULUM CONNECTION: Check the flags of Canadian provinces for lines of symmetry. POSSIBLE SOURCE:

G5-14
Comparing Shapes

Draw a regular hexagon on the blackboard. Ask your students if it has any right angles. How many pairs of parallel sides does it have? Is it an equilateral shape? How many lines of symmetry does it have? Have volunteers mark the parallel sides and the lines of symmetry. Then draw a hexagon with two right angles and make the comparison chart shown below. Ask volunteers to help you fill in the chart.

<table>
<thead>
<tr>
<th>Property</th>
<th>Shape 1</th>
<th>Shape 2</th>
<th>Same?</th>
<th>Different?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of vertices</td>
<td>6</td>
<td>6</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of edges</td>
<td>6</td>
<td>6</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of pairs of parallel sides</td>
<td>3</td>
<td>3</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of right angles</td>
<td>0</td>
<td>2</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of acute angles</td>
<td>0</td>
<td>0</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of obtuse angles</td>
<td>6</td>
<td>4</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Any lines of symmetry?</td>
<td>yes</td>
<td>yes</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of lines of symmetry</td>
<td>6</td>
<td>2</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Is the figure equilateral?</td>
<td>yes</td>
<td>no</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

Invite volunteers to write a comparison of the two shapes using the chart.

**Assessment**
Write a comparison of the two shapes. Be sure to mention the following properties:

- The number of vertices ✓
- The number of edges ✓
- The number of pairs of parallel sides ✓
- The number of right angles ✓
- The number of symmetry lines ✓
- Whether the figure is equilateral ✓
Give each student (or team of students) a deck of shape cards and a deck of property cards. (These cards are in the BLM section.) Let them play the first game in Activity 1 below. The game is an important preparation for Venn diagrams.

Draw a Venn diagram on the board and do the first exercise on the worksheet using volunteers. Remind your students that any letters that cannot be placed in either circle should be written outside the circles (but inside the box). FOR EXAMPLE, the answer to QUESTION 1 a) of the worksheet should look like this:

Also remind students that figures that share both properties, in this case A and H, should be placed in the overlap.

Let your students play the game in Activity 2. Then draw the following set of figures on the board:
Here are some properties students could use to sort the figures in a Venn diagram.

a) 1. At least two right angles
   2. Equilateral

b) 1. Equilateral
   2. Has exactly one line of symmetry
   (In this case the centre part contains only one irregular pentagon—figure J)

c) 1. Has more than one pair of parallel sides
   2. Has a right angle

d) 1. Is a pentagon
   2. Equilateral

Assessment
Use the same figures as above to make a Venn diagram with properties:

1. Exactly one line of symmetry
2. Has at least two right angles

Draw two figures on the board:

You may tell students that young wizards in a wizard school learn to transfigure figures. However, they have to be able to give a complete description of a figure before and after the transformation. Use volunteers to describe the figures completely, each figure separately. Do not ask: “How many vertices does the shape have?”, but rather let your students recall the properties themselves. The description should mention:

• Number of sides
• Number of right angles
• Number of vertices
• Number of lines of symmetry
• Number of pairs of parallel sides
• Is the figure equilateral?

After that ask them to write the comparison of the shapes.

Bonus
Write the “Transfiguration formula”—Change number of vertices from 4 to 5, etc.

Assessment
Name all properties the figures have in common. Then describe any differences:

Play the game called Always/Sometimes/Never True (Shapes)—see the BLM for the worksheet for this game. It will help students sharpen their understanding of two-dimensional shapes.
2-D Shape Sorting Game
Each student flips over a property card and then sorts their shape cards onto two piles according to whether a shape on a card has the property or not. Students get a point for each card that is on the correct pile. (If you prefer, you could choose a property for the whole class and have everyone sort their shapes using that property.)

Once students have mastered this sorting game they can play the next game.

2-D Venn Diagram Game
Give each student a copy of the Venn diagram sheet in the BLM section (or have students create their own Venn diagram on a sheet of construction paper or bristol board). Ask students to choose two property cards and place one beside each circle of the Venn Diagram. Students should then sort their shape cards using the Venn diagrams. Give 1 point for each shape that is placed in the correct region of the Venn diagram.

Extensions
1. A Game for Two: Player 1 draws a shape without showing it to Player 2, then describes it in terms of number of sides, vertices, parallel sides, right angles, lines of symmetry, etc. Player 2 has to draw the shape from the description.

2. Draw a Venn diagram for special quadrilaterals: trapezoids, parallelograms, rectangles, squares, rhombs. As an easier version you may provide the drawing of the diagram and ask to fill in the names, like in Extension 4 to G5-10.
## Problems and Puzzles

This worksheet reviews the properties of shapes and consolidates the knowledge of the **GEOMETRY** unit in Workbook 1.

### More Review Questions

<table>
<thead>
<tr>
<th>Questions</th>
<th>Possible Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) I have five sides and one line of symmetry.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>Two of my angles are right angles.</td>
<td></td>
</tr>
<tr>
<td>Draw me.</td>
<td></td>
</tr>
<tr>
<td>b) I have five sides and one line of symmetry.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>Three of my angles are right angles.</td>
<td></td>
</tr>
<tr>
<td>Draw me.</td>
<td></td>
</tr>
<tr>
<td>c) I have four sides and no lines of symmetry.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>My opposite sides are equal. Draw me.</td>
<td></td>
</tr>
<tr>
<td>d) I have four sides and two lines of symmetry that go through the vertices.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>Draw me. What is my name?</td>
<td></td>
</tr>
<tr>
<td>e) I am a quadrilateral with 4 lines of symmetry.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>What am I?</td>
<td></td>
</tr>
<tr>
<td>f) I have three sides, one line of symmetry and a right angle. Draw me and name me.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>g) I have three sides, one line of symmetry and one angle of 50°. Draw me and name me.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>h) I have three sides and two angles of 30°. Name me.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>Isosceles obtuse - angled triangle</td>
<td></td>
</tr>
</tbody>
</table>

Which two special quadrilaterals have exactly two lines of symmetry? What is the difference in the position of the lines of symmetry in these quadrilaterals?
G5 Part 1: BLM List

2-D Shape Sorting Game _________________________________________________2
Always/Sometimes/Never True (Shapes) _____________________________6
Paper Triangles ______________________________________________________7
Pattern Blocks _______________________________________________________8
Venn Diagram ______________________________________________________9
Word Search Puzzle (Shapes) ________________________________________10
2-D Shape Sorting Game

- Square
- Triangle
- Pentagon
- Hexagon
- Trapezoid
- Rectangle
2-D Shape Sorting Game (continued)
### 2-D Shape Sorting Game (continued)

<table>
<thead>
<tr>
<th>Three or more sides</th>
<th>Four or more vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three or more vertices</td>
<td>More than one line of symmetry</td>
</tr>
<tr>
<td>No lines of symmetry</td>
<td>Hexagon</td>
</tr>
</tbody>
</table>
## 2-D Shape Sorting Game (continued)

<table>
<thead>
<tr>
<th>Quadrilateral</th>
<th>No right angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>One right angle only</td>
<td>Equilateral</td>
</tr>
<tr>
<td>At least five sides</td>
<td>Two or more sides</td>
</tr>
</tbody>
</table>
### Always/Sometimes/Never True (Shapes)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hexagons have 6 equal sides.</td>
<td>A triangle can have 2 right angles.</td>
<td>Figures that have the same shape are congruent.</td>
</tr>
<tr>
<td>D</td>
<td>E</td>
<td>F</td>
</tr>
<tr>
<td>Rhombuses have 4 unequal sides.</td>
<td>A right-angle triangle does not have any right angles.</td>
<td>Parallelograms are rectangles.</td>
</tr>
<tr>
<td>G</td>
<td>H</td>
<td>I</td>
</tr>
<tr>
<td>Squares are equilateral.</td>
<td>A quadrilateral can have 3 right angles.</td>
<td>Hexagons and pentagons can be congruent.</td>
</tr>
<tr>
<td>J</td>
<td>K</td>
<td>L</td>
</tr>
<tr>
<td>A right-angle triangle can be similar to a triangle without a right angle.</td>
<td>Quadrilaterals always have at least 2 parallel lines.</td>
<td>A shape with 4 right angles is a rectangle or square.</td>
</tr>
<tr>
<td>M</td>
<td>N</td>
<td>O</td>
</tr>
<tr>
<td>Hexagons are equilateral.</td>
<td>Polygons have more than 2 lines of symmetry.</td>
<td>Pentagons have 5 lines of symmetry.</td>
</tr>
</tbody>
</table>

Choose a statement from the chart above and say whether it is **always** true, **sometimes** true, or **never** true. Give reasons for your answer.

1. What statement did you choose? Statement Letter ________
   This statement is...
   
   **Always True**                                         **Sometimes True**                                      **Never True**
   
   Explain: ________________________________________________
   
   If you said false, give an example that proves the statement is false:
   
   ________________________________________________________
   
   If the statement is sometimes true, rewrite the statement so that it is always true:
   
   ________________________________________________________

2. Choose a statement that is sometimes true, and reword it so that it is always true.
   What statement did you choose? Statement Letter ________
   Your reworded statement: ________________________________________________________

3. Repeat the exercise with another statement.
Paper Triangles
Pattern Blocks

Triangles

Squares

Rhombuses

Trapezoids

Hexagons
Venn Diagram
## Word Search Puzzle (Shapes)

Word search puzzle—can also be assigned as homework.

<table>
<thead>
<tr>
<th>I</th>
<th>a</th>
<th>o</th>
<th>l</th>
<th>g</th>
<th>i</th>
<th>o</th>
<th>l</th>
<th>d</th>
<th>l</th>
<th>r</th>
<th>l</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>e</td>
<td>n</td>
<td>e</td>
<td>e</td>
<td>a</td>
<td>e</td>
<td>g</td>
<td>l</td>
<td>g</td>
<td>r</td>
<td>o</td>
</tr>
<tr>
<td>r</td>
<td>n</td>
<td>i</td>
<td>l</td>
<td>l</td>
<td>i</td>
<td>s</td>
<td>e</td>
<td>v</td>
<td>c</td>
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<td>v</td>
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<td>v</td>
<td>t</td>
<td>p</td>
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<td>e</td>
<td>g</td>
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<td>a</td>
<td>c</td>
<td>l</td>
<td>g</td>
<td>l</td>
<td>e</td>
<td>i</td>
<td>a</td>
<td>y</td>
<td>a</td>
<td>v</td>
<td>p</td>
</tr>
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<td>a</td>
<td>a</td>
<td>r</td>
<td>s</td>
<td>o</td>
<td>i</td>
<td>i</td>
<td>r</td>
<td>e</td>
<td>r</td>
</tr>
<tr>
<td>i</td>
<td>e</td>
<td>e</td>
<td>t</td>
<td>a</td>
<td>p</td>
<td>e</td>
<td>e</td>
<td>i</td>
<td>i</td>
<td>i</td>
<td>v</td>
</tr>
<tr>
<td>r</td>
<td>o</td>
<td>e</td>
<td>n</td>
<td>i</td>
<td>p</td>
<td>n</td>
<td>r</td>
<td>t</td>
<td>t</td>
<td>t</td>
<td>t</td>
</tr>
<tr>
<td>d</td>
<td>x</td>
<td>s</td>
<td>e</td>
<td>c</td>
<td>i</td>
<td>t</td>
<td>r</td>
<td>e</td>
<td>v</td>
<td>n</td>
<td>x</td>
</tr>
<tr>
<td>a</td>
<td>i</td>
<td>o</td>
<td>p</td>
<td>o</td>
<td>l</td>
<td>y</td>
<td>g</td>
<td>o</td>
<td>n</td>
<td>t</td>
<td>a</td>
</tr>
<tr>
<td>u</td>
<td>a</td>
<td>q</td>
<td>q</td>
<td>a</td>
<td>e</td>
<td>s</td>
<td>n</td>
<td>q</td>
<td>s</td>
<td>a</td>
<td>e</td>
</tr>
<tr>
<td>q</td>
<td>l</td>
<td>n</td>
<td>n</td>
<td>x</td>
<td>s</td>
<td>r</td>
<td>g</td>
<td>q</td>
<td>e</td>
<td>s</td>
<td>t</td>
</tr>
</tbody>
</table>

### WORDS TO SEARCH:
- triangle
- vertices
- quadrilateral
- sides
- pentagon
- angle
- vertex
- polygon
PA5-24
Finding Rules for T-tables—Part 1

Make a simple design from pattern blocks, like the one in the picture.

Ask your students: How many pentagons did I use? How many triangles? How many triangles and how many pentagons will I need for two such designs? For three designs? Remind your students that they previously used T-tables to solve this type of question. Invite volunteers to draw a T-table and to continue it to 5 rows. **ASK:** I want to make 20 such designs. Should I continue the table to check how many pentagons and triangles I need? Can you find a more efficient way to find the number of pentagons and triangles? How many triangles are needed for one pentagon in the design? What do you do to the number of pentagons to find the number of triangles in a set of designs?

Ask volunteers how to derive the number of triangles in a particular row of the T-table from the number of pentagons. Students should see that the procedure they follow to find the number of triangles can be expressed as a general rule: “The number of triangles is 5 times the number of pentagons”. Mathematicians often use letters instead of numbers to express this type of rule. For example, they would use “p” for the number of pentagons and “t” for the number of triangles, and get the rule “5 × p = t”. This rule is called a “formula” or “equation”. Write these terms on the board beside the formula itself.

Write another formula, such as 4 × s = t. Explain that “s” represents the number of squares and “t” is the number of triangles as before. What rule does the formula express? What do you have to do to the number of squares to get the number of triangles? You can make a table of values that matches this formula. All you have to do is to write the actual number of squares instead of “s” and do the multiplication. The result of the multiplication is the number of triangles. Draw a T-table:

<table>
<thead>
<tr>
<th>Number of Squares (s)</th>
<th>Formula (4 × s = t)</th>
<th>Number of Triangles (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4 × 1 = 4</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>4 × 2 = 8</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4 × 3 = _</td>
<td></td>
</tr>
</tbody>
</table>

Ask volunteers to fill in the missing numbers and to add two more lines to the table. Ask them to find the number of triangles for 25 figures (25 squares). Let your students practice drawing T-tables for more formulas, like: 3 × t = s, 6 × s = t (t – number of triangles, s – number of squares).

Ask your students to create designs to go with the formulas above.
Students could make a design using concrete materials (as in QUESTIONS 3 and 4 in the worksheets) and predict how many of each element in their design they would need to make 8 copies.

**Extension**

Find the rules by which the following T-tables were made.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
</tr>
<tr>
<td>3</td>
<td>120</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250</td>
</tr>
<tr>
<td>2</td>
<td>500</td>
</tr>
<tr>
<td>3</td>
<td>750</td>
</tr>
</tbody>
</table>

**PA5-25**

**Finding Rules for T-Tables—Part II**

**GOALS**

Students will find simple additive, multiplicative or subtractive rules for T-tables.

**PRIOR KNOWLEDGE REQUIRED**

Increasing sequences

**VOCABULARY**

T-table, formula, input, output

Start with a simple problem: Rose invites some friends to a party. She needs one chair for each friend and one for herself. Can you provide Rose with a formula or equation for the number of chairs?

Ask your students to suggest a letter to use for the number of friends and a letter for the number of chairs. Given the number of friends how do you find the number of chairs? Ask your students to write a formula for the number of chairs. Suggest that your students make a T-table similar to the one they used in the last lesson for multiplicative rules.

Give your students several questions to practice writing rules and making T-tables, such as:

a) Lily and Rose invite some friends to a party. How many chairs do they need?

b) Rose, Lily and Pria invite some friends to a party. How many chairs they need? They invited 20 friends. How many chairs will they need?

c) A family invited several friends to a party. The number of chairs they need is 6 + f = c. How many people are in this family? If they invited 10 friends, how many chairs will they need?

Ask your students to write a problem for the formula 4 + f = c.

Explain to your students that the number that you put into a formula in place of a letter is often called the “input”. The result that the formula provides—
the number of chairs, for instance—is called the “output”. Write these terms on the board and ask volunteers to circle the input and underline the output in the formulas you have written on the board.

Draw several T-tables on the board, provide a rule for each and ask your students to fill in the tables. Start with simple inputs like 1, 2, 3 or 5, 6, 7 and continue to more complicated combinations like 6, 10, 14 and so on.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

The rules you provide should be additive (Add 4 to the input), multiplicative (Multiply the input by 5) and subtractive (Subtract 3 from the input).

Suggest that your students try a more complicated task—you write a table and they have to produce a rule for it. Ask them to think what was done to the input to get the output. Give them several simple tables, like:

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>18</td>
</tr>
</tbody>
</table>

Assessment

1. Complete the tables:

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>53</td>
<td></td>
</tr>
</tbody>
</table>

Add 13 to the input

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

Multiply the input by 11

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td></td>
</tr>
</tbody>
</table>

Subtract 5 from the input

2. Find the rules and the formulas (or equations) for the tables:

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>13</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>21</td>
</tr>
<tr>
<td>21</td>
<td>28</td>
</tr>
<tr>
<td>23</td>
<td>30</td>
</tr>
</tbody>
</table>
**Bonus**

Find the rule, the formula and the output for input = 7 and input = 10.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>111</td>
</tr>
<tr>
<td>2</td>
<td>222</td>
</tr>
<tr>
<td>3</td>
<td>333</td>
</tr>
</tbody>
</table>

Each student will need a spinner as shown and a die. Spin the spinner and roll the die. Write a formula for the rule given by the spinner and the die. For example, if the spinner shows “Multiply” and the die shows 3, the rule is “Multiply by 3”, and the formula is “3 × A = B”.

**ACTIVITY 1**

- Add
- Multiply
- Subtract

**A Game for Two**

Each pair of students will need a die and a spinner as in the previous activity. Player 1 spins a spinner and rolls the die as before, but so that Player 2 does not see the result. Player 1 writes the rule and the formula given by the spinner and the die and gives the other player 3 pairs of input and output numbers. Player 2 has to guess the formula.
Part 2
Patterns & Algebra

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PA5-26
Finding Rules for Patterns—Part I

Draw the following sequence of figures on the board and tell your students that the pictures show several stages in construction of a castle made of blocks.

How will they keep track of the number of blocks needed for the castle? Invite volunteers to make a T-table with two columns—the number of triangles and the number of squares.

Ask your students to find a verbal rule and a formula that tell how to get the number of squares from the number of triangles. Could they predict the formula before building a T-table? There are three squares for every triangle. So there are three times more squares than triangles, and we can write that as “Multiply the number of triangles by 3” or “3 × t = s”.

Repeat with the next block pattern:

Let your students practice with various patterns with multiplicative or additive rules.

Assessment

Make a T-table and write a formula and a rule for the patterns:

a) (use s for shaded squares and u for the un-shaded squares)

b) (use r for rhombuses and t for trapezoids)
Extension
A dragon has 44 teeth and 4 poisonous spikes on its tail. A dragon breeder uses 2 formulas: 
\[ t = 44 \times d, \quad s = 4 \times d. \]
What does each formula describe? Can you write a formula that relates the quantities \( t \) and \( s \), and does not involve \( d \)?

**HINT:** Make a T-table with columns: dragons, spikes, teeth. Students should derive the formula \( t = 11 \times s \). They should also recognize that not all inputs make sense. A dragon can’t have only one spike, for instance. Since a dragon has 4 spikes on its tail, only inputs that are multiples of 4 make sense.

PA5-27
Direct Variation

Show your students several sequences made of blocks with a multiplicative rule, such as:

![Image of blocks](image-url)

Invite volunteers to draw T-tables for the number of triangles and the number of squares:

<table>
<thead>
<tr>
<th>Figure Number (f)</th>
<th>Number of Squares (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure Number (f)</th>
<th>Number of Triangles (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of Squares (s)</th>
<th>Number of Triangles (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ask your students to write a rule and a formula for each table. Remind your students the meaning of the terms “input” and “output”.

Explain to your students that when the rule is “Multiply the input by ___”, we say that the output varies directly with the input. So in this pattern the number of squares varies directly with the Figure Number, and the Number of Triangles varies directly with both the Figure Number and the Number of Squares.

Draw several tables with columns “Input” and “Output” and ask your students to give examples of rules that will make the output vary directly with the input. (Multiply the input by ___.) Invite volunteers to fill in the tables according to the rules. Vary the input from 1, 2, 3 to combinations like 5, 6, 7 and then to 4, 6, 8. Next pass to more complicated input combinations, such as 12, 8, 4 and even 22, 7. After that write several rules such as “Multiply the input by 22”, “Add 7”, “Multiply by 5 and subtract 3” and ask your students to tell
which rules make the output vary directly with the input.

Draw several tables on the board and ask your students to tell in which of these tables the output is a direct variation of the input. **EXAMPLES:**

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
</tr>
</tbody>
</table>

**yes**

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
</tbody>
</table>

**no**

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
</tbody>
</table>

**no**

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>19</td>
</tr>
<tr>
<td>7</td>
<td>23</td>
</tr>
</tbody>
</table>

**no**

**PROMPTS:** What do you do to the input to get the output, when the output varies directly with the input? Is the output in the table a multiple of the input? (Check each row separately) What number did you multiply the input by in each row? Suggest that students write these numbers beside the table in the form if a multiplication statement. For instance, in the last table students could write:

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>45</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
</tr>
<tr>
<td>7</td>
<td>35</td>
</tr>
</tbody>
</table>

**yes**

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>24</td>
</tr>
</tbody>
</table>

**no**

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>22</td>
</tr>
<tr>
<td>11</td>
<td>121</td>
</tr>
<tr>
<td>30</td>
<td>330</td>
</tr>
</tbody>
</table>

**yes**

The numbers in bold (the second factors) are the same, so the output varies directly with the input.

Draw a sequence of squares with sides 1, 2, 3, etc. Ask your students to find the areas and the perimeters of the squares. Ask them to make a T-table for both and to check which quantity varies directly with the side length (perimeter). Encourage them to write a formula not only for the perimeter, but also for the area of the square.

Present a problem:

The number of feet (f) varies directly with the number of people (p) (2 people—4 feet, 3 people—6 feet, 4 people—8 feet, f = 2 × p). Does the number of paws vary directly with the number of cats? What is the formula?

A cat has five claws on each front paw and four claws on each back paw. Make a T-table showing the number of cats and the number of claws and then another T-table showing the number of paws (add one paw at a time!) and the number of claws. Does the total number of claws vary directly with the number of paws or with the number of cats?
Assessment

Circle the tables where the input varies directly with the output:

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>15</td>
</tr>
<tr>
<td>8</td>
<td>18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>18</td>
</tr>
</tbody>
</table>

Extension

Find a sequence of rectangles so that the area will vary directly with length. Does the perimeter vary directly with the length?
PA5-28
Finding Rules for Patterns—Part II

GOALS
Students will find the varying part of the pattern rule of type “t = 2n + a”, where n is the figure number.

PRIOR KNOWLEDGE REQUIRED
Increasing sequences  
T-tables  
Direct variation

VOCABULARY
T-table  
formula  
input  
output  
direct variation

Draw the following sequence of figures on the board and tell your students that the pictures show several stages in construction of a castle made of blocks.

![Castle Construction Figures]

How could they find a formula to keep track of the number of blocks needed for the castle? The castle has two towers and a gate between them. The gate does not change from figure to figure, but the towers grow. Ask your students to find the rule for the number of blocks in the towers (2 × Figure Number). They can use a T-table to find the rule if needed. After that ask them to find a T-table for the total number of blocks in the figures. In which T-table does the number of blocks in the output column vary directly with the figure number—the T-table showing the number of blocks in the towers or the one showing the total number of blocks?

Give your students several more block patterns, shade the part that varies directly with the figure number, as in QUESTION 1 of the worksheet, and ask them to find the rule for the number of shaded blocks.

Ask your students to create a sequence of figures that goes with the table and to shade the part of the figures that varies directly with the figure number. They should leave the part that does not change from figure to figure unshaded.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
</tbody>
</table>

Extension
A cab charges a $3.50 flat rate (that you pay just for using the cab) and $2 for every minute of the ride. Write a formula for the price of a cab ride. How much will you pay for a 4-minute cab ride? For a 5-minute ride?
PA5-29
Predicting the Gap Between Terms in a Pattern

GOALS
Students will understand the connection between the gap in the sequence and the part that varies directly.

PRIOR KNOWLEDGE REQUIRED
Increasing sequences
Difference in sequences
T-tables
Direct variations

VOCABULARY
T-table
input
direct variations
gap
formula
output
difference
gap

ACTIVITY
A game for pairs: The students will need pattern blocks and two dice of different colors. Player 1 rolls the dice so that player 2 does not see the result. He builds a pattern of figures according to the rule: The number of blocks = Figure number \times \text{ Result on the red die } + \text{ Result on the blue die}. Player 2 has to guess what the results on the dice were.

Give your students a pair of dice of different colors each and ask them to try the following game. Roll the dice, and write a sequence according to the rule: to find each term multiply the term number by the result on the red die and add the result on the blue die. Ask your students to find the difference between the terms of their sequences each time. After they have created several sequences, ask them what they have noticed. Likely, the students have noticed that the difference (gap) in the sequence equals the result of the red die, which is the multiplicative factor.

Draw or make the following sequence:

Ask students to describe what part of the pattern changes and what part stays the same? Draw a T-table for the number of blocks in each figure of the sequence as shown.

Ask students to predict what the gap between the terms in the output column (the “Number of Blocks” column) will be before you fill in the column. Students should see that the “gap” between terms in the T-table is simply the number of new blocks added to the pattern at each stage. **ASK:** What do you add to each figure to get the next one? How many blocks do you add? Students should also see that to find the number of shaded blocks in a particular figure they simply multiply the figure number by the “gap” (since the gap is the number of new shaded blocks added each time). In the next lesson, students will learn how to use this insight to find rules for patterns made by multiplication and addition (or multiplication and subtraction).
PA5-30
Finding Rules for T-tables—Part III

Explain to your students that today they will continue to find rules for patterns but their task will be harder—you will not show them a particular block pattern, you will only give them the number of blocks used to build the pattern. Remind your students that the patterns they have worked with usually consisted of two parts: the part that stays the same and the part that varies with the number of the figure.

Draw the following sequence of figures on the board:

Figure 1 Figure 2 Figure 3

Ask a volunteer to shade the part of each figure that varies directly with the figure number (the squares). Write the following T-table for the pattern on the board and ask students to fill in the gap between the numbers in the output column.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Figure Number × Gap</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td></td>
</tr>
</tbody>
</table>

Then ask students to fill in the middle column of the table:

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Figure Number × Gap</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 × 3 = 3</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>2 × 3 = 6</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>3 × 3 = 9</td>
<td>11</td>
</tr>
</tbody>
</table>

Ask your students what the numbers in the middle column of the T-table correspond to in the pattern. Students should see that the “gap” is simply the number of squares that are added at each stage in the pattern. So to find the number of squares in each figure, you simply multiply the gap by the figure number. (In other words, the numbers in the middle column—3, 6, 9—simply gave the number of squares in the successive figures.)
Students should also see that to find the total number of blocks (the number in the output column), they just need to add an adjustment factor (the number of triangles) to the number in the middle column.

\[
\begin{array}{|c|c|c|}
\hline
\text{Figure Number} & \text{Figure Number} \times \text{Gap} & \text{Number of Blocks} \\
\hline
1 & 1 \times 3 = 3 & 5 \\
2 & 2 \times 3 = 6 & 8 \\
3 & 3 \times 3 = 9 & 11 \\
\hline
\end{array}
\]

Hence the rule for the T-table is: Multiply the figure number by 3 (the gap) and add 2 (the adjustment factor). Students should notice that the adjustment factor corresponds to the number of triangles in each figure: this number is constant and does not change from figure to figure).

Give your students practice at finding rules for various T-tables.

**Example 1**

**STEP 1:** Input

<table>
<thead>
<tr>
<th>Input</th>
<th>Input × Gap</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>16</td>
</tr>
</tbody>
</table>

By what amount does the output increase? (In this case, the gap is 3.)

**STEP 2:** Multiply the gap by the input.

**STEP 3:** What must you add to each number in the second column to get the output? In the T-table above, the fixed amount of increase (gap) is 3. **NOTICE:** Multiplying the input 2 by 3 gives 6, one less than the output 7. Multiplying the input 3 by 3 gives 9, one less than the output 10. Multiplying the input 4 by 3 gives 12, one less than the output 13, etc.

**STEP 4:** The rule is: Multiply by 3 and add 1.

**Example 2**

**STEP 1:** Input

<table>
<thead>
<tr>
<th>Input</th>
<th>Input × Gap</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>19</td>
</tr>
</tbody>
</table>

The output increases by 4

**STEPS 2, 3:** **NOTICE:** Multiplying the input 2 by 4 gives 8, one more than the output 7. Multiplying the input 3 by 4 gives 12, one more than the output 11, etc.
STEP 4: The rule is: Multiply by 4 and subtract 1. If the input does not increase by 1 and the output does not increase by a fixed amount, your student should find the rule for the T-table by guessing and checking.

NOTE: It is extremely important that your students learn to find rules for T-tables by the method outlined above as all of the applications of T-tables in this unit involve finding such rules.

Let your students practice finding the rules for several patterns. In the first examples tell them whether you add or subtract something to get the output from the multiple input × gap, then use mixed questions. You may also use the activity below for practice.

After that you may draw (or build) several more complicated block patterns and ask the students to find the rules for them. Give them more practice questions, like:

Find the rules for the perimeter and the area of the following figures. Use your rules to predict the perimeter and the area of term 15.

For ASSESSMENT, you may ask students how many inner line segments the 20th figure has in this pattern.

For a more advanced activity, draw several block patterns like the one shown below (or use pattern blocks for each sequence) and ask your students to make a T-table for the number of blocks of each type, and for the total number of blocks in each figure.

Ask your students to find a rule for each T-table and to describe the parts of the pattern that change and the parts that stay the same.

**Bonus**

Can you draw or build a sequence of figures where the number of blocks in each figure is given by the rule

a) Number of Triangles = Figure Number + 3
b) Number of Squares = 3 × Figure Number – 2
c) Number of Squares = 2 × Figure Number – 1
A Game for Pairs
Each pair of students will need a coin with the signs + and – on its sides and two dice. Player 1 throws the dice and the coin so that Player 2 does not see the results. Suppose that the larger number rolled is L and the smaller is S. Player 1 writes the first three terms of the sequence of numbers according to the rule: The number of blocks = Figure number × L +/– S. The coin defines the sign. Player 2 has to guess the results of the dice roll.

ADVANCED: Instead of the coin, students may use the spinner from ACTIVITY 1 of PA5-25: Finding Rules for T-tables. If the spinner reads “Add” or “Subtract”, the rule is as before. If the spinner says “Multiply”, the player multiplies the Figure Number (or Input) by the product of the two numbers rolled, so that the sequence rule will be multiplicative only.

Once students master the first two games, they may give each other 4 random terms (two of them adjacent, like 2, 3) of the sequence, as in the table below, rather than giving the first three terms.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>29</td>
</tr>
<tr>
<td>6</td>
<td>21</td>
</tr>
</tbody>
</table>

Extension
Can you draw or build a sequence of figures using triangles, squares and trapezoids where the number of each type of block will vary according to all of these rules?

• Number of triangles: Figure number + 2
• Number of squares: 2 × Figure number – 1
• Number of trapezoids: 4 × Figure number – 4

Write a formula that tells you how to find the total number of blocks in a figure from the figure number.
PA5-31
Patterns Created Using One Operation

Let your students play a game in pairs. They will need a pair of dice and a spinner from Activity 1 of PA5-25: Finding Rules for T-tables. Player 1 rolls the dice and spins the spinner so that Player 2 does not see the result of the spinner. He adds, subtracts or multiplies the numbers rolled according to the instructions on the spinner, and presents both numbers and the result of the operation to Player 2. Player 2 has to tell what the spinner read. For example, if the dice give 2 and 6, and the spinner reads "Subtract", Player 1 has to write: 6 \( \bigcirc \) 2 = 4, and Player 2 has to deduce that the sign is "-".

After that you may write several sequences made by multiplication and ask your students to continue them. In the beginning tell them what the factor is:

a) \( \times 2 \quad 5, 10, \_, \_, \_ \)  
b) \( \times 4 \quad 2, 8, \_, \_, \_ \)  
c) \( \times 3 \quad 3, 9, \_, \_, \_ \)

Then tell your students that you are going to make the task harder: they will have to find the factor. Let them practice with questions like:

a) \( \times \quad 15, 30, 60, \_, \_, \_ \)  
b) \( \times \quad 2, 10, 50, \_, \_, \_ \)  
c) \( \times \quad 3, 12, 48, \_, \_, \_ \)

Explain to your students that since they are doing very well, you are sure they will succeed in the next task. You will give them several sequences made using one operation: addition, multiplication or subtraction. You will not tell them which operation was used, what number you added or subtracted from the term, or what number you multiplied the term by. They will have to continue the sequence and write a rule for each sequence. Remind your students of the way they write the rules for sequences: Start at __ and ________. Discuss with the class what strategies can be used to decide which operation and numbers were used. **ASK:** If the sequence was made by addition, is it increasing or decreasing? By subtraction? By multiplication? If the first two numbers in the sequence are 2 and 4, how can I tell whether the sequence was made by addition or multiplication? What should I check? The students might reason as follows:

Look at the difference between the first two numbers. Check if the sequence is increasing or decreasing. If it is decreasing, subtraction was used and I have to check the difference between the numbers. If the sequence is increasing, it is either addition or multiplication, so I should...
look at the difference between the first and the second terms, then at the difference between
the second and the third terms. If the differences are the same, the sequence was made by
addition. If not, it was made by multiplication.

Assessment
Continue the sequences and write the rule for each sequence.

a) 15, 30, 45, __, __

b) 52, 39, 26, __, __

c) 3, 17, 31, __, __

Extensions
1. The patterns below were made by multiplying successive terms by a fixed number and then
   adding or subtracting a fixed number. Find the missing terms and state the rule for making the
   pattern. Include the word term in your answer. (For instance, the rule for the first pattern below
   is “Start at 1. Multiply each term by 2 and add 1.”)
   a) 1, 3, 7, 15, 31, ____
   b) 1, 4, 13, 40, ____
   c) 2, 7, 22, 67, ____
   d) 2, 3, 5, 9, ____
   e) 1, 2, 5, 14, ____

2. Put a dot in the centre of a polygon and draw a line from the centre to each vertex of the
   polygon. How many line segments are there in each figure? Predict how many line segments
   there would be in a hexagon and an octagon. Test your prediction.
PA5-32

Patterns with Increasing and Decreasing Steps

Draw the following pattern on the board or build it with blocks:

Ask a volunteer to build the next term of the sequence. Ask your students to fill the T-table. Then ask: How many blocks are added each time? What are you adding to the structure? Another row. How many blocks are in the first row that you added? In the second row? This is the difference in the total number of blocks at each stage. Ask a volunteer to write the difference in the circles beside the table.

Ask your students if they can see a pattern in the differences. Ask a volunteer to add a term to the pattern of differences. After that ask another volunteer to fill in the next row of the table. Ask another volunteer to build another figure to check the result.

Give your students several questions to practice:

Find the differences between the terms of the sequences. Extend the sequence of the differences and then extend the sequence itself.

a) 5, 8, 12, 17, __, __

b) 3, 5, 11, 17, 25, __, __

c) 11, 15, 23, 39, __, __

d) 6, 8, 13, 21, 32, __, __

Let them also practice with decreasing sequences:

a) 65, 64, 62, 59, __, __

b) 73, 70, 64, 55, 43, __, __
Let your students build growing patterns of pattern blocks. For each sequence of patterns, find out how many blocks you need using a T-table.

**ACTIVITY**

Show the following geometrical pattern and ask how many triangles will be in the next design:

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Draw the T-table for the pattern. How many triangles do you add each time? We add a new row that is two triangles longer than the previous one. The new row is the difference. If your students have problems extending the sequence of differences, you can draw a second row of circles beside the first one to see the differences between the circles. Ask the volunteers to extend first the sequence of differences, then the sequence itself.

**Assessment**

a) 15, 18, 22, 27, ____, ____

b) 13, 16, 22, 31, 43, ____, ____

c) 101, 95, 87, 77, ____, ____

d) 88, 85, 78, 67, 52, ____, ____

Let your students build growing patterns of pattern blocks. For each sequence of patterns, find out how many blocks you need using a T-table.

**Extensions**

1. Janet is training for a marathon. On Monday she ran 5 km. Every day she ran 1 km more than on the previous day. How many kilometres did she run in the whole week?

Draw the T-table. The first few entries should appear as follows:

<table>
<thead>
<tr>
<th>Day</th>
<th>Total km Run from the Beginning of the Week</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Monday</td>
<td>5</td>
</tr>
<tr>
<td>2. Tuesday</td>
<td>11</td>
</tr>
<tr>
<td>3. Wednesday</td>
<td>18</td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>

Each day Janet runs 1 km more.
2. The pattern that is created in QUESTION 6 b) of the worksheet is called The Sierpinski Triangle. If you take a part of this pattern and magnify it, the picture will look exactly the same as the original. A picture with this property is called a fractal. Here is another example of a fractal, called the H-fractal.

Count the line segments at each stage of the construction. Make a T-table and try to predict the total number of line segments at the 7th step.

**STEP 1:** Draw a horizontal line of 12 cm.

**STEP 2:** Draw two vertical lines of 8 cm each, as shown.

**STEP 3:** Draw four horizontal lines of length $\frac{1}{2}$ of the previous horizontal line (6 cm here), so that the middle of each segment is at the end of the previous vertical segments.

**STEP 4:** Draw eight vertical lines of length $\frac{1}{2}$ of the previous vertical line (4 cm here), so that the middle of each segment is at the end of the previous horizontal segments.

After 10 steps the picture will look as shown below.

3. A restaurant has tables shaped like trapezoids. Two people can sit along the longest side of a table, but only one person can sit along each shorter side:

a) Draw a picture to show how many people could sit at 4 and 5 tables. Then, fill in the T-table.

<table>
<thead>
<tr>
<th>Number of Tables</th>
<th>Number of People</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Describe the pattern in the number of people. How does the step change?

c) Extend the pattern to find out how many people could sit at 8 tables.
Creating and Extending Patterns (Advanced)

Draw the following tree on the board and ask your students to count the number of branches in each level. Write the number of branches beside each level. Ask your students, “What happens to each branch?” It splits in two. This means that the number of branches doubles—multiplies by two—each time. So this sequence was made by multiplication.

Ask your students to find the sequence of differences between the numbers of branches in each level (1, 2, 4, 8…). What do they notice? They should see that the differences between terms in the sequence are the same as the numbers in the sequence.

Ask your students, “Which sequence grows faster—the one made by addition or the one made by multiplication?”

FOR EXAMPLE:

1, 11, 21, 31, … What is the rule for this sequence?

1, 2, 4, 8, … What is the rule for this sequence?

Start at 1 and multiply by 2.

You may call a vote: Which is larger—the 10th term of the first sequence or 10th term of the second sequence? Invite a volunteer to make a tally chart of the vote results. Have volunteers extend both sequences to the 10th term.

If students are engaged, repeat the activity with the second sequence above and the sequence 1, 101, 201… Check the 10th and the 11th terms of both sequences.

Show another sequence: 3, 8, 6, 11, 9, 14, 12, 17, 15…

You might write the sequence in this form...

... and ask students to describe the patterns they see.

Ask students to continue the sequence and to explain the rule by which it was made. Here are some possible answers:

1. The top row is the sequence 3, 6, 9, 12… The rule is “Start at three and add three each time”. The bottom row is 8, 11, 14, 17, and the rule is “Start at eight and add three each time”.

GOALS

Students will extend increasing and decreasing sequences using addition, subtraction and multiplication and alphabet patterns.

PRIOR KNOWLEDGE REQUIRED

Increasing and decreasing sequences
Skip counting
T-tables
Alphabet

VOCABULARY

increasing sequence
decreasing sequence
difference
2. The rule for the whole pattern (looking at all terms) is “Start at three and add five or subtract two alternatively”.

**ANOTHER EXAMPLE:** 101, 97, 98, 94, 95, 91, 92, ...

Provide students with a copy of the alphabet and suggest that they extend several sequences based on the alphabet:

- B, D, F, H, ...
- CZ, DZ, EY, FY, GX, ...
- A, Z, B, Y, C, X, ...
- A, G, M, S, ...

**Assessment**

Continue the sequences:

- 2, 4, 8, 14, 22, 32, ____, ____
- 10, 12, 9, 11, 8, 10, 7, ____, ____
- 2, 6, 18, 54, ____, ____
- 98, 95, 90, 83, 74, ____
- B, f, J, n, R, ...
- Ab, De, Gh, Jk, ...

**Bonus**

A mighty and courageous but somewhat short-sighted knight assaulted a wicked 12-headed dragon. When the knight hews off a dragon head, two new heads spring in place of the hewn one. The knight can only hew one head in a minute. Our knight has ingloriously left the battle field after 15 minutes of battle. How many heads does the monster have now?

**Extensions**

1. “The Legend of the Chess Board.” The same doubling sequence is used in the beginning of the lesson.

   **POSSIBLE SOURCE:**
   
   http://britton.disted.camosun.bc.ca/jbchessgrain.htm

2. Write the differences for the patterns. Identify the rule for the sequence of differences. Extend first the sequence of differences, then the sequence itself.

   a) 7, 10, 14, 19
   b) 12, 15, 20, 27
   c) 57, 54, 50, 45
   d) 32, 30, 26, 20
   e) 15, 18, 22, 25, 29
   f) 77, 72, 69, 64, 61
Tell your students that they have done so well with patterns that today you are going to give them patterns with **HUGE** numbers.

Present several problems and call for volunteers to solve them, using T-tables.

**A normal heartbeat rate is 72 times in a minute. How many times will your heart beat in five minutes?**

There are 60 minutes in an hour. George sleeps for six hours. How many minutes does he sleep? He asks his friend to wake him after 500 minutes. About how many hours is this? (Skip count to find out.)

A sprinting ostrich’s stride is 700 cm long. A publicity-loving ostrich spots a photographer 3 000 cm away and runs towards him. How far from the camera will it be after three strides?

Find the number of rhombuses in each of the following stars:

**HINT:** Count the number of rhombuses between the pair of heavy black lines and then multiply.

**Assessment**

An extinct elephant bird weighed about 499 kg. Make a T-table to show how much five birds would weigh. Do you see a pattern in the numbers (look at ones, tens and hundreds separately)? Can you write the weights of six, seven, and eight birds without actually adding?

**Extensions**

1. A regular year is 365 days long, a leap year (2000, 2004…) is 366 days long. Tom was born on Jan. 8, 2000. How many days old was he on January 8, 2003? January 8, 2005? When was he 2000 days old? (Finding the year, the month and the day are three different problems of increasing difficulty.)
2. Use the following pattern to figure out what $9 \times 999$ would be:

\[
\begin{align*}
2 \times 999 & \quad 3 \times 999 & \quad 4 \times 999 \\
\text{(from the Atlantic Curriculum)} & \\
\end{align*}
\]

**PA5-35**

**Introduction to Algebra**

Tell your students that today you will solve algebraic equations. Let them know that equations are like the scales people use to weigh objects (such as apples). But there’s a problem: A black box prevents you from seeing part of whatever object you are weighing. Solving an equation means figuring out what is inside the black box. Write the word “equation” on the board. Ask your students if they know any similar words (**EXAMPLES**: equal, equality, equivalence). So the word “equation” means “making the same,” or in other words, balancing the scales.

Draw a line down the middle of a desk. Put 7 apples on one side of the line and put 3 apples and a bag or box containing 4 more apples on the other side. Tell your students that there are the same number of apples on both sides of the line. (If you have a scale and a set of objects of equal weight, you might put the objects on the pans of the scale.) Students should be able to deduce that there are 4 apples in the box. They should also see that they can find the number of hidden apples either by counting up from 3 to 7, or by subtracting 3 from 7. Challenge students with several more examples.

Tell your students that it is easy to represent the problem they just solved with a picture:

\[
\begin{align*}
\text{+} & \quad \text{=}
\\
\text{+} & \quad \\
\text{+} & \\
\end{align*}
\]

Invite a student to draw the missing apples in the box.

After students have had practice with this sort of problem, explain that it is inconvenient to draw the apples all the time. So people use numbers to represent the hidden quantities. Let them practice writing an equation that represents a picture, as on the worksheet. For instance, the equation for the picture above is:

\[
\text{+} + 3 = 7
\]

Students can create models for equations that involve addition using blocks. A square could stand for the unknown and a set of circles could be used to model the numbers in the equation.
For instance the equation:

\[ \square + 2 = 7 \]

has the model:

\[ \square \bigcirc \bigcirc = \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \]

If the students think of the square as having a particular weight, then solving the equation becomes equivalent to finding the weight of the square in terms of the circles.

How many circles are needed to balance the square?

You can find the answer by removing all the circles from the left hand scale and an equivalent number of circles from the other side.

The square has the same weight as five circles.

Ask students to make a model with squares and circles to solve the following problems. (They should draw a picture of their model with a balance scale as in the figures above.) Then they should explain how many circles they would remove to find the weight of the square.

\[ 7 + \square = 11 \quad 6 + \square = 13 \quad 4 + \square = 10 \quad 9 + \square = 12 \]

Write an equation and draw the picture:

\[ \bigcirc \bigcirc + \square = \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \]

Ask if the picture makes sense. Why not? (You are a mathematician, not a magician. You cannot turn apples into bananas.) When you draw pictures like the ones students have worked with so far, the objects on both sides of the equation have to be the same. However, when you use numbers in an equation—rather than pictures—the numbers don’t always stand for the same thing.

For instance, the equation \( 10 = 3 + \square \) could represent the problem: “There are ten pets in a store. Three are cats. How many are dogs?” Here the number in the box and the number outside the box on the right side of the equation stand for exactly different things. However, in all equations, the amounts on both sides of the equal sign will be in the same category; for instance the animals in the question above are all pets.

Explain that the square is often replaced by a letter, say “n”, to signify that this is “the unknown”. Ask students to make a model with squares and circles to solve the following problems. (They should draw a picture of their model with a balance scale as in the figures above.) Then they should explain how many circles they would remove to find the weight of the square.

a) \( n + 2 = 8 \)  
b) \( n + 3 = 10 \)  
c) \( n + 5 = 9 \)
Ask students to translate the following pictures into equations using the letter “n” as the unknown.

a) 

b) 

Point out that the choice of the letter is arbitrary—both equations \( n + 3 = 5 \) and \( a + 3 = 5 \) have the same model (\( \square\text{OOO} = \text{OOOOO} \)) and the same solution (\( n \) or \( a = 2 \)).

Read several word problems. Invite volunteers to draw models, write equations using squares and numbers, and solve the equations:

There are ten trees in the garden. Three of them are apple trees. All the rest are cherry trees. How many cherry trees are in the garden?

Jane has 12 books. Three of them are fairy tales. All the rest are ghost stories. How many of her books are ghost stories?

**Assessment**

1. Solve the equations:
   
   \[
   5 + \square = 11 \\
   8 + \square = 15 \\
   3 + n = 13 \\
   7 + a = 13
   \]

2. Write an equation to solve the problem:

   There are 15 flowers in the flower-bed. Six are lilies. All the rest are peonies. How many peonies grow in the flower-bed?

**Math Bingo—Addition only**

Your students will each need a playing board (See: the BLM) and 16 tokens to mark the numbers. The teacher reads the card out loud. Players have to figure out the answer and then to place a token on the board, if they have the answer. The first player to fill a column, row or diagonal wins.

**Bonus**

1. There were 150 bloodthirsty pirates on a galleon and a schooner. 40 of them are on the schooner. How many pirates are on the galleon?

2. A multi-headed dragon has 15 heads. Some of them were cut off by a mighty and courageous knight. The dragon ran away from the knight with seven remaining heads. How many heads were hewn off by the knight?
Extensions
1. The same symbol in the equation means the same number. Solve the equations:

   a) \[ \square + \square = 12 \]
   b) \[ 5 + \diamond + \diamond = 13 \]
   c) \[ \bigcirc + \bigcirc + \bigcirc = 9 \]
   d) \[ 9 + \bigcirc + \bigcirc + \bigcirc = 15 \]

Remind your students that in the last lesson they drew models and wrote equations for word problems. The equations they drew all had a + sign. Today they will learn to write other kinds of equations.

Present a word problem: Sindi has a box of apples. She took two apples from the box and four were left. How many apples were in the box before she removed the apples?

Draw the box. There are some apples inside, but we do not know how many. Draw two apples and cross them to show that they are taken away. Four apples were left in the box, so draw them too. How many were there from the beginning? (Six).

Explain that when we write an equation, we draw it differently. We draw the apples that we took out, outside the box with the “–” sign, to show that they were taken away:

\[ \quad = \quad \quad \rightarrow \quad \quad = \quad \quad \]

So to solve the equation we have to put all the apples into the box—the ones that we took out and the ones that are outside.

Remind your students that they also learned to write equations in number form. Can they guess what the equation will look like:

\[ \square - 2 = 4? \]
Draw several models like the ones on the worksheets, and ask your students to write the equations for them. Ask volunteers to present the answers on the board.

Then ask your students to draw models for the equations and to solve them by drawing the original number of apples in the box:

\[
\begin{align*}
-6 &= 9 \\
-7 &= 12 \\
-5 &= 3 \\
-3 &= 10
\end{align*}
\]

Remind your students how they could use letters in equations and ask them to rewrite all the equations with letters.

Tell your students that they can also write equations for multiplication problems. Remind them that “2 ×” means that some quantity is taken two times.

**FOR EXAMPLE:**

\[
2 \times \begin{array}{ccc}
& & \\
& & \\
& & \\
\end{array} = \begin{array}{ccc}
& & \\
& & \\
& & \\
\end{array}
\]

Present the problem below and ask your students to draw the appropriate number of circles in the box:

\[
2 \times \begin{array}{cccc}
& & & \\
& & & \\
& & & \\
\end{array} = \begin{array}{cccc}
& & & \\
& & & \\
& & & \\
\end{array}
\]

Present more problems and ask your students to write and solve numerical equations for these problems. Encourage them to use letters instead of squares. Then show your students how to write an equation for a word problem involving multiplication:

Tony has four boxes of pears. Each box holds the same number of pears. He has 12 pears in total. How many pears are in each box?

The students might reason in the following way: we usually represent the thing that we do not know by a square (the “unknown”). So four times the “unknown” constitutes 12, and we have the equation:

\[
4 \times \begin{array}{cc}
& \\
& \\
\end{array} = 12.
\]

**PRACTICE:**

Three identical evil dragons have 15 heads together. How many heads does each dragon have?

Five heffalumps have 10 tails. How many tails does each heffalump have?

A cat has four paws. 16 cat paws are scratching in the basket. How many kittens are inside?

Here is a mixture of problems students could try involving addition, subtraction, or multiplication:

Katie has several dogs. They have 20 paws together. How many dogs does Katie have?

Jenny used three eggs to bake muffins. Seven eggs remained in the carton. How many eggs were in the carton?

Bob has nine pets. Three of them are snakes. All the rest are iguanas. How many iguanas does Bob have?
Assessment

1. Solve the equations:
   
   \[
   \begin{align*}
   3 + \square &= 8 \\
   3 \times \square &= 15 \\
   \square - 4 &= 11 \\
   2 \times \square &= 14
   \end{align*}
   \]

2. Draw models to solve the problems:
   
   Hamide has 12 stamps. Four of them are Canadian. How many foreign stamps does she have?
   
   Joe has 15 stamps. Five of them are French and the rest are German. How many German stamps does Joe have?
   
   Marylyn has 18 stamps. They come in three identical sheets. How many stamps are in each sheet?

Bonus

Solve the equations:

\[
\begin{align*}
243 + \square &= 248 \\
8 \times \square &= 56 \\
\square - 4 &= 461 \\
60 \times \square &= 240
\end{align*}
\]

Extensions

1. Ask students to translate each story problem below into an equation using a letter to stand for the unknown, rather than a box. They should also model problem a) with squares and circles.

   a) Carl has seven stickers. He has two more stickers than John. How many stickers does John have?

   \[\text{SOLUTION:}\] Let \(n\) stand for the number of stickers that John has. Carl has two more stickers, so you would have to add two to the number of stickers John has. So the correct equation is: \(n + 2 = 7\)

   b) Katie has ten stickers. She has three fewer stickers than Laura.

   \[\text{SOLUTION:}\] Let \(n\) be the number of stickers that Laura has. Katie has three fewer stickers than Laura so you need to subtract three from the number of stickers that Laura has. So the correct equation is: \(n - 3 = 10\)

2. Two birds laid the same number of eggs. Seven eggs hatched, and three did not. How many eggs did each bird lay?

3. 60 little crocodiles hatched from three crocodile nests of the same size. We know that only half of the eggs hatched. How many eggs were in each nest? \(\text{HINT:}\) How many eggs were laid in total?
Explain to your students that today they will learn to write equations the way mathematicians do it. In mathematics people usually use letters instead of drawing a square or a diamond for the unknown. Remind your students that they already used letters in formulas for patterns—for example, ask your students to tell how many triangles will be needed to make broaches using the design shown.

1 broach: \(1 \times 5\) triangles
2 broaches: \(2 \times 5\) triangles
7 broaches: ______

Remind your students that in previous lessons they used letters for the number of pentagons or triangles in a set of broaches.

Give them several problems, like the ones below, and ask them to write a formula or equation for the problem:

A boat travels at a speed of 10 km per hour. What distance will it cover in 2 hours? 5 hours? In \(h\) hours?

A house has 12 windows. How many windows do 3 houses have? 7 houses? \(X\) houses?

Ask a volunteer to write a formula/equation for a T-table:

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

Change the headings of the T-table to \(A\) and \(B\) and ask a volunteer to write the formula for the new table. Explain to your students, that even if the names of the columns change, the rule for the T-table will still have the same form. Previously the rule was Number of Blocks = Figure Number + 4, now it is \(B = A + 4\).

Let your students practice finding rules for T-tables with letter headings. Start with simple rules, involving one operation and continue to more complicated rules like \(B = 2 \times A + 3\).

Explain to your students that when letters are used in an equation, the multiplication sign “\(\times\)" is often omitted to avoid confusion with the letter \(X\) and to make the notation shorter. In this case instead of writing \(2 \times A\) people simply write \(2A\). Ask your students to rewrite the equations that they have written without the multiplication sign. After that give them several word problems, such as:
Ricardo made 12 sandwiches. Four of them are avocado sandwiches, and the rest are cheese sandwiches. How many cheese sandwiches does Ricardo have?

Ask your students to draw models for the problems, write equations with squares and rewrite the equations with a variable. Eventually students should find it easy to write the equation directly with a variable.

---

**PA5-38**

**Equations**

Give your students a mixed set of equations (like the ones on the worksheet) and ask them to solve the problems using guess and check. Tell them that sometimes they might see a way to solve the problem without guessing—for example, with a model. In this case they should draw the model and to solve the problem that way.

\[
\begin{align*}
&\text{a) } \boxed{x} \times 6 = 42 \\
&\text{b) } \boxed{+} + 6 = 8 \\
&\text{c) } \boxed{-} \div 5 = 10 \\
&\text{d) } \boxed{-} - 4 = 9 \\
&\text{e) } 9 - \boxed{=} = 7 \\
&\text{f) } \boxed{+} + 2 = 10 \\
&\text{g) } 17 - \boxed{=} = 14 \\
&\text{h) } 8 - \boxed{=} = 5 \\
&\text{i) } 2 \times \boxed{=} = 14 \\
&\text{j) } 20 + \boxed{=} = 4 \\
&\text{k) } 3 \times \boxed{=} = 24 \\
&\text{l) } \boxed{\div} \div 9
\end{align*}
\]

Ask your students to rewrite some of the equations using letter variables. Present the following problem: There are several hats and gloves in a box. All left-hand gloves have a matching right-hand glove, and there are nine objects in the box. How many gloves and how many hats are there?

Draw an equation on the board. Say that the circle will represent the number of left-hand gloves. Gloves come in pairs, so there will be two circles—one for the right hands and one for the left hands. The square will represent the number of hats.

\[
\boxed{\bigcirc} + \boxed{\bigcirc} + \boxed{\square} = 9
\]

Ask volunteers to try to fit some numbers into the equation. Suggest starting with the circle. Draw a T-table of the results:

<table>
<thead>
<tr>
<th>\boxed{\bigcirc}</th>
<th>Equation</th>
<th>Rewrite the Equation</th>
<th>\boxed{\square}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 + 1 + \boxed{\square} = 9</td>
<td>2 + \boxed{\square} = 9</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>2 + 2 + \boxed{\square} = 9</td>
<td>4 + \boxed{\square} = 9</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>3 + 3 + \boxed{\square} = 9</td>
<td>6 + \boxed{\square} = 9</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4 + 4 + \boxed{\square} = 9</td>
<td>8 + \boxed{\square} = 9</td>
<td>1</td>
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</tbody>
</table>

Use as many volunteers to help filling the table as possible. **ASK:** Why did we have to stop after the 4th row?

Ask your students to write down the sequences in the third and the fourth columns and to give the rules for the sequences.
Present several problems such as the problem below and ask your students to write the equations using letter variables and to solve the equations.

Timur threw 3 darts and scored 12 points. The dart in the centre is worth twice as much as the dart on the outside. How much is each dart worth?

**SOLUTION:** Let \( n \) be the value of a dart on the outside. The dart in the centre is worth twice as much the dart on the outside, so its worth is \( 2 \times n \), or \( 2n \). The total value of the darts is \( n + n + 2n = 12 \), or \( 4n = 12 \).

**Assessment**
1. Ask your students to solve several equations such as:
   a) \( \square \times 6 = 48 \)
   b) \( \square + 6 = 28 \)
   c) \( \square / 5 = 6 \)
   d) \( \square - 4 = 16 \)
   e) \( \square + \square + \bigcirc + \bigcirc = 10 \)

2. Sindi had 12 cookies. She shared 8 with her friends and ate the rest. How many cookies did she eat?

**Extensions**
1. In the magic trick below, the magician can always predict the result of a sequence of operations performed on any chosen number. Try the trick with students, then encourage them to figure out how it works using a block to stand in for the mystery number (give lots of hints).

<table>
<thead>
<tr>
<th>The Trick</th>
<th>The Algebra</th>
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<tbody>
<tr>
<td>Pick any number</td>
<td>( \square )</td>
<td>Use a square block to represent the mystery number</td>
</tr>
<tr>
<td>Add 4</td>
<td>( \square \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc )</td>
<td>Use 4 circles to represent the 4 ones that were added.</td>
</tr>
<tr>
<td>Multiply by 2</td>
<td>( \square \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc )</td>
<td>Create 2 sets of blocks to show the doubling</td>
</tr>
<tr>
<td>Subtract 2</td>
<td>( \square \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc )</td>
<td>Take away 2 circles to show the subtraction</td>
</tr>
<tr>
<td>Divide by 2</td>
<td>( \square \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc )</td>
<td>Remove one set of blocks to show the division.</td>
</tr>
<tr>
<td>Subtract the mystery number</td>
<td>( \bigcirc \bigcirc \bigcirc \bigcirc )</td>
<td>Remove the Square</td>
</tr>
</tbody>
</table>

The answer is 3!
No matter what number you choose, after performing the operations in the magic trick, you will always get the number 3. The model above shows why the trick works.

Encourage students to make up their own trick of the same type.

2. Give your students a copy of a times table. Ask them to write an equation that would allow them to find the numbers in a particular column of the times table given the row number. For instance, to find any number in the 5s column of the times table you multiply the row number by five: Each number in the 5s column is given by the algebraic expression $5 \times n$ where $n$ is the row number. Ask students to write an algebraic expression for the numbers in a given row.

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**PA5-39**

**Problems and Puzzles**

PA5-39 is a review worksheet, which can be used for practice.

**Extension**

Consecutive numbers are numbers that follow each other on the number line. You can sum a set of consecutive numbers quickly, by grouping the numbers as follows:

**STEP 1:** Add the first and last number, the second and the second to last number and so on. What do you notice?

\[ 1 + 2 + 3 + 4 + 5 + 6 = 7 + 7 + 7 \]

**STEP 2:** Rewrite the addition statement as a multiplication statement.

\[ 3 \times 7 = 21 \]

Add the following sets of numbers by grouping pairs that add to the same number and multiplying that number by the number of pairs.

- a) \[ 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = \]
- b) \[ 12 + 13 + 14 + 15 + 16 + 17 = \]
PA5 Part 2: BLM List

Math Bingo Game_______________________________________________________2
## Math Bingo Game

### Sample Boards

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# Math Bingo Game (continued)

## Cards (Addition Only)

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<th>Equation</th>
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<tr>
<td>$7 + \square = 11$</td>
<td>$6 + \square = 13$</td>
<td>$5 + \square = 10$</td>
<td>$9 + \square = 12$</td>
</tr>
<tr>
<td>$\square + 9 = 11$</td>
<td>$\square + 12 = 13$</td>
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<td>$7 + \square = 16$</td>
<td>$6 + \square = 16$</td>
<td>$4 + \square = 15$</td>
<td>$5 + \square = 17$</td>
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<tr>
<td>$\square + 7 = 21$</td>
<td>$\square + 6 = 19$</td>
<td>$\square + 3 = 18$</td>
<td>$\square + 3 = 19$</td>
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<td>$5 + \square = 22$</td>
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<td>$4 + \square = 23$</td>
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## Cards (Mixed Equations)

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<tbody>
<tr>
<td>$7 + \square = 21$</td>
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<td>$5 + \square = 24$</td>
<td>$9 + \square = 20$</td>
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<td>$\square - 9 = 11$</td>
<td>$\square - 1 = 11$</td>
<td>$\square - 4 = 22$</td>
<td>$\square - 4 = 16$</td>
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<tr>
<td>$7 \times \square = 21$</td>
<td>$6 \times \square = 30$</td>
<td>$4 \times \square = 16$</td>
<td>$3 \times \square = 18$</td>
</tr>
<tr>
<td>$\square + 7 = 15$</td>
<td>$\square + 6 = 21$</td>
<td>$\square - 4 = 13$</td>
<td>$\square - 5 = 11$</td>
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<tr>
<td>$9 \times \square = 18$</td>
<td>$4 \times \square = 36$</td>
<td>$8 \times \square = 80$</td>
<td>$21 + \square = 22$</td>
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NS5-61
Equal Parts and Models of Fractions

Ask your students if they have ever been given a fraction of something (like food) instead of the whole, and gather their responses. Bring a banana (or some easily broken piece of food) to class. Break it in two very unequal pieces. SAY: This is one of two pieces. Is this half the banana? Why not? Emphasize that the parts have to be equal for either of the two pieces to be a half.

Draw numerous examples of shapes with one of two parts shaded, some that are equal and some that are not, and ask volunteers to mark the diagrams as correct or incorrect representations of one half.

ASK: Which diagram illustrates one-fourth? What’s wrong with the other diagram? Isn’t one of the four pieces still shaded?

A sport played by witches and wizards on brooms regulates that the players must fly higher than 5 m above the ground over certain parts of the field (shown as shaded). Over what fraction of the field must the players fly higher than 5 m?

Explain that it’s not just shapes like circles and squares and triangles that can be divided into fractions, but anything that can be divided into equal parts. Draw a line and ask if a line can be divided into equal parts. Ask a volunteer to guess where the line would be divided in half. Then ask the class to suggest a way of checking how close the volunteer’s guess is. Have a volunteer measure the length of each part. Is one part longer? How much longer? Challenge students to discover a way to check that the two halves...
are equal without using a ruler, only a pencil and paper. [On a separate sheet of paper, mark the length of one side of the divided line. Compare that length with the other side of the divided line by sliding the paper over. Are they the same length?]

Have students draw lines in their notebooks and then ask a partner to guess where the line would be divided in half. They can then check their partner’s work.

**ASK:** What fraction of this line is double?

**SAY:** The double line is one part of the line. How many equal parts are in the whole line, including the double line? [5, so the double line is \( \frac{1}{5} \) of the whole line.]

Mark the length of the double line on a separate sheet of paper. Compare that length to the entire line to determine how many of those lengths make up the whole line. Repeat with more examples.

Then ask students to express the fraction of shaded squares in each of the following rectangles.

![Rectangles with shaded squares](#)

Have them compare the top and bottom rows of rectangles.

**ASK:** Are the same fraction of the rectangles shaded in both rows? Explain. If you were given the rectangles without square divisions, how would you determine the shaded fraction? What could be used to mark the parts of the rectangle? What if you didn’t have a ruler? Have them work as partners to solve the problem. Suggest that they mark the length of one square unit on a separate sheet of paper, and then use that length to mark additional square units.

Prepare several strips of paper with one end shaded, and have students determine the shaded fraction without using a pencil or ruler. Only allow them to fold the paper.

Draw several rectangles with shaded decimetres.

![Rectangles with shaded decimetres](#)

Have students divide the rectangles into the respective number of equal parts (3, 4 and 5) with only a pencil and paper. Have them identify the fractions verbally and orally. Then draw a 50 cm rectangle with two shaded decimetres (i.e. two-fifths), and challenge your students to mark units half the length of the shaded decimetres. How can this be done with only a pencil and paper?

**Fold the paper so that these markings meet. Draw a marking along the fold.**
Draw a shaded square and ask students to extend it so the shaded part becomes half the size of the extended rectangle.

![Square](image)

Repeat this exercise for squares becoming one-third and one-quarter the size of extended rectangles. **ASK:** How many equal parts are needed? [Three for one-third, four for one-quarter.] How many parts do you already have? [1.] So how many more equal parts are needed? [Two for one-third, three for one-quarter.]

**Bonus**

Extend the squares so that \(\frac{2}{5}\) of them are shaded. Repeat this exercise for \(\frac{1}{7}\), \(\frac{3}{7}\), etc.

Give your students rulers and ask them to solve the following puzzles.

- a) Draw a line 1 cm long. If the line represents \(\frac{1}{6}\) show what a whole line looks like.
- b) Line: 1 cm long. The line represents \(\frac{3}{5}\). Show the whole.
- c) Line: 2 cm long. The line represents \(\frac{1}{4}\). Show the whole.
- d) Line: 3 cm long. The line represents \(\frac{1}{2}\). Show the whole.
- e) Line: 1 \(\frac{1}{2}\) cm long. The line represents \(\frac{1}{4}\). Show the whole.
- f) Line: 1 \(\frac{1}{2}\) cm long. The line represents \(\frac{1}{2}\). Show the whole.
- g) Line: 3 cm long. The line represents \(\frac{1}{4}\). Show \(\frac{1}{2}\).
- h) Line: 2 cm long. The line represents \(\frac{1}{4}\). Show \(\frac{1}{4}\).

This is a good activity to do at the end of a day, so that students with extra time can play with the left-over play dough until the end of class.

Prepare enough small balls of coloured play dough for 3 for each student (they will only need two, but this allows students to choose their colours). Demonstrate to students how to make fractions using play dough. Tell them that you are going to roll one spoon of red play dough and three spoons of blue play dough into a ball. Explain the necessity of flattening the play dough on a spoon so that each spoonful is the same size. Demonstrate not leaving any play dough of the first colour on the spoon. Roll the play dough into a ball carefully mixing it so that it becomes a uniform colour. This has been tried with fresh play dough only; store-bought play dough may not produce the same effect and may be more difficult to mix thoroughly. For a recipe, **SEE:**

http://www.teachnet.com/lesson/art/playdoughrecipes/traditional.html

An alternative to play dough is to use small spoonfuls of food colouring.

**ASK:** How many spoons of play dough have I used altogether? How many spoons of red play dough have I used? What fraction of this ball is red? How many spoons of blue play dough have I used? What fraction of this ball is blue? Write the “recipe” on a triangular flag (see below) which can be made from a quarter of a regular sheet taped to a straw (insert the straw into your ball of play dough). The recipe is shown on the flag below:

(Continued on next page.)
ACTIVITY

Ask students to select two colours of play dough (red, blue, white or yellow). Then have them decide how many spoonfuls of each colour will be mixed into their recipe and recorded as fractions on their flag. The total number of spoons they use should be 2, 3, 4, 5 or 6 since they will need to compare the fractions later using the BLM “Fraction Strips”. (Advanced students can use three colours, but they should first make a ball with two colours, since they will need it for a later activity.) Have each student write their name on the back side of their flag.

After they have taped their flags to their straws, distribute one small measuring spoon to each student and have them prepare their play dough balls according to the recipes on their flags. The play dough balls should be mixed thoroughly to show only one colour.

Then have them insert their flags into their play dough balls and hand them in, as the flags will be needed later on when fractions are compared and ordered.

Extensions

1. The smaller angle is what fraction of the larger angle?

HINT: Use tracing paper.

2. Draw the whole angle if the given angle is $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$ of the whole angle.
   a) $\frac{1}{2}$
   b) $\frac{1}{3}$
   c) $\frac{1}{4}$

3. a) Sketch a pie and cut it into fourths. How can it be cut into eighths?
   b) Sketch a pie and cut it into thirds. How can it be cut into sixths?

4. Ask students to make and identify as many fractions in the classroom as they can, for instance:
   - $\frac{1}{4}$ of the blackboard is covered in writing.
   - $\frac{2}{5}$ of the counters are red.
   - $\frac{1}{2}$ the length of a 30 cm ruler is 15 cm.
   - About $\frac{1}{3}$ of the door is covered by a window.
   - $\frac{11}{25}$ of the class has black hair.
   - The room is about $\frac{4}{5}$ wide as it is long.
5. Ask students to make a model of a fraction from materials in the classroom.

6. (Adapted from Grade 4 Atlantic Curriculum)

Ask students to divide the rectangle...

a) into thirds two different ways.
b) into quarters three different ways.

NOTE: The division shown below may not be obviously divided into quarters until it is further divided into eighths.

---

**NS5-62**

**Equal Parts of a Set**

Review equal parts of a whole. Tell your students that the whole for a fraction might not be a shape like a circle or square. Tell them that the whole can be anything that can be divided into equal parts. Brainstorm with the class other things that the whole might be: a line, an angle, a container, apples, oranges, amounts of flour for a recipe. Tell them that the whole can even be a group of people. For example, the grade 5 students in this class is a whole set and I can ask questions like: what fraction of students in this class are girls? What fraction of students in this class are ten years old? What fraction of students wear glasses? What do I need to know to find the fraction of students who are girls? (The total number of students and the number of girls). Which number do I put on top: the total number of students or the number of girls? (the number of girls). Does anyone know what the top number is called? (the numerator) Does anyone know what the bottom number is called? (the denominator) What number is the denominator? (the total number of students). What fraction of students in this class are girls? (Ensure that they say the correct name for the fraction. For example, \( \frac{10}{25} \) is said “ten twenty fifths“.) Tell them that the girls and boys don’t have to be the same size; they are still equal parts of a set. Ask students to answer: What fraction of their family is older than 10? Younger than 10? Female? Male? Some of these fractions, for some students, will have numerator 0, and this should be pointed out. Avoid asking questions that will lead them to fractions with a denominator of 0 (For example, the question “What fraction of your siblings are male?” will lead some students to say 0/0).

Then draw pictures of shapes with two attributes changing:
a) △ ○ □ ○ ○

**ASK:** What fraction of these shapes are shaded? What fraction are circles? What fraction of the circles are shaded?

b) □ △ □ □ △

**ASK:** What fraction of these shapes are shaded? What fraction are unshaded? What fraction are squares? Triangles? What fraction of the triangles are shaded? What fraction of the squares are shaded? What fraction of the squares are not shaded?

Then ask students for each picture below to write in their notebooks:

a) What fraction are circles?
b) What fraction are shaded?
c) What fraction are squares?
d) What fraction are triangles?

i) △ □ ○ ○ ○ ○

ii) △ ○ △ ○ □ △ △ △

iii) △ ○ △ ○ ○ ○ △ △

**Bonus**
(pictures with 3 attributes changing)

□ ○ ○ △ △ △ □ □ □

Have students answer the same questions as above and then more complicated questions like: What fraction of the triangles are shaded? (So the triangles are now the whole set). What fraction of the shaded shapes are triangles? **ASK:** Now what is the whole set?

Have students make up questions to ask each other.

Draw on the board:

c) ○ △ □ □ △ △ △ □ □

Have students volunteer questions to ask and others volunteer answers.

Then have students write fraction statements in their notebooks for similar pictures.
Ask some word problems:

A basketball team played 5 games and won 2 of them. What fraction of the games did the team win?

A basketball team won 3 games and lost 1 game. How many games did they play altogether? What fraction of their games did they win?

A basketball team won 4 games, lost 1 game and tied 2 games. How many games did they play? What fraction of their games did they win?

Also give word problems that use words such as “and,” “or” and “not”: Sally has 4 red marbles, 2 blue marbles and 7 green marbles.

a) What fraction of her marbles are red?
b) What fraction of her marbles are blue or red?
c) What fraction of her marbles are not blue?
d) What fraction of her marbles are not green?

**Bonus**

Which two questions have the same answer? Why?

Challenge your students to write another question that uses “or” that will have the same answer as: What fraction of her marbles are not blue?

Look at the shapes below:

![Shapes](image)

a) What fraction of the shapes are shaded and circles?
b) What fraction of the shapes are shaded or circles?
c) What fraction of the shapes are not circles?

Write a question that has the same answer as c):

What fraction of the shapes are ______ or ______?

Have students create their own questions and challenge a partner to answer them. Find the number of each item given the fractions.

a) A team played 5 games. They won \( \frac{2}{5} \) of their games and lost \( \frac{3}{5} \) of their games. How many games did they win? Lose?

b) There are 7 marbles. \( \frac{2}{7} \) are red, \( \frac{4}{7} \) are blue and \( \frac{1}{7} \) are green. How many blue marbles are there? Red marbles? Green marbles?

Then tell your students that you have five squares and circles. Some are shaded and some are not. Have students draw shapes that fit the puzzles. (Students could use circles and sequences of two different colours to solve the problem.)

a) \( \frac{2}{5} \) of the shapes are squares. \( \frac{3}{5} \) of the shapes are shaded. One circle is shaded.

**SOLUTION:**

![Shaded Shapes](image)
b) \( \frac{3}{5} \) of the shapes are squares. \( \frac{2}{5} \) of the shapes are shaded. No circle is shaded.

c) \( \frac{3}{5} \) of the shapes are squares. \( \frac{3}{5} \) of the shapes are shaded. \( \frac{1}{5} \) of the squares are shaded.

**Extensions**

1. Have students investigate:
   
   a) What fraction of the squares on a Monopoly board are “Chance” squares? What fraction of the squares are properties that can be owned?

   b) On a Snakes and Ladders board, on what fraction of the board would you be forced to move down a snake?

   Have students make up their own fraction question about a board game they like and tell you the answer next class.

2. Draw a picture to solve the puzzle. There are 7 triangles and squares. \( \frac{2}{7} \) of the figures are triangles. \( \frac{5}{7} \) are shaded. 2 triangles are shaded.
3. Give your students harder puzzles by adding more attributes and more clues:

   There are 5 squares and circles. \( \frac{1}{3} \) of the squares are big.
   \( \frac{3}{5} \) of the shapes are squares. \( \frac{2}{5} \) of the squares are shaded.
   \( \frac{3}{5} \) of the shapes are shaded. No shaded shape is big.
   \( \frac{2}{5} \) of the shapes are big.

**SOLUTION:**

```
□ □ □ ○ ○
```

4. Give your students red and blue counters (or any other pair of colours) and ask them to solve the following problems by making a model.

   a) Half the counters are red. There are 10 red counters. How many are blue?
   b) Two fifths of the counters are blue. There are 6 blue counters. How many are red?
   c) \( \frac{3}{4} \) of the counters are red. 9 are red. How many are blue?

5. What fraction of the letters of the alphabet is...
   a) in the word “fractions”
   b) not in the word “fractions”

6. For the month of September, what fraction of all the days are...
   a) Sundays
   b) Wednesdays
   c) School days
   d) Not school days
   e) What fraction of the days are divisible by 5?

7. What fraction of an hour has passed since 9:00? Reduce if possible.
   a) 9:07
   b) 9:15
   c) 9:30
   d) 9:40

8. a) There are 5 circles and triangles. Can you draw a set so that:
   i) \( \frac{3}{5} \) are circles and \( \frac{2}{5} \) are striped?

   Have volunteers show the different possibilities before moving on. Ask questions like:
   How many are striped circles? How many different answers are there?
   ii) \( \frac{3}{5} \) are circles and \( \frac{2}{5} \) are triangles?

   What is the same about these two questions? (the numbers are the same in both) What is different? [one is possible, the other is not; one uses the same attribute (shape) in both, the other uses two different attributes (shape and shading)].

   b) On a hockey line of 5 players, \( \frac{4}{5} \) are good at playing forward and \( \frac{2}{5} \) are good at playing defense. How many could be good at playing both positions? Is there only one answer? Which question from 4 a) is this similar to? What is similar about it?
NS5-63
Parts and Wholes

GOALS
Students will make divisions not already given to form equal-sized parts.

PRIOR KNOWLEDGE REQUIRED
A fraction of an area is a number of equal-sized parts out of a total number of equal-sized parts. The whole a fraction is based on can be anything.

VOCABULARY
fraction numerator whole denominator part

Draw on the board the shaded strips from before:

ASK: Is the same amount shaded on each strip? Is the same fraction of the whole strip shaded in each case? How do you know? Then draw two hexagons as follows:

ASK: Is the same amount shaded on each hexagon? What fraction of each hexagon is shaded?

Then challenge students to find the fraction shaded by drawing their own lines to divide the shapes into equal parts:

Give your students a set of tangram pieces. Ask students what fraction of each of the tangram pieces a small triangle represents. ASK: How many small triangles cover a square? What fraction of the square is the small triangle? How many small triangles cover the large triangle? What fraction of the large triangle is the small triangle? What fraction of the medium triangle is the small triangle? What fraction of the parallelogram is the small triangle?

Ask students to make as many rectangles or trapezoids as they can using the pieces and determine the fraction of the shape that is covered by the small triangle. (For some sample rectangles and trapezoids, see questions 1 and 3 on the worksheet.)
Then draw shapes on the board and have students decide what fraction of the shape the small triangle is:

If \( \square = \text{red} \), and \( \square = \text{blue} \), approximately what fraction of each flag or banner is red and what fraction is blue:

For parts b) and c) students should subdivide the flag as shown below:

(From Atlantic Curriculum A3.6) Have students prepare a poster showing all the equivalent fractions they can find using a set of no more than 30 pattern blocks.

**Extensions**

1. a) What fraction of a tens block is a ones block?  
   b) What fraction of a tens block is 3 ones blocks?  
   c) What fraction of a hundreds block is a tens block?  
   d) What fraction of a hundreds block is 4 tens blocks?  
   e) What fraction of a hundreds block is 32 ones blocks?  
   f) What fraction of a hundreds block is 3 tens blocks and 2 ones blocks?
2. On a geoboard, show 3 different ways to divide the area of the board into 2 equal parts.

**EXAMPLES:**

![Geoboard Diagram]

3. Give each student a set of pattern blocks. Ask them to identify the whole of a figure given a part.

   a) If the pattern block triangle is $\frac{1}{6}$ of a pattern block, what is the whole?
   **ANSWER:** The hexagon.

   b) If the pattern block triangle is $\frac{1}{3}$ of a pattern block, what is the whole?
   **ANSWER:** The trapezoid.

   c) If the pattern block triangle is $\frac{1}{2}$ of a pattern block, what is the whole?
   **ANSWER:** The rhombus.

   d) If the rhombus is $\frac{1}{6}$ of a set of pattern blocks, what is the whole?
   **ANSWER:** 2 hexagons or 6 rhombuses or 12 triangles or 4 trapezoids.

4. Students can construct a figure using the pattern block shapes and then determine what fraction of the figure is covered by the pattern block triangle.

5. What fraction of the figure is covered by...

   a) The shaded triangle
   b) The small square

![Shape Diagram]

6. To prepare students for comparing and ordering fractions, ask them to guess what fraction with numerator 1 is closest to the answer to parts c) and d): one half, one third, one fourth or one fifth. Students can then check their estimates on an enlarged version of these drawings by using counters.

![Flag Diagram]

**ASK:** How many counters cover the red part? How many counters cover the whole flag? About how many times more counters cover the whole flag than cover the red part? About what fraction of the flag is red? If students guess for example, that the fraction of the starred flag in the bonus that is red is one half, they could cover the red star with red counters, the blue boundary with (same sized) blue counters and see if the number of red counters is close to the number of blue counters.
GOALS
Students will understand that as the numerator increases and the denominator stays the same, the fraction increases and that as the numerator stays the same and the denominator increases, the fraction decreases.

PRIOR KNOWLEDGE REQUIRED
Naming fractions
Fractions show same-sized pieces

VOCABULARY
part  numerator
whole denominator
fraction

Draw on the board:

Have students name the fractions shaded and then have them say which circle has more shaded. ASK: Which is more: one fourth of the circle or three fourths of the circle?

ASK: Is three quarters of something always more than one quarter of the same thing? Is three quarters of a metre longer or shorter than a quarter of a metre? Is three quarters of a dollar more money or less money than a quarter of a dollar? Is three fourths of an orange more or less than one fourth of the orange? Is three fourths of the class more or less people than one fourth of the class? If three fourths of the class have brown eyes and one quarter of the class have blue eyes, do more people have brown eyes or blue eyes?

Tell students that if you consider fractions of the same whole—no matter what whole you’re referring to—three quarters of the whole is always more than one quarter of that whole, so mathematicians say that the fraction \( \frac{3}{4} \) is greater than the fraction \( \frac{1}{4} \). Ask students if they remember what symbol goes in between:

\[
\frac{3}{4} \quad \quad \quad \quad \quad \quad \frac{1}{4}
\]

Remind them that the inequality sign is like the mouth of a hungry person who wants to eat more of the pasta but has to choose between three quarters of it or one quarter of it. The sign opens toward the bigger number:

\[
\frac{3}{4} > \frac{1}{4} \quad \text{or} \quad \frac{1}{4} < \frac{3}{4}
\]

Have students decide which is more and to write the appropriate inequality in between the numbers:

\[
\frac{2}{5} \quad \quad \quad \quad \quad \quad \frac{3}{5}
\]
Repeat with several examples, eventually having students name the fractions as well:

\[
\begin{align*}
\frac{2}{7} & \quad \frac{3}{7} & \quad \frac{5}{7} & \quad \frac{4}{7} & \quad \frac{6}{7} \\
\end{align*}
\]

Draw the following pictures on the board:

\[
\begin{align*}
\text{ASK:} \quad \text{Which is greater: one quarter or two quarters? One eighth or two eighths? One sixth or two sixths? Show students a pie cut into sixths on the board and ask: If Sally gets one sixth and Tony gets two sixths, who gets more? If Sally gets three sixths and Tony gets two sixths, who gets more? Which is greater:}
\end{align*}
\]

\[
\begin{align*}
\frac{1}{9} & \quad \frac{2}{9} \quad ? \quad \frac{1}{12} & \quad \frac{2}{12} \quad ? \quad \frac{1}{53} & \quad \frac{2}{53} \quad ? \quad \frac{1}{100} & \quad \frac{2}{100} \quad ? \quad \frac{1}{807} & \quad \frac{2}{807} \quad ? \\
\frac{2}{9} & \quad \frac{5}{9} \quad ? \quad \frac{3}{11} & \quad \frac{4}{11} \quad ? \quad \frac{9}{11} & \quad \frac{8}{11} \quad ? \quad \frac{35}{87} & \quad \frac{43}{87} \quad ? \quad \frac{91}{102} & \quad \frac{54}{102} \quad ? \\
\end{align*}
\]

**Bonus**

\[
\begin{align*}
\frac{1}{21} & \quad \frac{11}{21} & \quad \frac{8}{21} & \quad \frac{19}{21} & \quad \frac{6}{21} & \quad \frac{12}{21} & \quad \frac{5}{21} \\
\end{align*}
\]

Then ask students to order a list of fractions with the same denominator (EXAMPLE: \(\frac{2}{7}, \frac{5}{7}, \frac{4}{7}\)) from least to greatest, eventually using bigger numerators and denominators and eventually using lists of 4 fractions.

**Bonus**

\[
\begin{align*}
\frac{4}{11}, \frac{9}{11}, \frac{3}{12}, \frac{9}{12}, \frac{4}{16}, \frac{13}{16}, \frac{21}{48}, \frac{25}{48}, \frac{67}{131}, \frac{72}{131} \\
\end{align*}
\]

Then write on the board:

Ask students to think of numbers that are in between these two numbers. Allow several students to volunteer answers. Then ask students to write individually in their notebooks at least one fraction in between:

\[
\begin{align*}
a) \frac{4}{11} & \quad \frac{9}{11} & \quad b) \frac{3}{12} & \quad \frac{9}{12} & \quad c) \frac{4}{16} & \quad \frac{13}{16} & \quad d) \frac{21}{48} & \quad \frac{25}{48} & \quad e) \frac{67}{131} & \quad \frac{72}{131} \\
\end{align*}
\]

**Bonus**

\[
\begin{align*}
\frac{104}{18301} & \quad \frac{140}{18301} \\
\end{align*}
\]
Draw on the board:

\[ \frac{1}{2} \]

\[ \frac{1}{3} \]

\[ \frac{1}{4} \]

Have a volunteer colour the first part of each strip of paper and then ask students which fraction shows the most: \( \frac{1}{2} \), \( \frac{1}{3} \), or \( \frac{1}{4} \). **ASK:** Do you think one fifth of this fraction strip will be more or less than one quarter of it? Will one eighth be more or less than one tenth?

**ASK:** Is one half of something always more than one quarter of the same thing? Is half a metre longer or shorter than a quarter of a metre? Is half an hour more or less time than a quarter of an hour? Is half a dollar more money or less money than a quarter of a dollar? Is half an orange more or less than a fourth of the orange? Is half the class more or less than a quarter of the class? If half the class has brown eyes and a quarter of the class has green eyes, do more people have brown eyes or green eyes?

Tell students that no matter what quantity you have, half of the quantity is always more than a fourth of it, so mathematicians say that the fraction \( \frac{1}{2} \) is greater than the fraction \( \frac{1}{4} \). Ask students if they remember what symbol goes in between: \( \frac{1}{2} \) \( \frac{1}{4} \) (\(<\) or \(>\)).

Tell your students that you are going to try to trick them with this next question so they will have to listen carefully. Then **ASK:** Is half a minute longer or shorter than a quarter of an hour? Is half a centimetre longer or shorter than a quarter of a metre? Is half of Stick A longer or shorter than a quarter of Stick B?

Stick A: 

Stick B: 

**ASK:** Is a half always bigger than a quarter?

Allow everyone who wishes to attempt to articulate an answer. Summarize by saying: A half of something is always more than a quarter of the same thing. But if we compare different things, a half of something might very well be less than a quarter of something else. When mathematicians say that \( \frac{1}{2} > \frac{1}{4} \), they mean that a half of something is always more than a quarter of the same thing; it doesn’t matter what you take as your whole, as long as it’s the same whole for both fractions.

Draw the following strips on the board:

Ask students to name the fractions and then to tell you which is more.

Have students draw the same fractions in their notebooks but with circles instead of strips. Is \( \frac{3}{4} \) still more than \( \frac{3}{8} \)? (yes, as long as the circles are the same size)
**Bonus**

Show the same fractions using a line of length 8 cm.

Ask students: If you cut the same strip into more and more pieces of the same size, what happens to the size of each piece?

Draw the following picture on the board to help them:

```
1 big piece
2 pieces in one whole
3 pieces in one whole
4 pieces in one whole
```

Many pieces in one whole

**ASK:** Do you think that 1 third of a pie is more or less pie than 1 fifth of the same pie? Would you rather have one piece when it's cut into 3 pieces or 5 pieces? Which way will you get more? Ask a volunteer to show how we write that mathematically ($\frac{1}{3} > \frac{1}{5}$).

Do you think 2 thirds of a pie is more or less than 2 fifths of the same pie? Would you rather have two pieces when the pie is cut into 3 pieces or 5 pieces? Which way will you get more? Ask a volunteer to show how we write that mathematically ($\frac{2}{3} > \frac{2}{5}$).

If you get 7 pieces, would you rather the pie be cut into 20 pieces or 30? Which way will get you more pie? How do we write that mathematically? ($\frac{7}{20} > \frac{7}{30}$).

Give students several problems similar to QUESTION 2 in the workbook. Use bigger numerators and denominators as bonus, always keeping the numerators the same.

**Extra Bonus**

Have students order the following list of numbers:

```
\frac{21}{28} \frac{21}{22} \frac{8}{200} \frac{19}{105} \frac{13}{200} \frac{19}{28} \frac{13}{105} \frac{19}{61}
```

**SAY:** Two fractions have the same numerator and different denominators. How can you tell which fraction is bigger? Why? Summarize by saying that the same number of pieces gives more when the pieces are bigger. The numerator tells you the number of pieces, so when the numerator is the same, you just look at the denominator. The bigger the denominator, the more pieces you have to share between and the smaller the portion you get. So bigger denominators give smaller fractions when the numerators are the same.

**SAY:** If two fractions have the same denominator and different numerators, how can you tell which fraction is bigger? Why? Summarize by saying that if the denominators are the same, the size of the pieces are the same. So just as 2 pieces of the same size are more than 1 piece of that size, 84 pieces of the same size are more than 76 pieces of that size.

Emphasize that students can’t do this sort of comparison if the denominators are not the same.

**ASK:** Would you rather 2 fifths of a pie or 1 half? Draw the following picture to help them:
Tell your students that when the denominators and numerators of the fractions are different, they will have to compare the fractions by drawing a picture or by using other methods that they will learn later.

Have students draw several hundreds blocks on grid paper (or provide them with several already made) and ask them to show the following fractions: \( \frac{32}{100}, \frac{47}{100}, \frac{82}{100}, \frac{63}{100} \).

**ASK:** Which fractions are greater than \( \frac{1}{2} \)? Which fractions are less than \( \frac{1}{2} \)? Which fraction with denominator 100 would be exactly \( \frac{1}{2} \)?

Write the two fractions \( \frac{3}{4} \) and \( \frac{4}{5} \) on the board. **ASK:** Do these fractions have the same numerator? The same denominator? (no, neither) Explain that we cannot compare these fractions directly using the methods in this section, so we need to draw a picture.

Have a volunteer shade the fraction 3 fourths on the first strip and the fraction 4 fifths on the second strip. **ASK:** On which strip is a greater area shaded? On which strip is a smaller area unshaded? How many pieces are not shaded? What is the fraction of unshaded pieces in each strip? (\( \frac{1}{4} \) and \( \frac{1}{5} \)) Can you compare these fractions directly? (yes, they have the same numerator). Emphasize that if there is less left unshaded, then there is more shaded. So \( \frac{4}{5} \) is a greater fraction than \( \frac{3}{4} \) because the unshaded part of \( \frac{4}{5} \) (i.e. \( \frac{1}{5} \)) is smaller than the unshaded part of \( \frac{3}{4} \) (i.e. \( \frac{1}{4} \)).

Repeat for other such pairs of fractions:

- \( \frac{7}{8} \) and \( \frac{8}{9} \), \( \frac{7}{8} \) and \( \frac{5}{6} \), \( \frac{89}{90} \) and \( \frac{74}{75} \), \( \frac{3}{5} \) and \( \frac{4}{6} \), \( \frac{72}{74} \) and \( \frac{34}{36} \), \( \frac{56}{76} \) and \( \frac{39}{59} \)

(draw the pictures when the numerators and denominators are small).

**ACTIVITY 1**

Give students their play dough flags made in a previous activity. Have them organize themselves into groups with people who chose the same two colours they did. For example, suppose 5 people chose red and blue as their two colours. Have those 5 students order their colours in terms of most red to least red and then order the fractions for red from the recipes in order from greatest to least. All students in the group should individually check their results by using fraction strips. Before distributing the BLM “Fraction Strips”, ask the class as a whole why they might expect slight disagreements with the fraction strip results and the play dough results. Which order of fractions do they think will be the correct order? What mistakes may have been made when making the play dough balls? (Some spoons may have had some red play dough still in them when making blue; the play dough may not have been completely flattened all the time.)
Give each student three strips of paper. Ask them to fold the strips to divide one strip into halves, one into quarters, one into eighths. Use the strips to find a fraction between

a) $\frac{3}{8}$ and $\frac{5}{8}$ (one answer is $\frac{1}{2}$ )

b) $\frac{1}{4}$ and $\frac{2}{4}$ (one answer is $\frac{3}{8}$ )

c) $\frac{5}{8}$ and $\frac{7}{8}$ (one answer is $\frac{3}{4}$ )

Have students fold a strip of paper (the same length as they folded in ACTIVITY 2) into thirds by guessing and checking. Students should number their guesses.

EXAMPLE:

Is $\frac{1}{3}$ a good answer for any part of Activity 2? How about $\frac{2}{3}$?

![Try folding here too short, so try a little further](image)

1st guess

$\frac{1}{3}$

next guess

$\frac{2}{3}$

Extensions

1. Write the following fractions in order from least to greatest. Explain how you found the order.

$\frac{1}{3}, \frac{1}{2}, \frac{5}{6}, \frac{2}{3}, \frac{1}{8}$

2. Why is $\frac{2}{3}$ greater than $\frac{2}{5}$? Explain.

3. Why is it easy to compare $\frac{2}{5}$ and $\frac{2}{3}$? Explain.

4. Have students compare $\frac{13}{87}$ and $\frac{14}{86}$ by finding a fraction with the same numerator as one of them and the same denominator as the other that is in between both fractions. For instance, $\frac{13}{86}$ is clearly smaller than $\frac{14}{86}$ and bigger than $\frac{13}{87}$. Another way to compare the two given fractions is to note that the second fraction has more pieces (14 instead of 13) and each piece is slightly bigger, so it must represent a bigger fraction.

5. (Atlantic Curriculum A10.4) If you know that $\frac{2}{5}$ > $\frac{2}{7}$, what do you know about □?

6. (Atlantic Curriculum A10.6) Connection to probability: Have students conduct an experiment by rolling 2 coloured dice (one red and one blue) and making a fraction with numerator from the red die and denominator from the blue die) Have students predict whether the fraction will usually be less than half and to verify their prediction by rolling the dice several times.
Mixed Fractions

Introduce mixed fractions by drawing the following picture on the board:

![Mixed Fraction Diagram]

Tell students that some friends ordered 3 pizzas with 4 pieces each. The shaded pieces show how much they have eaten. They ate two whole pizzas plus a quarter of another one. Draw the following pictures on the board.

a) ![Mixed Fraction Diagram]

b) ![Mixed Fraction Diagram]

c) ![Mixed Fraction Diagram]

**ASK:** How many whole pizzas are shaded? What fraction of the last pizza is shaded? Write the mixed fraction for the first picture and have volunteers write the fractions for the second and third pictures.

Draw models of several mixed fractions, asking students to name them in their notebooks. Use a variety of shapes for the whole piece, such as rectangles and triangles.

Write a fraction such as $3 \frac{1}{4}$ on the board. Draw a series of circles subdivided into the same number of parts, as given by the denominator of the fraction (since the denominator of the fraction in this example is 4, each pie has 4 pieces). Ask your students to shade the correct number of pieces in the pies to represent the fraction.

Tell your students that you have drawn more circles than they need so they have to know when to stop shading.

**EXAMPLE:**

$3 \frac{1}{4}$

They should shade the first 3 circles and 1 part of the fourth circle.

Have students sketch the pies for given fractions in their notebooks.

**EXAMPLES:** $2 \frac{1}{4}$, $3 \frac{1}{5}$, $1 \frac{6}{5}$, $2 \frac{1}{5}$, $3 \frac{2}{5}$.

If students have trouble, give them practice drawing the whole number of pies drawn with the correct number of pieces.
Extensions

1. Teach students how to count forwards by halves, thirds, quarters, and tenths beyond 1.
   Ask students to complete the patterns:
   a) \( \frac{1}{2}, \frac{2}{4}, \frac{3}{4}, \_\_\_, \_\_\_, \_\_\_, \_\_\_, \_\_\_ \)
   b) \( 2\frac{1}{2}, 2\frac{2}{4}, \_\_\_, \_\_\_, \_\_\_, \_\_\_, \_\_\_, \_\_\_, \_\_\_ \)
   c) \( \frac{1}{3}, \frac{2}{3}, \_\_\_, \_\_\_, \_\_\_, \_\_\_, \_\_\_, \_\_\_ \)
   d) \( \frac{7}{10}, \frac{8}{10}, \_\_\_, \_\_\_, \_\_\_, \_\_\_, \_\_\_, \_\_\_ \)

2. What model represents \( 2\frac{1}{2} \)? How do you know?

3. ASK: Which fractions show more than a whole? How do you know?
   \( 2\frac{3}{8}, \frac{5}{7}, 1\frac{5}{8}, 2\frac{1}{3}, \frac{2}{5} \)

4. Ask students to order these fractions from least to greatest: \( 3\frac{1}{6}, 1\frac{5}{6}, 7\frac{1}{11} \)
   ASK: Did they need to look at the fractional parts at all or just the whole numbers? Why?

Bonus
Order the following list of numbers: \( 3\frac{1}{6}, 5\frac{2}{7}, 2\frac{1}{11}, 6\frac{1}{8}, 8\frac{5}{7}, 4\frac{3}{10} \)

5. Ask students to order mixed fractions where some of the whole numbers are the same, and the fractional parts have either the same numerator or the same denominator.
   EXAMPLES: a) \( 3\frac{1}{6}, 5\frac{2}{7}, 3\frac{1}{5} \) b) \( 5\frac{3}{6}, 5\frac{2}{8}, 6\frac{1}{11} \)

Bonus
Give students longer lists of numbers that they can order using these strategies.
NS5-66
Improper Fractions

GOALS
Students will name improper fractions and fractions representing exactly one whole.

PRIOR KNOWLEDGE REQUIRED
Mixed fractions

VOCABULARY
mixed fraction fraction improper proper fraction

Draw the following shapes on the board:

Have students name the fractions shaded. **ASK:** How many parts are shaded? How many parts are in one whole? Tell them that they are all 1 whole and write $1 = \frac{4}{4}$ and $1 = \frac{6}{6}$.

Then have student volunteers fill in the blanks:

$$1 = \frac{9}{9} \quad 1 = \frac{7}{7}$$

Then tell them that sometimes they might have more than 1 whole—they might have two whole pizzas, for **EXAMPLE:**

**ASK:** Which number goes on top—the number of parts that are shaded or the number of parts in one whole? Tell them to look at the pictures. Ask them how many parts are in one whole circle and how many parts are shaded.

Then write:

$$2 = \frac{2}{2} \quad \text{and} \quad 2 = \frac{3}{3}$$

Then ask a volunteer to come and write the number of shaded pieces.

**ASK:** How are the numerator and denominator in each fraction related? (you double the denominator to get the numerator). Have students fill in the missing numbers:

$$2 = \frac{4}{2} \quad 2 = \frac{28}{14} \quad 2 = \frac{76}{38}$$

(Always use an even number when giving the numerator.)

**ASK:** How are the fractions above different from the fractions we’ve seen so far? Tell them these fractions are called improper fractions because the numerator is larger than the denominator. Challenge students to guess what a fraction is called if its numerator is smaller than its denominator. (proper fractions)

Draw on the board:
ASK: How many pieces are shaded? (9)

SAY: I want to write a fraction for this picture. Should 9 be the numerator or the denominator? (numerator) Do I usually put the number of shaded parts on top or on bottom? (top) How many equal parts are in 1 whole? (4) Should this be the numerator or the denominator? (denominator) Do we usually put the number of parts in 1 whole on top or on bottom? (bottom) Tell your students that the fraction is written \( \frac{9}{4} \).

Have volunteers write improper fractions for these pictures.

a) ![Fraction A]

b) ![Fraction B]

c) ![Fraction C]

ASK: How many parts are shaded? How many parts are in one whole?

Draw models of several improper fractions, asking students to name them in their notebooks. Use a variety of shapes such as rectangles and triangles for the whole.

Write a fraction such as \( \frac{15}{4} \) on the board. Draw a series of circles subdivided into the same number of parts, as given by the denominator of the fraction (since the denominator of the fraction in this example is 4, each pie has 4 pieces). Ask your students to shade the correct number of pieces in the pies to represent the fraction.

Tell your students that you have drawn more circles than they need so they have to know when to stop shading.

EXAMPLE:

\[
\frac{15}{4}
\]

They should shade the first 3 circles and 3 parts of the fourth circle.

Have students sketch the pies for given fractions in their notebooks.

EXAMPLE: \( \frac{11}{4} , \frac{15}{8} , \frac{19}{8} , \frac{10}{3} , \frac{12}{5} \).

ASK: Which fractions show more than a whole? How do they know?

\( \frac{13}{5} , \frac{2}{9} , \frac{14}{3} , \frac{12}{7} \).
GOALS
Students will relate mixed and improper fractions.

PRIOR KNOWLEDGE REQUIRED
Mixed fractions
Improper fractions

VOCABULARY
mixed fraction
improper fraction

ASK: What is a mixed fraction? What is an improper fraction? Have a volunteer write a mixed fraction for this picture and explain their answer:

Have another volunteer write an improper fraction for the same picture and explain their answer.

Draw several models of fractions larger than 1 on the board and have students write both the mixed fraction and the improper fraction. Use several different shapes other than circles.

Draw several more such models on the board and ask students to write an improper fraction if the model contains more than 2 whole pies, and a mixed fraction otherwise. This will allow you to see if students know the difference between the terms “mixed” and “improper”.

Tell students to draw models for the following mixed fractions and to write the corresponding improper fraction that is equal to it (this may be done in 2 different steps if students need it broken down).

a) \(2 \frac{3}{4}\)  b) \(3 \frac{1}{3}\)  c) \(2 \frac{1}{6}\)  d) \(1 \frac{5}{6}\)  e) \(3 \frac{2}{5}\)  f) \(2 \frac{7}{8}\)

Then tell students to draw models for the following improper fractions and then to write the mixed fraction that is equal to it.

a) \(\frac{13}{4}\)  b) \(\frac{7}{3}\)  c) \(\frac{11}{6}\)  d) \(\frac{19}{5}\)  e) \(\frac{27}{8}\)

SAY: You have written many numbers as both a mixed and an improper fraction. What is the same in both? What is different? (the denominators are the same because they tell you how many parts are in a whole, but the numerators will be different because the mixed fraction counts the pieces that make up the wholes separately from the pieces that only make up part of the whole; improper fractions count them all together)

ACTIVITY
Students can make models of the fractions on the worksheet by placing the smaller pattern blocks on top of the hexagon (or on top of whatever block is being used to represent the whole).
Extensions

1. Have students write their answers to these questions as both mixed and improper fractions.

   - What fraction of a tens block is 7 ones blocks? 17 ones blocks? 32 ones blocks?
   - What fraction of a hundreds block is 32 tens blocks? 43 tens blocks and 5 ones blocks?
   - How many metres are in 230 cm? 571 cm?
   - How many decimetres are in 54 centimetres? 98 cm?
   - What fraction of a dime is a quarter?

2. Ask students to solve these problems with pattern blocks or by sketching their answers on triangular grid paper.

   a) Which two whole numbers is \(\frac{23}{6}\) between?

   b) What mixed fraction of a pie would you have if you took away \(\frac{1}{6}\) of a pie from 3 pies (and what would the improper fraction be)?

---

**NS5-68**

**Mixed Fractions (Advanced)**

**GOALS**

Students will use multiplication to find the improper fraction equivalent to a given mixed fraction.

**PRIOR KNOWLEDGE REQUIRED**

- Mixed fractions
- Improper fractions
- Using pictures to see that mixed and improper fractions can represent the same amount
- Multiplication

**VOCABULARY**

- mixed fraction
- improper fraction

Draw on the board:

SAY: How many parts are in 1 pie? There are 4 quarters in one pie. How many quarters are in 2 pies? (8)

What operation can we use to tell us the answer? (multiplication) How many quarters are there in 3 pies? (4 \(\times\) 3 = 12)

ASK: How many quarters are in \(3\frac{3}{4}\) pies?

\[
3 \times 4 = 12 \\
\frac{3}{4} \leftarrow 3 \text{ extra pieces}
\]

So there are 15 pieces altogether.

How many halves are in 1 pie? (2) In 2 pies? (4) In 3 pies (6) In 17 pies? (34)

How do you know? What operation did you use to find that? (17 \(\times\) 2)
ASK: How many halves are in 1 \( \frac{1}{2} \) pies? Have a volunteer draw the picture on the board.

ASK: How many halves are in 2 \( \frac{1}{2} \) pies? In 3 \( \frac{1}{2} \) pies? In 4 \( \frac{1}{2} \) pies? In 20 \( \frac{1}{2} \) pies? What operations do you use to find the answer? \((20 \times 2 + 1)\)

\[40 \text{ pieces in 20 whole pies} \quad \text{1 extra piece}\]

Draw 8 \( \frac{1}{2} \) pies on the board and ASK: How many halves are in 8 \( \frac{1}{2} \)?

Emphasize that the extra half is just one more piece, so once they know how many halves are in 8 pies, they just add one to find how many are in 8 \( \frac{1}{2} \).

Have students write in their notebooks how many halves are in...

\[\begin{align*}
a) \ 2 \frac{1}{2} & \quad b) \ 5 \frac{1}{2} & \quad c) \ 11 & \quad d) \ 11 \frac{1}{2} & \quad \text{BONUS:} \ 49 \frac{1}{2} & \ 84 \frac{1}{2}
\end{align*}\]

ASK: How many thirds are in 1 pie? (3) Have a volunteer come to the board and divide a circle into thirds. ASK: How many thirds are in 2 pies? In 3 pies? In 10 pies? In 100 pies? In 1000 pies? How many thirds are in 1013 pies? In 1023 pies? In 523 pies?

Have students write in their notebooks how many thirds are in...

\[\begin{align*}
a) \ 2 \frac{1}{3} & \quad b) \ 5 \frac{1}{3} & \quad c) \ 11 & \quad d) \ 11 \frac{5}{3} & \quad \text{BONUS:} \ 49 \frac{2}{3} & \ 84 \frac{1}{3}
\end{align*}\]

Include questions with denominator 4.

Then introduce problems with a context. SAY: I have boxes that will hold 4 cans each. What fraction of a box is each can? (one fourth) How many fourths are in 2 wholes? How many cans will 2 boxes hold? How are these questions the same? How are they different?

A box holds 4 cans. How many cans will:

\[\begin{align*}
a) \ 1 \frac{1}{4} & \text{ boxes hold.} & b) \ 2 \frac{3}{4} & \text{ boxes hold.} & c) \ 1 \frac{1}{4} & \text{ boxes hold.} & d) \ 1 \frac{3}{4} & \text{ boxes hold.}
\end{align*}\]

To help your students, encourage them to rephrase the question in terms of fourths and wholes. For example, since a can is one fourth of a whole, a) becomes “How many fourths are in 1 \( \frac{1}{4} \)?”

Next, students will have to rephrase the question in terms of fractions other than fourths, depending on the number of items in each package.

\[\begin{align*}
a) \ \text{A box holds 6 cans. How many cans will } 1 \frac{5}{6} & \text{ boxes hold?} \\
b) \ \text{A box holds 8 cans. How many cans will } 2 \frac{3}{8} & \text{ boxes hold?} \\
c) \ \text{BONUS: A box holds 326 cans. How many cans will } 1 \frac{5}{326} & \text{ boxes hold?} \\
d) \ \text{Tennis balls come in cans of 3. How many balls will } 7 \frac{1}{3} & \text{ cans hold?} \\
e) \ \text{A bottle holds 100 mL of water. How many mL of water will } 7 \frac{63}{100} & \text{ bottles hold?}
\end{align*}\]

Teach students how to change mixed fractions to improper fractions: To change 2 \( \frac{3}{4} \) to an improper fraction, start by calculating how many pieces are in the whole pies \((2 \times 8 = 16)\) and add on the remaining pieces \((16 + 3 = 19)\), so \(2 \frac{3}{4} = \frac{19}{8}\). Give students several problems of this sort, where they convert from mixed to improper form.
Teach your students to explain how to turn a mixed fraction into an improper fraction, using a concrete model (for instance some pizzas or circles—cut into parts) as an example:

I multiply $3 \times 4$ because there are 3 whole pies which each have 4 pieces in them: this gives 12 pieces altogether.

$$3 \frac{1}{4} \quad \frac{12}{3 \times 4 = 12}$$

I add 1 more piece because the remaining pie has 1 piece in it: this gives 13 pieces altogether.

$$3 \frac{1}{4} \quad \frac{12 + 1}{12} = 13 \text{ pieces}$$

I keep the denominator the same because it tells the size of the pieces in each pie (which doesn’t change).

$$3 \frac{1}{4} = \frac{13}{4} \quad \text{13 equal parts shaded} \quad \text{4 equal parts in one whole}$$

Have students write the following mixed fractions as improper fractions and explain how they found the answer:

a) $3 \frac{1}{7}$  
\hspace{1cm} b) $5 \frac{1}{6}$  
\hspace{1cm} c) $4 \frac{3}{8}$  
\hspace{1cm} d) $7 \frac{5}{6}$  
\hspace{1cm} e) $6 \frac{5}{6}$
**NS5-69**

**Mixed and Improper Fractions (Advanced)**

Have each student write in their notebooks the mixed and improper fractions for several pictures displaying area:

Then provide examples involving length and capacity as well, as shown in QUESTION 4 of the worksheet.

How long is the line? How many litres are shown?

Have students write each whole number below as an improper fraction with denominator 2 and show their answer with a picture and a multiplication statement:

<table>
<thead>
<tr>
<th>a) 3 = $\frac{6}{2}$</th>
<th>b) 4</th>
<th>c) 2</th>
<th>d) 7</th>
<th>e) 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 × 2 = 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**ASK:** If I have the improper fraction $\frac{10}{2}$, how could I find the number of whole pies it represents? Draw on the board:

whole pies = $\frac{10}{2}$ pies

Ask students how many whole pies 10 half-sized pieces would make? Show students that they simply divide 10 by 2 to find the answer:

5 whole pies. **ASK:** How many whole pies are in $\frac{8}{2}$ pies? In $\frac{10}{2}$ pies? In $\frac{20}{2}$ pies? In $\frac{12}{2}$ pies? In $\frac{14}{2}$ pies? In $\frac{16}{2}$ pies? In $\frac{18}{2}$ pies?

Then have students write the mixed fractions below as improper fractions and show their answer with a picture. Students should also write a statement for the number of half-sized pieces in the pies.

<table>
<thead>
<tr>
<th>a) $3\frac{1}{2} = \frac{7}{2}$</th>
<th>b) $4\frac{1}{2}$</th>
<th>c) $2\frac{1}{2}$</th>
<th>d) $5\frac{1}{2}$</th>
<th>e) $8\frac{1}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \times 2 + 1 = 7$ halves</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

There are 3 pies with 2 halves each.
SAY: If I have the improper fraction $\frac{15}{2}$, how can I know how many whole pies there are and how many pieces are left over? I want to divide 15 into sets of size 2 and I want to know how many full sets there are and then if there are any extra pieces. What operation should I use? (division)
What is the leftover part called? (the remainder)

Write on the board: $15 \div 2 = 7$ Remainder 1, SO: $\frac{15}{2} = 7 \frac{1}{2}$.

15 ÷ 2 = 7 R 1

Draw the following picture on the board with three number statements.

$\frac{15}{4} = 3 \frac{3}{4}$

$4 \times 3 + 3 = 15$

$15 \div 4 = 3$ Remainder 3

Have students discuss the three interpretations of the picture and what they mean. When we divide 15 into sets of size 4, we get 3 sets and then 3 extra pieces left over. This is the same as dividing pies into fourths and seeing that 15 fourths is the same as 3 whole pies (with 4 pieces each) and then 3 extra pieces.

Repeat for several pictures, having volunteers write the mixed and improper fractions as well as the multiplication and division statements. Then have students do similar problems individually in their notebooks.

Then give students improper fractions and have them draw the picture, write the mixed fraction, and the multiplication and division statements.

Then show students how to change an improper fraction into a mixed fraction:

$2 \frac{1}{4} = 2 \times 4 + 1$ quarters = 9 quarters = $\frac{9}{4}$

Starting with $\frac{9}{4}$, we can find: $9 \div 4 = 2$ Remainder 1, SO:

$\frac{9}{4} = 2$ wholes and 1 more quarter = $2 \frac{1}{4}$

Have students change several improper fractions into mixed fractions without using pictures.

Tell them that $\frac{7}{2}$ pies is the same as 3 whole pies and another half a pie. ASK: Is this the same thing as 2 whole pies and three halves? Do we ever write $2 \frac{3}{2}$? ASK: When we find $7 \div 2$, do we write the answer as 3 Remainder 1 or 2 Remainder 3? Tell your students that as with division, we want to have the fewest number of pieces left over.

**Extension**

Write the following fractions in order: 3 $\frac{1}{4}$, 2 $\frac{7}{4}$, 1 $\frac{11}{4}$, 2 $\frac{1}{4}$, 3 $\frac{36}{4}$, 4 $\frac{3}{4}$
**NS5-70**

Investigating Mixed & Improper Fractions

**GOALS**

Students will make mixed and improper fractions using different shapes as the whole.

**PRIOR KNOWLEDGE REQUIRED**

- Familiarity with pattern blocks
- The relationship between mixed and improper fractions

**VOCABULARY**

- mixed fraction
- improper fraction

Give students pattern blocks or a copy of the pattern blocks BLM (and have them cut out the shapes).

Tell students that a hexagon represents one whole pie and ask them to show you a whole pie. If some students don’t know which piece is the hexagon, **ASK:** How many sides does a hexagon have? (6) When all students have shown you the hexagon, ask them how many triangles they would need to cover an entire hexagon. What fraction of the hexagon is a triangle? (1/6) Find a shape that is half of the hexagon. How many trapezoids would you need to make $\frac{1}{2}$ hexagons? How many trapezoids would they need to make $\frac{5}{2}$ hexagons? What fraction of a hexagon is a rhombus? How many rhombuses would you need to make 2 hexagons? What improper fraction is equal to 2 wholes? Tell them that there is no mixed fraction for 2 wholes—it is just a whole number.

**NOTE:** Students can make models of the fractions on the worksheet by placing the smaller pattern blocks on top of the hexagon block (or on top of whatever block is being used to represent the whole).

I want to make $\frac{1}{6}$ hexagons. How many equal-sized pieces should the hexagon be divided into? What shape should I use for my equal-sized pieces? Why? (I should use triangles because 6 triangles can be put together to make a hexagon) Show how to make $\frac{1}{6}$ hexagons using triangles.

What shape would you use to make $\frac{2}{5}$ hexagons? Why? Show how to make $\frac{2}{5}$ hexagons using trapezoids.

What shape would you use to make $\frac{13}{3}$ hexagons? Why? Show how to make $\frac{13}{3}$ hexagons using rhombuses.

**SAY:** So far, we have used only hexagons as a whole pie. Let’s use trapezoids as a whole pie. What shape would you use to make $\frac{2}{3}$ trapezoids? Why? What piece is one third of the trapezoid? Show how to make $\frac{2}{3}$ trapezoids using triangles.

Have students show how to make each fraction below using triangles and to draw pictures of their models in their notebooks:

- a) $\frac{1}{2}$
- b) $\frac{4}{3}$
- c) $\frac{8}{5}$
- d) $\frac{11}{3}$
- e) $\frac{2}{5}$
- f) $\frac{3}{1}$

Be sure everyone has done at least parts a) and b) above. **ASK:** Which 2 fractions from the list above are the same? How can you tell this from your pictures? Use your pictures to order the fractions above from least to greatest.

Write the following problem on the board.
Figure A is a model of 1 whole and Figure B is a model of \( \frac{5}{2} \).

a) Ori says that the shaded area in Figure A is more than the shaded area in Figure B. Is he correct? How do you know?

b) Ori says that because the shaded area of Figure A is greater than the shaded area of Figure B, 1 whole must be more than \( \frac{5}{2} \). What is wrong with his reasoning?

Extensions

1. If \( \frac{4}{9} \) of a structure looks like this: 

   What could the whole look like?

2. If the triangle represents a whole, draw \( 2 \frac{1}{2} \).

3. (Atlantic Mathematics Curriculum) Break egg cartons into sections (1 through 11), and use complete cartons as well. Distribute at least one of the sections to each of the students and say, “If this (whole carton) is one, what is \( \frac{1}{2} \)? If this (9 section piece) is one whole, show me one third. If this (2 sections) is one, show me \( 2 \frac{1}{2} \),” etc. Students should realize that any one section can have many different names depending on the size of the whole. It is also beneficial for students to frame these types of questions for their classmates.

4. What fraction of a metre is a decimetre? 12 decimetres?

5. Ask questions of the form: Stick B is what fraction of Stick A? Stick A is what fraction of Stick B? Write your answers as proper or improper fractions; not as mixed fractions.

Demonstrate with the following example.

A: 

B: 

Stick B is \( \frac{3}{5} \) of Stick A since putting it on top of Stick A will look like:

If Stick A is the whole, then the denominator is 5, because Stick A has 5 equal-sized parts. How many of those parts does Stick B take up? (3). So Stick B is \( \frac{3}{5} \) of Stick A.

What fraction of Stick B is Stick A?

If Stick B is the whole, what is the denominator? (3) Why? (because Stick B has 3 equal-sized pieces) How many of those equal-sized pieces does Stick A take up? (5) So stick A is \( \frac{5}{3} \) of Stick B.
As an improper fraction, Stick A is $1\frac{2}{3}$ of Stick B since it takes up one whole Stick B plus 2 more of those equal-sized pieces, but we will write our answers in terms of improper fractions instead of mixed fractions.

Let your students investigate with several examples. In this way, your students will discover reciprocals. If you know what fraction Stick A is of Stick B, what fraction is Stick B of Stick A? (just turn the fraction upside down!)

**REFLECT:** Why was it convenient to use improper fractions instead of mixed fractions?

## NS5-71
### Equivalent Fractions

## NS5-72
### Models of Equivalent Fractions

**GOALS**

Students will understand that different fractions can mean the same amount. Students will find equivalent fractions by using pictures.

**PRIOR KNOWLEDGE REQUIRED**

Comparing fractions
Fractions as area
Fractions of a set

**VOCABULARY**

equivalent fractions

Show several pairs of fractions on fraction strips and have students say which is larger, for **EXAMPLE**:

\[
\begin{array}{c}
\text{\includegraphics[width=3cm]{fraction1.png}} \\
\text{\includegraphics[width=3cm]{fraction2.png}}
\end{array}
\]

Include many examples where the two fractions are equivalent. Tell your students that when two fractions look different but actually show the same amount, they are called equivalent fractions. Have students find pairs of equivalent fractions from the pictures you have on the board. Tell them that we have seen other examples of equivalent fractions from previous classes and ask if anyone knows where. (There are 2 possible answers here: fractions that represent 1 whole are all equivalent, and the same for fractions representing 2 wholes; also, mixed fractions have an equivalent improper fraction).

Then have students find equivalent fractions by shading the same amount in the second strip as in the first strip and writing the shaded amount as a fraction:

\[
\begin{array}{c}
\text{\includegraphics[width=3cm]{fraction3.png}} \\
\text{\includegraphics[width=3cm]{fraction4.png}}
\end{array}
\]

Show students a fraction strip chart and have volunteers fill in the blank areas.
Shade the fraction \( \frac{1}{2} \) and then **ask**: What other fractions can you see that are equivalent to \( \frac{1}{2} \)? (\( \frac{2}{4} \) or \( \frac{4}{8} \) or \( \frac{5}{10} \)). What other fraction from the chart is equivalent to \( \frac{3}{4} \)? Repeat with \( \frac{8}{10} \), \( \frac{3}{5} \), \( \frac{1}{5} \), and \( \frac{4}{10} \). What fractions on the chart are equivalent to 1 whole?

Have students find as many fractions as they can that are all equivalent to the fraction shown in the picture.

Draw several copies of a square on the board with half shaded:

Have a volunteer draw a line to cut the square into 4 equal parts. Have another volunteer draw 2 lines to cut the square into 6 equal parts; have another cut the square into 8 equal parts and another into 10 equal parts.

Have volunteers name the equivalent fractions shown by the pictures (\( \frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} \)).

Then write on the board:

\[
\begin{align*}
\frac{1}{2} \times \square &= \frac{2}{4} \\
\frac{1}{2} \times \square &= \frac{3}{6} \\
\frac{1}{2} \times \square &= \frac{4}{8} \\
\frac{1}{2} \times \square &= \frac{5}{10}
\end{align*}
\]

For each picture, **ask**: How many times more shaded pieces are there? How many times more pieces are there altogether? Emphasize that if each piece (shaded or unshaded) is divided into 4 pieces, then, in particular, each shaded piece is divided into 4 pieces. Hence if the number of pieces in the figure is multiplied by 4, the number of shaded pieces will also be multiplied by 4: that is why you multiply the top and bottom of a fraction by the same number to make an equivalent fraction.

For the pictures below, have students divide each piece into equal parts so that there are a total of 12 pieces. Then have them write the equivalent fractions with the multiplication statements for the numerators and denominators:
Ask students to draw 4 boxes of equal length on grid paper and shade 1 box.

Point out to students that \( \frac{1}{4} \) of the area of the boxes is shaded. Now ask students to draw the same set of boxes, but in each box to draw a line dividing the box into 2 parts.

Now \( \frac{2}{8} \) of the area is shaded. Repeat the exercise, dividing the boxes into 3 equal parts, (roughly: the sketch doesn’t have to be perfectly accurate), then 4 parts, then five parts.

Point out to your students that while the appearance of the fraction changes, the same amount of area is represented.

\[ \frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20} \] all represent the same amount: They are equivalent fractions.

Ask students how each of the denominators in the fractions above can be generated from the initial fraction of \( \frac{1}{4} \). ANSWER: each denominator is a multiple of the denominator 4 in the original fraction:

\[ 8 = 2 \times 4; \quad 12 = 3 \times 4; \quad 16 = 4 \times 4; \quad 20 = 5 \times 4; \]

Then ask students how each fraction could be generated from the original fraction. ANSWER: multiplying the numerator and denominator of the original fraction by the same number:

\[ \frac{1 \times 5}{4 \times 5} = \frac{5}{20} \]

Point out that multiplying the top and bottom of the original fraction by any given number, say 5, corresponds to cutting each box into that number of pieces.

The fractions \( \frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16} \ldots \) form a family of equivalent fractions. Notice that no whole number
greater than 1 will divide into both the numerator and denominator of \( \frac{1}{4} \), so \( \frac{1}{4} \) is said to be reduced to lowest terms. By multiplying the top and bottom of a reduced fraction by various whole numbers, you can generate an entire fraction family. For instance \( \frac{2}{5} \) generates the family

\[
\begin{align*}
\frac{2 \times 2}{5 \times 2} &= \frac{4}{10} \\
\frac{2 \times 3}{5 \times 3} &= \frac{6}{15} \\
\frac{2 \times 4}{5 \times 4} &= \frac{8}{20}
\end{align*}
\]

and so on.

**ACTIVITY 1**

Use the play dough activity described in **NS4-71**: Equal Parts and Models of Fractions. Have them investigate what happens when they use different sizes of spoons. Have them make a ball that is one third red and two thirds white using half a teaspoon and then another ball with the same fractions, but using a whole teaspoon. Did they get the same colour? (Yes.) What if they used 2 half teaspoons of red and 4 half teaspoons of white—how is this the same as using one teaspoon of red and 2 teaspoons of white? What pair of equivalent fractions does this show?

**ACTIVITY 2**

Ask your students to make a model (using concrete materials such as cubes or beads) of a fraction that can be described in two ways. Ask students to describe these fractions. For instance, if the student makes the following model:

They might say “I can say \( \frac{3}{6} \) of the counters are white or \( \frac{1}{2} \) of the counters are white.”

**ACTIVITY 3**

Give your students 10 counters of one colour and 10 counters of a different colour. Ask them to make a model of a fraction that can be described in at least 3 different ways.

Here are two solutions:

\[
\begin{align*}
\frac{1}{2} &= \frac{2}{4} = \frac{4}{8} \\
\frac{6}{12} &= \frac{2}{4} = \frac{1}{2}
\end{align*}
\]

**ACTIVITY 4**

Give students blocks of 2 colours and have them make models of fractions of whole numbers using the method described in Exercise 5 of the Worksheet **NS5-72**: Models of Equivalent Fractions. Here are some fractions they might try:

a) \( \frac{3}{5} \) of 15  

b) \( \frac{3}{4} \) of 16  

c) \( \frac{3}{5} \) of 20  

d) \( \frac{2}{3} \) of 21
Extensions

1. Draw a ruler on the board divided into mm and cm, with a certain number of cm shaded, and write two fractions:

\[
\begin{array}{c|c}
\text{number of mm shaded} & \text{number of cm shaded} \\
\hline
\text{number of mm in total} & \text{number of cm in total}
\end{array}
\]

Are these fractions equivalent? They can also use the metre stick and write triples of equivalent fractions using cm, mm and dm.

2. Write as many equivalent fractions as you can for each picture.

a) 
\[
\begin{array}{c}
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\end{array}
\]

b) 
\[
\begin{array}{c|c|c}
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\\hline
\end{array}
\]

3. List 3 fractions between \(\frac{1}{2}\) and 1. HINT: Change \(\frac{1}{2}\) to an equivalent fraction with a different denominator (EXAMPLE: \(\frac{4}{8}\)) and then increase the numerator or decrease the denominator. Show your answers on a number line (this part is easier if they increase the numerator instead of decrease the denominator).

4. (Atlantic Curriculum A3.1) Ask the students to use their fingers and hands to show that \(\frac{1}{2}\) and \(\frac{5}{10}\) are equivalent fractions.

5. Ask students to use the patterns in numerators and denominators of the equivalent fractions below to fill in the missing numbers.

a) \(\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15}\)

b) \(\frac{1}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20} = \frac{15}{25}\)

c) \(\frac{3}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20}\)

Ask students if the same patterning method will work to find equivalent fractions in these patterns:

a) \(\frac{2}{3} = \frac{3}{4} = \frac{4}{12} = \frac{5}{15}\)

b) \(\frac{1}{2} = \frac{3}{5} = \frac{5}{8} = \frac{10}{16}\)

Have students discuss what is different about these questions than the ones above. Ensure that students understand that all the fractions in a sequence of equivalent fractions must be obtained by multiplying the numerator and denominator of a particular fraction by the same number. In these examples, the patterns are only obtained through adding, not multiplying, so the pattern will not produce a sequence of equivalent fractions.

6. Divide students into groups. Give each group a fraction with numerator 1.

EXAMPLES: \(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{10}\). Have each group find as many fractions as they can that are equivalent to their given fraction. Then ask each group to find, for each fraction, how many times more is the denominator than the numerator. Encourage students to notice that the multiplicative relationship between numerators and denominators is constant for equivalent fractions. This can also be explored for fractions with larger numerators (EXAMPLES: \(\frac{2}{3}, \frac{2}{5}\)) after students have seen how to describe multiplicative relationships using simple fractions (See Extension 4 of NS5-73: Fractions of Whole Numbers).
Brainstorm the types of things students can find fractions of (circles, squares, pies, pizzas, groups of people, angles, hours, minutes, years, lengths, areas, capacities, apples).

Brainstorm some types of situations in which it wouldn’t make sense to talk about fractions. For EXAMPLE: Can you say $3 \frac{1}{2}$ people went skiing? I folded the sheet of paper $4 \frac{1}{4}$ times?

Explain to your students that it makes sense to talk about fractions of almost anything, even people and folds of paper, if the context is right. EXAMPLE: Half of her is covered in blue paint; half the fold is covered in ink. Then teach them that they can take fractions of numbers as well. ASK: If I have 6 hats and keep half for myself and give half to a friend, how many do I keep? If I have 6 apples and half of them are red, how many are red? If I have a pie cut into 6 pieces and half the pieces are eaten, how many are eaten? If I have a rope 6 metres long and I cut it in half, how long is each piece? Tell your students that no matter what you have 6 of, half is always 3. Tell them that mathematicians express this by saying that the number 3 is half of the number 6.

Tell your students that they can find $\frac{1}{2}$ of 6 by drawing rows of dots. Put 2 dots in each row until you have placed 6 dots. Then circle one of the columns:

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>● ●</td>
<td>● ●</td>
<td>● ●</td>
</tr>
<tr>
<td>● ●</td>
<td>● ●</td>
<td>● ●</td>
</tr>
</tbody>
</table>

The number of dots in one column is half of 6. Have students find $\frac{1}{2}$ of each number using this method:

a) $\frac{1}{2}$ of 4 b) $\frac{1}{2}$ of 8 c) half of 10 d) half of 14

**Bonus**

Use this method to find $\frac{1}{3}$ of each of the following numbers. HINT: Put 3 dots in each row.

a) $\frac{1}{3}$ of 12 b) $\frac{1}{3}$ of 15 c) one third of 18 d) one third of 3

ASK: If you want to find $\frac{1}{3}$ of 12, how many dots in a row would you draw? (4) You would draw 4 columns and circle the dots in 1 column. Now draw 4 sets and share the dots into the sets (1 dot to each of the sets, 4 in total, then another dot to each of the sets, continue till you get 12 dots in total). What fraction of the 12 dots does each set represent? What would you do if you needed $\frac{3}{4}$ of 12? (You have to take 3 sets).
\[ \frac{1}{2} \text{ of } 8 = 2 \]
\[ \frac{3}{4} \text{ of } 8 = 6 \]

Invite volunteers to find \( \frac{3}{4} \) of 16 and \( \frac{2}{5} \) of 15 using this method. After that draw several pictures yourself and ask them to find the fractions they represent.

Teach your students to see the connection between the fact that 6 is 3 twos and the fact that \( \frac{1}{3} \) of 6 is 2. The exercise below will help with this:

Complete the number statement using the words “twos”, “threes”, “fours” or “fives”. Then draw a picture and complete the fraction statements. (The first one is done for you.)

<table>
<thead>
<tr>
<th>Number Statement</th>
<th>Picture</th>
<th>Fraction Statements</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 6 = 3 twos</td>
<td></td>
<td>( \frac{1}{3} \text{ of } 6 = \underline{\phantom{000}} ) ( \frac{2}{3} \text{ of } 6 = \underline{\phantom{000}} )</td>
</tr>
<tr>
<td>b) 12 = 4 _______</td>
<td></td>
<td>( \frac{1}{4} \text{ of } 12 = \underline{\phantom{0000}} ) ( \frac{2}{4} \text{ of } 12 = \underline{\phantom{0000}} ) ( \frac{3}{4} \text{ of } 12 = \underline{\phantom{0000}} )</td>
</tr>
<tr>
<td>c) 15 = 3 _______</td>
<td></td>
<td>( \frac{1}{3} \text{ of } 15 = \underline{\phantom{0000}} ) ( \frac{2}{3} \text{ of } 15 = \underline{\phantom{0000}} )</td>
</tr>
</tbody>
</table>

Then draw the following picture on the board:

\[ \begin{array}{cccc}
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & & \\
\end{array} \]

Tell students that one person said the picture represented \( \frac{5}{8} \), and another person said the picture represented \( \frac{5}{4} \). Ask your students to explain what both people were thinking. (The picture could represent \( \frac{5}{8} \) because \( \frac{5}{8} \) of the squares are shaded. The picture could represent \( \frac{5}{4} \) because \( \frac{5}{4} \) of one whole block is shaded.) To say what fraction of a figure is shaded, you first have to know what part of the fraction is being taken as the whole.

Ask your students what they would do to find \( \frac{1}{3} \) of 15 dots? (Divide the dots into 3 columns and count how many were in each column). Ask them to find a division statement that would suit the model they drew. (15 ÷ 3 = 5)

Write on the board: “\( \frac{1}{3} \) of 15 = 5 means 15 ÷ 3 = 5”. Give your students several more statements to rewrite as normal division statements, like \( \frac{1}{6} \) of 12, \( \frac{1}{5} \) of 15, \( \frac{1}{4} \) of 20, \( \frac{1}{2} \) of 16, \( \frac{1}{3} \) of 10, and so on.
Ask your students how they could use the exercise they finished to solve the following problem: circle \( \frac{1}{2} \) of a set of dots: ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ . Draw several sets of dots (such as 12 dots, 18 dots, 24 dots) in a line (not in two rows!) and ask them to circle half and then a third for each set.

Ask your students to tell you several ways to find \( \frac{2}{3} \) of 6 dots (or small circles). If the following two solutions do not arise, present them to your students:

1) To find two thirds of 6, you could find one third of six and multiply by 2
2) Circle two out of every 3 dots and count the total number of dots circled.

Have students find fractions of sets of objects (you can give the same number of objects arranged in different ways, for example, have your students find \( \frac{3}{4} \) of 16 boxes with the boxes arranged as \( 4 \times 4, 8 \times 2 \) or \( 16 \times 1 \) arrays.)

(Activity 1) (Adapted from Atlantic Curriculum A4.3) Give students 18 counters and a large circle divided into 3 equal parts. Ask students to use the circle and the counters to find \( \frac{2}{3} \) of 18. Then ask them to explain how they would use the circle to find \( \frac{2}{3} \) of 33. Then have them draw a circle they would use to find \( \frac{3}{5} \) of 20.

(Activity 2) Before doing this activity, ensure that students are comfortable finding fractions of numbers such as: \( \frac{13}{15} \) of 15. Teach this by saying: What is \( \frac{3}{4} \) of 4? If I divide 4 dots into 4 columns, how many do I have in each column? In 3 columns? What is \( \frac{5}{7} \) of 7? If I divide 7 dots into 7 columns, how many are in each column? In 5 columns? Repeat until students can tell you that \( \frac{13}{15} \) of 15 is 13 without having to divide 15 dots into 15 columns. Then use the BLM “Math Bingo Game (Sample boards)” with the BLM “Cards (Fractions of numbers)”.

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Extensions

1. How many months are in:
   a) \( \frac{1}{2} \) year?  
   b) \( \frac{2}{3} \) year?  
   c) \( 1 \frac{1}{2} \) years?

2. How many minutes are in:
   a) \( \frac{2}{3} \) of an hour?  
   b) \( \frac{1}{4} \) of an hour?  
   c) \( 1 \frac{1}{10} \) of an hour?

3. \( \frac{5}{6} \) of a day is how many hours?

4. Teach students how to describe multiplicative relationships between quantities by using simple fractions. Explain to your students that if you have 4 marbles and I have 4 marbles, then I have as many marbles as you have and you have as many marbles as I have.

   **ASK:** If I have 4 marbles and you have 8 marbles, how many times more marbles do you have than I have? If I have 4 marbles and you have 3 times as many marbles as I have, how many marbles do you have? If I have 4 marbles and you have 10 marbles, how many times more marbles do you have than I have? Guide students by saying: Two times as many marbles would be 8 and three times as many would be 12. If you have 10, that is halfway between two times as many and three times as many. What number is halfway between two and three? (two and a half) So you have \( 2 \frac{1}{2} \) times as many marbles as I have.

   Ask students to decide how many times more marbles they have than you if:
   
   a) You have 4 marbles and they have 6 marbles. (1 \( \frac{1}{2} \))
   b) You have 3 marbles and they have 8 marbles. (2 \( \frac{2}{3} \))
   c) You have 10 marbles and they have 25 marbles. (2 \( \frac{1}{2} \))
   d) You have 8 marbles and they have 12 marbles. (1 \( \frac{1}{2} \))
   e) You have 6 marbles and they have 9 marbles. (1 \( \frac{1}{2} \))
   f) You have 6 marbles and they have 8 marbles. (1 \( \frac{1}{2} \))

5. By weight, about \( \frac{1}{3} \) of a human bone is water and \( \frac{1}{4} \) is living tissue. If bone weighs 120 grams, how much of the weight is water and how much is tissue?

6. Explain to your students that you can have a fraction of a fraction! Demonstrate finding half of a fraction by dividing it into a top half and a bottom half:

   ![Diagram of fraction](image)

   \( \frac{1}{2} \) of \( \frac{3}{5} \) is \( \frac{3}{10} \)

   Have them find:
   
   a) \( \frac{1}{2} \) of \( \frac{2}{5} \)
   
   b) \( \frac{1}{2} \) of \( \frac{5}{7} \)

   Then have them draw their own pictures to find:
   
   c) \( \frac{1}{2} \) of \( \frac{3}{7} \)
   d) \( \frac{1}{2} \) of \( \frac{2}{5} \)
   e) \( \frac{1}{2} \) of \( \frac{5}{6} \)
   f) \( \frac{1}{2} \) of \( \frac{4}{7} \)

   **Bonus**

   Find two different ways of dividing the fraction \( \frac{4}{7} \) in half.
ANSWER:

\[
\begin{align*}
\text{Divide in half top to bottom.} \\
\frac{1}{2} \text{ of } \frac{4}{7} & \quad \text{is} \quad \frac{4}{14} \\
\text{Divide the 4 pieces in half from left to right.} \\
\frac{1}{2} \text{ of } \frac{4}{7} & \quad \text{is} \quad \frac{2}{7}
\end{align*}
\]

ASK: Are the two fractions \(\frac{2}{7}\) and \(\frac{4}{14}\) equivalent? How do you know?

7. Revisit the worksheet NS5-63. Ask students if they remember how they estimated the red section in Question 4 and then to explain their thinking. Then ask them to estimate the fraction of each flag that is blue.

For instance, for the flag of Chile in Part a), a student might say:

Half the flag is not red.

About \(\frac{1}{3}\) of the part that is not red is covered by the blue square with the star in it.

If you take half of a flag and cut it into thirds, you are left with \(\frac{1}{6}\) of the flag (\(\frac{1}{2}\) of \(\frac{1}{3}\) is \(\frac{1}{6}\)).

So \(\frac{1}{6}\) of the flag is covered by the blue square with a star in it.

About half of that square is covered by a white star and the other half is blue. So \(\frac{1}{2}\) of \(\frac{1}{6}\) of the flag is blue, but \(\frac{1}{2}\) of \(\frac{1}{6}\) is \(\frac{1}{12}\).

So altogether about \(\frac{1}{12}\) of the flag is blue.

Ask your students to determine by drawing a picture (as shown above) what the following amounts would be: \(\frac{1}{3}\) of \(\frac{1}{2}\), \(\frac{1}{4}\) of \(\frac{1}{2}\), \(\frac{1}{5}\) of \(\frac{1}{2}\), \(\frac{1}{6}\) of \(\frac{1}{2}\), \(\frac{1}{7}\) of \(\frac{1}{2}\), \(\frac{1}{8}\) of \(\frac{1}{2}\), \(\frac{1}{9}\) of \(\frac{1}{2}\), and so on.

Then have them find some flags in an atlas and estimate what fraction of each flag is covered by a particular colour.

Have students choose a sports team or other logo of their choice and decide what fraction of the logo is one colour by using counters. Students could revisit this exercise when they compare and order fractions.
NS5-74
Fractions of Whole Numbers (Advanced)

Remind your students of the problems they solved in NS5-73: Fractions of Whole Numbers.

Ask them to present as many methods to find the missing fraction in the statement 2 is ____ of 10 as they can. This time, they have to draw the picture themselves. They should keep making groups of 2 until they reach 10. Have them find the missing fraction for the following statements: 4 is ____ of 12, 4 is ____ of 20, 4 is ____ 16, 5 is ____ of 20, etc.

**ASK**: How many months are in a year? How many months are in \( \frac{2}{3} \) of a year? Which is longer? 9 months or \( \frac{3}{4} \) of a year? Which is longer—21 months or 1 \( \frac{3}{4} \) years? **HINT**: How many months are in \( \frac{5}{6} \) years? In 1 \( \frac{1}{2} \) years?

How many minutes are in an hour? How many minutes are in \( \frac{3}{5} \) of an hour? If Rita studied for 35 minutes and Katie studied for \( \frac{2}{3} \) of an hour, who studied longer? Katie started studying at 7:48 PM. Her favourite television show starts at 8:30 PM. Did she finish on time?

Tell students that you want to change word problems about fractions into mathematical sentences. Ask what symbol they would replace each word or phrase by: more than (>), is (=), half (\( \frac{1}{2} \)), three quarters (\( \frac{3}{4} \)).

Write on the board:

Calli’s age is half of Ron’s age.
Ron is twelve years old.
How old is Calli?

Teach students to replace each word they do know with a math symbol and what they don’t know with a blank:

\[
\text{Calli’s age} = \frac{1}{2} \text{ of } 12 \\
\text{Calli’s age} \text{ is } \text{ half } \text{ of } \text{ Ron’s age.}
\]

Have them do similar problems of this sort.

a) Mark gave away \( \frac{3}{4} \) of his 12 stamps. How many did he give away? 
(____ = \( \frac{3}{4} \) of 12)

b) There are 8 shapes. What fraction of the shapes are the 4 squares? 
(____ of 8 = 4)

c) John won three fifths of his five sets of tennis. How many sets did he win? (____ = \( \frac{3}{5} \) of 5)
Then have students change two sentences into one, replacing the underlined words with what they’re referring to:

a) Mark has 12 stamps. He gave away \( \frac{3}{4} \) of them. (Mark gave away \( \frac{3}{4} \) of his 12 stamps)
b) A team played 20 games. They won 11 of them.

Then have students solve several word problems, for **EXAMPLE:**

Anna had 10 strawberries. She ate two of them. What fraction of her strawberries did she eat?

**Literacy Connection**

Read *Alice in Wonderland* with them and give them the following problems.

a) The gardeners coloured red \( \frac{2}{3} \) of 15 roses. How many roses are red now? How many roses remained blank?

b) Alice ate 12 of 20 shrinking cakes. Which fraction of the shrinking cakes did she eat? Is that more than a half?

**Extension**

The chart shows the times of day when the Eyed Lizard is active:

- Awake but inactive
- Asleep
- Awake and active

a) What fraction of the day is the lizard…
   - i) awake but inactive
   - ii) asleep
   - iii) awake and active

b) How many hours a day is the lizard…
   - i) awake but inactive
   - ii) asleep
   - iii) awake and active
NS5-75
Comparing and Ordering Fractions

**GOALS**
Students will compare fractions where the denominator of the one fraction divides evenly into the denominator of the other fraction.

**PRIOR KNOWLEDGE REQUIRED**
Comparing fractions with the same denominator

Draw the following chart on the board.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 whole</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Have students fill in the remaining boxes. Then ask volunteers to name fractions that are:

- a) less than one third
- b) greater than two thirds
- c) between one half and two thirds
- d) between three fifths and four fifths
- e) between one quarter and one half
- f) equivalent to one half
- g) greater than three fifths
- h) equivalent to one whole

Have students write $>$ (greater than) or $<$ (less than) in between the fractions as appropriate:

- a) $\frac{2}{3}$ $\frac{3}{4}$
- b) $\frac{2}{3}$ $\frac{3}{5}$
- c) $\frac{1}{3}$ $\frac{2}{5}$
- d) $\frac{1}{2}$ $\frac{2}{5}$

**ASK:** Which fraction is larger: $\frac{2}{3}$ or $\frac{3}{7}$? How do you know? What makes these fractions easy to compare?

Tell your students that you would like to compare $\frac{1}{3}$ and $\frac{5}{12}$. How can you turn this problem into a problem like the last one? Emphasize that turning one problem into a different one that they already know how to do is a tool that mathematicians use every day. Is $\frac{1}{3}$ equivalent to a fraction with denominator 12? Which fraction? $\frac{4}{12}$. Demonstrate this with a picture:

\[
\frac{1}{3} \times 4 = \frac{4}{12}
\]

Which fraction is larger—$\frac{4}{12}$ or $\frac{5}{12}$? Which fraction is larger—$\frac{1}{3}$ or $\frac{5}{12}$? How do you know?

Have students compare (and demonstrate using a picture):

- a) $\frac{1}{3}$ and $\frac{4}{15}$
- b) $\frac{2}{3}$ and $\frac{5}{6}$
- c) $\frac{2}{5}$ and $\frac{5}{10}$
- d) $\frac{1}{2}$ and $\frac{7}{12}$
Bonus

\( \frac{4}{10} \) and \( \frac{7}{12} \). HINT: Use the answers to a) and d).

Then have students compare two fractions, where the denominator of the second divides evenly into the denominator of the first (EXAMPLES: \( \frac{3}{8} \) and \( \frac{1}{4} \), \( \frac{7}{10} \) and \( \frac{3}{5} \), \( \frac{3}{10} \) and \( \frac{1}{2} \)).

Finally, mix up the order of the fractions, so that sometimes the larger denominator is first and other times, it is second. Always make sure that the smaller denominator divides evenly into the larger denominator. Challenge students not to need a picture.

Extensions

1. Have students place these fractions (0, \( \frac{1}{5} \), \( \frac{2}{5} \), \( \frac{3}{4} \), \( \frac{1}{10} \), \( \frac{3}{40} \), \( \frac{1}{2} \), \( \frac{11}{40} \), \( \frac{19}{20} \)) on a number line by finding equivalent fractions over 40.

2. (Adapted from Atlantic Curriculum A2) Ask students if they think that \( \frac{1}{40} \) is a small fraction or a large fraction. Then discuss how \( \frac{1}{40} \) can represent a small number or a large number. For example, discuss how long \( \frac{1}{40} \) m is. It is 4 times smaller than a tenth of a m, so it is 4 times smaller than 10 cm. Have students hold their fingers apart the distance they think that represents. ASK: Is that a small distance or a large distance? Then ask students whether \( \frac{1}{40} \) of the Canadian population is a small number or a large number. ASK: If you take 1 out of every 40 people, how many of every 1 000 people is that? Write on the board:

\[
\frac{1}{40} \times \frac{1}{40} = \frac{1}{1000}
\]

Have students find the numerator. Then say: If we take 1 out of every 40 people in Canada, we are taking 25 out of every 1 000. How many out of every 10 000 people is that? (250) Out of every 100 000? (2 500) Out of every 1 000 000? (25 000) If Canada has about 30 000 000 people, how many people is 1 out of 40? (750 000) Tell students that this is about the number of people in all of New Brunswick; quite a large number even though it is a small fraction.
GOALS
Students will compare fractions with different denominators by changing both fractions to have the same denominator.

PRIOR KNOWLEDGE REQUIRED
Comparing fractions with the same denominator
Changing fractions to an equivalent fraction with a different denominator
Lowest common multiples

VOCABULARY
lowest common multiple
lowest common denominator

Review finding lowest common multiples (see PA5-16) and drawing lines to cut pies into more pieces to make an equivalent fraction (see NS5-72).

Draw two pies on the board with a different fraction shaded in each. (EXAMPLES: \(\frac{1}{3}\) and \(\frac{1}{2}\), or \(\frac{1}{4}\) and \(\frac{1}{7}\), or \(\frac{3}{7}\) and \(\frac{1}{6}\), or \(\frac{1}{2}\) and \(\frac{3}{5}\), or \(\frac{2}{5}\) and \(\frac{3}{5}\), or \(\frac{5}{8}\) and \(\frac{3}{8}\), or \(\frac{2}{5}\) and \(\frac{3}{5}\).

Have volunteers find i) the number of pieces in each pie and ii) the LCM of these two numbers. Then have volunteers draw lines to divide the pies into that many pieces. Students should then individually write the new fractions in their notebooks and compare them. Which fraction is larger?

Teach students how they can compare two fractions with different denominators without using pictures:

STEP 1: Find the lowest common multiple of the two denominators

STEP 2: Change both fractions to equivalent fractions with that denominator

STEP 3: Compare the two fractions that have the same denominator (the fraction with the larger numerator will be larger)

Be sure to include examples where the larger denominator is first and examples where the larger denominator is second.

Bonus
Have students compare a list of 3 or 4 fractions by finding the lowest common multiple of all the denominators.

Extensions

1. Tom uses \(\frac{7}{3}\) of a cup of flour to make bread and Allan uses 2 \(\frac{1}{2}\) cups. Who uses more flour? (Encourage students to do this problem first by comparing improper fractions and then by comparing mixed fractions. Which method is easier?)

2. Fill in the missing numbers in the sequence:

\[
\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \square, \frac{5}{6}, \square
\]

HINT: Change all fractions to equivalent fractions with denominator 6.

3. Put the numbers into the correct boxes to complete the sequence:

\[
\frac{1}{10}, \square, \frac{3}{10}, \square, \square, \frac{7}{10}, \frac{6}{10}, \square, \square
\]

\[
\frac{4}{10}, \frac{1}{2}, \frac{3}{5}, \frac{9}{10}, \frac{1}{5}, 1
\]
NS5-77
Adding and Subtracting Fractions

Draw two large circles representing pizzas (you can use paper pizzas as well) and divide them into 4 pieces each, shading them as shown.

Explain that these are two plates with several pizza pieces on each. How much pizza do you have on each plate? Write the fractions beneath the pictures. Tell them that you would like to combine all the pieces onto one plate, so put the “+” sign between the fractions and ask a volunteer to draw the results on a different plate. How much pizza do you have now?

Draw on the board:

\[
\frac{1}{4} + \frac{2}{4} = \]

Tell your students that you would like to regroup the shaded pieces so that they fit onto one circle. SAY: I shaded 2 fourths of one circle and 1 fourth of another circle. If I move the shaded pieces to one circle, what fraction of that circle will be shaded? How many pieces of the third circle do I need to shade? Tell them that mathematicians call this process adding fractions. Just like we can add numbers, we can add fractions too.

Do several examples of this, like \( \frac{1}{5} + \frac{2}{5} \), \( \frac{1}{3} + \frac{1}{3} \), never extending past 1 whole circle. Ask your students: You are adding two fractions. Is the result a fraction too? Does the size of the piece change while we transport pieces from one plate to the other? What part of the fraction reflects the size of the piece—top or bottom? Numerator or denominator? When you add fractions, which part stays the same, the top or the bottom; the numerator or the denominator? What does the numerator of a fraction represent? (The number of shaded pieces) How do you find the total number of shaded pieces when you moved them to one pizza? What operation did you use?

Show a couple more examples using pizzas, and then have them add the fractions without pizzas. Assign lots of questions like \( \frac{3}{5} + \frac{1}{5} \), \( \frac{2}{7} + \frac{3}{7} + \frac{2}{11} + \frac{4}{11} \), etc. Enlarge the denominators gradually.

**Bonus**

Add:

\[
\frac{12}{134} + \frac{45}{134} \quad \frac{67}{567} + \frac{78}{567} \quad \frac{67}{456} + \frac{49}{456}
\]

**Bonus**

Add more fractions:

\[
\frac{3}{17} + \frac{1}{17} + \frac{2}{17} \quad \frac{5}{94} + \frac{4}{94} + \frac{7}{94} \quad \frac{3}{19} + \frac{2}{19} + \frac{5}{19} + \frac{1}{19} + \frac{3}{19}
\]
Return to the pizzas and say that now you are taking pieces of pizza away. There was \( \frac{3}{4} \) of a pizza on a plate. You took away \( \frac{1}{4} \). Show on a model the one piece you took away:

[Diagram of a pizza with a piece taken away]

How much pizza is left? Repeat the sequence of exercises and questions you did for addition using subtraction.

### Extensions

1. Tell your students that sometimes adding fractions can result in more than one whole.
   
   Draw on the board:

   ![Diagram of two circles with shaded areas]

   Ask how many parts are shaded in total and how many parts are in 1 whole circle. Tell your students that, when adding fractions, we like to regroup the pieces so that they all fit onto 1 circle. **ASK:** Can we do that in this case? Why not? Tell them that since there are more pieces shaded than in 1 whole circle, the next best thing we can do is to regroup them so that we fit as many parts onto the first circle as we can and then we put only the leftover parts onto the second circle.

   Draw on the board:

   ![Diagram of two circles with shaded areas]

   Ask how many parts are shaded in the first circle and how many more parts do we need to shade in the second circle. Ask a volunteer to shade that many pieces and then tell them that mathematicians write this as:

   \[
   \frac{3}{4} + \frac{2}{4} = \frac{5}{4} = 1 + \frac{1}{4}
   \]

   Do several examples of this where the sum of the fractions is more than 1 whole, using more complex shapes to make it look harder as students get used to the new concept and then continue with examples where the sum is more than 2 wholes.

2. Teach your students the role of 0 in adding and subtracting fractions.

   a) \( \frac{3}{5} - \frac{3}{5} \)  
   b) \( \frac{5}{7} + \frac{0}{7} \)  
   c) \( \frac{3}{8} - \frac{0}{8} \)  
   d) \( \frac{6}{7} - 0 \)  
   e) \( 1 \frac{5}{3} + 0 \)

   Tell them that 0 is the same number with any denominator. This is very different from fractions with any other numerator, where the denominator matters a lot.

3. Once students know how to change a fraction to an equivalent fraction, they can, as an enriched exercise, quickly learn to add fractions. See the JUMP fractions unit, available from our website www.jumpmath.org for material on adding fractions.
4. Revisit Exercise 4 d) of worksheet NS5-63. Have your students estimate what fraction of the flag is red and then do the subdividing necessary (the subdividing should reveal that \( \frac{12}{40} \) of the flag is red). Now that students know how to compare fractions, they should check how close their answer is to the real fraction. Have your students compare the fractions guesses such as \( \frac{1}{3} \), \( \frac{1}{4} \), and \( \frac{2}{5} \) to \( \frac{12}{40} \) by finding equivalent fractions with denominator 120: \( \frac{40}{120} \), \( \frac{30}{120} \), \( \frac{48}{120} \) and \( \frac{36}{120} \). **ASK:** Which of the three guesses (\( \frac{1}{3} \), \( \frac{1}{4} \), and \( \frac{2}{5} \)) are closest to \( \frac{36}{120} \)?

---

**NS5-78**

**Fractions Review**

This worksheet is a review of the fractions section and can be used as consolidating homework.

**EXTENSIONS**

1. **ASK:** Which is greater, eight thirds or twelve fifths? (2 and two thirds or 2 and two fifths) and have students write the fractions as improper fractions and then as mixed fractions. Which way makes it easier to compare the fractions? Why?

**REFLECT:** Recall that when learning reciprocals in Extension 5 of NS5-70: Investigating Mixed and Improper Fractions, it was better to use improper fractions. When comparing 8 thirds to 12 fifths, it is better to change the fractions to mixed fractions. Emphasize that sometimes mixed fractions are more convenient and sometimes improper fractions are more convenient. As students gain more experience they will learn to predict which will be more convenient. It is important to understand both forms so that they can choose which one is more convenient for their purpose.

2. Give pairs of students cards with the following fractions on them:

\[
\frac{3}{6}, \quad \frac{2}{3}, \quad \frac{1}{4}, \quad \frac{7}{8}, \quad \frac{5}{12}, \quad \frac{5}{6}
\]

Ask them to arrange the cards in order from least to greatest (\( \frac{1}{4}, \frac{5}{12}, \frac{3}{6}, \frac{2}{3}, \frac{5}{6}, \frac{7}{8} \)) and to give reasons for their arrangement.

Possible reasons could include: \( \frac{2}{3} \) is only one step away from reaching the end of a number line divided into 6 parts. Similarly, \( \frac{7}{8} \) is only one step away from reaching a number line divided into 8 parts, but the steps in \( \frac{7}{8} \) are smaller, so the person who is \( \frac{7}{8} \) of the way across is closer to the end than the person who is only \( \frac{2}{3} \) of the way along the line; 6 steps out of 12 would be a half, and \( \frac{5}{12} \) is almost a half, so it is more than \( \frac{1}{2} \).
Part 2
Number Sense

NOTE: In this and subsequent lessons, blank hundreds charts/hundreds blocks are used extensively to illustrate decimals and decimal concepts. If you have an overhead projector or interactive white board, you can use the BLM “Blank Hundreds Charts” to project and work with hundreds blocks on a board, wall, or screen. Alternatively, you can (a) work with enlarged photocopies of the BLM or (b) draw an oversized hundreds block on chart paper and laminate it.

Remind students that there are 10 tenths in every whole—10 tenths in a whole pie, 10 tenths in a whole tens block, 10 tenths in a whole chocolate bar.

ASK: What is \( \frac{1}{10} \) of a centimetre? (a millimetre) How many millimetres are in a centimetre? (10) What is \( \frac{1}{10} \) of a dollar? (a dime or 10 cents) How many dimes do you need to make a dollar? (10) What is one tenth of 1000? (one hundred) How many hundreds do you need to make a thousand? What is one tenth of 100? (ten) What is one tenth of 10? (one) What is one tenth of 1? (one tenth)

WRITE:

\[
\begin{align*}
1000 &= 1\text{ thousand } + 0\text{ hundreds } + 0\text{ tens } + 0\text{ ones} \\
100 &= 1\text{ hundred } + 0\text{ tens } + 0\text{ ones} \\
10 &= 1\text{ ten } + 0\text{ ones} \\
1 &= 1\text{ one}
\end{align*}
\]

Tell students that just as we have a place value for thousands, hundreds, tens, and ones, mathematicians have invented a place value for tenths. Write the number 47 on the board and ASK: If we read the digits from left to right, do we start at the largest place value or the smallest? Does the 4 mean 4 tens or 4 ones? So if the largest place value is on the left and the place values get smaller as we move right, where should we put the place value for the tenths—to the left of the 4 or to the right of the 7? If I have 47 pies and one tenth of a pie, I could try writing it like this:

\[
\begin{array}{c}
tens \quad 471 \\ ones
\end{array}
\]

Does anyone see a problem here? What number does this look like?
We need a way to tell the difference between “four hundred seventy-one” and “forty-seven and one tenth.” We need a way to separate the ones from the tenths so that we know how many whole units we have. We could actually use anything.

DRAW:

\[
\begin{array}{c}
tens \quad 4 \quad 7 \quad 1 \\
ones
\end{array}
\]

\[
\begin{array}{c}
tens \quad 470 \\
ones
\end{array}
\]

\[
\begin{array}{c}
tens \quad 47 \ast 1 \\
ones
\end{array}
\]
**ASK:** How do we show the difference between whole dollars and tenths of a dollar, between dollars and dimes? What do we put in between the number of dollars and the number of dimes? Have a volunteer show four dollars and thirty cents on the board: $4.30 Tell them that the dot between the 4 and the 3 is called a **decimal point** and **ASK:** What if the decimal point wasn’t there—how much money would this look like?

Ask students to identify the place value of the underlined digits:

<table>
<thead>
<tr>
<th></th>
<th>2.5</th>
<th>34.1</th>
<th>6.3</th>
<th>10.4</th>
<th>192.4</th>
<th>37.2</th>
<th>8</th>
<th>073.2</th>
<th>100.5</th>
</tr>
</thead>
</table>

On a transparency or enlarged photocopy of a hundreds block, have a volunteer shade one tenth of the block. If the student shades a row or column, point out that the student chose an organized way of shading one tenth rather than randomly shading ten squares. If the student did the latter, have another volunteer show an organized way of shading one tenth. Then ask another volunteer to circle or otherwise highlight one tenth of the shaded part. **ASK:** What fraction of the hundreds block is the circled part? What coin is one tenth of a loonie? What coin is one tenth of a dime? What fraction of a loonie is that coin? What fraction of a whole anything is one tenth of a tenth? (one hundredth)

Write on the board: 4 183.25

Cover up all but the 4 and tell students that the 4 is the thousands digit. Uncover the 1 and **ASK:** What place value is the 1? How do you know? (PROMPT: What is one tenth of a thousand?) Continue in this way, uncovering each digit one at a time until you uncover the 5 and **SAY:** We know the 2 is the tenths digit. What is one tenth of a tenth? What place value is the 5?

Write on the board:

```
ones         8.04 ← hundredths
          ↑
tenths
```

In their notebooks, have students write the place value of the underlined digit in these numbers:

<table>
<thead>
<tr>
<th></th>
<th>2.71</th>
<th>3.42</th>
<th>7.65</th>
<th>8.46</th>
<th>9.01</th>
<th>0.83</th>
<th>1.04</th>
<th>.97</th>
<th>.45</th>
<th>.45</th>
</tr>
</thead>
</table>

Students can refer to the board for the spelling of “tenths” and “hundredths.”

Have students identify the place value of the digit 5 in each of these numbers:

<table>
<thead>
<tr>
<th></th>
<th>5.03</th>
<th>8.05</th>
<th>50.03</th>
<th>30.05</th>
<th>74.35</th>
<th>743.5</th>
<th>7435</th>
</tr>
</thead>
</table>

**Bonus**

432.15 34 521.08

**Extensions**

1. Remind students that a hundred is ten times more than ten. Now ask them to picture a square divided into ten parts and a square divided into a hundred parts. Each little part in the second square (each hundredth) is ten times smaller than a part in the first square (a tenth). Another way to describe this is to say that there are ten hundredths in each tenth. **ASK:** What is ten times a hundred? (a thousand) What is one tenth of a hundredth? (a thousandth) What is ten times a thousand? (ten thousand) What is one tenth of a thousandth? (one ten thousandth)
Have students write the place value of the digit 5 in each number:

41.015   32.6752   872.0105   54 321.02679   867 778.3415

How many times more is the first 5 worth than the second 5:

a) 385.5072   3 855.072   38.55072   3.855072   .3855072
b) 525   52.5   5.25   .525   .0525
c) 5.5   51.5   512.5   5127.5   51270.5

How many times more is the 6 worth than the 2?

6.2   6.32   6.72   6.02   6.102   61.02   610.2   674.312

How many times more is the 2 worth than the 5?

2.5   2.35   2.75   2.05   2.105   21.05   210.5   274.315

2. Draw a decagon on the board and ask students to count the number of sides. Write “decagon” underneath it. Then write the word “decade” on the board. Ask if anyone can read the word and if anyone knows what the word means. **ASK:** Do these words have any letters in common? (yes; deca) What do the meanings of the words have in common? (Both words refer to 10 of something: the number of sides, the number of years.) What do you think “deca” means? Then **ASK:** How many events do you think are in a decathlon? Which month do you think was originally the tenth month when the calendar was first made? (December) Is it still the 10th month? (No, it’s now the 12th month.) Then write the word “decimal” on the board. Underline the first 3 letters and ask students to think about how 10 is important for decimal numbers. (Each digit has 10 times larger place value than the digit on the right. There are 10 single digits from 0 to 9.) Allow several students to answer in their own words.

3. Have students fill in the blanks in questions where you mix up the order of the words ones, tenths, and hundredths.

**EXAMPLES:**

5.02   ___ tenths ___ hundredths ___ ones

89.13   ___ hundredths ___ ones ___ tenths ___ tens
NS5-80
Decimal Hundredths

**GOALS**
Students will translate fractions with denominator 10 or 100 to decimals.

**PRIOR KNOWLEDGE REQUIRED**
- Fractions
- Thinking of different items (EXAMPLE: a pie, a hundreds block) as a whole
- Decimal place value
- 0 as a place holder

**VOCABULARY**
- decimal
- decimal tenth
- decimal point

Draw the following pictures on the board and ask students to show the fraction \( \frac{5}{10} \) in each picture:

Tell students that mathematicians invented decimals as another way to write tenths: One tenth \( (\frac{1}{10}) \) is written as 0.1 or just .1. Two tenths \( (\frac{2}{10}) \) is written as 0.2 or just .2. Ask a volunteer to write \( \frac{7}{10} \) in decimal notation. (.7 or 0.7)

Ask if there is another way to write it. (0.7 or .7) Then have students write the following fractions as decimals:

- a) \( \frac{3}{10} \)
- b) \( \frac{8}{10} \)
- c) \( \frac{9}{10} \)
- d) \( \frac{5}{10} \)
- e) \( \frac{6}{10} \)
- f) \( \frac{4}{10} \)

**Bonus**
- Have students convert to an equivalent fraction with denominator 10 and then to a decimal:
  - a) \( \frac{2}{5} \)
  - b) \( \frac{1}{2} \)
  - c) \( \frac{4}{5} \)

In their notebooks, have students rewrite each addition statement using decimal notation:

- a) \( \frac{3}{10} + \frac{1}{10} = \frac{4}{10} \)
- b) \( \frac{2}{10} + \frac{3}{10} = \frac{5}{10} \)
- c) \( \frac{2}{10} + \frac{3}{10} = \frac{5}{10} \)
- d) \( \frac{4}{10} + \frac{2}{10} = \frac{6}{10} \)

**Bonus**
- a) \( \frac{1}{2} + \frac{1}{5} = \frac{7}{10} \)
- b) \( \frac{1}{2} + \frac{1}{5} = \frac{8}{10} \)

Draw on the board:

**ASK:** What fraction does this show? \( (\frac{5}{10}) \) What decimal does this show? \( (0.4 \) or \( .4 \))

Repeat with the following pictures:

Have students write the fractions and decimals for similar pictures independently, in their notebooks.
Then ask students to convert the following decimals to fractions, and to draw models in their notebooks:

- a) 0.3  
- b) .8  
- c) .9  
- d) 0.2

Demonstrate the first one for them:

0.3 = \frac{3}{10}  

Tell students that we use 1 decimal place to write how many tenths there are, and we use 2 decimal places to write how many hundredths there are. Show this picture:

SAY: There are 13 hundredths shaded. We can write this as 0.13 or .13 or \frac{13}{100}.

Have students write both a fraction and a decimal for each of the following pictures:

Tell your students that writing decimals is a little trickier when there are less than 10 hundredths. **ASK:** What if we have only 9 hundredths—would we write .9? If no one recognizes that .9 is 9 tenths, **ASK:** How do we write 9 tenths? Is 9 tenths the same as 9 hundredths? Which is larger?

Put up 2 hundreds blocks (with the grid drawn in) and have one volunteer shade 9 tenths and another volunteer shade 9 hundredths. Tell students that we write 9 tenths as .9 and 9 hundredths as .09; write each decimal beneath the corresponding picture.

Put up the following pictures:

- a)  
- b)  
- c)  
- d)  
- e)  
- f)  

---
Have volunteers write a decimal and a fraction for the first two and then have students do the same for the others independently, in their notebooks.

Put up several more hundreds blocks and tell the students that you are going to mix up the problems. Some pictures will show less than ten hundredths and some will show more than ten hundredths. Students should write the correct decimal and fraction for each picture in their notebooks.

EXAMPLES:

One row and one column are shaded. How many squares are shaded? What fraction is shaded?

Discuss various strategies students may have used to count the shaded squares. Did they count the squares to each side of the point at which the row and column overlap (2 + 7 in the row, 4 + 5 in the column, 1 for the square in the middle)? Did they add 10 + 10 for the row and column and then subtract 1 for the one square they counted twice? Did they count the squares in the column (10) and then add 2 + 7 for the squares not yet counted in the row? Did they add the 4 rectangles of white squares (8 + 10 + 28 + 35) and then subtract from 100? Did they push the 4 white rectangles together and see that it forms a 9 by 9 square?

Have students write decimals and fractions for the following pictures.

Encourage students to count the shaded squares using different strategies. Students might count some squares twice and adjust their answers at the end; they might subtract the non-shaded squares from the total; they might divide the shaded parts into convenient pieces, count the squares in the pieces separately, and add the totals. Have students show and explain various solutions to their classmates, so that all students see different ways of counting the squares. Tell students that when they can solve a problem in two different ways and get the same answer, they can know that they’re right. They won’t need you to check the answer because they can check it themselves!
Extensions

1. Put the following sequence on the board: .1, .3, .5, _____

   Ask students to describe the sequence (add .2 each time) and to identify the next number (.7).

   Even though the numbers are not in standard dollar notation, students can think of them in terms of dollars and dimes: .3 is 3 dimes, .5 is 5 dimes, .7 is 7 dimes, and so on. **ASK:** What is 1.3 in terms of dollars and dimes? (1 dollar and 3 dimes)

   Have students complete the following sequences by thinking of the numbers in terms of dollars and dimes and counting out loud. This will help students to identify the missing terms, particularly in sequences such as h) and i). Students should also state the pattern rules for each sequence (**EXAMPLE:** start at .1 and add .3).

   a) .1, .4, .7, _____
   b) 3.1, 3.4, 3.7, _____
   c) 1, .9, .8, _____
   d) 1, .8, .6, _____
   e) 3, 2.9, 2.8, _____
   f) 4.4, 4.2, 4, _____
   g) .2, .3, .4, _____, _____, _____
   h) .7, .8, .9, _____, _____, _____
   i) 2.7, 2.8, 2.9, _____, _____, _____
   j) 1.4, 1.3, 1.2, _____, _____, _____

2. Have students draw a line 25 squares long on grid paper and mark the ends as shown:

   ![Number Line](image)

   Have them mark the location of 4.8, 5.0, and 5.8.

   Then repeat with endpoints 42 and 67 and have them mark the locations of 48, 50, and 58. **ASK:** How are these two questions similar and how are they different?

   Notice that 4.2 is just 42 tenths and 6.7 is 67 tenths, so the number line with endpoints 42 and 67 can be regarded as counting the number of tenths.
GOALS
Students will practise identifying and distinguishing between tenths and hundredths.

PRIOR KNOWLEDGE REQUIRED
- Tenths and hundredths
- Decimal notation
- Fractional notation
- Reading 2-digit whole numbers such as 35 as 3 tens and 5 ones or 35 ones

Put up 2 blank hundreds blocks. Have one volunteer shade one tenth of one block, and have a second volunteer shade one hundredth of the other block. Invite other volunteers to write the corresponding decimals and fractions for each block:

- 0.1 = \[\frac{1}{10}\]
- 0.01 = \[\frac{1}{100}\]

Point to the block showing one tenth and SAY: How many hundredths does this block show? How else can we write that as a decimal? (We can write 0.10.) Emphasize that 0.1 is equal to 0.10, and tell students that 0.10 can be read as “10 hundredths” or “1 tenth and 0 hundredths.”

Draw on the board:

- 0.43

ASK: How many squares are shaded? (43) How many hundredths does this picture show? (43) How many full rows are shaded? (4) Since 1 full row is 1 tenth, how many tenths are shaded? (4) How many more squares are shaded? (3) SAY: There are 4 full rows and 3 more squares shaded. So there are 4 tenths and 3 hundredths. Tell students that we can write 4 tenths and 3 hundredths as follows:

\[
\begin{align*}
\text{ones} & \quad 0.43 \quad \text{hundredths} \\
\text{tenths} & \quad \uparrow
\end{align*}
\]

We can read this as “43 hundredths” or “4 tenths and 3 hundredths.”

ASK: How is this similar to the different ways we can read the 2-digit number 43? (we can read 43 as 4 tens and 3 ones or just as 43 ones)

On grid paper, have students draw a hundreds block, shade in a given fraction, and then write the fraction as a decimal. (Students can also work on a copy of the BLM “Blank Hundreds Charts.”) When students are done, have them read the decimal number in terms of hundredths only and then in terms of tenths and hundredths. Give students more fractions to illustrate and write as decimals:

- a) \(\frac{26}{100}\)
- b) \(\frac{81}{100}\)
- c) \(\frac{14}{100}\)
- d) \(\frac{41}{100}\)
- e) \(\frac{56}{100}\)
- f) \(\frac{72}{100}\)
Draw the following figure on the board:

AKS: How many full rows of ten are there? (none) So how many tenths do we have? (0) How many hundredths are there? (4). Write on the board:

ones $\rightarrow$ 0.04 $\leftarrow$ hundredths

tenths

ASK: How can we read this number? (4 hundredths or 0 tenths and 4 hundredths)

Have students draw the following fractions on grid paper and write the corresponding decimals:

a) $\frac{5}{100}$  

b) $\frac{7}{100}$  

c) $\frac{1}{100}$  

d) $\frac{2}{100}$  

e) $\frac{9}{100}$  

f) $\frac{3}{100}$

Finally, draw the following figure on the board:

ASK: How many full rows of ten are there? (4) So how many tenths do we have? (4) How many hundredths are in 4 tenths? (40) Are any other hundredths shaded? (no, just the 4 full rows)

Write on the board:

ones $\rightarrow$ 0.40 $\leftarrow$ hundredths

tenths

Tell students that we can read this as “4 tenths and 0 hundredths” or “40 hundredths.”

Have students draw the following fractions and write the corresponding decimals.

a) $\frac{60}{100}$  

b) $\frac{70}{100}$  

c) $\frac{80}{100}$  

d) $\frac{30}{100}$  

e) $\frac{50}{100}$  

f) $\frac{10}{100}$

Now have students illustrate and rewrite fractions that have either 0 tenths or 0 hundredths:

a) $\frac{6}{100}$  

b) $\frac{60}{100}$  

c) $\frac{90}{100}$  

d) $\frac{3}{100}$  

e) $\frac{20}{100}$  

f) $\frac{8}{100}$

Finally, give students a mix of all 3 types of fractions:

a) $\frac{36}{100}$  

b) $\frac{40}{100}$  

c) $\frac{5}{100}$  

d) $\frac{18}{100}$  

e) $\frac{46}{100}$  

f) $\frac{9}{100}$

Extension

Decimals are not the only numbers that can be read in different ways. Show students how all numbers can be read according to place value. The number 34 can be read as “34 ones” or “3 tens and 4 ones.” Similarly, 7.3 can be read as “73 tenths” or “7 ones and 3 tenths.” Challenge students to find 2 ways of reading the following numbers:
a) 3,500 (3 thousands and 5 hundreds or 35 hundreds)
b) 320 (3 hundreds and 2 tens or 32 tens)
c) 5.7 (5 ones and 7 tenths or 57 tenths)
d) 1.93 (19 tenths and 3 hundredths or 193 hundredths)
e) 0.193 (19 hundredths and 3 thousandths or 193 thousandths)

**NS5-82**

**Changing Tenths to Hundredths**

**GOALS**
Students will understand that whole decimal tenths can be written with a 0 in the hundredths position to form an equivalent decimal.

**PRIOR KNOWLEDGE REQUIRED**
Equivalent fractions
Decimal notation
Tenths and hundredths

**VOCABULARY**
equivalent fraction
equivalent decimal

Draw the following figure on the board:

ASK: What fraction of the first square is shaded? What fraction of the second square is shaded? Are these equivalent fractions? How do you know?

Draw the following figure on the board:

ASK: What fraction of the square is shaded? How many of the 100 equal parts are shaded? How many of the 10 equal rows are shaded? Are these equivalent fractions?

Have students use the following pictures to find equivalent fractions with denominators 10 and 100.

Then have students find equivalent fractions without using pictures:

a) \( \frac{80}{100} = \frac{4}{5} \)
b) \( \frac{2}{10} \)
c) \( \frac{40}{100} = \frac{2}{5} \)
d) \( \frac{1}{10} \)
When students can do this confidently, ask them to describe how they are getting their answers. Then remind students that a fraction with denominator 100 can be written as a decimal with 2 decimal places. **ASK:** What decimal is equivalent to \( \frac{80}{100} \)? (0.80) Remind them that a fraction with denominator 10 can be written as a decimal with 1 decimal place and **ASK:** What decimal is equivalent to \( \frac{8}{10} \)? (0.8) Tell them that mathematicians call 0.80 and 0.8 equivalent decimals and ask if anyone can explain why they are equivalent. (They have the same value; the fractions they are equivalent to are equivalent).

Have students rewrite these equivalent fractions as equivalent decimals:

a) \( \frac{90}{100} = \frac{9}{10} \)  
b) \( \frac{20}{100} = \frac{2}{10} \)  
c) \( \frac{40}{100} = \frac{4}{10} \)  
d) \( \frac{10}{100} = \frac{1}{10} \)

Tell students that saying “0.9 = 0.90” is the same as saying “9 tenths is equal to 90 hundredths or 9 tenths and 0 hundredths.” **ASK:** Is “3 tenths” the same as “3 tenths and 0 hundredths?” How many hundredths is that? Have a volunteer write the equivalent decimals on the board. (.3 = .30)

Have students fill in the blanks:

a) \( .3 = \frac{3}{10} = \frac{30}{100} = .\Box \Box \Box \)  
b) \( \Box \Box = \frac{7}{10} = \frac{70}{100} = .70 \)  
c) \( .4 = \frac{4}{10} = \frac{40}{100} = .\Box \Box \Box \)  
d) \( \Box \Box \Box \Box = \frac{50}{100} = .\Box \Box \Box \Box \)  
e) \( \Box \Box \Box \Box \Box = \frac{2}{10} = \frac{20}{100} = .\Box \Box \Box \Box \Box \)  
f) \( .9 = \frac{9}{10} = \frac{90}{100} = .\Box \Box \Box \Box \Box \)

**Extension**

**ASK:** Do you think that 5 hundredths is the same as 5 hundredths and 0 thousandths? How would we write the decimals for those numbers? (.05 = .050) Do you think that 7 ones is the same as 7 ones and 0 tenths? How would you write that in decimal notation? (7 = 7.0) Have students circle the equivalent decimals in each group of three:

a) 8  0.8  8.0  
b) 0.04  0.40  0.4  
c) 0.03  0.030  0.3  
d) 0.9  9.0  9  
e) 2.0  0.2  0.20
GOALS
Students will relate tenths and hundredths of whole numbers to tenths and hundredths of dollars, that is, to dimes and pennies. Students will use this understanding to compare and order decimals having one and two decimal places.

PRIOR KNOWLEDGE REQUIRED
Pennies, dimes, and dollars
Decimal notation
Writing values such as 3 tenths and 5 hundredths as 35 hundredths

Tell students that a dime is one tenth of a dollar then ASK: Does this mean I can take a loonie and fit 10 dimes onto it? Does it mean 10 dimes weigh the same as a loonie? What does it mean? Make sure students understand that you are referring not to weight or area, but to value—a dime has one tenth the value of a dollar, it is worth one tenth the amount. Ask students what fraction of a dollar a penny is worth. (one hundredth)

Have students find different ways of making $0.54 using only dimes and pennies. (5 dimes and 4 pennies, 4 dimes and 14 pennies, and so on until 54 pennies)

Tell students that since a dime is worth a tenth of a dollar and a penny is worth a hundredth of a dollar, they can write 5 dimes and 4 pennies as 5 tenths and 4 hundredths. ASK: How else could you write 4 dimes and 14 pennies? (4 tenths and 14 hundredths) Continue rewriting all the combinations of dimes and pennies that make $0.54. Tell students that they have seen 2 of these combinations before. Do they remember which ones and why they are special? Prompt by asking: Which way uses the most tenths (or dimes)? The most hundredths (or pennies)?

Then say you have 6 tenths and 7 hundredths and ask students to say this in terms of just hundredths. Have students rewrite both expressions in terms of dimes and pennies. (6 dimes and 7 pennies; 67 pennies)

Tell students that since a dime is worth a tenth of a dollar and a penny is worth a hundredth of a dollar, they can write 6 tenths and 7 hundredths as 67 hundredths. Then give students a decimal number (EXAMPLE: .18) and have students express it in hundredths only (18 hundredths), tenths and hundredths (1 tenth and 8 hundredths), pennies only (18 pennies), and dimes and pennies (1 dime and 8 pennies). Repeat with several decimals, including some that have no tenths or no hundredths.

EXAMPLES:

| .94 | .04 | .90 | .27 | .70 | .03 | .60 | .58 | .05 |

ASK: Which is worth more, 2 dimes and 3 pennies or 8 pennies? How many pennies are 2 dimes and 3 pennies worth? Which is more, 23 or 8? How would we write 2 dimes and 3 pennies as a decimal of a dollar? (.23) How would we write 8 pennies as a decimal? (.08) Which is more, .23 or .08?

ASK: How would you make $0.63 using pennies and dimes? How would you make .9 dollars using pennies and dimes? Which is more, .63 or .9?

Give students play-money dimes and pennies and ask them to decide which is more between:

a) .4 and .26 b) .4 and .62 c) .3 and .42 d) .3 and .24

Tell the class that you once had a student who said that .41 is more than .8 because 41 is more than 8. ASK: Is this correct? What do you think the student was thinking? Why is the student wrong? (The student was thinking...
of the numbers after the decimal point as whole numbers. Since 41 is more than 8, the student thought that .41 would be more than .8. But .8 is 8 tenths and .41 is 41 hundredths. The tenths are 10 times greater than the hundredths, so comparing .8 to .41 is like comparing 8 tens blocks to 41 ones blocks, or 8 cm to 41 mm. There might be more ones blocks, but they’re worth a lot less than the tens blocks. Similarly, there are more mm than there are cm, but 8 cm is still longer than 41 mm.

Students often make mistakes in comparing decimals with 1 and 2 decimal places. For instance, they will say that .17 is greater than .2. This activity will help students understand the relationship between tenths and hundredths.

Give each student play-money dimes and pennies. Remind them that a dime is a tenth of a dollar (which is why it is written as $0.10) and a penny is a hundredth of a dollar (which is why it is written as $0.01). Ask students to make models of the amounts in the left-hand column of the chart below and to express those amounts in as many different ways as possible by filling in the other columns. (Sample answers are shown in italics.) You might choose to fill out the first row together.

<table>
<thead>
<tr>
<th>Amount</th>
<th>Amount in Pennies</th>
<th>Decimal Names (in words)</th>
<th>Decimal Names (in numbers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 dimes</td>
<td>20 pennies</td>
<td>2 tenths (of a dollar)</td>
<td>.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20 hundredths</td>
<td>.20</td>
</tr>
<tr>
<td>3 pennies</td>
<td>3 pennies</td>
<td>3 hundredths</td>
<td>.03</td>
</tr>
<tr>
<td>4 dimes and 3</td>
<td>43 pennies</td>
<td>4 tenths and 3 hundredths</td>
<td>.43</td>
</tr>
<tr>
<td>pennies</td>
<td></td>
<td>43 hundredths</td>
<td>.43</td>
</tr>
</tbody>
</table>

When students have filled in the chart, write various amounts of money on the board in decimal notation and have students make models of the amounts. (EXAMPLES: .3 dollars, .27 dollars, .07 dollars) Challenge students to make models of amounts that have 2 different decimal representations (EXAMPLES: .2 dollars and .20 dollars both refer to 2 dimes).

When you feel students are able to translate between dollar amounts and decimal notation, ASK: Would you rather have .2 dollars or .17 dollars? In their answers, students should say exactly how many pennies each amount represents; they must articulate that .2 represents 20 pennies and so is actually the larger amount.

For extra practice, ask students to fill in the right-hand column of the following chart and then circle the greater amount in each column. (Create several such charts for your students.)

<table>
<thead>
<tr>
<th>Amount (in dollars)</th>
<th>Amount (in pennies)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.2</td>
<td></td>
</tr>
</tbody>
</table>
Extension

Have students create a hundredths chart by filling in a blank hundreds chart with hundredths, beginning with .01 in the top left corner and moving across and then down each row.

Ask students to find the following patterns in their charts. They should describe the patterns of where the numbers occur using the words “column” and “row.”

(Start in the fifth row and fifth column, move down one row and then repeat; Start in the seventh row and eighth column, move down one row and then repeat; Start in the second row and fourth column, move one row down and one column right, then one row down and one column left, and then repeat; Start in the first row and first column, move one row down and one column right, then repeat.)

Bonus

Fill in the missing numbers in the patterns below without looking at your chart.
NS5-84
Changing Notation: Fractions and Decimals

GOALS
Students will translate between fractional and decimal notation.

PRIOR KNOWLEDGE REQUIRED
Decimal notation
Fractions
Tenths and hundredths

Have students copy the following chart, with room for rows a) to e), in their notebooks:

<table>
<thead>
<tr>
<th></th>
<th>Tenths</th>
<th>Hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Draw on the board:

a)

b)

c)

d)

e)

Have students fill in their charts by recording the number of tenths in the first column and the number of hundredths left over (after they’ve counted the tenths) in the second column. When students are done, have them write in their notebooks the fractions shown and the corresponding decimals. Remind them that hundredths are shown with 2 decimal places: 9 hundredths is written as .09, not .9, because .9 is how we write 9 tenths and we need to make 9 hundredths look different from 9 tenths. (More on the differences later in the lesson.)

Point out, or ask students to describe, the relationship between the chart and the decimal numbers: the tenths (in the first column) are recorded in the first decimal place; the extra hundredths are recorded in the second decimal place.

Have students make 5 (non-overlapping!) hundreds blocks (10 by 10) on grid paper and label them a) to e). (You could also hand out copies of the BLM “Blank Hundreds Charts.”)

While they are doing this, write on the board:

a) .17  b) .05  c) .20  d) .45  e) .03

Tell students to shade in the correct fraction of each square to show the decimal number. When they are done, students should translate the decimals to fractions.

Remind students that the first decimal place (to the right of the decimal point) counts the number of tenths and ASK: What does the second decimal place count? Then write .4 on the board and ASK: What does this number mean—is it 4 tenths, 4 hundredths, 4 ones, 4 hundreds—what? How do you know?
(It is 4 tenths, because the 4 is the first place after the decimal point). **ASK:** If you write .4 as a fraction, what will the denominator be? (You may need to remind students that the denominator is the bottom number.) What will the numerator be? Write $\frac{4}{10}$ on the board.

Then write on the board: 0.28. **ASK:** How many tenths are in this number? How many hundredths are there in each tenth? How many more hundredths are there? How many hundredths are there altogether? What is the numerator of the fraction? The denominator? Write $\frac{28}{100}$ on the board.

Have students fill in the numerator of each fraction:

a) $\frac{.6}{10}$  

b) $\frac{.7}{10}$  

c) $\frac{.2}{10}$  

d) $\frac{.63}{100}$  

e) $\frac{.97}{100}$  

f) $\frac{.48}{100}$  

g) $\frac{.50}{100}$  

h) $\frac{.07}{100}$  

i) $\frac{.8}{10}$  

j) $\frac{.09}{100}$  

k) $\frac{.90}{100}$  

l) $\frac{.9}{10}$  

Then tell students that you are going to make the problems a bit harder. They will have to decide whether the denominator in each fraction is 10 or 100. Ask a volunteer to remind you how to decide whether the fraction should have denominator 10 or 100. Emphasize that if there is only 1 decimal place, it tells you the number of tenths, so the denominator is 10; if there are 2 decimal places, it tells you the number of hundredths, so the denominator is 100. Students should be aware that when we write $\frac{.30}{10}$, we are saying that 30 hundredths are the same as 3 tenths. The 2 decimal places in .30 tell us to count hundredths, whereas the denominator of 10 in $\frac{3}{10}$ tells us to count tenths.

Have students write the fraction for each decimal in their notebooks:

a) $.4$  

b) $.75$  

c) $.03$  

d) $.3$  

e) $.30$  

f) $.8$  

g) $.09$  

h) $.42$  

i) $.2$  

j) $.5$  

k) $.50$

**Bonus**

$.789$  

$.060$  

$.007$  

$.053$  

$.301$  

$.596$  

$.0102507$

Write on the board: $\frac{28}{100}$. **ASK:** How would we change this to a decimal? How many places after the decimal point do we need? (2) How do you know? (Because the denominator is 100, so we’re counting the number of hundredths). Ask volunteers to write decimals for the following fractions:

<table>
<thead>
<tr>
<th>3</th>
<th>24</th>
<th>8</th>
<th>80</th>
<th>8</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

Have them change the following fractions to decimals in their notebooks:

a) $\frac{29}{100}$  

b) $\frac{4}{10}$  

c) $\frac{4}{100}$  

d) $\frac{13}{10}$  

e) $\frac{6}{10}$  

f) $\frac{70}{100}$  

g) $\frac{3}{10}$  

h) $\frac{30}{100}$  

i) $\frac{67}{100}$  

j) $\frac{7}{100}$

**Bonus**

a) $\frac{399}{1000}$  

b) $\frac{48}{1000}$  

c) $\frac{4}{1000}$

**Bonus**

Have students rewrite the following addition statements using decimal notation:

a) $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$  

b) $\frac{40}{100} + \frac{5}{100} = \frac{45}{100}$  

c) $\frac{21}{100} + \frac{31}{100} = \frac{52}{100}$  

d) $\frac{22}{100} + \frac{7}{10} = \frac{92}{100}$

Put the following equivalencies on the board and **SAY:** I asked some students to change decimals to fractions and these were their answers. Which ones are incorrect? Why are they incorrect?

a) $.37 = \frac{37}{100}$  

b) $.68 = \frac{68}{10}$  

c) $.4 = \frac{4}{10}$  

d) $.90 = \frac{90}{10}$  

e) $.9 = \frac{9}{10}$  

f) $.08 = \frac{8}{10}$
Extensions

1. Cut the pie into more pieces to show \( \frac{2}{5} = .4 \)

   ![Pie diagram](image)

2. Draw a letter E covering more than .3 and less than .5 of a 10 × 10 grid.

3. **ASK:** When there is 1 decimal place, what is the denominator of the fraction? When there are 2 decimal places, what is the denominator of the fraction? What do you think the denominator of the fraction will be when there are 3 decimal places? How would you change .00426 to a fraction? How would you change \( \frac{9823}{1000000} \) to a decimal?

4. Write the following decimals in order by changing the decimals to fractions with a denominator of 100 and then comparing the fractions:
   
   \( .8, \ .4, \ .07, \ .17, \ .32, \ .85, \ .3 \)

---

**NS5-85**

Decimals and Fractions Greater Than One

**GOALS**

Students will write mixed fractions as decimals. Students will use pictures to compare and order decimals having 1 decimal place with decimals having 2 decimal places.

**PRIOR KNOWLEDGE REQUIRED**

Translating between decimal and fractional notation for numbers less than 1

**VOCABULARY**

mixed fraction

**ASK:** If I use a hundreds block to represent a whole, what can I use to show one tenth? (a tens block) What fraction does a ones block show? (one hundredth)

Draw on the board:

- 2 wholes
- 4 tenths
- 3 hundredths

Tell students that this picture shows the decimal 2.43 or the fraction \( \frac{43}{100} \). Mixed fractions can be written as decimals, too! Have volunteers write the mixed fraction and the decimal shown by each picture:

a) ![Mixed fraction A]

b) ![Mixed fraction B]

c) ![Mixed fraction C]
Have students draw base ten models for these mixed fractions in their notebooks:

\[
\begin{align*}
a) & \quad 1 \frac{30}{100} \\ b) & \quad 2 \frac{8}{100} \\ c) & \quad 4 \frac{21}{100}
\end{align*}
\]

**Bonus**
First change these fractions to mixed fractions and then draw models:

\[
\begin{align*}
a) & \quad 1 \frac{103}{100} \\ b) & \quad 2 \frac{214}{100} \\ c) & \quad 3 \frac{280}{100}
\end{align*}
\]

Then have students draw a model for each decimal:

\[
\begin{align*}
a) & \quad 3.14 \\ b) & \quad 2.53 \\ c) & \quad 4.81
\end{align*}
\]

Put up the following pictures and have volunteers write a mixed fraction and a decimal for each. Remind students that each fully shaded hundreds block is a whole.

\[
\begin{align*}
a) & \quad \\ b) & \quad \\ c) & \quad \\ d) & \quad \\ e) & \quad \\ f) & \quad 
\end{align*}
\]

**ASK:** How are these different from the base ten pictures? How are they the same? (The difference is that we haven’t pulled the rows (tenths) and squares (hundredths) apart; they’re both lumped together in a shaded square.)

Put up more pictures and have students write mixed fractions and decimals in their notebooks. Include pictures in which there are no tenths or no more hundredths after the tenths are counted.

**EXAMPLES:**

\[
\begin{align*}
a) & \quad \\ b) & \quad \\ c) & \quad \\ d) & \quad \\ e) & \quad \\ f) & \quad 
\end{align*}
\]

Have students draw pictures on grid paper and write decimals for each mixed fraction:

\[
\begin{align*}
a) & \quad 1 \frac{95}{100} \\ b) & \quad 2 \frac{39}{100} \\ c) & \quad 2 \frac{4}{100} \\ d) & \quad 3 \frac{85}{100} \\ e) & \quad 1 \frac{9}{10} \\ f) & \quad 1 \frac{9}{100}
\end{align*}
\]

Have students change these fractions to decimals without using a picture:

\[
\begin{align*}
a) & \quad 3 \frac{18}{100} \\ b) & \quad 12 \frac{3}{10} \\ c) & \quad 25 \frac{4}{100} \\ d) & \quad 34 \frac{8}{10} \\ e) & \quad 11 \frac{98}{100} \\ f) & \quad 41 \frac{19}{100} \quad \text{BONUS:} \quad 5 \frac{138}{100}
\end{align*}
\]

Then have students draw pictures on grid paper to show each decimal:

\[
\begin{align*}
a) & \quad .53 \\ b) & \quad .03 \\ c) & \quad .30 \\ d) & \quad .19 \\ e) & \quad .8 \\ f) & \quad .08
\end{align*}
\]

Finally, have students draw pictures on grid paper to show each pair of decimals and then to decide which decimal is greater:

\[
\begin{align*}
a) & \quad 3.04 \text{ or } 3.17 \\ b) & \quad 1.29 \text{ or } 1.7 \\ c) & \quad 1.05 \text{ or } 1.50 \\ d) & \quad 5 \text{ tenths or } 5 \text{ hundredths}
\end{align*}
\]

**Bonus**
Have students draw pictures on grid paper to show each decimal and then to put the decimals
in order from smallest to largest. What word do the corresponding letters make?

E. 7.03  S. 7.30  I. 2.15  L. 2.8  M. 2.09

NOTE: The correct word is “miles.” (You might need to explain to students what a mile is.) If any students get “smile” or “slime,” they might be guessing at the correct answer (this is much more likely than having made a mistake in the ordering of the numbers). Encourage them to put the numbers in order before trying to unscramble the letters. Then they can be reasonably confident that their word is the right one. If the numbers are in order and the word makes sense, they don’t need you to confirm their answer. It is important to foster this type of independence.

Extensions

1. Students can make up their own puzzle like the one in the last Bonus questions. Partners can solve each other’s puzzles.

   **STEP 1:** Choose a 3- or 4-letter word whose letters can be used to create other words.

   **EXAMPLE:** The letters in the word “d are” can be rearranged to make “read” or “dear.”

   **NOTE:** This step is crucial. Be sure students understand why it is important for the letters they choose to be able to make at least 2 different words.

   **STEP 2:** Choose one decimal number for each letter and write them in order.

   **EXAMPLE:**
   
   d. 1.47  a. 1.52  r. 2.06  e. 2.44

   **STEP 3:** Scramble the letters, keeping the corresponding numbers with them.

   **EXAMPLE:**
   
   a. 1.52  e. 2.44  d. 1.47  r. 2.06

   **STEP 4:** Give the scrambled letters and numbers to a partner to put in the correct order. Did your partner find the word you started with?

2. Show .2 in each of these base ten blocks:

   a) 
   
   thousands block

   b) 
   
   hundreds block

   c) 
   
   tens block

3. (Adapted from Atlantic Curriculum A6.7 and A7)

   Teach your students that the decimal point is read as “and.” For example, 13.7 is read as “thirteen and seven tenths.”

   Have students list:

   a) whole numbers that take exactly 3 words to say

   **EXAMPLES:** 9 000 080, 600 000, 403

   b) decimal numbers that take exactly 6 words to say

   **EXAMPLES:** 403.08 (four hundred three and eight hundredths)
   600 000.43 (six hundred thousand and forty-three hundredths)
   9 000 080.09 (nine million eighty and nine hundredths)
If students are familiar with thousandths and ten thousandths, they might come up with:

- 3.542 (three and five hundred forty-two thousandths)
- 500.000 1 (five hundred and one ten thousandth)

4. (Atlantic Curriculum A7) Teach students to interpret whole numbers written in decimal format

**EXAMPLE:** 5.1 million as 5 100 000

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### NS5-86

**Decimals and Fractions on Number Lines**

**GOALS**

Students will place decimal numbers and mixed fractions on number lines. Students will also write the words for decimals and fractions (proper, mixed, or improper).

**PRIOR KNOWLEDGE REQUIRED**

Decimal numbers with up to 2 decimal places and their equivalent fractions (proper or mixed)

Translating between mixed and improper fractions

Number lines

Draw on the board:

```
0 |---|---|---|---|---|---|---|---|---|---|---|
  |  \ |  \ |  \ |  \ |  \ |  \ |  \ |  \ |  \ |  \ |
 1/10 2/10 3/10 4/10 5/10 6/10 7/10 8/10 9/10 1
```

Have students count out loud with you from 0 to 1 by tenths: zero, one tenth, two tenths, … nine tenths, one.

Then have a volunteer write the equivalent decimal for \( \frac{1}{10} \) on top of the number line:

```
0 |---|---|---|---|---|---|---|---|---|---|---|
  |  \ |  \ |  \ |  \ |  \ |  \ |  \ |  \ |  \ |  \ |
 0.1 |---|---|---|---|---|---|---|---|---|---|---|
```

Continue in random order until all the equivalent decimals have been added to the number line.

Then have students write, in their notebooks, the equivalent decimals and fractions for the spots marked on these number lines:

- a) \( 0.1 \)
- b) \( 0.9 \)
- c) \( 0.4 \)
- d) \( 0.7 \)

Have volunteers mark the location of the following numbers on the number line with an X and the corresponding letter.

- A. 0.7
- B. 2 \( \frac{7}{10} \)
- C. 1.40
- D. \( \frac{8}{10} \)
- E. 1 \( \frac{9}{10} \)
Invite any students who don’t volunteer to participate. Help them with prompts and questions such as: Is the number more than 1 or less than 1? How do you know? Is the number between 1 and 2 or between 2 and 3? How do you know?

Review translating improper fractions to mixed fractions, then ask students to locate the following improper fractions on a number line from 0 to 3 after changing them to mixed fractions:

A. $\frac{17}{10}$  B. $\frac{23}{10}$  C. $\frac{14}{10}$  D. $\frac{28}{10}$  E. $\frac{11}{10}$

When students are done, ASK: When the denominator is 10, what is an easy way to tell whether the improper fraction is between 1 and 2 or between 2 and 3? (Look at the number of tens in the numerator—it tells you how many ones are in the number.)

ASK: How many tens are in 34? (3)  78? (7)  123? (12)  345? (34)

ASK: How many ones are in $\frac{34}{10}$? (3)  $\frac{78}{10}$? (7)  $\frac{123}{10}$? (12)  $\frac{345}{10}$? (34)

ASK: What two whole numbers is each fraction between?

a) $\frac{29}{10}$  b) $\frac{4}{10}$  c) 12  d) $\frac{81}{10}$  e) $\frac{127}{10}$  f) $\frac{318}{10}$

Invite volunteers to answer a) and b) on the board, then have students do the rest in their notebooks. When students are finished, ASK: Which 2 fractions in this group are equivalent?

Tell students that there are 2 different ways of saying the number 12.4. We can say “twelve decimal four” or “twelve and four tenths”. Both are correct. (Note that “twelve point four” is also commonly used.) Point out the word “and” between the ones and the tenths, and tell students that we always include it when a number has both ones and tenths (and/or hundredths).

Have students place the following fractions on a number line from 0 to 3:

A. three tenths  B. two and five tenths  C. one and seven tenths
D. one decimal two  E. two decimal eight

Draw a number line from 0 to 3 on the board. Mark the following points with an X—no numbers—and have students write the number words for the points you marked:

1.3  .7  2.4  .1  2.8  2.1

Draw a line on the board with endpoints 0 and 1 marked:

[-------------------]

0  1

Ask volunteers to mark the approximate position of each number with an X:

a) .4  b) $\frac{6}{10}$  c) 0.9

Then draw a number line from 0 to 3 with whole numbers marked:

[-------------------]

0  1  2  3

Ask volunteers to mark the approximate position of these numbers with an X:

a) 2.1  b) 1 $\frac{3}{10}$  c) $\frac{29}{10}$  d) .4  e) 2 $\frac{2}{10}$
Continue with more numbers and number lines until all students have had a chance to participate. (EXAMPLE: Draw a number line from 0 to 2 with whole numbers marked and have students mark the approximate position of 0.5, 1.25, and others.)

**Bonus**

Use larger whole numbers and/or longer number lines.

**Extensions**

1. Use a metre stick to draw a line that is 2 metres long and ask students to mark 1.76 metres.

2. Mark the given decimals on the number lines:
   
   a) Show .4
   
   b) Show 1.5

---

**NS5-87**

**Comparing & Ordering Fractions and Decimals**

Draw on a transparency:

[Diagram of a number line from 0 to 1 with decimal markers from 0 to .9 labeled]

(NOTE: If you don’t have an overhead projector, tape the transparency to the wall and invite students, in small groups if necessary, to gather around it as you go through the first part of the lesson.) Have a volunteer show where \( \frac{1}{2} \) is on the number line. Have another number line the same length divided into 2 equal parts on another transparency and superimpose it over this one, so that students see that \( \frac{1}{2} \) is exactly at the .5 mark. **ASK:** Which decimal is equal to \( \frac{1}{2} \)? Is 0.2 between 0 and \( \frac{1}{2} \) or between \( \frac{1}{2} \) and 1? Is 0.7 between 0 and \( \frac{1}{2} \) or between \( \frac{1}{2} \) and 1? What about 0.6? 0.4? 0.3? 0.9?

Go back to the decimal 0.2 and **ASK:** We know that 0.2 is between 0 and \( \frac{1}{2} \), but is it closer to 0 or to \( \frac{1}{2} \)? Draw on the board or the transparency:

[Diagram of a number line from 0 to 1 with decimal markers from 0 to .9 labeled]

**ASK:** Is .6 between 0 and \( \frac{1}{2} \) or between \( \frac{1}{2} \) and 1? Which number is it closest to, \( \frac{1}{2} \) or 1? Have a volunteer show the distance to each number with arrows. Which arrow is shorter? Which number is .4 closest to, 0, \( \frac{1}{2} \) or 1? Which number is .8 closest to? Go through all of the remaining decimal tenths between 0 and 1.
On grid paper, have students draw a line 10 squares long. Then have them cut out the line—leaving space above and below for writing—and fold it in half so that the two endpoints meet. They should mark the points 0, $\frac{1}{2}$, and 1 on their line. Now have students fold the line in half again, and then fold it in half a second time. Have them unfold the line and look at the folds. **ASK:** What fraction is exactly halfway between 0 and $\frac{1}{2}$? How do you know?

($\frac{1}{4}$ because the sheet is folded into 4 equal parts so the first fold must be $\frac{1}{4}$ of the distance from 0 to 1) What fraction is halfway between $\frac{1}{2}$ and 1? How do you know? ($\frac{3}{4}$ because the sheet is folded into 4 equal parts so the third fold must be $\frac{3}{4}$ of the distance from 0 to 1).

Have students mark the fractions $\frac{1}{4}$ and $\frac{3}{4}$ on their number lines. Then have students write the decimal numbers from .1 to .9 in the correct places on their number lines (using the squares on the grid paper to help them).

Tell students to look at the number lines they’ve created and to fill in the blanks in the following questions by writing “less than” or “greater than” in their notebooks.

a) 0.4 is _______ $\frac{1}{4}$ b) 0.4 is _______ $\frac{1}{2}$ c) 0.8 is _______ $\frac{3}{4}$

d) 0.2 is _______ $\frac{1}{4}$ e) 0.3 is _______ $\frac{1}{2}$ f) 0.7 is _______ $\frac{3}{4}$

Have students rewrite each statement using the “greater than” and “less than” symbols: > and <.

**ASK:** Which whole number is each decimal, mixed fraction, or improper fraction closest to?

<table>
<thead>
<tr>
<th>a) 0.7</th>
<th>b) $1 \frac{4}{10}$</th>
<th>c) 2.3</th>
<th>d) $\frac{18}{10}$</th>
<th>e) $2 \frac{6}{10}$</th>
<th>f) 1.1</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>a) 17.2</th>
<th>b) 16.8</th>
<th>c) $16 \frac{3}{10}$</th>
<th>d) $\frac{174}{10}$</th>
<th>e) 15.9</th>
<th>f) 15.3</th>
</tr>
</thead>
</table>

**ASK:** Which decimal is halfway between 1 and 2? Halfway between 17 and 18? Halfway between 31 and 32? Between 0 and 3? Between 15 and 18? Between 30 and 33? Between 25 and 28?

**Bonus**

Which whole number is each decimal closest to?

a) 23.4 b) 39.8 c) 314.1 d) 235.6 e) 981.1 f) 999.9

**Extension**

(Adapted from Atlantic Curriculum A6.3) Have students rearrange the following words to create different numbers. What are the smallest and largest numbers that can be made using these words:

hundredths hundred thousand two five nine thirty-seven and

**ANSWER:**

Largest number: Thirty-seven thousand nine hundred five and two hundredths
Smallest number: Two thousand five hundred nine and thirty-seven hundredths
GOALS
Students will compare and order decimals and fractions by first changing them all to fractions with denominator 10 or 100.

PRIOR KNOWLEDGE REQUIRED
Ordering mixed fractions with the same denominator
Decimal place value
Translating between fractions with denominator 10 or 100 and decimals
Equivalent fractions

To ensure students have the prior knowledge required, ask them to do the following questions in their notebooks.

1. Write each decimal as a fraction or mixed fraction with denominator 10.
   a) 0.7  b) 0.4  c) 1.3  d) 2.9  e) 7.4  f) .6

2. Put the fractions in order from smallest to largest.
   a) $\frac{7}{10}$, $\frac{4}{10}$, $\frac{9}{10}$  b) $\frac{4}{10}$, $\frac{3}{10}$, $\frac{1}{10}$
   c) $\frac{7}{10}$, $\frac{4}{10}$, $\frac{5}{10}$

Circulate while students work and assist individuals as required. You can also review these concepts by solving the problems together as a class. Think aloud as you work and invite students to help you. For example, SAY: I want to turn 1.3 into a mixed fraction. What should I do first? Why?

Then write on the board: $2\frac{4}{10}$, 2.3, 3.7

Tell students you want to order these numbers from smallest to largest. ASK: How is this problem different from the problems we just did? (Not all the numbers are fractions with denominator 10; some are decimals.) Can we change it into a problem that looks like the ones we just did? How? (Yes, by changing the decimals to fractions with denominator 10.) Have one volunteer change the decimals as described and another volunteer put all 3 fractions in the correct order.

Have students order the following numbers in their notebooks. Volunteers should do the first two or three problems on the board.

a) 0.8, 0.4, $\frac{7}{10}$  b) 1.7, .9, $1\frac{3}{10}$  c) 8.4, 5.5, 7.7

   d) 2.8, 1.5, $\frac{7}{10}$  e) 3.7, 3.9, $3\frac{3}{10}$  f) 8.4, 9.5, 9.7

**Bonus**
Provide problems where students have to change improper fractions to mixed fractions:

   g) 3.7, 2.9, $\frac{36}{10}$  h) 3.9, 3.4, $\frac{36}{10}$  i) 12.1, $1\frac{16}{10}$, 12

**Bonus**
Provide problems where one of the fractions has denominator 2 or 5, so that students have to find an equivalent fraction with denominator 10:

   j) 3.7, 2.9, $\frac{5}{2}$  k) 3.9, 3.4, $\frac{7}{2}$  l) 12.3, 12 $\frac{5}{6}$, 12

Have volunteers say how many hundredths are in each number:

   a) .73  b) .41  c) .62  d) .69  e) .58  f) .50

For each pair of numbers, ask which number has more hundredths and then which number is larger:

   a) .73, .68  b) .95, .99  c) .35, .42  d) .58, .57
Now have volunteers say how many hundredths are in each of these numbers:

a) .4    b) .6    c) .5    d) .7    e) .1    f) .8

**ASK:** How can we write 0.4 as a number with 2 decimal places? Remind students that “4 tenths” is equivalent to “4 tenths and 0 hundredths” or “40 hundredths.” This means 0.4 = 0.40.

For each pair of numbers below, **ASK:** Which number has more hundredths? Which number is larger? Encourage students to change the number with 1 decimal place to a number with 2 decimal places.

a) .48 .5    b) .71 .6    c) .73 .9    d) .2 .17

Put on the board:

Ask volunteers to write 2 different fractions for the amount shaded in the pictures. Have other volunteers change the fractions to decimals. **ASK:** Do these 4 numbers all have the same value? How do you know? What symbol do we use to show that different numbers have the same value? (the equal sign) Write on the board: .9 = .90 = \( \frac{90}{100} \).

Have students change more decimals to fractions with denominator 100:

a) .6    b) .1    c) .4    d) .8    e) .35    f) .42

Have students put each group of numbers in order by first changing all numbers to fractions with denominator 100:

a) .3 .7 .48    b) \( \frac{38}{100} \) \( \frac{4}{10} \) 0.39    c) 2 \( \frac{17}{100} \) 2 \( \frac{3}{10} \) 2.2

Now show a hundreds block with half the squares shaded:

**SAY:** This hundreds block has 100 equal squares. How many of the squares are shaded? (50) So what fraction of the block is shaded? (\( \frac{50}{100} \)) Challenge students to give equivalent answers with different denominators, namely 10 and 2. **PROMPTS:** If we want a fraction with denominator 10, how many equal parts do we have to divide the block into? (10) What are the equal parts in this case and how many of them are shaded? (the rows; 5) What fraction of the block is shaded? (\( \frac{5}{10} \)) What are the equal parts if we divide the block up into 2? (blocks of 50) What fraction of the block is shaded now? (\( \frac{1}{2} \))

Ask students to identify which fraction of each block is shaded:
Challenge them to find a suitable denominator by asking themselves: How many equal parts the size of the shaded area will make up the whole? Have students convert their fractions into equivalent fractions with denominator 100.

\[
\frac{1}{5} = \frac{20}{100}, \quad \frac{1}{4} = \frac{25}{100}, \quad \frac{1}{20} = \frac{5}{100}
\]

Write on the board: \(\frac{2}{5} = \frac{100}{\phantom{00}}\) and \(\frac{3}{4} = \frac{100}{\phantom{00}}\), and have volunteers fill in the blanks. Then have students copy the following questions in their notebooks and fill in the blanks.

a) \(\frac{2}{5} = \frac{100}{\phantom{00}}\)  \(\frac{3}{5} = \frac{100}{\phantom{00}}\)  \(\frac{4}{5} = \frac{100}{\phantom{00}}\)  \(\frac{5}{5} = \frac{100}{\phantom{00}}\)

b) \(\frac{2}{20} = \frac{100}{\phantom{00}}\)  \(\frac{3}{20} = \frac{100}{\phantom{00}}\)  \(\frac{4}{20} = \frac{100}{\phantom{00}}\)

**BONUS:** \(\frac{17}{20} = \frac{100}{\phantom{00}}\)

Have students circle the larger number in each pair by first changing all numbers to fractions with denominator 100:

a) \(\frac{1}{2}\) or .43  
b) \(\frac{3}{5}\) or 1.6  
c) 3.7 or 3 \(\frac{1}{2}\)  
d) \(\frac{1}{2}\) or .57  
e) \(\frac{1}{4}\) or .23  
f) \(\frac{3}{5}\) or .7

Give students groups of fractions and decimals to order from least to greatest by first changing all numbers to fractions with denominator 100. Include mixed, proper, and improper fractions. Start with groups of only 3 numbers and then move to groups of more numbers.

**Extensions**

1. Students can repeat Extension 1 from **NS5-85**: Decimals and Fractions Greater Than One using a combination of decimals and fractions. Remind students to start with a word whose letters can be used to create other words, and review why this is important. Students can now assign either a decimal or a fraction to each letter, scramble the letters and numbers, and invite a partner to order the numbers to find the original word.

2. Write a decimal for each fraction by first changing the fraction to an equivalent fraction with denominator 100.

   a) \(\frac{6}{5}\)  
b) \(\frac{1}{2}\)  
c) \(\frac{1}{4}\)  
d) \(\frac{3}{5}\)

3. Compare without using pictures, and determine which number is larger:

   a) 2 \(\frac{3}{4}\) and 17 sevenths  
b) 1.7 and 17 elevenths  
c) 1.5 and 15 ninths  
d) 2.9 and 26 ninths

   Students will have to convert all of the numbers into fractions in order to compare them. Is it best to use improper or mixed fractions? You can invite students to try both and see which types of fractions are easier to work with in this case (mixed works better for parts a and d, improper works better for parts b and c).

4. Draw a picture to show a decimal between .3 and .4. Explain why more than one answer is possible.

5. Write digits in the boxes that will make the statement true.

   \[
   \underline{\phantom{.3}} .5 < \underline{\phantom{.3}} .3
   \]

6. Teach students to describe multiplicative quantities by using simple decimals. For example, 6 is exactly halfway between 1 times 4 and 2 times 4, so 6 is 1.5 times 4. So if I have 4 marbles and you have 6 marbles, then you have 1.5 times as many marbles as I have.
NS5-89
Adding and Subtracting Tenths

To ensure students have the prior knowledge required for the lesson, complete the blanks in the following questions as a class or have students complete them independently in their notebooks.

a) 5.3 = ____ tenths  
b) .8 = ____ tenths  
c) 1.5 = ____ tenths  
d) ____ = 49 tenths  
e) ____ = 78 tenths  
f) ____ = 4 tenths  

g) ____ = 897 tenths  
h) ____ = 54301 tenths  
i) ____ = 110 tenths  

Review concepts with individual students or the whole class as required.

Now tell students that you want to add some decimals. Write on the board:

```
2.1  
+ 1.0  
3.1
```

**ASK:** How many tenths are in 2.1? (21) How many tenths are in 1.0? (10) How many tenths are there altogether? (31) **SAY:** There are 31 tenths in the sum. What number is that? (3.1) Write the answer below the addends, being careful to state that you are lining up the tenths under the tenths and the ones under the ones:

```
2.1  
+ 1.0  
3.1
```

Do a second problem together. This time, write out the number of tenths and add them using the standard algorithm for addition.

<table>
<thead>
<tr>
<th>Numbers to add</th>
<th>Numbers of tenths in those numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>14 tenths</td>
</tr>
<tr>
<td>+ 2.3</td>
<td>+ 23 tenths</td>
</tr>
<tr>
<td>3.7</td>
<td>37 tenths</td>
</tr>
</tbody>
</table>

Have students add the following decimals in their notebooks using this method, that is, by adding the whole numbers of tenths first and then turning the answer into a decimal:

```
a) 3.4 + 1.5  
b) 2.6 + 4.1  
c) 8.5 + 1.2  
d) 3.7 + 4.2  
e) 134.3 + 245.5
```

When students are done, tell them that Sonia adds decimal numbers by lining up the ones digits with the ones digits, the tenths digits with the tenths digits, and then adding each digit separately. Show them an example by re-doing question a) this way. **ASK:** Does Sonia get the right answer with this method? Why do you think that is? How is what Sonia does similar to what you did? Did you line up the digits when you added the whole numbers? Did you add each digit separately?
Write on the board:

\[
\begin{array}{cccc}
a) & 3.5 & b) & .7 \\
+ & 4 & + & 3.5 \\
c) & 192.8 & d) & 4.8 \\
+ & 154 & + & 12.1 \\
e) & 154.7 & & + & 16.3 \\
\end{array}
\]

**ASK:** In which questions are the digits lined up correctly? How can you tell? In all the questions where digits are lined up correctly, what else is also lined up? (the decimal point) Is the decimal point always going to be lined up if the digits are lined up correctly? (yes) How do you know? (The decimal point is always between the ones and the tenths, so if those digits are lined up correctly, then the decimal point will be as well.)

Demonstrate using this method to solve 12.1 + 4.8:

\[
\begin{array}{c}
12.1 \\
+ 4.8 \\
\hline
16.9
\end{array}
\]

Tell students that when you add the ones digits, you get the ones digit of the answer and when you add the tenths digits, you get the tenths digit of the answer. If the digits are lined up, then the decimal points are lined up, too—the decimal point in the answer must line up with the decimal points in the addends. Ask students why this makes sense and give several individuals a chance to articulate an answer.

Tell students that as long as they line up the numbers according to the decimal points, they can add decimals just like they add whole numbers. Demonstrate with a few examples, including some that require regrouping and carrying:

\[
\begin{array}{cccc}
3.5 & .7 & 192.8 & 154.7 \\
+ & 4 & + & 3.5 \\
7.5 & 4.2 & 346.8 & 171.0
\end{array}
\]

Give students lots of practice adding decimals (with no more than 1 decimal place). Working on grid paper will help students to line up the digits and the decimal points. Include examples with regrouping. Bonus problems could include larger numbers (but still only 1 decimal place). Emphasize that the decimal point is always immediately after the ones digit, so a whole number can be assumed to have a decimal point **(EXAMPLE:** 43 = 43.0) It is also important that students line up the digits around the decimal point carefully, perhaps by using grid paper.

To emphasize this, have students identify the mistake in:

\[
\begin{array}{c}
341.7 \\
+ 5216.2 \\
\hline
867.9
\end{array}
\]

The decimals are technically lined up properly, but the remaining digits are not.

**ASK:** What happened?

**ASK:** If we can add decimals the way we add whole numbers as long as we line up the decimal points, do you think we can subtract decimals the same way we subtract whole numbers? Use the following problem to check:

\[
\begin{array}{c}
4.5 \\
- 2.3 \\
\hline
2.2
\end{array}
\]
Do the problem together two ways—first rewrite the decimals as tenths and subtract the whole numbers, then subtract using Sonia’s method—and compare the answers. Then subtract more numbers (with no more than 1 decimal place) together. **EXAMPLES:**

\[
\begin{array}{ccc}
6.7 & 4.9 & 392.6 \\
- 3 & - 1.7 & - 45.3 \\
3.7 & 3.2 & 347.3
\end{array}
\]

As with addition, give student lots of practise subtracting decimals. Remind them to line the numbers up according to the decimal point!

Then introduce adding decimals on a number line:

\[
\begin{array}{ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
NS5-90
Adding Hundredths

GOALS
Students will add hundredths by lining up the decimal points.

PRIOR KNOWLEDGE REQUIRED
Adding whole numbers with regrouping
Adding tenths
Adding fractions with the same denominator
Knowing how many hundredths are in a number with 2 decimal places
Knowing which number has a given number of hundredths

Have students add these fractions:

a) \( \frac{30}{100} + \frac{24}{100} \)   b) \( \frac{20}{100} + \frac{39}{100} \)   c) \( \frac{32}{100} + \frac{16}{100} \)   d) \( \frac{25}{100} + \frac{24}{100} \)

Ask students to rewrite their addition statements in terms of decimals.
(EXAMPLE: \( .3 + .24 = .54 \))

Then give students addition problems that require regrouping. (EXAMPLE: \( \frac{36}{100} + \frac{17}{100} \)) Again, have students rewrite their addition statements in terms of decimals. (EXAMPLE: \( .36 + .17 = .53 \))

Now have students do the opposite: add decimals by first changing them to fractions with denominator 100. Invite volunteers to do some of the following problems on the board, then have students do the rest independently.

a) \( .32 + .57 \)   b) \( .43 + .16 \)   c) \( .81 + .17 \)   d) \( .44 + .44 \)   e) \( .40 + .33 \)
   f) \( .93 + .02 \)   g) \( .05 + .43 \)   h) \( .52 + .20 \)   i) \( .83 + .24 \)   j) \( .22 + .36 \)

Have a volunteer do a sum that requires regrouping (.54 + .28), then have students add the following independently:

a) \( .37 + .26 \)   b) \( .59 + .29 \)   c) \( .39 + .46 \)   d) \( .61 + .29 \)

Finally, add decimals whose sum is more than 1 by changing them to fractions first. Have volunteers do 2 examples on the board (.36 + .88 and .45 + .79). Students can then add the following independently:

a) \( .75 + .68 \)   b) \( .94 + .87 \)   c) \( .35 + .99 \)   d) \( .46 + .64 \)   e) \( .85 + .67 \)
   f) \( .75 + .50 \)   g) \( .65 + .4 \)   h) \( .7 + .38 \)   i) \( .9 + .27 \)

ASK: How many hundredths are in .9? (90) How many hundredths are in .27? (27) How many hundredths is that altogether? (117) What number has 117 hundredths? (1.17) What number has 234 hundredths? 5682 hundredths? 48 hundredths? 901 hundredths? 800 hundredths? 8 hundredths? 8 hundredths?

Have students add more decimal numbers by identifying the number of hundredths in each number and then adding the whole numbers:

a) \( .78 + .4 \)   b) \( .37 + .49 \)   c) \( .85 + .65 \)   d) \( .43 + .34 \)
   e) \( .25 + .52 \)   f) \( .14 + .41 \)   g) \( .76 + .67 \)   h) \( .89 + .98 \)
   i) \( .43 + .87 \)   j) \( .55 + .55 \)   k) \( 1.43 + 2.35 \)   l) \( 3.5 + 2.71 \)
   m) \( 4.85 + 3.09 \)

Remind students that we were able to add tenths by lining up the digits and decimal points. ASK: Do you think we can add hundredths the same way? Do an example together:

\[
\begin{array}{r}
4.85 \\
+ 3.09 \\
\hline
7.94 \\
\end{array}
\]
Invite students to help you add the numbers. Tell them to pretend the decimal point isn’t there and to add as though they are whole numbers. (PROMPTS: What do the hundredths digits add to? Where do I put the 4? the 1?) ASK: Why can we add as though the decimal points are not there? How many hundredths are in 4.85? (485) In 3.09? (309) In both numbers altogether? (485 + 309 = 794) How does this get the same answer? (794 hundredths = 7.94 ones, so we just line up the decimal points and proceed as though the decimal point is not there)

ASK: How can we check our answer? If no one suggests a method, invite a volunteer to add the numbers after rewriting them as fractions with denominator 100:

Method 1: 4.85 + 3.09 = 4 \frac{85}{100} + 3 \frac{9}{100} = 7 \frac{94}{100} = 7.94

Method 2: 4.85 + 3.09 = \frac{485}{100} + \frac{309}{100} = \frac{794}{100} = 7.94

Emphasize that all methods of addition reduce the problem to one students already know how to do: adding whole numbers. When they add using decimals, students know where to put the decimal point in the answer by lining up the decimal points in the addends. When they add using fractions, students know where to put the decimal point by looking at the denominator. If the denominator of the fraction is 10, they move the decimal point 1 place left. If the denominator of the fraction is 100, they move the decimal point 2 places left.

Give students lots of practice adding decimal hundredths by lining up decimal places. Include numbers that do and do not require regrouping.

EXAMPLES: .34 + .28 .65 + .21 .49 + .7 1.3 +.45 2.86 + .9

Bonus

12.3 + 1.23 354.11 + 4 672.6

Give students base ten blocks and tell them to use the hundreds block as a whole. This makes the tens block a tenth and the ones block a hundredth. Assign the following problems to pairs or individuals and invite students to share their answers.

1. Start with these blocks:

• What decimal does this model represent? (ANSWER: 2.3)

• Add 2 blocks so that the sum, or total, is between 3.3 and 3.48. (ANSWER: add or add )

• Write a decimal for the amount you added. (ANSWER: 1.1 or 1.01)

2. Start with these blocks:

• What decimal does this model represent? (ANSWER: 3.3)

• Add 2 blocks so that the sum (or total) is between 3.4 and 3.48

• Write a decimal for the amount you added (ANSWER: add 0.11).
3. Take the same number of blocks as number 1 above:

\[
\begin{array}{ccc}
\square & \square & \square & \square & \square & \square & \square & \square & \square & \square & \square
\end{array}
\]

• Add 2 blocks so that the sum is between 2.47 and 2.63. (\textbf{Answer}: 2 tens blocks)

• Write a decimal for the amount you added (\textbf{Answer}: 0.2).

4. Take these blocks:

\begin{align*}
\text{a) or b)}
\end{align*}

• What decimal does this model represent? (\textbf{Answer}: a) 1.42 b) 2.42)

• Add 2 blocks so that the sum is between 2.51 and 2.6. (\textbf{Answer}: a) add \[ \square \] b) add \[ \square \])

• Write a decimal for the amount you added. (\textbf{Answer}: a) 1.1 b) 0.11)

\section*{Extensions}

1. Write the numbers as decimals and add: \(2 + \frac{3}{10} + \frac{7}{100}\).

2. \textbf{Ask}: What if the denominator of a fraction is 1000—how do we know where to put the decimal point in the decimal number? What is \(0.437 + 0.021\)? Turn the decimals into fractions to add them: \(\frac{437}{1000} + \frac{21}{1000} = \frac{458}{1000}\). \textbf{Say}: The sum is 458 thousandths, or 0.458. Since the denominator of the fraction is 1000, we know to move the decimal point 3 places left. \textbf{Ask}: Can we add the decimals by lining up the decimal points? Do we get the same answer? Add the decimals this way to find out.
Tell your students that today they will learn to subtract hundredths. As in the last lesson, begin by subtracting hundredths written as fractions. (Start with problems that do not require regrouping and then move to problems that do.)

**EXAMPLES:**

\[
\begin{align*}
a) \quad & \frac{34}{100} - \frac{14}{100} \\
& \frac{47}{100} - \frac{28}{100} \\
& \frac{43}{100} - \frac{37}{100} \\
b) \quad & \frac{58}{100} - \frac{11}{100} \\
c) \quad & \frac{47}{100} - \frac{28}{100} \\
d) \quad & \frac{43}{100} - \frac{19}{100}
\end{align*}
\]

Have students rewrite the subtraction statements in terms of decimals. (EXAMPLE: \(.34 - .14 = .20\))

Then have students subtract decimals by first changing them into fractions (proper, mixed, or improper). Include numbers greater than 1.

**EXAMPLES:**

\[
\begin{align*}
a) \quad & .45 - .14 \\
b) \quad & .53 - .1 \\
c) \quad & .85 - .3 \\
d) \quad & 1.23 - .11
\end{align*}
\]

Tell students that we can subtract hundredths the same way we add them: by lining up the digits and decimal points. Solve \(1.93 - .22\) together and have students check the answer by rewriting the decimals as fractions.

\[
\begin{align*}
1.93 & \\
- .22 & \\
1.71 &
\end{align*}
\]

Give students a chance to practice subtracting decimal hundredths. Include numbers that require regrouping and numbers greater than 1. **EXAMPLES:**

\[
\begin{align*}
a) \quad & .98 - .42 \\
b) \quad & 2.89 - .23 \\
c) \quad & 3.49 - 1.99
\end{align*}
\]

Remind students of the relationship between missing addends and subtraction. Write on the board: \(32 + 44 = 76\). **SAY:** If \(32 + 44 = 76\), what is \(76 - 44\)? What is \(76 - 32\)? How can I find the missing addend in \(32 + ___ = 76\)? (find \(76 - 32\)) What is the missing addend in \(.32 + ____ = .76\)?

Have students find the missing addend in:

\[
\begin{align*}
.72 + ___ & = .84 \\
.9 & = ____ + .35 \\
.87 & = ____ + .5 \\
.65 & + ____ = .92
\end{align*}
\]

Have students demonstrate their answers using a model of a hundreds block. For example, students might fill in 72 hundredths with one colour and then fill in squares with another colour until they reach 84 to see that they need 12 more hundredths to make .84 from .72.
As in the previous lesson, give students base ten blocks and tell them to think of the hundreds block as a whole, the tens block as a tenth, and the ones block as a hundredth. Have pairs or individuals solve the following problems:

1. Take these blocks:

   - What decimal does this model represent?
   - Take away 2 blocks so the result (the difference) is between 1.21 and 1.35.
   - Write a decimal for the amount you took away.

2. Take these blocks:

   - What decimal does this model represent?
   - Take away 3 blocks so the result (the difference) is between 1.17 and 1.43.
   - Write a decimal for the amount you took away.
   - Repeat so that the difference is between 2.17 and 2.43.

Make up similar problems for students to solve independently or have students make up their own problems and exchange them with a partner.

**Extensions**

1. Show students how to subtract decimals from 1 by first subtracting from .99:

   \[ 1 - .74 = .01 + .99 - .74 = .01 + .25 = .26 \]

2. Teach students to estimate sums and differences of decimal numbers. Students can round to the nearest ten, one, tenth, hundredth and so on. (**EXAMPLE**: $15.87 - $11.02 is about $16 - $11 = $5). Also teach students to reflect on whether their estimate will be higher or lower than the actual answer. In the example above, the actual difference will be just under $5. You might also teach students to estimate sums by grouping numbers that add to about 10, 100 or 1000. For example, 3.96 + 6.1 + 98.4 + 63 + 35 is about 10 + 100 + 100 = 210.
NS5-93
Multiplying Decimals by 10 and

NS5-94
Multiplying Decimals by 100

Tell your students that a hundreds block represents 1 whole. **ASK:** What does a tens block represent? (one tenth) What does a ones block represent? (one hundredth)

**ASK:** How many tens blocks do we need to make a hundreds block? How many tenths do we need to make one whole? Can we write this as a multiplication statement? If we use four 3s to make 12, what multiplication statement do we have? (4 \times 3 = 12) If we use ten 0.1s to make one whole, what multiplication statement do we have? (10 \times 0.1 = 1) Show this using base ten materials (or a model of base ten materials):

\[
\begin{align*}
\text{Ten of these:} & \quad \text{equals} \quad \text{one of these:} \\
10 \times 0.1 & \quad = \quad 1
\end{align*}
\]

**ASK:** If 0.1 is represented by a tens block, how would you represent 0.2? How would you show 10 \times 0.2? (2 hundreds blocks) What is 10 \times 0.2? (2) Repeat for 10 \times 0.3. Have students predict 10 \times 0.7, 10 \times 0.6 and 10 \times 0.9. Students should then show their answers using base ten blocks.

**ASK:** If 0.1 is represented by a tens block, how would you represent 0.01? What block is ten times smaller than a tens block? (a ones block) How would you show 100 \times 0.01? (a hundreds block) What is 100 \times 0.01? (1) Repeat for 100 \times 0.02 and 100 \times 0.03. Have students predict 100 \times 0.07, 100 \times 0.06 and 100 \times 0.09. Students should then show their answers using base ten blocks.

Repeat the exercise above with pennies, dimes and loonies in place of ones, tens and hundreds blocks.

Draw a ruler on the board that shows cm and mm:

\[
\begin{align*}
0 & \quad 10 & \quad 20 & \quad 30 & \quad 40 & \quad 50 \text{ (mm)} \\
0 & \quad 1 & \quad 2 & \quad 3 & \quad 4 & \quad 5 \text{ (cm)}
\end{align*}
\]

**ASK:** If you know that the length of an object is 4 cm, how can you find the length of the object in mm? How can you obtain the number of mm from the number of cm? (multiply by 10) Why does this work? (because mm are ten times smaller than cm)
ASK: How many mm are in 0.7 cm? Then have a volunteer show this on the ruler you drew. Repeat for other numbers (0.9 cm, 0.3 cm, and so on)

Show a metre stick and repeat the exercise using cm and m.

Have students skip count by 0.3 to finish the pattern:

0.3, 0.6, 0.9, 1.2, _____, _____, _____, _____, _____, _____.

ASK: What is 3 × 0.3? 5 × 0.3? 8 × 0.3? 10 × 0.3? Repeat with skip counting by 0.2 and 0.5.

Ask students to think about how they can multiply 325 × 10 by just adding a 0. **ASK:** Where do you add the zero when you multiply by 10? How does the values of each digit change when you multiply a number by 10?

When you multiply by 10: 325 becomes 3 250

300, 20, and 5 become 3 000, 200, and 50

Write the number 346.51 and **ASK:** What is each digit worth? (300, 40, 6, 0.5 and 0.01) How can you move the decimal point so that each digit is worth ten times more (i.e., the ones digit becomes the tens digit, the tens digit becomes the hundreds digit, and so on)? The digits become worth 3000, 400, 60, 5 and 0.1, so the number becomes 3465.1. This is obtained by simply moving the decimal point one place to the right.

**ASK:** How would you find 32.5 × 10? Where should you move the decimal point to make every digit worth ten times more?

**ASK:** How is multiplying a decimal number by 10 different from multiplying a whole number by 10? Can we just add a zero to 32.5 to multiply by 10? What would we get if we added 0? (32.50) Does adding 0 to the decimal number change it at all? (No, but adding 0 to the end of a whole number changes the ones digit to 0, and every other digit becomes worth 10 times more.) Tell students that when they multiply a decimal number by 10, they move the decimal point one place to the right. Point out that they are, in fact, doing the same thing when they multiply whole numbers by 10. For example, 17 = 17.0, so when you multiply by 10 by adding a 0 to the end, you’ve also moved the decimal point over to get 170.

Have students try these problems in their notebooks:

a) 0.5 × 10   b) 10 × 0.8   c) 1.3 × 10   d) 10 × 2.4   e) 134.6 × 10   f) 10 × 12.45

**ASK:** How would you find 32.5 × 100? Where should you move the decimal point to make every digit worth a hundred times more? How much is each digit worth right now? (30, 2 and 0.5) How much will each digit be worth when you multiply by 100? (3000, 200 and 50) What number is that? (3 250) Explain to students that this is like moving the decimal point two places to the right:

\[ 32.5 \rightarrow 3250. \]

Have students try these problems in their notebooks:

a) 0.54 × 100   b) 0.92 × 100   c) 0.3 × 100   d) 1.4 × 100   e) 432.789 × 100

Remind students that multiplying by 100 is the same as multiplying by 10 and then by 10 again. How do you do multiplying by 10 (move decimal point one place right). So what do you do to multiply by 10 and then by 10 again? (move decimal point two places right)
GOALS
Students will discover the connection between multiplying decimals by whole numbers and multiplying whole numbers by whole numbers.

PRIOR KNOWLEDGE REQUIRED
Place value up to hundredths
Multiplying 3- and 4-digit numbers by 1-digit numbers

VOCABULARY
divisor dividend quotient

Using the hundreds block as 1 whole, have volunteers show:

a) 1.23 and then 2 × 1.23 b) 4.01 and then 2 × 4.01

c) 3.12 and then 3 × 3.12

Have students individually solve the following problems by multiplying each digit separately:

a) 4.12 = ____ ones + ____ tenths + ____ hundredths
2 × 4.12 = ____ ones + ____ tenths + ____ hundredths = ____

b) 3.11 = ____ ones + ____ tenths + ____ hundredths
3 × 3.11 = ____ ones + ____ tenths + ____ hundredths = ____

c) 1.02 = ____ ones + ____ tenths + ____ hundredths
4 × 1.02 = ____ ones + ____ tenths + ____ hundredths = ____

Have students multiply mentally:

a) 4 × 2.01  b) 3 × 2.31  c) 3 × 1.1213

d) 2 × 1.114312  e) 3 × 1.1212231

Bonus

Have students solve the following problems by regrouping when necessary:

a) 3 × 4.42 = ____ ones + ____ tenths + ____ hundredths
= ____ ones + ____ tenths + ____ hundredths
= ____

b) 4 × 3.32 = ____ ones + ____ tenths + ____ hundredths
= ____ ones + ____ tenths + ____ hundredths
= ____

c) 3 × 3.45 = ____ ones + ____ tenths + ____ hundredths
= ____ ones + ____ tenths + ____ hundredths
= ____ ones + ____ tenths + ____ hundredths
= ____

Then have students solve the following problems by regrouping when necessary:

a) 3 × 442 = ____ hundreds + ____ tens + ____ ones
= ____ hundreds + ____ tens + ____ ones
= ____

b) 4 × 332 = ____ hundreds + ____ tens + ____ ones
= ____ hundreds + ____ tens + ____ ones
= ____
c) \(3 \times 345 = \) ______ hundreds + ______ tens + ______ ones

= ______ hundreds + ______ tens + ______ ones

Discuss with students the similarities and differences between these problems and solutions. Remind students about the standard algorithm for multiplying 3-digit by 1-digit numbers and ask students if they think they can use the standard algorithm for multiplying decimal numbers. Emphasize that none of the digits in the answer changes when the question has a decimal point; only the place value of the digits changes. The key to multiplying decimals, then, is to just pretend the decimal point isn’t there, and then add it back in at the end. The only tricky part is deciding where to put the decimal point at the end.

Demonstrate using \(442 \times 3\):

\[
\begin{array}{llll}
1 & 1 & \frac{442}{4.42} & \times 3 & \frac{442}{4.42} \\
\times 3 & \times 3 & & & \\
1326 & 1326 & & & \\
\end{array}
\]

Tell students that now you need to know where to put the decimal point. Will the answer be closer to 1 or 13 or 132 or 1326? **ASK:** How many whole ones are in 4.42? How many ones are in 3? About how many ones should be in the answer? (\(4 \times 3 = 12\)) What is closest to 12: 1, 13, 132 or 1326? Have a volunteer guess where the decimal point should go and ask the class to explain why the volunteer chose the answer or to agree or disagree with the choice.

Repeat with several problems. (**EXAMPLES:** \(3.35 \times 6; \) \(41.31 \times 2; \) \(523.4 \times 5; \) \(9.801 \times 3\))

**Bonus**

\(834\ 779.68 \times 2; \) \(5\ 480.63 \times 7\)

**Extension**

(Adapted from Atlantic Curriculum A2.6) Teach students that just as they can take fractions of whole numbers, they can take decimals of whole numbers. Ask students whether they get the same answer when they take a quarter of a number (say 8) by dividing the number into 4 equal parts and when they take a quarter of the same number by multiplying the number by 0.25. Tell your students that \(\frac{1}{4}\) of the people in this class is the same as 0.25 of the people in this class. They can find 0.25 of a number by multiplying that number by 0.25.

Ask students to find a context in which 0.25 represents a small amount and one in which it represents a large amount.
NS5-96
Dividing Decimals by 10

Tell students that a hundreds block represents one whole. **ASK:** When you divide one whole into 10 equal parts, what do you get? (a tens block or one tenth)

Write on the board:

\[
\begin{align*}
\phantom{a} & \quad \div 10 = \\
1.0 & \quad \div 10 = 0.1
\end{align*}
\]

Then have a volunteer divide 0.1 by 10 using pictures:

\[
\begin{align*}
\phantom{a} & \quad \div 10 = \\
0.1 & \quad \div 10 = 0.01
\end{align*}
\]

Draw several pictures on the board and have students write the corresponding division statements individually: **EXAMPLES:**

\[
\begin{align*}
\phantom{a} & \quad \div 10 = \\
\phantom{a} & \quad \div 10 = \\
\phantom{a} & \quad \div 10 = \\
\phantom{a} & \quad \div 10 =
\end{align*}
\]

Then give students the following problems and have them draw the models themselves:

a) \(3 \div 10\)  
 b) \(0.2 \div 10\)  
 c) \(2 \div 10\)  
 d) \(0.4 \div 10\)  
 e) \(12 \div 10\)  
 f) \(1.2 \div 10\)

Write the number 43.5 on the board and **ASK:** How much is each digit worth? (40, 3 and 0.5). If we divide 43.5 by 10, how much does each digit become worth? (4, 0.3 and 0.05) What is \(43.5 \div 10\)? (4.35) Repeat this exercise with several examples (567.8 \(\div 10\), 43.57 \(\div 10\), 89.312 \(\div 10\), 41 325.5 \(\div 10\))  
**ASK:** How do you move the decimal point when dividing a number by 10? How can you make each digit worth ten times less? (move the decimal point one place to the left)

Teach students the connection between ten times more and the measurement units we use. For example, a dm is ten times larger than a cm. **ASK:** If an object is 3 dm long, how many cm long is it? What operation did you do? If an object is 40 cm long, how many dm long is it? What operation did you do? If an object is 7 cm long, how many dm long is it?
9 cm = _____ dm, 13 cm = _____ dm, 702 cm = _____ dm. Repeat the exercise with mm and cm.

(EXAMPLES: 8.4 mm = _____ cm, 39.2 mm = _____ cm)

**Extensions**

1. Teach students how to divide by 100 or 1 000 by moving the decimal point 2 or 3 places to the left. **ASK:** What is 4 531.2 ÷ 100? What is each digit worth? (4 000, 500, 30, 1 and 0.2) When we divide by 100, what will each digit be worth? (40, 5, 0.3, 0.01 and 0.002) What number is that? (45.312) How did the decimal point move when you divided by 100? Why does this make sense? How is dividing a number by 100 the same as dividing the number by 10 and then dividing it by 10 again? Emphasize that dividing something into 10 equal parts and then dividing those 10 equal parts into 10 parts again leaves the original object divided into 100 equal parts. Notice that moving the decimal point 2 places left is the same as moving the decimal point one place left and then another place left. Challenge students to predict how they would divide a decimal number by 1000.

**Bonus**

Find 31 498.76532 ÷ 1 000 000.

2. The wind speed in Vancouver was 26.7 km/h on Monday, 16.0 km/h on Tuesday and 2.4 km/h on Wednesday. What was the average wind speed over the 3 days?

3. A pentagonal box has a perimeter of 3.85 m. How long is each side?

4. (From Atlantic Curriculum A7) Draw a number line with only the endpoints 2 and 4 marked. Have students mark where the following numbers would be and to defend their positions: 2.3, 2.51, 2.999, 3.01, 3.75, 3.409, 3.490.

5. (Adapted from Atlantic Curriculum A7.1) Have students roll a die three times and ask them to use the digits to represent tenths, hundredths and thousandths. Have students make the smallest number they can and then to say how much would need to be added to make one whole.

6. (Atlantic Curriculum A7)

   a) If gas is priced at 56.9¢ per litre, what part of a dollar is this?
   b) If you drank 0.485 L of juice, how much more would you have to drink to equal 0.5 L?

7. (Atlantic Curriculum A7.9) Have students write a report on the use of 0.5 and ½. Students could survey adults and check newspapers and magazines to find when each is used.
NS5-97
Dividing Decimals by Whole Numbers

**GOALS**
Students will use long division to divide decimal numbers by single-digit whole numbers.

**PRIOR KNOWLEDGE REQUIRED**
Long division of 3- and 4-digit whole numbers by single-digit whole numbers. Decimal place value up to hundredths

**VOCABULARY**
tenths hundredths dividend divisor quotient

Tell students that a hundreds block represents one whole and draw on the board:

```
  1.0
  0.1
  0.01
```

Have volunteers draw the base ten models for: 5.43, 8.01, 0.92. Then have students do similar problems in their notebooks.

Then tell students that you would like to find 6.24 ÷ 2. Have a volunteer draw the base ten model for 6.24. Then draw 2 circles on the board and have a volunteer show how to divide the base ten materials evenly among the 2 circles. What number is showing in each circle? What is 6.24 ÷ 2?

Repeat for other numbers where each digit is divisible by 2 (or 3).

**EXAMPLES:** 46.2 ÷ 2, 3.63 ÷ 3, 4.02 ÷ 2, 6.06 ÷ 2, 6.06 ÷ 3.

Then write on the board 3.54 ÷ 2. **ASK:** In what way is this problem different from the previous problems? (each digit is not divisible by 2). Have a volunteer draw the base ten model for 3.54.

Draw 2 circles on the board and ask why you chose to draw 2 circles rather than a different number of circles?

Then lead students through the steps of long division, as in **NS5-41**: Long Division—3- and 4-Digit by 1-Digit. Then do a comparison of the steps for 3.54 ÷ 2 and 354 ÷ 2. Emphasize that students can just pretend that the decimal point does not exist and then put the decimal point in the correct place at the end. For example, students will see that 354 ÷ 2 = 177. To find 3.54 ÷ 2, they can round 3.54 to 4 and estimate that 3.54 ÷ 2 is about 4 ÷ 2 = 2. Where should they put the decimal point so that the answer is close to 2? **(ANSWER:** 1.77) Have students do several problems where they figure out the answer this way, first by long division of whole numbers and then by estimating to find where to put the decimal place.

Then, to ensure students are doing this last step correctly, give problems where students do not need to do the long division, but only this last step of putting the decimal point in the correct place:

a) 856.1 ÷ 7 = 1 2 2 3 **(ANSWER:** 122.3 since the answer will be close to but more than 100)

b) 8922.06 ÷ 6 = 1 4 8 7 0 1 **(ANSWER:** 1487.01 since the answer will be close to but more than 1000)

To ensure that students do not simply count decimal places, but actually estimate their answers, give some examples where you add an extra zero to the answer. For example, 23.28 ÷ 6 = 3 8 8 0 has answer: 3.880 or just 3.88.
Students should be encouraged to discover their own rule regarding where to put the decimal point when they have finished the long division (put the decimal point above the decimal point, as shown on the worksheet.) You can explain why you line up the decimal points when doing long division as follows: When you divide 342 by 2 by long division, the answer is aligned as shown:

\[
\begin{array}{c|cc}
\phantom{1}\phantom{7} & \phantom{1} & \phantom{7} \\
\hline
2 & 3 & 4 & 2 \\
\end{array}
\]

If you divide 3.42 by 2 the answer will be 100 times smaller than the answer to 342 ÷ 2. Just as I need 100 times fewer 2s to make 8 as I need to make 800, I need 100 times fewer 2s to make 3.42 as I need to make 342. Hence the answer to 3.42 ÷ 2 by long division should look like:

\[
\begin{array}{c|cc}
\phantom{1}\phantom{7} & \phantom{1} & \phantom{7} \\
\hline
2 & 1 & 7 & 1 \\
\end{array}
\]

Notice that when the decimal in the dividend (3.42) shifts 2 left, the decimal in the quotient (1.71) shifts 2 left, so the decimals in the dividend and the quotient line up.

**Extensions**

1. Teach students to divide decimals by single-digit decimals (EXAMPLE: 86.4 ÷ 0.9) by treating both the dividend and the divisor as whole numbers (864 ÷ 9 = 96) and then estimating by rounding each number to the nearest whole number (86 ÷ 1 is about 90) to decide where to put the decimal point.

2. Compare multiplying and dividing decimals to adding and subtracting decimals. You can multiply 3 × 4.2 by removing the decimal point pretending they are whole numbers and then put the decimal point in the correct place. Can you add 3 + 4.2 using this same strategy? Why not? Discuss.
NS5-98
Thousandths

Review place value up to hundredths, then write the number 6.142 on the board. Cover up all but the first 6 and the decimal point. ASK: What is the place value of the 6? How do you know? (The 6 is the ones digit because the decimal point is right next to it). Repeat the exercise, uncovering the 1 and then the 4. Emphasize that each digit is worth ten times less than the previous one. ASK: What is ten times less than a tenth? What is ten times less than a hundredth? Uncover the 2 and ask what its place value is.

Ensure that students can identify the place value of any given underlined digit. (EXAMPLES: 3.407, 6.015, 32.809)

Remind students of the connection between fractions and decimal numbers. Ask volunteers to write each decimal number as a fraction or mixed fraction with denominator 10 or 100: 0.7, 0.91, 0.09, 1.5, 1.07. Then ASK: How did you know whether the denominator was 10 or 100? (look at the number of decimal places) What if there are 3 decimal places? Then what will the denominator of the fraction be? Have students write each of these decimals as fractions or mixed fractions with denominator 10 or 100 or 1000: 0.9, 0.98, 0.987, 0.098, 0.009, 0.07, 0.652, 0.073.

Show students how to write a decimal number in expanded form by writing the place value of each digit: 3.241 = 3 ones + 2 tenths + 4 hundredths + 1 thousandth. Have students write the place value of each digit for various numbers (EXAMPLES: 2.4, 2.04, 2.004, 20.04, 200.4, 21.35, 2.135, 42.135).

Have students convert the following fractions to decimals:

\[
\begin{align*}
a) \frac{32}{100} &= .32 \\
b) \frac{3}{100} &= .03 \\
c) \frac{324}{1000} &= .324 \\
d) \frac{324}{100} &= 3.24 \\
e) \frac{32}{1000} &= .032 \\
f) \frac{3}{1000} &= .003
\end{align*}
\]

Have students compare the decimals by first changing them to fractions with the same denominator:

\[
\begin{align*}
a) .298 & \quad .32 \\
b) .65 & \quad .541 \\
c) .8 & \quad .312 \\
d) .68 & \quad .07
\end{align*}
\]

Have students compare these same decimals by adding zeroes when necessary to make both decimals have the same number of digits. (EXAMPLE: compare .298 to .320) ASK: How is this the same as changing to fractions with the same denominator? How is this different?

Have students practice ordering decimals that have different numbers of decimal places.

EXAMPLES:

\[
\begin{align*}
a) 6.53 & \quad 18.2 \\
b) 456.73 & \quad 21.72006 \\
c) 85.7601 & \quad 112.03 \\
d) 13.54 & \quad 13.5
\end{align*}
\]
Extensions

1. Have students compare numbers with more decimal places and with more digits before the decimal point. **(EXAMPLE: 32.4167 and 298.345)**

2. (From Atlantic Curriculum A2.4) Show the students cards on which decimals have been written **(EXAMPLE: 0.75 , and 0.265 m)**. Ask students to place the cards appropriately on a metre stick.

3. Challenge students to write decimals for the following fractions:
   a) \(\frac{1}{10000}\)  
   b) \(\frac{1}{100000}\)  
   c) \(\frac{27}{10000}\)  

   and then fractions for the following decimals:
   a) 0.0003  
   b) 0.12074  
   c) 0.000981004

4. (From the Atlantic Curriculum)
   Have students use each of the digits from 0 to 9 once to fill in the 10 spaces and make these statements true.
   \[
   \_ \_ \_ \_ \_ \_ \_ < \_ \_ \_ \_ \_ \_ \_ \\
   \_ \_ \_ \_ \_ \_ > \_ \_ \_ \_ \_ \_ \_ 
   \]

5. Have students add decimal thousandths by using these different methods:
   i) Count the number of thousandths, add the whole numbers and then change the number back to a decimal. **(EXAMPLE: 1.487 + 0.23 = 1 487 thousandths + 230 thousandths = 1 717 thousandths = 1.717)**
   ii) Change both decimals to fractions with denominator 1 000 and then add the fractions. **(EXAMPLE: 1.487 + 0.23 =1 \frac{487}{1000} + \frac{230}{1000} = \frac{1717}{1000} = 1.717)**
   iii) Line up the decimal points. **(EXAMPLE: 1.487 + 0.23 = \frac{1487}{1000} + \frac{230}{1000} = \frac{1717}{1000} = 1.717)**

**SAMPLE EXERCISES:**

a) 0.81 + 0.081  
   b) 5.7 + 4.32 + 5.014  
   c) 0.23 + 6.3 + 306.158  
   d) 19.41 + 173.2 + 6.471

6. Relate the ordering of numbers to the alphabetical ordering of words. When we put words in alphabetical order, we compare first the left most letters, then the next letters over, and so on. Ask students to put these pairs of words in alphabetical order by identifying the first letter that’s different:
   mouse, mice  
   noun, none  
   room, rope  
   snap, snip  
   trick, trim  
   sun, fun  
   pin, tin  
   spin, shin  

Students can write the words one above the other and circle the first letter that’s different:
Tell students that a blank always comes first. Use the words “at” and “ate” to illustrate this. The first two letters in “at” and “ate” are the same. There is no third letter in “at”—there is nothing, or a blank, after the “t.” The blank comes before the “e” at the end of “ate,” so “at” comes before “ate.” Have students use this knowledge to put the following pairs in alphabetical order:

- mat, mate
- an, a
- no, noon
- bath, bat
- kit, kite

Point out the difference between lining up numbers and words. The numbers 61435 and 7384 would be lined up:

<table>
<thead>
<tr>
<th>61435</th>
<th>not</th>
<th>61435</th>
</tr>
</thead>
<tbody>
<tr>
<td>7384</td>
<td>7384</td>
<td></td>
</tr>
</tbody>
</table>

But the words “at” and “ate” would be lined up like this:

<table>
<thead>
<tr>
<th>ate</th>
<th>not</th>
<th>ate</th>
</tr>
</thead>
<tbody>
<tr>
<td>at</td>
<td>at</td>
<td></td>
</tr>
</tbody>
</table>

When ordering words, you line up the leftmost letters. When ordering numbers, you line up the ones digits, whether they’re on the right, the left, or anywhere in between. For both words and numbers, you start comparing from the left.

Another important difference is that 5-digit whole numbers are always greater than 4-digit whole numbers, but 5-letter words can be before or after 4-letter words.

Point out that in numbers, as in words, a space is always less than a number. When we compare:

- 6.53
- 8.2
- 17.1

the blank is really a 0 and is less than, or comes before, any number.
NS5-99

Differences of 0.1 and 0.01

Have your students add the following numbers in their notebooks:

a) .48 + .1  b) .48 + .01  c) .52 + .1  d) .52 + .01

e) .63 + .1  f) .63 + .01  g) 4.32 + .01  h) 4.32 + 1

i) 4.32 + .1  j) 7.38 + .1  k) 7.38 + .01  l) 7.38 + 1

ASK: How do you add .1 to a number? (add 1 to the tenths digit) When you added .1 above, how many digits changed? (only 1, the tenths digit)

Then have students find .94 + .1. PROMPTS: How many hundredths are in .94? (94) How many hundredths are in .1? (10) How many hundredths is that altogether? (104) So the answer is 1.04. ASK: How is adding .1 to .94 different from adding .1 to the numbers above? What else changes besides the tenths digit? (the ones digit changes—from 0 to 1—because the answer is more than 100 hundredths)

Have students do the following problems and tell you when they just add 1 to the tenths digit and when they have to change the ones digit, too:

.49 + .1  .86 + .1  .93 + .1  .97 + .1  .36 + .1

Now have students add .01 to various decimals:

.49 + .01  .94 + .01  .86 + .01  .49 + .01  .94 + .01  .28 + .01

ASK: What number is .1 more than 9.3? Ask students to identify the number that is .1 more than: .7, 8.4, .6, .9, 5.9. Prompt students with questions such as: How many tenths are in 5.9? What is one more tenth? What number has 60 tenths?

What number is .01 more than:

8.47  8.4  .3  .39  .86  .89

What number is 1 more than:

9.3  .7  1.43  9.8  9.09  9.99

Repeat with subtraction, asking students to find numbers that are .01, .1, and 1 less than various numbers with 1 and 2 decimal points. Include examples where students need to borrow/regroup.

Have a volunteer add the missing decimal numbers to this number line:

| 4.0 | | | | | | | | 5.0 |

Students can refer to the number line to complete these sequences in their notebooks:

4.3, 4.4, 4.5, ____  4.1, 4.4, 4.7, ____

4.0, 4.2, 4.4, ____  4.9, 4.7, 4.5, ____
Have students give the rule for each sequence. (EXAMPLE: start at 4.3 and add .1)

Have another volunteer add the missing decimal numbers to this number line:

```
7.3   ______   ______   ______   ______   ______   ______   8.3
```

Students should complete and describe these sequences in their notebooks:

- 7.7, 7.8, 7.9, ____  7.2, 7.5, 7.8, ____
- 7.5, 7.7, 7.9, ____  8.3, 8.2, 8.1, ____, ____

Have students fill in the blanks in their notebooks:

- 5.9 + .1 = ____  8.9 + .1 = ____  .9 + .1 = ____
- 6.49 + .1 = ____  6.49 + .01 = ____  6.49 + 1 = ____
- 8.93 + .1 = ____  8.99 + .1 = ____  8.99 + .01 = ____

**Extensions**

1. Ask students to count forward from the following numbers by tenths, orally.
   - a) 6           b) 17.8          c) 123.2

2. a) Count forwards by hundredths from 0.96:
   - 0.96, _____, _____, _____, _____, _____, _____
   - b) Count forwards by cm from 96 cm:
   - 96 cm, _____cm, _____cm, _____cm, _____m, _____m _____cm,
   - _____m _____cm
   - c) Count forward by cents from 96 cents:
   - 96¢, _____¢, _____¢, _____¢, $______, $______, $______
   - d) How are questions a) to c) above the same? How are they different?
   - e) What connections can you make between counting by hundredths and measuring lengths in centimeters and metres?
   - f) What connections can you make between counting by hundredths and counting money in cents and dollars?
GOALS
Students will review concepts in decimals.

PRIOR KNOWLEDGE REQUIRED
Base ten materials
Units of measurement: metres (m), centimetres (cm), and millimetres (mm)
Fractions and equivalent decimals

NS5-100
Concepts in Decimals

Draw on the board: 

Ask a volunteer to shade one tenth of the picture. Then have students draw one tenth of each of the following pictures in their notebooks:

Tell your students that the fraction of a measurement depends on which unit is the whole. Ask: What is one hundredth of a meter? (1 cm) How would you write this as a fraction? (1/100 m) How would you write this as a decimal? (.01 m) Have students write each of the following measurements as a fraction and a decimal in metres.

2 cm = _____ m = _____ m
20 cm = _____ m = _____ m
25 cm = _____ m = _____ m
132 cm = _____ m = _____ m

Have your students write these measurements as a fraction and decimal in centimetres.

8 mm = ____ cm = ______ cm
4 mm = ____ cm = ______ cm
17 mm = ____ cm = ______ cm
29 mm = ____ cm = ______ cm

Bonus

54 mm = ____ m = ____ m
135 mm = ____ m = ____ m
6054 mm = ____ m = ____ m

Have your students add the measurements by changing the one in smaller units to a measurement in the larger units.

a) 3 cm + 9.46 m 

b) 24 cm + .12 m 

 c) 584 cm + 2.3 m

Have students check their answers by changing the number in larger units to one in smaller units, adding, and then changing the answer back to the larger units. Did they get the same answer?
Extensions

Give each student or pair a set of base ten blocks. Tell students the hundreds block is the whole, so the tens block represents .1 and the ones block represents .01.

1. Ask students to show and write all the decimals they can make with these 3 blocks.

(SOLUTION: 1.0, 1.1, 1.01, 1.11, 0.1, 0.01, 0.11)

NOTE: One way to prepare students for this exercise is to hold up combinations of blocks and ask them to write the corresponding decimal in their notebook. For instance, if you hold up the hundreds block (which represents one unit) and the tens block (which represents a tenth) they should write 1.1.

2. Use base ten blocks to make a decimal
   a) greater than .7.
   b) less than 1.2.
   c) between 1 and 2.
   d) between 1.53 and 1.55.
   e) with tenths digit equal to its ones digit.
   f) with hundredths digit one more than its tenths digit.

3. Create models of 2 numbers such that
   a) one number is 4 tenths greater than the other.
   b) one number has tenths digit 4 and is twice as large as the other number.

4. One decimetre (1 dm) is 10 cm. Explain how you would change 3.2 dm into centimetres.

5. (Atlantic Curriculum A9.4) Ask students to explain why you cannot compare two decimal numbers by simply counting the number of digits of each.
NS5-101
Word Problems with Decimals

Review word problems with your students.

Extensions

1. The chart shows how many times stronger (or weaker) gravity is on the given planets than on earth.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Saturn</th>
<th>Jupiter</th>
<th>Mars</th>
<th>Mercury</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravity Factor</td>
<td>1.15</td>
<td>2.34</td>
<td>0.83</td>
<td>.284</td>
</tr>
</tbody>
</table>

   a) On which planets is gravity less strong than on Earth?
   b) How much would a 7 kg infant weigh on each planet?
   c) How much more would the infant weigh on Jupiter than on Mars?
   d) John can jump 1 metre on Earth. How high can he jump on Mercury?

2. Tell students that information in a word problem can be redundant; sometimes you don’t need all the information given to solve the problem.

   EXAMPLE: Find the secret number:

   A. I am an odd number between 20 and 30.
   B. My ones digit is not 4.
   C. I am divisible by 3.
   D. My ones digit is greater than my tens digit.

   Have students try to figure out which 3 of the clues they need to answer the question.

   By changing only the last clue slightly, challenge your students to solve the puzzle using only two of the clues.

   A. I am an odd number between 20 and 30.
   B. My ones digit is not 4.
   C. I am divisible by 3.
   D. My tens digit is greater than my ones digit.

   Students can make up their own number puzzle with one piece of redundant information and have a partner solve the puzzle and identify the unneeded information.

3. Have students make up rhyming math poems as puzzles. EXAMPLE:

   A 2-digit number named Todd
   Has tens digit odd.
   But he’s even you see
   And his digits add to three.
   Which two numbers can he be?
NS5-102
Unit Rates and
NS5-103
Scale Diagrams

GOALS
Students will understand simple multiplicative relationships involving unit rates.

PRIOR KNOWLEDGE REQUIRED
Money (dollars and cents)
Distance (km, m and cm)
Time (weeks, hours)

VOCABULARY
rate
unit rate

Explain that a rate is the comparison of two quantities in different units. For example, “3 apples cost 50¢” is a rate. The units being compared are apples and cents. Have students identify the units being compared in the following rates.

5 pears cost $2.
$1 for 3 kiwis.
4 tickets cost $7.
1 kiwi costs 35¢.
Sally is driving at 50 km/hour.
On a map, 1 cm represents 3 m.
She earns $6 an hour for babysitting.
The recipe calls for 1 cup of flour for every teaspoon of salt. (NOTE: for this last example the units are not merely cups and teaspoons, they are cups of flour and teaspoons of salt.)

Explain that one of the quantities in a unit rate is always equal to one. Give several examples of unit rates, and have students identify the unit which makes it a unit rate.

1 kg of rice per 8 cups of water. (1 kg makes it a unit rate)
1 apple costs 30¢. (1 apple)
$1 for 2 cans of juice. ($1)
1 can of juice costs 50¢. (1 can of juice)
The speed limit is 40 km per hour. (1 hour)
She runs 1 km in 15 minutes. (1 km)

Explain that knowing a unit rate can help to determine other rates. **ASK:** If one book costs $3, how much do two books cost? … three books? … four books?

Draw a map with two cities joined by a line. Assuming that 1 cm represents 10 km, have volunteers determine the actual distance between the cities by measuring the line with a metre stick. Then have them explain their calculation for the class.

If 1 cm on a map represents 2 km, how much does 3 cm represent? How much does 7 cm represent? 4.5 cm? Two schools are 6 km apart. How far apart should they be drawn on the map? Two buildings are 11 km apart. How far apart should they be drawn on the map?

**ASK:** If you know that two books cost $6, how can you determine the cost for three books? What makes this problem different from the other problems in this lesson? (Instead of starting with the cost of 1 book, we are now starting with the cost of 2 books; we are not given a unit rate) How does working with unit rates make it easier to calculate other rates?
Working in pairs, have your students change this problem into a unit rate and then share their procedures with the class. Explain that higher rates can be determined through multiplication of the given rate, but the single unit rate can only be determined through division:

If 1 peach costs 25¢, then 3 peaches cost 75¢ (3 × 25¢).
If 3 peaches cost 75¢, then 1 peach costs 25¢ (75¢ ÷ 3).

Assign several problems that require your students to determine unit rates. Be sure the answers are whole numbers.

a) 4 pears cost 80¢. How much does 1 pear cost?

b) 24 cans of juice cost $24. How much does 1 can of juice cost?

c) 2 books cost $14. How much does 1 book cost?

d) 3 teachers supervise 90 students on a field trip. How many students does each teacher supervise?

Extensions

1. Have students determine the unit rates and then solve the following problems.

   a) If 4 books cost $20, how much do 3 books cost?
   
   b) If 7 books cost $28, how much do 5 books cost?
   
   c) If 4 L of soy milk costs $8, how much do 5 L cost?

2. Bring in some flyers from a grocery store, and ask students to determine unit prices and calculate the cost of quantities greater than one. For instance, if the unit price is $2.75 per item, how much will three items cost? If they do not know how to multiply a decimal number with a single-digit number, challenge them to select and use an alternate unit to dollars—they should use cents. **ASK:** How many cents are in $2.75? If each item costs 275¢, how many cents will three items cost? What does that equal in dollars?

   Ask students to calculate the unit price of an item using division. For instance, if three items cost $1.62, how much will one item cost? Again, have them convert dollars to cents and then back to dollars.
NS5-104
Proportions

GOALS
Students will use ratios to determine how many times greater one quantity is than another and will write this answer as a decimal.

PRIOR KNOWLEDGE REQUIRED
Comparing decimals and fractions
Writing the fractions with denominator 4 as decimals (0.25, 0.5, 0.75, 1, 1.25 and so on)
Equivalent fractions
Unit rates

Tell your students that you have a pancake recipe that calls for 7 cups of flour and 4 cups of bananas. Remind them that this is called a rate because it compares 2 quantities in different units (cups of flour to cups of bananas). Remind them that last time, we found the unit rate and ask what a unit rate is. Then ASK: What makes the unit rate harder to find in this case than in the last lesson? How many cups of flour do we need for 1 cup of bananas? (As an improper fraction: \( \frac{7}{4} \), and as a mixed fraction: \( 1 \frac{3}{4} \)) ASK: How would you write \( 1 \frac{3}{4} \) as a decimal? (1.75) Tell students that they would need 1.75 cups of flour for every 1 cup of bananas. ASK: How does that make it easy to tell how many cups of flour they would need for 5 cups of bananas? (multiply 5 \( \times \) 1.75) Have students do this calculation. Emphasize that the number of cups of flour is always 1.75 times the number of cups of bananas, so if they know how many cups of bananas they have, then they can deduce the number of cups of flour. Have students perform this calculation for various amounts of bananas (2 cups, 7 cups, 6 cups, 3 cups).

Repeat with several problems.

Emphasize that in the examples above, they compared numbers through multiplication rather than through addition. If we had said: The recipe calls for 7 cups of flour and 4 cups of bananas, so it calls for 3 more cups of flour than cups of bananas, this would be comparing through addition and subtraction rather than through multiplication. ASK: If I want to make the same recipe, but with 5 cups of bananas instead, should I use 8 cups of flour since that is 3 more than 5? Could that mess up the recipe? Explain to students that in this situation, we want the ratio of flour to bananas to remain the same; that is, we want how many times more cups of flour than bananas to remain the same—in this case, the cups of flour is 1.75 times the cups of bananas.

Extensions
1. Ask students to find many real-life examples of ratios:
   a) For every ____ months, there is 1 year, so the ratio of months to years is ____ : 1.
   b) For every ____ days, there is ____ week, so the ratio of days to weeks is ____ : ____.
   c) For every ____ dozen, there are ____ items, so the ratio of dozens to items is ____ : ____.
   d) For every ____ mm, there are ____ cm, so the ratio of mm to cm is ____ : ____.
   e) A recipe calls for 5 cups of flour and 2 cups of sugar. A double recipe calls for ____ cups of flour and ____ cups of sugar. The ratio of cups of flour to cups of sugar is ____ : ____ or ____ : ____.
Encourage students to fill in the blanks in more than one way so that they can find equivalent ratios.

2. Someone gets paid $25 for 3 hours worked. What is the ratio of dollars earned to hours worked. How much would the person get paid for working 6 hours?

---

**NS5-105**

**Numbers in Careers and the Media**

This lesson combines concepts from number sense, measurement and data management and can be used as an assessment. No lesson plan is associated with this worksheet, although activities and extensions are described below.

### GOALS

Students will see connections between numbers and real-life situations.

### PRIOR KNOWLEDGE REQUIRED

- Decimal numbers
- Times as many
- Multiplying decimals
- Bar graphs

### ACTIVITY

Ask students to find an advertisement or article in the paper in which numbers are a significant part of the content of the piece. Ask students to say which numbers in the piece are exact and which are estimates. Ask whether any numbers may have been used or represented in a misleading way. (For instance, an advertisement that says “Many items reduced to half price” doesn’t tell you what proportion of items in the store have been reduced to half price.) Have students make up a word problem using the numbers in the article or advertisement.

### Extensions

1. Doctors study the body. Here are some facts a doctor might know:
   
   a) **FACT:** “The heart pumps about 0.06 L of blood with each beat.” How much blood would the heart pump in 3 beats?
   
   b) **FACT:** “The heart beats about 80 times a minute.” How long would it take the heart to beat 240 times?
   
   c) **FACT:** “All of the blood passes through the heart in a minute.” How many times would the blood pass through the heart in a day?
d) FACT: “\(\frac{55}{100}\) of human blood is a pale yellow liquid called plasma.”

How much plasma would there be in 2 L of blood? (ANSWER: \(\frac{55}{100} = \frac{50}{100} + \frac{5}{100} = \frac{1}{2} + \frac{1}{20}\).

\(\frac{1}{2}\) of 2 L is 1 L. \(\frac{1}{20}\) of 2 L is 1 tenth of \(\frac{1}{2}\) or 0.1 L. So \(\frac{55}{100}\) of 2 L is 1 L + 0.1 L = 1.1 L).

\(\frac{1}{10}\) of \(\frac{1}{2}\) is \(\frac{1}{20}\):

\[\text{ANSWER: } \frac{55}{100} = \frac{50}{100} + \frac{5}{100} = \frac{1}{2} + \frac{1}{20}\]

\(\frac{1}{2}\) of \(\frac{1}{2}\) is \(\frac{1}{4}\):

\(\frac{1}{10}\) of \(\frac{1}{2}\) is \(\frac{1}{20}\):

\[\text{ANSWER: } \frac{55}{100} = \frac{1}{2} - \frac{1}{20}, \text{ so } \frac{45}{100} \text{ of 24 is about } 12 - 1 = 11.\]

2. Meteorologists study the weather. The world’s highest temperature in the shade was recorded in Libya in 1932. The temperature reached 58°C.

a) How long ago was this temperature recorded?

b) On an average summer day in Toronto, the temperature is 30°C. How much higher was the temperature recorded in Libya?

---

**NS5-106**

**Word Problems**

This worksheet is a review and can be used as an assessment.

**Extension**

Ask students to estimate their answers in QUESTIONS 1, 2, 3, and 4 to check the reasonableness of their answers. Here are some types of calculations they can practice estimating with:

a) \(271 \times 3.8\)  

b) \(278 + 77.2\)  

c) \(585 - 27.7\)  

d) \(38.5 \div 5\)
PRESENT THE FOLLOWING PROBLEM: You need to program a machine that will sell candy bars for 45¢. Your machine accepts only dimes and nickels. It does not give change. But you have to teach your machine to recognize when it’s been given the correct change. The simplest way to do this is to give your machine a list of which coin combinations to accept and which to reject. So you need to list all combinations of dimes and nickels that add up to 45¢.

Explain that there are two quantities—the number of dimes and the number of nickels. The best way to find the possible combinations is to list one of the denominations (usually the larger—dimes) in increasing order. You start with no dimes, then with one dime, etc. Where do you stop? How many dimes will be too much? Then next to each number of dimes in the list, you write down the number of nickels needed to make 45¢. Make a list and ask volunteers to fill it in. You can also use a table, as shown below.

<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Let students practice making lists of dimes and nickels (for totals such as 35¢, 75¢, 95¢), nickels and pennies (for totals such as 15¢, 34¢, 21¢), quarters and nickels (for totals such as 60¢, 85¢, 75¢).

**Bonus**

Make 135¢ and 175¢ using dimes and nickels.

**Assessment**

List all the combinations of dimes and nickels that make 55¢.

Now let your students solve a more complicated riddle:

Dragons come in two varieties: the Three-Headed Fearsome Forest Dragons and the Nine-Headed Horrible Hill Dragons. A mighty and courageous knight is fighting these dragons, and he has slain 5 of them. There are 27 heads in the pile after the battle. Which dragons did he slay?

Remind your students that it is convenient to solve such problems with a chart and to start with dragons that have the largest number of heads in the first column. How many nine-headed dragons could there be? Not more than 3—otherwise there are too many heads. How many
three-headed dragons could there be? There are 5 dragons in total. Make a list and ask the volunteers to fill in the numbers:

<table>
<thead>
<tr>
<th>Nine-Headed Horrible Hill Dragons</th>
<th>Three-Headed Fearsome Forest Dragons</th>
<th>Total Number of Heads</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ask another volunteer to pick out the right number from the table—there were 2 nine-headed dragons and 3 three-headed dragons slain.

Give your students additional practice, with more head numbers: 12 heads, 2 dragons; 20 heads, 4 dragons. What is the least and the most number of heads for 6 dragons?

Tell students that you want to find all pairs of numbers that multiply to 4.

<table>
<thead>
<tr>
<th>First Number</th>
<th>Second Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
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<tr>
<td>3</td>
<td></td>
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<td>4</td>
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<td>5</td>
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<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

Ask students how large can the first number be. Can I stop now? How do I know when I can stop? (the first number cannot be more than 4 because $5 \times 1$ is already more than 4, so $5 \times$ any number will also be more than 4). Then erase the 5 and 6 from the “First Number” column and complete the chart with your students, noting that 3 does not have a second number. There is no number that multiplies with 3 to give 4. Then list all the pairs: $1 \times 4$, $2 \times 2$, $4 \times 1$.

Have students create similar T-charts for pairs of numbers that multiply to 5, 6, 7, 8, 9 and 10 and then to list the pairs in order of increasing first numbers.

5: $1 \times 5$, $5 \times 1$
6: $1 \times 6$, $2 \times 3$, $3 \times 2$, $6 \times 1$
7: $1 \times 7$, $7 \times 1$
8: $1 \times 8$, $2 \times 4$, $4 \times 2$, $8 \times 1$
9: $1 \times 9$, $3 \times 3$, $9 \times 1$
10: $1 \times 10$, $2 \times 5$, $5 \times 2$, $10 \times 1$

Have students look for any patterns among the lists. Do they notice any symmetry? Why is there a symmetry in the list? (Multiplication is commutative, so if one pair multiplies to a given number, so does its “reverse” pair—the same numbers written in reverse order).
Then, as a whole class, find all the pairs of numbers that multiply to 36. Make a large T-chart with 36 numbers.

Check with the whole class if the numbers 1 through 8 are factors of 36 by using skip counting if necessary. (For example, skip count by 3s to 36 to see that $3 \times 12 = 36$, skip count by 5s to see that 5 does not have a “partner” that multiplies to 36—see NS5-35: Dividing by Skip Counting).

<table>
<thead>
<tr>
<th>First Number</th>
<th>Second Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
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<td>4</td>
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<td>7</td>
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<td>8</td>
<td>–</td>
</tr>
</tbody>
</table>

Then, ask students how they can use the commutativity of multiplication to decide which numbers have “partners.” Does 9 have a partner number to make 36? How do you know? (yes, because we have already found that $4 \times 9 = 36$) Does 10? Tell your students that the partner for 9 is 4. If 10 has a partner, will it be more than 4 or less than 4? How do you know? (Since 9 and 4 multiply to 36, 10 and any number more than 4 will multiply to higher than 36, so 10’s partner must be less than 4). ASK: Is 10 the partner of any number less than 4? (no, the partners for 1, 2 and 3 are 36, 18 and 12, respectively). Does 11 have a partner? (no, same reason) Does 12 have a partner? (yes, $3 \times 12 = 36$ is already found) Does 13 have a partner? (no, its partner has to be less than 3, but 1 and 2 have different partners) In fact, the only numbers higher than 9 that have partners will already be found. Once we find a pair that is repeated, we can stop, so the pairs are:

1, 36  2, 18  3, 12  4, 9  6, 6  9, 4  12, 3  18, 2  36, 1

Have students practise this skill of listing all the factors of a number until they see a pair of factors that is repeated.

When students finish the worksheet, ask them to explain why the charts in QUESTIONS 6 and 7 stop when they do.

**Extensions**

1. List all the combinations of quarters and dimes to make 60¢, 75¢.

2. Dragons come in three varieties: One-Headed Woe-of-Woods Dragons, Three-Headed Danger-of-Dale Dragons, and Ever-Quarrelling-Nine-Headed-Tundra Dragons. A mighty and courageous knight fought a mob of 13 dragons and cut off all of their heads. There were 27 heads in the pile after the battle (and none of them belonged to the knight). How many of each type of dragon did the knight slay? Find all possible solutions. (ANSWERS: 7 three-headed and 6 one-headed or 1 nine-headed, 3 three-headed and 9 one-headed).
3. The dragons from the previous question all have 4 legs each. When the knight next fought the dragons, he produced a pile with 48 legs and 86 heads. Which dragons were destroyed? **HINT:** There are many heads in the pile. Start with the largest number of nine-headed dragons possible.

4. a) One-headed dragons have 2 tails and four-headed dragons have 3 tails. The knight must cut off both the heads and the tails, as dragons with a tail can still breed other dragons, even without a head. After the battle, the knight counts 31 heads and 37 tails. How many dragons of each type did he slay? **HINT:** Students will need to be organized. Suggest the following chart to struggling students:

<table>
<thead>
<tr>
<th>Four-Headed Dragons</th>
<th>One-Headed Dragons</th>
<th>Total Dragons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dragons</td>
<td>Heads</td>
<td>Tails</td>
</tr>
<tr>
<td>1</td>
<td></td>
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<td>7</td>
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</tbody>
</table>

Ask students the following question: if there are 31 heads, what is the maximum number of four-headed dragons? Why can't there be eight dragons of this sort? Challenge students to fill in the first column and then the second and third columns from the “Four-Headed Dragons” section of the chart. Can they fill in any columns from the “Total Dragons” section? How does this help with the columns for one-headed dragons? Students will see that there are 11 one-headed dragons and 5 two-headed dragons.

b) Change the question slightly, so that four-headed dragons have 5 tails and one-headed dragons still have 2 tails. Now, the knight counts 23 heads and 34 tails. From that, he knows immediately that there are 11 dragons altogether. How did he figure that out? **(ANSWER:** Each dragon has one more tail than head, so if there are 11 more tails than heads in the pile, and 34 is 11 more than 23, then there must have been 11 dragons.)

c) Using the information from b), find the number of each type of dragon slain. Why is this question less work than part a) was? **SOLUTION:** Since we know the total number of dragons, the only possibilities are: 1 four-headed and 10 one-headed, 2 four-headed and 9 one-headed, and so on until 7 four-headed and 4 one-headed. Checking all possibilities shows that in fact there are 7 four-headed and 4 one-headed dragons.
NS5-108
Arrangements and Combinations

GOALS
Students will learn to problem solve.

PRIOR KNOWLEDGE REQUIRED
Comparing and ordering numbers
Concept of “closeness” for numbers
Counting by small numbers

VOCABULARY
product
difference
sum

Ask students to list in an organized way:

a) all the pairs of numbers that multiply to 10
b) all the pairs of numbers that add to 7

ASK: Can you find two numbers that:

a) multiply to 10 and add to 7
b) multiply to 12 and add to 7
c) multiply to 12 and add to 8

SAY: I want to make a two-digit number with the digits 1 and 2. How many different numbers can I make? [Two: 12 and 21.] Which two-digit numbers can I make with the digits 3 and 5? … 4 and 7? … 2 and 9?

Ask students to find a three-digit number with the digits 1, 2 and 3. Record their answers, stopping when they have listed all six. Have them write the six different numbers in an organized list. Repeat with the digits 2, 5 and 7, then with the digits 3, 4, and 8, and then with the digits 4, 6 and 7. Ask them to explain how their list is organized (for example, they might organize the numbers by the hundreds digit), and if organization makes it easy or not to know when all of the three-digit numbers have been listed.

Then write two sets of numbers: a) 4, 9, 7 and b) 6, 3, 10.

Have students find the product of all pairs of numbers, with one number taken from each set. ASK: Which pair of numbers has the smallest product? … the largest? … the product closest to 50? Did they need the entire set of products to find the pair with the largest product? … the smallest product? … the product closest to 50? The entire set of products might be needed to find the product closest to 50, but only the largest and smallest numbers in each set are needed to find the largest and smallest products. Repeat this exercise for sums and then differences.

Draw:

Show your students the two ways to arrange these circles in a row.

Then draw a third circle and have your students find all six ways to arrange three circles in a row.

Have volunteers illustrate, then ask if there is an organized way to find all six arrangements. Start by deciding which circle to place first in the arrangement.
—white, for example. The second circle in the arrangement can then be solid or striped. Have a volunteer illustrate these two arrangements. Have additional volunteers illustrate the arrangements with the solid circle and the striped circle placed first in the arrangement, respectively.

Have your students start exercise 3 of the worksheet, but first explain that mathematicians will often solve simple problems before they solve similar, more complicated problems, as a way to comprehend the problems. How can exercise 3 be simplified? Can fewer than four boxes be used to simplify the problem?

Tell students that tennis balls are sold in packages of 3 and 4. **ASK:** Can we buy exactly 5 tennis balls? … 6 tennis balls? … 7? … 8? … 9? … 10? Students might solve these by randomly guessing and finding solutions: e.g. 7 tennis balls can be bought by buying a can of 3 and a can of 4. Then **ASK:** Can we buy 42? How is this problem different from the previous problems? [The solution is harder to guess randomly.] Have them write an organized list of all of the possible ways of adding threes and fours to total 42. Explain that solving this problem for smaller numbers first will help them to solve the problem for larger numbers, like 42. **ASK:** Can you find ways to use only threes and fours to total 10? What is the greatest number of threes that can be used in the solution? Why can’t four threes be used in the solution? [4 × 3 = 12, which is greater than 10.] Then write

<table>
<thead>
<tr>
<th>No threes</th>
<th>1 three</th>
<th>2 threes</th>
<th>3 threes</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 + 10</td>
<td>3 + 7</td>
<td>6 + 4</td>
<td>9 + 1</td>
</tr>
</tbody>
</table>

Point to the circled numbers and explain that these individual amounts have to be totalled with fours only. **SAY:** If a solution has exactly one three, and the solution is composed of only threes and fours, the remaining numbers have to be fours. Can you use only fours to total 7? Can you use only fours to total 10? Can you use only fours to total 4? To total 1? The only solution is to buy two cans containing three tennis balls and one can containing four tennis balls. Have students repeat this exercise for numbers such as 17 (see below), 35 or 42.

<table>
<thead>
<tr>
<th>0 + 17</th>
<th>3 + 14</th>
<th>6 + 11</th>
<th>9 + 8</th>
<th>12 + 5</th>
<th>15 + 2</th>
</tr>
</thead>
</table>

The only solution is three cans containing three tennis balls and two cans containing four tennis balls.

Then tell students that pencils are sold in packages of 5 and 6. Can exactly 13 pencils be bought? … 28 pencils? How can they systematically solve the problem? Repeat this exercise with pencils sold in packages of 4 and 6.

Note that there are various answers for the worksheet’s bonus questions.

**Extensions**

1. a) How many paths can be drawn from point A to point B using right and up directions only?

   ![Diagram](https://via.placeholder.com/150)

   Have students extend the pattern and determine if the number pattern continues.
b) Repeat part a) for this pattern of pictures:

- i) A \[\text{B} \]
- ii) A \[\text{B} \]
- iii) A \[\text{B} \]
- iv) A \[\text{B} \]

2. How many four-digit numbers can be made with the digits 1, 2, 3 and 4 and a thousands digit of 1? … a thousands digit of 2? … a thousands digit of 3? … a thousands digit of 4? How many four-digit numbers can be made altogether?

3. Find all triples of numbers that multiply to 24. Find three numbers that multiply to 24 and add to 10.

4. Show your students the following card trick. Take 6 cards and place them in pairs. Ask a volunteer to choose any pair without telling you. Then pick up the cards, keeping the pairs together. Then place the cards in the following order starting from the top of the deck:

   1  2  3
   4  5  6

   Ask your students which rows the cards appear in. If both cards are in the first row, tell your students that the pair was cards 1 and 2. If both cards are in the second row, tell your students that the pair was 5 and 6 and if the cards are in rows 1 and 2, tell them that the pair was cards 3 and 4 (you can just pick the cards up and show them). Repeat the trick until students see the pattern. Allow pairs of students to try this trick out on each other.

   Then show them the same trick, but using 12 cards instead. Again arrange the cards in pairs and have a volunteer choose a pair. Keeping the pairs together, pick up the cards and arrange them in this order:

   1  2  3  5
   4  7  8  9
   6 10 11 12

   Now, ask which row or rows the pair of cards appear in.

   1st row only ? cards 1 and 2
   1st and 2nd row ? cards 3 and 4
   1st and 3rd row ? cards 5 and 6
   2nd row only ? cards 7 and 8
   2nd and 3rd row ? cards 9 and 10
   3rd row only ? cards 11 and 12.

   The only memorization this trick requires in the order you used to lay the cards down. Luckily, this is done in an organized way. Can students figure it out? Repeat the trick several times and then have students investigate to see if they can perform the trick themselves. The organization is by the row numbers that the pairs go into. The top two cards both go to the first row. The next pair goes to the first and second rows and so on.

   Ask students if they think this trick can be performed with more cards. Have students investigate.
The trick can be performed with $6 = 2 \times 3$ cards or $12 = 3 \times 4$ cards or $20 = 4 \times 5$ cards, and so on. The number of pairs is then half of these numbers ($3$, $6$, $10$, and so on). Each pair of cards corresponds to a unique pair of rows and these pairs of rows can be listed in a very organized way:

For 3 pairs: $(1, 1), (1, 2), (2, 2)$

For 6 pairs: $(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)$

For 10 pairs: $(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)$

For 15 pairs: $(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 2), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 4), (4, 5)$

Another pattern that your students might notice is how the gap in the number of pairs increases ($3, 4, 5$, and so on).

5. As a cumulative review of many number sense concepts, provide the BLM “Always, Sometimes, or Never True (Numbers).”
NS5 Part 2: BLM List

Always, Sometimes or Never True (Numbers) ________________________________2
Blank Hundreds Charts___________________________________________________3
Cards (Fractions of Numbers)______________________________________________4
Fraction Strips___________________________________________________________5
Math Bingo Game_________________________________________________________6
Pattern Blocks___________________________________________________________7
### Always, Sometimes or Never True (Numbers)

<table>
<thead>
<tr>
<th></th>
<th>Statement</th>
<th>Letter</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>If you multiply a 3-digit number by a one-digit number, the answer will be a three-digit number.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>If you subtract a three-digit number from 999 you will not have to regroup.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>The product of two numbers is greater than the sum.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>If you divide a number by itself the answer will be 1.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>The product of 0 and a number is 0.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>Mixed fractions are larger than improper fractions.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>The product of 2 even numbers is an even number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>The product of 2 odd numbers is an odd number.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>A number that ends with an even number is divisible by 4.</td>
<td></td>
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</tr>
<tr>
<td>J</td>
<td>When you round to the nearest thousands place, only the thousands digit changes.</td>
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</tr>
<tr>
<td>K</td>
<td>When you divide, the remainder is less than the number you are dividing by.</td>
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<tr>
<td>L</td>
<td>The sum of the digits of a multiple of 3 is divisible by 3.</td>
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</tr>
<tr>
<td>M</td>
<td>The multiples of 5 are divisible by 2.</td>
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<td></td>
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<tr>
<td>N</td>
<td>Improper fractions are greater than 1</td>
<td></td>
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</tr>
<tr>
<td>O</td>
<td>If you have two fractions the one with the smaller denominator is the larger fraction.</td>
<td></td>
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</tr>
</tbody>
</table>

1. Choose a statement from the chart above and say whether it is **always** true, **sometimes** true, or **never** true. Give reasons for your answer.

   **What statement did you choose? Statement Letter ________**

   **This statement is…**

   **Always True** | **Sometimes True** | **Never True**

   **Explain:**

   1.  
   2.  

2. Choose a statement that is sometimes true, and reword it so that it is always true.

   **What statement did you choose? Statement Letter ________**

   **Your reworded statement:**

   1.  
   2.  

3. Repeat the exercise with another statement.
### Cards (Fractions of Numbers)

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Denominator</th>
</tr>
</thead>
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<tr>
<td>( \frac{1}{10} )</td>
<td>10</td>
</tr>
<tr>
<td>( \frac{1}{4} )</td>
<td>20</td>
</tr>
<tr>
<td>( \frac{2}{3} )</td>
<td>12</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>26</td>
</tr>
<tr>
<td>( \frac{17}{20} )</td>
<td>20</td>
</tr>
</tbody>
</table>
Fractions Strips
# Math Bingo Game

## Sample Boards

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<td>5</td>
<td>11</td>
<td>7</td>
<td>4</td>
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</table>
Pattern Blocks

Triangles

Squares

Rhombuses

Trapezoids

Hexagons
ME5-8
Centimetres

GOALS
Students will measure and draw items to a specific length in centimetres.

PRIOR KNOWLEDGE REQUIRED
The ability to use a ruler

VOCABULARY
ruler
grid paper

Give your students several objects of standard length (paper clips, unused pencils, etc.) and ask them to measure the lengths.

Demonstrate how to draw a line 2 cm in length with a ruler. Have your students practice these steps—separately, if necessary—in their notebooks.

**STEP 1:** Find the zero mark on a ruler/number line. Draw a vertical line to mark the zero.

**STEP 2:** Count forward from zero by two hops.

**STEP 3:** Draw a vertical line to mark the two.

**STEP 4:** Draw a line connecting the two vertical marks.

Have your students draw lines of several lengths with a ruler. If the class does not have a standard set of rulers, have students trade rulers with their classmates so that each measurement is made using a different ruler. This reinforces the idea that rulers have equal measurement markings, even when they look different.

Ask your students to draw another line 2 cm in length, but to start it at 3 on the number line. Following the steps taught earlier in this lesson, but beginning at 3 and ending at 5 on the number line, have a volunteer demonstrate how the line is drawn. Have students reproduce the problem in their workbooks and solve several others using this method (**EXAMPLE:** a line 8 cm in length beginning at 3 on the number line, a line 11 cm in length beginning at 2 on the number line, a line 14 cm in length beginning at 1 on the number line, etc.).

**Assessment**

Draw a line 4 cm long and a pencil 8 cm long.

Distribute centimetre grid paper to your students and have them estimate the width of the squares. Then have them measure the width with their rulers. Demonstrate how to draw items of specific length by using the centimetre grid as a guide. On your grid, draw a pencil (or some other item) that is 5 cm long. Explain that you selected a starting point and then hopped forward to 5. Draw vertical marks at the start and end points, then draw the item to fill the length between the marks.
ACTIVITY 1
Create a box to collect benchmark items that correspond with each unit of measurement introduced. Everyone can then refer to and compare these items as the lessons progress.

ACTIVITY 2
Draw each object to the given measure.
   a) A shoe 6 cm long          b) A tree 5 cm high          c) A glass 3 cm deep

ACTIVITY 3
Draw a triangle on grid paper and measure its sides to the nearest cm.

ACTIVITY 4
Draw a collection of long items.
   a) Draw a collection of alligators, each one being 1 cm longer than the previous.
   b) A pencil shrinks when it is sharpened. Draw a collection of pencils, each one being 1 cm shorter than the previous.
   c) Draw a sequence of toboggans where each one is 2 cm longer than the last.
   d) A carrot shrinks when it is eaten. Draw a collection of carrots, each one being 2 cm shorter than the previous.

ACTIVITY 5
Write a story about one of the growing or shrinking items in Activity 4 (or invent your own!). Tell the story of how the item grows or shrinks. How does the carrot get eaten? Who wants to ride the toboggans? Write the story to go with the pictures that you have drawn.

Extension
Estimate the height of a classmate in cm. Then measure their height using your hand. (Your hand with fingers spread slightly should be about 10 cm wide.) Finally, use a metre stick to check your result. How close were you?

HINT: Measure them against a wall to get an accurate result.

   Estimate _____ cm  Hand Measurement _____ cm  Actual Measurement _____ cm
ME5-9

**Millimetres and Centimetres**

Set out a number of items that range in size from 1 mm to 5 cm. Ask students to select the items that are about 1 mm wide. Measure the selected items with a ruler to verify their widths, then add the items measuring 1 mm to the measurement box (see Activity 1 of ME5-8).

Remind your students that they can use their index fingers as centimetre rulers. If there are 10 mm in a cm, how many mm are there in 2 cm?

Have students select several objects from their desks or backpacks (an eraser, a pencil, a pencil sharpener, etc.), measure three objects with their index fingers and then complete the following sentence for both objects.

- The _______ measures about _______ index fingers, so it is about _______ mm long.

Explain to your students that counting every millimetre in a measurement can take a long time, but there is a quick way to do it. Draw a ruler representing 30 mm and tell the class that you want to count 26 mm. Then demonstrate how to skip-count by 10s to the tens value preceding the amount, and continue to count by one (**EXAMPLE**: 10, 20, 21, 22, 23, 24, 25, 26).

Have volunteers demonstrate this shortcut method by counting to several different numbers.

Ask your students to sort the items they chose into piles of “greater than 50 mm” and “less than 50 mm.” Once they have sorted all of the items, have them measure each to verify the exact length in millimetres. Suggest that your students make a T-table with headings mm and cm. Ask them to record the lengths of the objects they have in millimetres. **ASK**: If an eraser is 30 mm long, how many centimetres long is it? What do you do to measurement in millimetres to convert it into centimetres?

Suggest that your students make a T-table with headings “Objects”, “Length in mm” and “Length in cm”. Ask them to record the lengths of the objects they measured in millimetres. **ASK**: An eraser is 40 mm long. How many centimetres long is it? What do you do to a measurement in millimetres to convert it into centimetres? (divide by 10) A sharpener is 23 mm long. How many centimetres long is it? Review division by 10 with decimal results, such as 25÷10 = 2.5, 37÷10 = 3.7, and so on. Ask your students to convert the lengths of their objects from millimetres to centimetres. Add several more measurements to convert, such as: 2.4 cm, .6 cm, 80 cm, 34 mm, 307 mm, 307 cm.

Write several pairs of measurements on the board, like:

- A: 8 cm and 9 cm  
- B: 9 cm and 10 cm  
- C: 10 cm and 11 cm

Ask your students to say which pair of centimetre measures (A, B or C) the measurement 87 mm lies between. Repeat the question for 93 mm. Can they find another millimetre measure between each pair of lengths?
Now write several measurements in mm, and ask your students to find a cm measure that is between the lengths in each pair:

56 mm and 63 mm, 78 mm and 86 mm, 102 mm and 114 mm.

### Assessment

Complete the following conversions.

<table>
<thead>
<tr>
<th>mm</th>
<th>230</th>
<th>134</th>
<th>800</th>
<th>45</th>
<th>57</th>
<th>6</th>
<th>21.2</th>
<th>0.2</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>cm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>21.2</td>
<td>0.2</td>
<td>70</td>
</tr>
</tbody>
</table>

### Bonus

1. Draw a line between 5 cm and 6 cm long and is…
   a) closer to 5 cm than to 6 cm.
   b) halfway between 5 cm and 6 cm.
   c) closer to 6 cm than to 5 cm.

   Give the lengths of the lines in mm.

2. Without using a ruler, give a measurement in mm that is between 7 cm and 8 cm and is…
   a) closer to 7 cm than to 8 cm.
   b) halfway between 7 cm and 8 cm.
   c) closer to 8 cm than to 7 cm.

### Extensions

1. A toonie is about 2 mm thick. Josie has a stack of toonies 10 mm high. How much money does Josie have?

2. A nickel is about 1 1/2 mm thick. How many stacked nickels equal 9 mm high?

3. Assume Carl has a pen that he can use to mark lengths on the sticks in QUESTION 17 on the worksheet. Explain how he could use the sticks and the pen to measure.
   a) 2 cm        b) 4 cm        c) 1 cm

4. How many millimetres are in… a) a metre? b) a decimetre?

5. Draw a line $\frac{17}{100}$ of a metre long. How many cm long is the line?
ME5-10
Decimetres

Tell your students that today they will learn about another unit of measurement, the decimetre. Write the word “decimetre” on the board. Circle the letters “d” and “m” and explain that they form the abbreviation for decimetre, dm. Write the abbreviation next to the word.

Explain that a decimetre is equal to 10 cm. Remind your students that the span of their hand is about 10 cm. This is equal to approximately 1 dm. Ask your students to name 3 objects that are shorter than 1 dm, 3 objects that are longer than 1 dm, and 3 objects that are about 1 dm. Students can measure the objects that are about 1 dm to check their estimates.

This would be a good time to do the Activity (see below).

Remind students of their work in the previous lesson, converting centimetres to millimetres. Ask: How many millimetres are in 1 cm? If a paper clip is 2 cm long, how many millimetres is that? What did you do to the measure in centimetres to get the measure in millimetres? If an eraser is 40 mm long, how many centimetres is that? What do you do to the measure in millimetres to get the measure in centimetres?

Draw this diagram on the board:

\[
\begin{array}{c}
\text{cm} \\
\times 10 \\
\downarrow \\
\text{mm} \\
\div 10 \end{array}
\]

SAY: I want to add decimetres to the diagram. Where should I write dm? (on the top step) How many centimetres are in 1 dm? If a pencil is 2 dm long, how many centimetres is that? If a book is 30 cm long, how many decimetres is that? Invite volunteers to add the dm and the corresponding arrows and operations (x 10, ÷ 10) to the diagram. Invite a volunteer to fill in the blanks: 1 dm = ___ cm = ____ mm.

Have students convert more measures in centimetres to decimetres and vice versa. **EXAMPLES:**

- 12 dm = ___ cm
- 50 cm = ___ dm
- 102 dm = ____ cm
- 200 cm = ___ dm
- 100 dm = ______ cm

**ASK:** Can someone give me a measure in whole decimetres that is between 33 cm and 46 cm? Then ask for a measure in centimetres that is between 13 dm and 14 dm. (Several answers are possible—anything from 131 cm through 139 cm.) **ASK:** Is the second measure more than 1 m or less than 1 m? How do you know?

**GOALS**
Students will take measurements in decimetres and will convert measurements between centimetres and decimetres.

**PRIOR KNOWLEDGE REQUIRED**
Centimetres and millimetres
Measuring with a ruler
Using body parts as approximate units of measurement
Using tables to organize data
Multiplication and division by 10
Converting between millimetres and centimetres

**VOCABULARY**
decimetre (dm)
centimetre (cm)
millimetre (mm)
conversion
Assessment

1. Write a measurement in centimetres that is between 5 dm and 6 dm.
2. Write a measurement in decimetres that is between 20 cm and 95 cm.
3. Complete the table:

<table>
<thead>
<tr>
<th>cm</th>
<th>20</th>
<th>71</th>
</tr>
</thead>
<tbody>
<tr>
<td>dm</td>
<td>6</td>
<td>40</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>mm</th>
<th>300</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>dm</td>
<td>.8</td>
<td>5.1</td>
</tr>
</tbody>
</table>

Extensions

1. If 1 dm is equal to 10 cm, how many decimetres are in 100 cm? Where would you add metres to the step-diagram used in the lesson?
2. Write a measurement in decimetres that is between...
   
   a) 320 and 437 mm   
   b) 507 and 622 mm   
   c) 1 \( \frac{1}{2} \) metres and 1 \( \frac{3}{4} \) metres

3. John has a strip of paper 1 dm long. He folds the strip of paper so that it has a crease in its centre. What measurements can John make in centimetres using the strip?
ME5-11
Metres and Kilometres

Remind your students that 1 km equals 1000 m. Ask your students to think of objects that can act as benchmarks for estimating large lengths, heights, and distances. You may measure the actual length or height of some of these objects if they are available. Here are some possible benchmarks:

- A (very tall) adult and a door are about 2 m tall.
- A level, or storey, in a building (viewed from outside the building) is about 2 doors tall, so about 4 m tall.
- A school bus is about 10 m long.
- A typical car is about 3 m long.
- You can walk about 1 km in 15 minutes.

Invite students to use these benchmarks to estimate greater lengths and distances, such as:

- A basketball field is about 9 cars long. How long is a basketball field in metres?
- Two minivans are as long as 1 school bus. How long is each minivan?
- A playground is about 10 cars long. How many minivans can be parked along the playground?
- Daniel lives in an apartment building with 18 storeys. About how many school buses standing end-to-end, one on top of the other, are as tall as Daniel’s building?
- How many minivans parked end-to-end would it take to form a line 100 m long? How many would you need to form a line 1 km long?
- How many school buses can be parked along 1 km?

Extensions

1. How would you change a measurement in kilometres into centimetres?
2. Rita wants to estimate the height of a tree that grows near the school building. The tree is 3.5 storeys tall. Rita multiplies 3.5 by 4 m (the height of a school storey). How tall is the tree?
3. Remind your students that things in the distance appear smaller than they really are. Draw a line about 40 cm long on the blackboard. A pencil is far less than 40 cm long, but depending on where you stand, the line on the blackboard can appear to be the same length as the pencil! Ask your students to test this by holding a pencil in an outstretched hand. Ask them to move around the room, towards and away from the blackboard,
until the line appears to be as long as the pencil. Can they move so that the line appears to be half as long as the pencil?

4. Rita wants to estimate the height of a tree growing in the middle of a park. Rita uses a pencil and a friend. Rita positions herself far enough from the tree so that the tree appears smaller than the pencil. She asks her friend Sindi to stand by the tree. Rita holds the pencil vertically in her outstretched hand (she keeps her hand outstretched at all times) and compares the size of the pencil and the tree. For example, the tree appears to be as long as three quarters of the pencil. She holds the pencil so that the point is level with the top of the tree and her fingers are level with the bottom of the tree. She turns the pencil horizontally (with her fingers as the center of rotation) and asks her friend Sindi to walk in the direction that is at a right angle to the line between Rita and the tree. Rita asks Sindi to stop when the distance between Sindi and the tree seems to her the same as the distance she marked on the pencil. (Rita holds the pencil so that her finger is still level with the bottom of the tree and the point of the pencil is level with Sindi’s feet.)

After that, Rita measures the distance between Sindi and the tree with giant steps. This is the height of the tree. Can you explain why Rita’s method works? **HINT:** Use congruent triangles, one vertical and the other horizontal.

**EXPLANATION:** There are two pairs of congruent right-angled triangles. In the first pair, one triangle has Rita, the bottom and the top of the tree as vertices, and the other triangle has Rita, Sindi and the bottom of the tree as vertices. The other pair of congruent triangles has Rita’s eye and arm in common and the pencil as the side opposite to Rita’s eye in both triangles. The smaller triangles at the picture on the right are congruent, because two of their sides are same (arm and pencil) they both have a right angle. The larger triangles are also right-angled, and their angles are the equal to the angles of the smaller triangles. This means that the angles of the larger triangles are equal, and they also have a common side, so the larger triangles have to be congruent.
ME5-12
Speed

Work through this set of word problems with your students.

- Joanna can walk 1 km in 15 minutes. How far can she walk in 1 hour? (PROMPTS: How many minutes are in an hour? How many times will 15 minutes pass within an hour? So how many kilometres can Joanna walk in an hour?)

- Joanna’s school is 2 km from her home. How long will it take her to walk to school?

- This seems like a long walk, so Joanna bikes to school instead. It takes her 12 minutes to bike to school. How far can Joanna bike in an hour? In 3 hours? How long does it take Joanna to ride 1 km? 15 km? NOTE: If students have difficulty with the last two questions, ASK: How far is the school? (2 km) Which part of this distance is 1 km? (½) How much of the time interval will pass while she bikes 1 km? (it will take half the time to cover half the distance) What is half of 12 minutes? So how many minutes will pass while Joanna bikes 1 km?

- Joanna has to be at school at 8:45 a.m. When should she leave her house to be at school on time? School ends at 3:20 p.m. When does Joanna get home?

- After school Joanna bikes to a drawing class that is 3 km from school. When does she get to her drawing class?

To solve the last problem, break it into steps. ASK: What do you have to know to figure out when Joanna gets to the drawing class? Do you need to know when she gets out of school? When does that happen? Do you need to know how long she bikes? Do you know that? (Not yet.) If you knew how long she bikes, what would you do? (Add the time of the ride to the starting time, 3:20.) So this is the last step of the problem. Suggest your students leave some space to fill in the unknown information (the time of the ride) and write: “3:20 + Time of the ride = Time Joanna gets to her drawing class.” ASK: How can you find the time of the ride? How far does Joanna ride? (3 km) What is her speed? Your students can present her speed in different ways: 2 km in 12 minutes, 1 km in 6 minutes, 10 km in an hour. ASK: Which way is the most common way to present speed? Which way is most convenient for this problem? (1 km in 6 minutes) If we know how long it takes Joanna to ride 1 km, what should we do to get the time of the ride? Write in the space left in the equation above: “Time of the ride = Time to ride 1 km × Distance.” Ask your students to calculate the time of the ride and the time Joanna will arrive at her drawing class.

MORE PROBLEMS:

- It takes an antelope 35 seconds to run 700 m. How many metres can it run in a second? In a minute? If an antelope were able to run an hour at
that speed (which never happens), how far would it run? How many kilometres is that?

• It takes an ostrich 5.5 minutes to run 5 km. How long does it take an ostrich to run 1 km?

ASK: How many seconds does it take an ostrich to run 5 km? How many metres is that? About how many metres does an ostrich run in 1 second? (Remind your students that 5 000 ÷ 330 = 500 ÷ 33, which is easier. Also, for an estimate, they can use the fact that 33 × 3 is about 100). Who runs faster—an antelope or an ostrich?

Students will need more practice with problems of the last type. Here are two more problems to try:

• Yesterday the wind speed was 500 metres in a minute. Today the wind speed is 5 metres per second. Was the wind yesterday stronger than today?

• The world record of train speed is 581 km per hour. The speed of sound is 340.3 metres in a second. Did the train achieving the record speed go faster than sound?

Assessment

1. It takes a cheetah 7 seconds to run 210 m. How many metres does a cheetah run in a second? In 10 seconds? If it were able to run that fast for a minute, how far would a cheetah run in 1 minute?

2. A car drives 90 km in 1 hour. How far will this car go in 1 minute? What has greater speed—a cheetah or a car? Explain.

Extensions

1. The graph shows the distance covered by a cat over a period of 12 seconds.

   ![Graph](image.png)

   a) How fast was the cat running during the first 6 seconds?
   b) What was the cat doing after 10 seconds?

2. Karla can run at a speed of 5 metres per second and Bonnie can cycle at a speed of 12 metres per second. If Bonnie passed Karla on her bike and both children were moving at top speed how far apart would they be after 3 seconds?
ME5-13
Changing Units

GOALS
Students will take measurements in metres, and convert measurements between metres and centimetres.

PRIOR KNOWLEDGE REQUIRED
Measuring with a ruler
Skip-counting
Concepts of millimetres, centimetres, metres, decimetres
Converting between cm and mm
Using tables to organize data
Decimals

VOCABULARY
metre conversion
centimetre metre stick

Review the units of measurement: mm, cm, dm and m. Ask your students to name several objects that they would measure in each of these units.

Remind your students how to record measurements as 1 metre and 25 centimetres, as 1 m and 25 cm, and as 125 cm, then ask them to record their measurements in all three styles.

Show your students how to develop an estimate: First, estimate the blackboard’s length in whole metres (EXAMPLE: estimate: 5 m). Then place a metre stick beside the blackboard and ask your students if they would like to change their estimate. Mark the points that are 1 m and 2 m from the end of the blackboard. Ask the students if they would change their estimate now. Continue until the remaining length is less than a metre. Ask your students to estimate the remaining distance in centimetres (EXAMPLE: Estimate: 5 m 25 cm).

Ask your students to estimate and measure other distances in the classroom, like the width of the classroom, the distance from the window to the blackboard, etc. You might also use the 3rd activity below at this point.

Let your students complete the QUESTIONS 1 and 2 of the worksheet ME5-13.

Ask volunteers to convert, step by step, several measurements in metres and centimetres to centimetres. For example,

- 2 m 30 cm = 2 m + 30 cm;
- 2 m = 200 cm;
- 2 m + 30 cm = 200 cm + 30 cm = 230 cm

For a measurement like 370 cm, reverse the step-by-step process. (Students could start by skip counting by hundreds to determine how many metres are in 370 cm.)

Complete several more examples, then allow your students to practice with questions like:

145 cm = _____ m _____ cm  
354 cm = _____ m _____ cm  
789 cm = _____ m _____ cm

3 m 78 cm = _____ cm  
4 m 64 cm = _____ cm

9 m 40 cm = _____ cm

Ask your students how many metres are in 70 cm. Then ask them to convert to metres such measurements as 1 m and 40 cm, 2 m and 35 cm, and so on.
Assessment
1. 12 m = _____ cm = _____ dm = _________ mm
   3 000 mm = _____ cm = _____ dm = ____ m
   3 456 mm = _____ cm = _____ dm = ____ m
   3 400 mm = _____ cm = _____ dm = ____ m

2. Convert the following units of measurements.
   a) 2 m 72 cm = ______ cm
   b) 3 m 56 cm = ______ cm
   c) 348 cm = ______ m ______ cm

Bonus
Convert the measurements:
123 456 789 mm into cm, dm, m and km
987.654 m to cm, dm and mm
0.987654321 km to m, dm, cm and mm

ACTIVITY 1
The Great Metre Hunt
Cut up a ball of string into numerous lengths, ranging from 5 to 20 centimetres. Hide these lengths of string around the classroom. Divide students into groups—pairs work well—and inform them that each group must find ten pieces of string. The goal is to find ten pieces of string that, when laid end to end, will come closest to measuring 1 metre. Explain that finding all of the long pieces will probably total much more than one metre in length. Don’t forget to hide or remove the metre sticks from the classroom, to prevent students from using them to measure their string during the hunt.

Allow students to roam around collecting string. Once every group has found ten pieces, have them lay out their string end to end. Use a metre stick to measure the lengths. The group with the combined length of string closest to one metre wins!

Then, ask students to trade pieces of string with other groups so that each group has collected a metre-long length of string pieces. Allow them to use metre sticks.

ACTIVITY 2
Ask students to measure and compare the lengths of various body parts using a string and a ruler.

EXAMPLE:
   a) Is your height greater than your arm span?
   b) Is the distance around your waist greater than your height?
   c) Is your leg longer than your arm?

Encourage your students to predict the answers before they perform the measurements. Ask your students to record the measurements in three ways: 1 metre and 25 cm, 1 m and 25 cm and 125 cm.
ACTIVITY 3

Estimate the width of the school corridor: First, try to compare the length to some familiar object. For example, a minivan is about 5 m long. Will a minivan fit across the school corridor? Then, estimate and measure the width with giant steps or let several students stand across the corridor with arms outstretched. Estimate the remaining length in centimetres. Measure the width of the corridor with a metre stick or measuring tape to check your estimate.

ACTIVITY 4

Hold up a metre stick and point to various positions on the stick. Ask students to express if the positions are closer to 0 metre, .5 metre, or 1 metre.

ACTIVITY 5

Measure your shoe. Use this natural benchmark to measure long lengths: the classroom, the width of the hallway, or the length of the whole school!

ACTIVITY 6

Say whether each measurement is closer to 0 metres, $\frac{1}{2}$ metre, $\frac{1}{4}$ metre, $\frac{3}{4}$ metre or 1 metre.

a) 10 cm  
b) 52 cm  
c) 37 cm  
d) 82 cm  
e) 2 cm  
f) 90 cm

Extension

Measure your stride: Draw a line on the floor. Position yourself with your toes on the line. Walk normally for ten strides (not giant steps!) from the line and mark the place where your front leg’s toes are. Measure the distance you walked with a metre stick and divide it by 10 to get the average length of your stride.

Now that you have measured your stride, create a map that shows directions in paces to a buried pirate treasure (EXAMPLE: 15 paces north, 3 paces east, etc.). Calculate the actual distance you need to walk to find the treasure.
ME5-14
Problem Solving with Kilometres

Present the following problem:

Michael is travelling from Calgary, AB to Prince George, BC. The total distance of the trip is 780 km. Michael makes some short stops along the way. His average speed is 80 km per hour. About how long will the trip take?

Suggest to students that they round the distance to the nearest 100 km to get an estimate of Michael’s travelling time. ASK: Is that the time Michael actually spent behind the wheel? If his average speed was 80 km per hour, does this mean that he was driving at 80 km per hour all the time? No, 80 km per hour is his average speed; he might have driven faster for part of the time and slower for another part of the time. He also made some stops along the way—at those times, his speed would have been 0 km per hour!

Michael decided to make longer stops along the way to take some photos. His average speed is now only 50 km per hour. How long will his trip take at this speed?

How long will the trip take if Michael is biking at a speed of 13 km per hour?

The distance from Calgary to Banff National Park is 108 km. Then Michael drives through Banff National Park and Jasper National Park. When he leaves Jasper National Park, he still has 346 km to go. How many kilometres did he travel through Banff and Jasper National Parks?

To solve this problem, students might find it helpful to draw a line and mark the distances:

Another way to solve this problem is to use a number line, as in the Geometry unit.

Michael turns into the Icefields Parkway that passes through the Banff and Jasper National Parks after driving 182 km from Calgary. The Parkway is 226 km long. How far from the end of his trip will Michael be when he gets to the end of the Icefields Parkway? How far is the beginning of the Parkway from the entrance to Banff National Park?

Invite students to make up their own problems based on the information in the previous problems.
**ACTIVITY**

Students will need a ruler, a piece of string, and the map on Worksheet ME5-14 or a large-scale, detailed map of Canada’s North that includes the Dempster highway. Ask your students to lay the string carefully along the line that represents the highway on the map, measure the length of the string, then figure out a scale for the map, knowing that the length they measured with the string (in centimetres) represents about 700 km (the approximate length of the actual highway). If you are using a large-scale map, use the actual length of 736 km.

**Extensions**

1. Find two cities on a map and use a ruler to measure the distance between the cities. Use the scale on the map to change your measurement to kilometres. Estimate how long it would take to travel between the cities at a speed of 100 km per hour.

2. A car travels with a speed of 90 km per hour. How far will it travel in half an hour? In 10 minutes? In 2 minutes?

3. Michael drives from Calgary to the boundary of Banff National Park at a speed of 90 km per hour. He drives through Banff and Jasper National Parks at an average speed of 60 km per hour. How long does his trip (from Calgary through both parks) take?
ME5-15
Changing Units (Advanced)

NOTE: There is a typo in the Workbook in QUESTION 3 of ME5-15, parts a), b) and d). The last line of each question should read:

a) 3.5 cm = ___35 mm  b) 2.7 cm = ___ mm  d) 3 m = ___ cm

Make sure your students are familiar with multiplication and division of decimals by 10, 100, and 1 000. They should have completed sections NS5-93 to NS5-100.

Draw a line on the board and ask students to measure it in decimetres and in centimetres. ASK: Which unit is larger, the centimetre or the decimetre? Will the same line hold more centimetres or more decimetres? If the line is 4 dm long, only 4 decimetres are needed to cover it, but it takes 40 centimetres to cover the same length. So the smaller the unit, the larger the number of the units you need. ASK: What do you do to a measurement in centimetres to get the measurement in decimetres? What do you do to measurements in decimetres to convert them to centimetres?

Draw this diagram on the board:

\[ \text{dm} \quad \times 10 \quad \text{cm} \quad \div 10 \]

ASK: Where should I put millimetres? (on the bottom step) Where should I put metres? (on the top step) Where should I add arrows and how should I label them?

ASK: A line is 5 dm long. How long is it in millimetres? Encourage your students to show more one than one solution to the problem. (For example, they could show 5 dm = 50 cm = 500 mm OR they could explain that since 1 dm = 100 mm, then 5 dm = 500 mm.)

Present a harder problem: Change 4.7 cm to millimetres. ASK: Which unit is larger, the centimetre or the millimetre? Will a line 4.7 cm long hold more or fewer millimetres? Should you multiply or divide? By how much? Point out that the length in centimetres has 1 digit after the decimal point and the length in millimetres is a whole number.

Ask your students to perform several more complicated changes of measurements, such as:

- 6.7 dm to mm
- .35 m to dm
- .24 m to mm
- 345 mm to dm
- 789 mm to m
- .35 mm to dm
- .54 cm to m
- .789 mm to m

Do the Activity (see below) to give students more practise expressing measurements in different units.
Ask students to tell which measurement is longer:

- 324 mm or 3 dm
- 5432 mm or 54.32 m
- 654 cm or 65 dm
- 87 dm or 86543 mm

You may also give your students word problems that require changing units, such as:

- A fence board is 2 dm wide. Three boards cost $10. How much will 3 m of fence cost?
- One metre of ribbon costs 12 cents. How much would 900 mm of ribbon cost?

**Assessment**

1. Fill in the blanks:

   - .7 dm = ___ mm
   - .35 m = ___ dm
   - .24 m = ___ mm
   - 345 mm = ___ dm
   - 789 mm = ___ m
   - .35 mm = ___ dm
   - 5.67dm = ___ cm
   - .05 dm = ___ m
   - .27 cm = ___ mm
   - 364 cm = ___ dm

2. Circle the largest length and underline the smallest length in each column above.

**Extensions**

1. What would you do to a measurement in millimetres to get a measurement in kilometres?
   A snake is 0.0105 km long. How long is it in mm?

2. Draw a line that is...

   a) 0.11 m long  
   b) 1.7 dm long  
   c) 134 mm long  
   d) 0.003 m long  
   e) .15 dm long
**ME5-16**
**Ordering and Assigning Appropriate Units**

Present the items from the measurement box individually (see ME5-8) and have your students express the measurement that each item represents. Review the full name and abbreviation for each unit.

Ask your students to tell you how far it is from Halifax to Calgary in millimetres. Then ask them to calculate the thickness of a coin in kilometres.

Explain that it is important to choose the appropriate unit of measurement for the length/distance being measured. Ask your students to tell you which unit of measurement will best express the distance from Halifax to Calgary. Which unit of measurement will best express the thickness of a coin?

Practice this by displaying a variety of objects (a book, a stapler, a coin, etc.) and asking your students to tell you which unit of measurement will best express the length, width, thickness or perimeter of the object. Is the length of a stapler expressed best by a centimetre or a metre? Is the thickness of a coin expressed best by a millimetre or a centimetre?

Have your students select five items in the classroom and guess which unit of measurement will best express each item’s height, width or length.

Have a volunteer measure one of the practice items (the stapler, for example), but do not specify a unit of measurement. After the volunteer relates the measurement to the class, identify the unit of measurement used and ask your students why the volunteer chose that unit of measurement. Students will likely respond that centimetres were used because the stapler is about the right size. It would be many millimetres long, and it’s much smaller than a metre.

Demonstrate that the stapler is, for example, 12 cm in length. That’s 120 mm, 0.12 m and 0.00012 km. Out of all those numbers, 12 is the nicest and the simplest.

Have your students measure their five objects. Which units of measurement do they automatically use? Do these units of measurement lend themselves easily to the task? Would alternate units of measurement offer simpler measurements?

Ask your students to list in their notebooks five things that could be measured and are not in the room (a bicycle, a video game, a rocket ship, anything). Ask students to arrange the five things in order from smallest to largest. Then ask them to indicate which unit of measurement will give the simplest measurement for each item.

Review the relationships between units of length measurement. Ask your students: Which number is larger, 2 or 35? What length is larger, 2 m or 35 cm? Why?

Explain that the easiest way to compare measurements that are expressed in different units is to convert all measurements to the small unit. For instance,
to compare 3 m and 250 cm, convert 3 m to 300 cm, and it becomes clear that 3 m is greater than 250 cm.

Let your students practice with questions like:

Circle the greater amount:

1 m or 80 cm  
6 m or 79 cm  
450 cm or 5 m  
230 cm or 2 m  
2 m or 38 cm

Ask students to order these lengths from shortest to longest and ask them to mark these lengths on a number line:

```
0 cm  100 cm  200 cm  300 cm  400 cm  500 cm  600 cm
```

**Assessment**

1. Which unit of measurement would you use to measure the…

   - Thickness of a book?
   - Length of a chocolate bar?
   - Height of the school?
   - Height of a tree?
   - Height of a mountain?
   - Distance from a window to a door?
   - Distance from your school to your home?
   - Distance from your nose to your toes?
   - Distance from the Earth to the Moon?
   - Distance between your eyes?

2. Convert the distances to cm and then order them from greatest to smallest.

   a) 75 cm  
   b) 85 m  
   c) 230 cm  
   d) 7 m  
   e) 4 cm

**ACTIVITY 1**

On a school walking trip, ask students to say when they think they have walked half a kilometre or 1 kilometre.

**ACTIVITY 2**

Ask students if they have taken any trips to nearby towns. Ask them to estimate how many kilometres away the towns are and then have them check the actual distances by measuring the distances on a map with a ruler, and then converting their measurements to kilometres using the scale on the map.

**ACTIVITY 3**

Divide your class into four groups and assign one unit of measurement (mm, cm, m or km) to each group. Give each group a sheet of chart paper, and ask them to write the full name of their unit of measurement and the abbreviation at the top of the page. Have them list as many things as they can that could be measured with that unit of measurement. Set a target quantity (maybe twenty) and ask students to try and list more than that quantity. Have each group share their ideas. Display the lists in the classroom until the instruction of ME5-18 is complete.
Ask your students to list as many words as they can that start with the unit of measurement prefixes. Some examples include centipede (“hundred feet”), centurion (a commander in the Roman army in charge of 100 soldiers), million (one thousand thousands), decibel, decimal, decimate (historically meaning to kill or remove one in every ten). Students might suggest other metric units, such as kilograms or kilowatts, for “kilo”.

Ask your students what numbers they know in French or any of the other Romance languages. Draw the following table and illustrate the patterns and similarities to your students.

<table>
<thead>
<tr>
<th>Language</th>
<th>10</th>
<th>100</th>
<th>1 000</th>
</tr>
</thead>
<tbody>
<tr>
<td>French</td>
<td>dix</td>
<td>cent</td>
<td>mille</td>
</tr>
<tr>
<td>Spanish</td>
<td>diez</td>
<td>cien</td>
<td>mil</td>
</tr>
<tr>
<td>Italian</td>
<td>dieci</td>
<td>cento</td>
<td>mille</td>
</tr>
<tr>
<td>Portuguese</td>
<td>dez</td>
<td>cem</td>
<td>mil</td>
</tr>
</tbody>
</table>

Check out The Metric Song by Kathleen Carroll:

http://www.songsforteaching.com/kathleencarroll/metricsong.htm

This song reinforces the distinction between milli and kilo.

**Extensions**

1. Introduce your students to the principle of metric prefixes. Explain to them that the metric system is comprised of many different units of measurement, all dependent on the dimensions being measured—distance is measured in metres, volume is measured in litres, weight is measured in kilograms.

   Then explain that each unit of measurement has a base unit (for these lessons, metres) that shares prefixes with the other base units. Write the word “metre” and ask your students to provide you with the prefixes.

   Write the prefixes and explain a bit about the etymology of each. For example, centi means one hundredth. That’s why there are 100 centimetres in a metre. Also, there are 100 cents in a dollar. There are 100 years in a century.

   Milli means one thousandth. That’s why there are 1 000 millimetres in a metre. Also, there are 1000 years in a millennium.

   Deci means one tenth. That’s why there are 10 decimetres in a metre. Also, there are 10 years in a decade.

   Kilo means 1 000. That’s why there are 1 000 m in a kilometre.

2. Have your students research (and prepare reports or descriptive posters on) alternative systems of measurement, such as Ancient Egyptian, Chinese or Old English systems. How long were their respective units of measurement, and what were they called? Are they still in use?
3. Compare the lengths of non-metric units of measurement (example: inches, feet, yards, etc.) to the metric units of measurement. What is the difference between an inch and a centimetre or a yard and a metre?

4. Big Numbers. Here are some questions you could assign or discuss with students to give them practice calculating and estimating with large numbers. Your students will need to know how to multiply whole numbers by multiples of 10.

   a. (Warm Up): There are 60 seconds in 1 minute. How many seconds are there in 1 year?

      SOLUTION: Start with seconds:

      How many seconds in one minute? 60 seconds
      How many minutes in a day?
      
      60 minutes = 1 hour; 1 day = 24 hours
      24 × 60 = 1440 minutes in one day
      1440 × 60 = 86 400 seconds in one day

      Finally, multiply the number of seconds in one day by the number of days in one year.

      86 400 seconds in one day × 365 days in one year = 31 536 000 seconds in one year! In one year there are more than 31 million seconds!

   b. About how many minutes have you been alive?

      SOLUTION: Start with the date the student was born and the current date.

      The student is 10 years and several days old. There are 365 days in a regular year.

      How many minutes in one day?
      
      24 × 60 = 1440 minutes in one day = 1440 minutes in one day.

      How many minutes are in 1 non-leap year?
      
      1440 minutes in one day × 365 days in one year = 525 600 minutes in one non-leap year.
      525 600 minutes in one year × 10 years; 5 256 000 minutes in 10 years

      How many days should we add to these 10 years? From August 29, 1998 to August 29, 2008 is 10 years. How many of them were leap years?
      2000, 2004 and 2008 were leap years, so we have to add three days.
      From August 29 to the end of the month we add three more days. From September 1, 2008 to the current date (September 13, 2008) we add: 3 days + 3 days + 13 days = 19 days to add. How many minutes are in 19 days?
      1440 × 19 = 27 360 minutes in 19 days

      How many minutes has the student been alive for?
      
      5 256 000 minutes in 10 non-leap years
      + 27 360 minutes in 19 additional days
      5 283 360 minutes in the student's life so far!
c. According to the Guinness Book of World Records, the tallest tree ever measured was an Eucalyptus tree discovered in Watt’s River in Victoria, Australia in 1872 by a forester named William Ferguson. The Eucalyptus tree was 132.6 m tall.

The Dyerville Giant, a coastal redwood tree, found in the Humboldt Redwoods State Park in California, USA is named as the tallest tree of modern times. This tree was 1 600 years old and 113.4 metres high when it fell in March 1991.

The height of a dime is 1 mm. How many dimes would need to be stacked to be as high as each of the tallest trees in history? How much would all these dimes be worth?

**SOLUTION:**

Convert each tree height from metres to millimetres.

\[
\begin{align*}
132.6 \text{ m} \times 1000 &= 132 600 \\
113.4 \text{ m} \times 1000 &= 113 400 \\
\end{align*}
\]

The height of a dime is 1 mm. We can stack 132 600 dimes to be as high as the Eucalyptus tree and 113 400 dimes to be as high as the Redwood tree.

The total values are 1 326 000¢ and 1 134 000¢ or $13 260 and $11 340.

d. Ellen MacArthu from France is the first woman to travel around the world on a sailboat, by herself. Her boat was named the Kingfisher and it could only carry one person. Ellen had dehydrated food and a few changes of clothing, as well as e-mail access on board her boat. It took her a total of 94 days, 4 hours, 25 minutes, and 40 seconds to complete her journey around the world.

Ellen had to be alert at all times so she could not take breaks to sleep. Amazingly, she would take 40 minute long naps, 10 times a day during her trip.

About how many hours of Ellen’s trip sailing around the world was she sleeping?

**SOLUTION:**

\[
40 \text{ minutes} \times 10 \text{ naps per day} = 400 \text{ minutes of sleep per day}
\]

400 minutes of sleep ÷ 60 minutes per hour = approximately 7 hours of sleep each day

7 hours × 94 days = 658 hours

Ellen slept approximately 660 hours during her trip around the world.

e. “Wranny” is what Andy Martell from Toronto, Ontario, Canada calls his creation made from Saran wrap. He is marked down in the Guinness Book of World Records for having created the largest ball made entirely from Saran wrap. Measured in February 2003, the ball had a circumference of 137 cm and weighed 20.4 kg.

A jelly bean weighs 0.5 grams. How many jelly beans do we need to equal the weight of “Wranny”, Andy’s famous Saran wrap ball?

**SOLUTION:**

The Saran wrap ball weighs 20.4 kg = 20.4 × 1000 = 20 400 grams

One jelly bean weights 0.5 grams, so 2 jelly beans would weigh 1 gram. Therefore 20 400 × 2 = 40 800 jelly beans would weight the same as the Saran wrap ball.
f. The world’s longest pencil was created in Malaysia by the company Faber-Castell. The pencil is 19.75 m long, and has a diameter of 0.8 m.

   a) How many regular pencils do you need to stack end to end to be as high as the largest pencil in the world?

   b) Could you put your hand around the largest pencil in the world?

SOLUTION:

   a) Convert the height of the largest pencil to cm: $19.75 \times 100 = 1975$ cm

      A regular pencil is about 20 cm long. $1975 \div 20 = 99$

      It would take about 99 pencils to equal the length of the world’s longest pencil.

   b) Have your students brainstorm how to answer this question. Measure the length of the palm of their hand (this should be around 10 cm to 12 cm). Convert the diameter of the pencil to cm:

      $0.8 \times 100 = 80$ cm

      Then approximate the circumference of a pencil. The circumference of a circle is approximately 3 times its diameter: $3 \times 80 = 240$ cm


g. The largest flag created was flown in Brasilia, Brazil in August 1998. The flag has a width of 70 m and a height of 100 m.

   What is the area of the largest flag?

   Measure the area of a regular piece of paper. How many regular pieces of paper would be needed to cover the entire flag?

SOLUTION:

   The area of the largest flag is: $70 \times 100 = 7000$ m²

   The dimensions of a regular piece of paper are: about $30 \text{ cm} \times 20 \text{ cm} = 0.3 \times 0.2 = 0.06$ m²

   The area of a regular piece of paper is: $3 \times 0.2 = 0.06$ m²

   But $100000 \times 0.06 = 6000$ (which is fairly close to 7000).

   At least 100,000 pieces of regular paper would be needed to cover the largest flag in the world.

NOTE: All the data collected for questions c to g is from the Guinness Book of World Records website: www.guinnessworldrecords.com
ME5-17
Mathematics and Architecture

GOALS
Students will use scale to determine the actual dimensions of historic buildings in pictures.

PRIOR KNOWLEDGE REQUIRED
m, mm, cm
Multiplication and division of decimals by 10, 100, and 1000
Converting between metres, millimetres, and centimetres

VOCABULARY
metre
millimetre
centimetre


ASK YOUR STUDENTS TO CALCULATE THE LENGTH OF THE PYRAMID BASE FROM THEIR MEASUREMENTS AND THE SCALE. (IT IS ABOUT 60 M.) THE STUDENTS MAY ALSO COUNT THE NUMBER OF LARGE "STEPS" IN THE PYRAMID (ON EITHER SIDE OF THE STAIRCASES) TO FIND THE APPROXIMATE HEIGHT OF EACH ONE.

ACTIVITY
Students could look up the measurements (base and height) of another pyramid in a book or on the Internet and then draw a scale diagram of the pyramid (as in QUESTION 1 on the worksheet) with a scale of their choice.

Extension
Over the last 4000 years, the Pyramid at Giza has lost 10 metres from its original height. If the pyramid continues to erode at the same rate, how much shorter will it be in 12000 years?
ME5-18
Problem Solving with Measurement

Start with the diagnostic test below. Don’t continue until all your students have passed the test.

1. Multiply:
   a) $100 \times 80 = \quad$ b) $17 \times 100 = \quad$ c) $32 \times 1\,000 = \quad$ d) $13.8 \times 1\,000 =$

2. Divide:
   a) $270 \div 100 = \quad$ b) $3\,700 \div 100 = \quad$ c) $1\,870 \div 100 =$
   d) $360 \div 1\,000 = \quad$ e) $87\,000 \div 1\,000 = \quad$ f) $6\,420 \div 1\,000 =$
   g) $5.45 \div 1\,000 = \quad$ h) $923.47 \div 100 =$

Present this series of word problems:

- The Jacques Cartier Bridge over the St. Lawrence River in Montreal is 3.4 km long. How long is that in metres?

- The bridge actually consists of two smaller bridges with an island in the middle. The total bridge structure (excluding the support sections) is 2687.42 m long. How many centimetres is that?

- The river beneath the bridge is divided into 3 parts, one about 400 m wide, another about 200 m wide, and the third about 66 m wide. How wide is the total span of water beneath the bridge?

- The bridge goes across Sainte-Hélène Island but passes right over Notre Dame Island. Sainte-Hélène Island is about 600 m wide and Notre Dame Island is about 230 m wide at the point the bridge crosses them. What is the distance between the shores of the St. Lawrence River at that point?
• The main bridge is 590.35 m long. Its centre span is 33 435 cm long. There is a support structure leading to the centre span on each side of the main bridge. Both support structures have the same length. How long is each support structure?

• There are 5 lanes along the bridge. Suppose that there is a traffic jam, and about 5 cars are standing along every 20 metres in each lane of the bridge. How many cars are stuck in the jam along the whole length of the bridge (3.4 km)?

Extensions

1. Name two places near where you live that are approximately as far apart as the Confederation Bridge is long. Use a map if necessary.

2. About 27 000 tonnes of asphalt were used to pave the surface of the Confederation Bridge. Approximately how many tonnes of asphalt were used in each kilometre of the bridge?

   HINT: Round the length of the bridge to the nearest ten kilometres.
ME5-19
Perimeter

GOALS
Students will measure the perimeter of given and self-created shapes.

PRIOR KNOWLEDGE REQUIRED
Adding sequences of 1-digit numbers

VOCABULARY
perimeter
grid paper

Write the word “perimeter” and explain to your students that perimeter is the measurement around the outside of a shape. Illustrate the perimeters of some classroom items; run your hand along the perimeter of a desk, the blackboard or a chalkboard eraser. Write the phrase “the measurement around the outside of a shape.”

Demonstrate the method for calculating perimeter by counting the entire length of each side and creating an addition statement. Write the length of each side on the picture.

Draw several figures on a grid and ask your students to find the perimeter of the shapes. Include some shapes with sides one square long, like the shape in the assessment exercise, as students sometimes overlook these sides in calculating perimeter. Ask your students to draw several shapes of their own design on grid paper and exchange the shapes with a partner.

For the last exercise, suggest that students draw a letter, or simple word (like CAT), or their own names.

Assessment
Write the length of each edge beside each edge and count the perimeter of this shape.

Do not miss any sides—there are ten!

Distribute one piece of string, about 30 cm long, and a geoboard to each student. Have them tie the ends of the string together to form a loop, and then create a variety of shapes on the geoboard with the string. Explain that the shapes will all have the same perimeter because the length of the string, which forms the outside edges, is fixed. How many different shapes can all have the same perimeter?
Extensions

1. Explain that different shapes can have the same perimeter. Have your students draw as many shapes as they can with a given perimeter (say ten units).

2. Can a rectangle be drawn with sides that measure a whole number of units and have a perimeter…

   a) of seven units?
   b) with an odd number of units?

   [Both are impossible.]

   Students can use a geoboard rather than grid paper, if preferred.

3. Create three rectangles with perimeters of 12 cm. (REMEMBER: A square is a rectangle.)

4. Distribute pentamino pieces (a set of twelve shapes each made of five squares, see the BLM) to your students and have them calculate the perimeter of each shape. Create a table and order the perimeters from smallest to greatest. Have students also calculate the amount of square edges inside each shape. Can they notice a pattern emerging in the table?

<table>
<thead>
<tr>
<th>Shape</th>
<th>Perimeter</th>
<th>Number of Inside Edges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Draw this figure:

and have your students demonstrate the calculation of the perimeter by totalling the outside edges.

Then draw the same rectangle without the inside edges:

and ask them how they could calculate the perimeter again. Explain that it can be measured, and then have volunteers measure each side with a metre stick.

Draw an irregular figure with some edges that are neither horizontal nor vertical:

Have a volunteer use a ruler or metre stick to measure each side, and then count the sides to confirm that they've all been measured.

Assign the first two exercises of the worksheet.

**Assessment**

Calculate the perimeter.

**TEACHER:** Draw the lengths of all sides in whole centimetres.

Ask your students to estimate the lengths of the shapes with their bodies (remind them that the widths of their hands are about 10 cm).

Review appropriate units. Have students determine the best units of measurement for calculating the perimeters of the schoolyard, a chalkboard eraser, a sugar cube, etc.

Have them complete the worksheet. As a possible homework assignment, have your students estimate then measure the perimeters of their bedrooms.
Assessment
The length of a minivan is 5 m. The house is a rectangle six minivans long and four minivans wide.
1. Sketch the house.
2. Measure the edges (in minivans and in metres).
3. Calculate the perimeter of the house.

Extensions
1. The sides of a regular pentagon are all 5 cm long. What is the pentagon’s perimeter?
2. The perimeter of a regular hexagon is 42 cm. How can the length of its sides be determined?
3. Measure the perimeter of this star.

4. Can 500 toothpicks line the entire perimeter of your school? Calculate an estimate.
5. Sally wants to arrange eight square posters into a rectangle. How many different rectangles can she create? She plans to border the posters with a trim. For which arrangement would the border be least expensive? Explain how you know.
ME5-21
Exploring Perimeters

Review the meaning of perimeter and the methods of finding it (counting the edges or adding the side lengths). Draw the following shape:

```
  +---+
  |   |
  +---+
```

Ask a volunteer to find the perimeter. Then ask additional volunteers to add a square to all possible perimeter positions and identify how the perimeter measurement changes. Summarize the results in a table. (It is also good to mark congruent shapes. Do they have the same perimeter?) Why does the perimeter change the way it does? How many edges that had previously been on the outside are now inside?

**ADVANCED:** Try to guide your students toward developing a formula (new perimeter = old perimeter + 4 – twice the number of sides that become inside edges).

Draw a rectangle on the board. Write a length on one of the longer sides. Ask your students what the length of the opposite side should be. Add the length of one of the shorter sides and ask your students what the length of the last side should be. What is the perimeter of the rectangle? Ask a volunteer to write the addition statement for the perimeter.

Draw another rectangle and say that its perimeter is 14 m. Say that the length of one side is 3 m. Ask your students to find the lengths of the other sides. Encourage them to present more than one solution to the problem (For example, they could say that 14 – 3 – 3 = 8, so the sum of the other two sides is 8, so each side is 4 m. Another solution would be to use the fact that the sum of two adjacent sides is exactly half of the perimeter: half of 14 is 7, so 3 + something = 7, which means each of the other sides is 4 m.)

Ask your students to find all the rectangles with a perimeter of 14 units and sides that have length in whole units.

Draw another rectangle on the board and say that the length is L and the width is 3 cm. What is the perimeter of this rectangle? What happens if the width is W and the length is 4 cm? How would you express the perimeter of a rectangle with length L and width W?

**ACTIVITY 1**

Students could try **QUESTION 3** on the worksheet with a geoboard.
ACTIVITY 2

Students enjoy creating complex shapes and finding their perimeter (particularly if you are impressed with their work). You might suggest that they draw and find the perimeter of a letter of the alphabet such as L, H, C, and A, or even a simple word like “CAT.”.

ACTIVITY 3

Point out to your students that different shapes can have the same perimeter. Ask them to draw as many shapes with a given perimeter (e.g., 10 units) as they can.

Extensions

1. A parallelogram has one side that is 3 cm long, and a perimeter of 16 cm. What are the lengths of the other sides?

2. An isosceles triangle has a perimeter of 15 units. The length of each side is a whole number. Find all possible triangles that meet these requirements. (ANSWER: 5, 5, 5; 7, 4, 4; 3, 6, 6; 1, 7, 7) Why can the length of the base not be an even number?

3. A school bus is 10 m long and 2.5 m wide. Karen’s school is 25 m long and 11 m wide.
   a) How many school buses can park along the entire perimeter of the school, so that each bus has its entire length along a school wall?
   b) How many school buses can park along the entire perimeter of the school if a bus is allowed to park so that it has its length along a school wall or along the width of another bus?

   Allowed in b) only

   Not allowed

   SOLUTION: a) Though the perimeter of the school is 72 m, only two buses will be able to park along its length, so the answer is 6 buses. In b), one can park 8 buses as shown below:
ME5-22
Circles and Irregular Polygons

Draw a square 10 cm by 10 cm on the board. Invite a volunteer to draw and to measure the diagonals of the square. (They will be about 14 cm long.) Ask your students to draw a square 1 cm by 1 cm in their notebooks. Ask them to predict the length of the diagonal in their squares. Then ask students to draw and to measure the diagonal. (It will be about 1.4 cm long.) Explain to your students that the length they have measured is an approximation, and they can use it to find the perimeter of the following shapes:

![Shapes diagram]

The horizontal and vertical distance between adjacent dots is 1 cm. The diagonal distance between dots is, as students just learned, about 1.4 cm. Ask your students to find the perimeter of the various shapes by adding together the lengths of the sides. Encourage them to use multiplication where possible, as a quicker way to add multiples of the same number. For example, in the diamond shape (second from the left), there are 4 equal sides. Instead of adding the length of each side 4 times, students could multiply the length by 4.

Now draw 3 circles—with radiuses of 15, 20, 25 cm—on the board, mark the centres, and explain that the distance around the outside of each circle is called the **circumference**. Write the term on the board. Then draw several horizontal lines through one of the circles such that one of them passes through the centre of the circle. Tell students that the width of the circle is measured by the longest line. **ASK:** Which line is the longest? (Another way to ask this: Suppose these circles are coins and you need to make slots, to push each coin through. Which line represents the minimal length of a slot for each “coin”? ) Help students to identify the line that goes through the centre of the circle as the longest one. You may tell students that the width of a circle is also called the **diameter**. Now measure the diameters of the remaining circles and fill in the first column in this T-table:

<table>
<thead>
<tr>
<th>Diameter</th>
<th>Circumference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Using a measuring tape (the type tailors use), string, or pieces of paper, have volunteers measure the circumference of each circle and fill in the second column in the table. Then look at the numbers and ask students if they can see a pattern. How does the circumference relate to the width?
How many times larger is the circumference than the width in each case? Students can either divide the numbers, or do the Activity, to find out. You might wish to explain that the circumference of a circle is always approximately 3 times greater than the width (or diameter). The exact number (3.14, rounded to 2 decimal places) is a very important number in mathematics and is called “pi.”

Before students try QUESTION 2 on the worksheets they could measure the diameter and circumference of several cylinders in the classroom (using string to find the circumference). Suggest that, for each of their measurements, they add the diameter repeatedly to see approximately how many diameters are needed to make the circumference. They will find that the answer is always about 3. (The ratio of the circumference to the diameter is called pi, and is about 3.14.)

Extensions

1. **PROJECT:** What were the ancient estimates of pi? **POSSIBLE SOURCES:**
   
   http://www-history.mcs.st-and.ac.uk/HistTopics/Pi_through_the_ages.html
   
   http://library.thinkquest.org/C0110195/history/history.html

2. Lee wants to arrange some shells around a circular flowerbed in her garden. The flowerbed is 1 m wide. Each shell is 5 cm long. About how many shells will Lee need? **HINT:** Use the width of the flowerbed (in centimetres or in number of shells) and the pattern from the T-table you made during the lesson to estimate the circumference.
ME5-23
Area in Square Centimetres

GOALS
Students will find the area in centimetres squared (cm²) of shapes drawn on grid paper.

PRIOR KNOWLEDGE REQUIRED
Drawing lines with a ruler
Measuring sides with a ruler
Perimeter

VOCABULARY
2-dimensional area
square centimeter
centimeters squared (cm²)
perimeter
rectangle

Remind students that area is often measured in units called “centimetres squared” or cm². Show students an example of a square centimetre, that is, a square whose sides are all 1 cm long.

Draw several rectangles and other shapes (EXAMPLE: L-shape, E-shape) on the board and subdivide them into squares. Ask volunteers to count the number of squares in each shape and write the area in cm².

Then draw several more rectangles and mark their sides at regular intervals, as shown below.

Ask volunteers to divide the rectangles into squares by joining the marks using a metre stick. Ask more volunteers to calculate the area of these rectangles.

Ask students to draw their own shapes on grid paper and to find the area and perimeter for each one.

Students work in pairs. One student draws a shape on grid paper, and the other calculates the area and the perimeter. ADVANCED VARIATION: One student draws a rectangle so that his/her partner does not see it, calculates the perimeter and the area, and gives them to the partner. The partner has to draw the rectangle with the given area and perimeter.

Students could try to make as many shapes as possible with an area 6 units (or squares) on grid paper or a geoboard. For a challenge, students could try making shapes with half squares. For an extra challenge, require that the shapes have at least one line of symmetry. For instance, the shapes below have area 6 units and a single line of symmetry.
Extensions

1. Sketch the shape below (at left) on centimetre grid paper. What is its area in cm²? (16) Now calculate the area using a different unit: 2 cm × 2 cm square (see below right). What is the area in 2 cm × 2 cm squares? (4) What happens to your measurement of area when you double the length of the sides of the square you are measuring with? (The area measurement decreases by a factor of 4.)

![Grid Paper Shape](image)

The new unit: 2 cm × 2 cm = 4 cm²

2. If the area of a shape is 20 cm², what would its area be in 2 cm × 2 cm squares? Sketch a rectangle with area 20 cm² to check your answer.

3. Draw a rectangle that has the same area (in cm²) as it does perimeter (in cm).

\[ 18 = 3 \times 6 \text{ (area)} = 2 \times 3 + 2 \times 6 \text{ (perimeter)} \]

4. Copy the figure onto grid paper. Draw a straight line to divide the shape into 2 parts of equal area.

![Grid Paper Figure](image)
ME5-24
Area of Rectangles

Draw an array of dots on the board. **ASK:** How many dots are in the array? Invite a volunteer to explain how he or she counted the dots. Ask another volunteer to write the corresponding multiplication statement on the board.

Now draw a rectangle on the board. (Draw it on a grid or subdivide it into squares.) Ask volunteers to write the length and width of the rectangle on the board, where length and width are measured in numbers of squares. Draw a dot in each square of the rectangle and **ASK:** How can a multiplication statement help us to find the area of the rectangle? Ask students to write and solve the multiplication statement for the area of the rectangle.

Draw several rectangles (again, subdivided into squares) and ask volunteers to write the length and the width and use them to calculate the area.

Draw rectangles with various lengths and widths (such as $20 \times 30$ cm, $40 \times 30$ cm, $30 \times 50$ cm) but don’t subdivide them into squares. Ask volunteers to measure the sides with a metre stick and calculate the area of the rectangles in cm$^2$.

Once students are comfortable finding the area of a rectangle by multiplying length and width, ask them to write the relationship as a verbal rule or a formula using letters: $\text{Area} = \text{Length} \times \text{Width}$, or $A = L \times W$. You might also encourage students to develop a rule or a formula for perimeter ($P = 2 \times L + 2 \times W$).

**Assessment**
1. Calculate the area of the rectangle:

```
  +---+---+---+---+
  |   |   |   |   |
  +---+---+---+---+
  |   |   |   |   |
  +---+---+---+---+
  |   |   |   |   |
  +---+---+---+---+
```

   Width: _____

   Length: ___

2. Measure the sides and calculate the area:

```
  +---+---+---+
  |   |   |   |
  +---+---+---+
  |   |   |   |
  +---+---+---+
```

   Width: _____

   Length: ___

**GOALS**
Students will find the area of rectangles in square units and in cm$^2$.

**PRIOR KNOWLEDGE REQUIRED**
Drawing lines with a ruler
Measuring sides with a ruler
Centimetres squared (cm$^2$)
Multiplication

**VOCABULARY**
2-dimensional
Area
Square centimeter
Perimeter
Rectangle
Length
Width
Ask students to construct rectangles of the same area, but with different lengths and widths. They can work on a geoboard or with square tiles on grid paper. How many different rectangles can they make with area 8 cm$^2$?

**ACTIVITY 2**

Ask students to create various rectangles and record their length, width, and area. Ask partners to give each other the length and width of some of their rectangles so they can calculate the area and check each other’s work.

**Extensions**

1. Calculate the area of the figure by adding the area of the rectangles:

![Diagram of two rectangles]

2. Divide the figure into rectangles and calculate the area:

![Diagram of a composite figure]
ME5-25
Exploring Area

GOALS
Students will find the area of rectangles with lengths and widths given in centimetres (cm), metres (m), and kilometres (km). Students will determine length (or width) given area and width (or length).

PRIOR KNOWLEDGE REQUIRED
Measuring sides with a ruler
Multiplication
Area of rectangle
Centimetres squared (cm²)

VOCABULARY
2-dimensional area
perimeter
rectangle
length
width
square centimetre and centimetre squared (cm²)
square kilometre and kilometre squared (km²)
square metre and metre squared (m²)

Draw a rectangle on the board and ASK: How can we calculate the area of this rectangle? Invite volunteers to help you solve the problem (one student could measure, another could write the measurements, a third could do the calculation and write the answer).

Tell students that a city block is 2 km long on every side. ASK: What shape is the block? What is its area? What units should we use for the area—is it square centimetres? Why not? If students do not infer the right answer and explanation, explain that a square kilometre is a square whose sides are 1 km long. When you multiply the length of the city block (in kilometres) by the width, you find out how many one-kilometre squares are in the block, so the area is in square kilometres, or km². ASK: What if the city block is 8 m long on every side? What is its area? What units do we use?

Then draw several rectangles on the board, write the length and the width using different units of measurement, and ask volunteers to find the area.

EXAMPLES:
8 cm × 9 cm   3 m × 7 m   6 km × 10 km
20 cm × 7 cm   8 m × 7 m

ASK: Which rectangle has the greatest area? Which rectangle has the smallest area?

ASK: Which units would you use to measure the area of these objects or places—square centimetres, square metres, or square kilometres:
- Canada
- your classroom
- a book
- your city or town
- school yard
- a field
- a table

Draw another rectangle on the board and mark the length: 3 m. SAY: I know that the area of the rectangle is 6 m². How can I calculate the width of the rectangle? (PROMPTS: If you knew the length and the width, how would you calculate the area? What do you have to multiply 3 (the length) by to get 6 (the area)? How do you know? What did you do to 6 to get 2?)

Give several more problems of this kind. EXAMPLES:
- Length 4 cm, area 20 cm², find the width.
- Width 3 m, area 27 m², find the length.
- A square has area 16 km². What is its width? (What can you say about the length and the width of a square? Students can try various lengths—1 × 1, 2 × 2, and so on—until they find the answer.)

Ask students to draw a rectangle that has an area of 24 cm². How many rectangles of different proportions can they draw? Prompt them to start with a width of 1 cm, then try 2 cm, 3 cm, and so on. Does 5 cm work? Why not? What about 7 cm?
Assessment
1. What is the area of the rectangles?
   A: Length 5 m, width 4 m.
   B: Length 6 km, width 7 km.
   C: Length 20 cm, width 15 cm.

   Order the rectangles from least to greatest.
2. A city square has area 800 ____. Its length is 40 ____. Fill in the appropriate units of measurement and calculate the width of the square.
3. Draw 3 different rectangles that have area 30 cm².

Extensions
1. Find the area of the shape by dividing it into smaller rectangles.
   HINT: First you will have to fill in the missing side lengths.

   ![Diagram]

   2. A rectangle has perimeter 12 cm and length 4 cm. What is its area?
### ME5-26

#### Area of Polygons

**GOALS**

Students will find the area of irregular polygons made up of squares, rectangles, and triangles.

**PRIOR KNOWLEDGE REQUIRED**

Area

Adding sequences of halves and whole numbers

Comparing mixed fractions with the denominator 2

**VOCABULARY**

area

congruent

triangle

rectangle

---

Draw a 1 × 2 rectangle on a grid on the board. Draw a diagonal and shade one of the two triangles formed. **ASK:** Which part of the rectangle is shaded? How do you know? What are triangles like the halves of this rectangle called? (congruent triangles) What is the area of the rectangle? What is the area of the shaded triangle? Repeat with rectangles 1 × 4, 1 × 6, then 1 × 3 and 2 × 3.

Draw several right-angled trapezoids and ask your students to divide them—by drawing a line—into a rectangle and a triangle. Then make right-angled trapezoids on a grid or geoboard and ask students to find the area of the trapezoids. Challenge them to find the area of a general, not-right-angled and not isosceles trapezoid, such as:

![Diagram of a trapezoid divided into a rectangle and two triangles.](image)

Make sure your students can add sequences of halves by grouping before you assign the worksheet:

**EXAMPLE:**

\[
\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 3 \frac{1}{2}
\]

1 whole 1 whole 1 whole 3 wholes and a half left over

Also, make sure students understand that in order to find the area of the triangular parts of the figures on the worksheet, they simply have to view the triangle as covering half the area of a rectangle:

![Diagram of a triangle covering half a rectangle.](image)

The triangle covers half the area of a 2 by 3 rectangle.

The area of the rectangle is 2 × 3 = 6. So the area of the triangle is 3 (half of 6).

Ask your students to find a pair of congruent triangles in the picture below:

![Diagram of two congruent triangles.](image)

What is the area of the shaded triangle then?
Students might argue the following way:

1. Triangles A and C are congruent, so the shaded area is the same as the area of triangles A and B together, which is one half of a square.

2. Triangles B, C, and D form a triangle of area one square. Triangle D has area of one half of the square, so B and C together have area of half the square.

Let your students find the area of larger shapes containing triangles congruent to the shaded triangles, such as:

![Image of a geometric shape]

**Assessment**

Find the area of the shapes:

![Images of various irregular shapes]

Let your students create various irregular polygons on geoboards and find the area of the shapes they produce.
**ME5-27**

**Area of Irregular Shapes and Polygons**

Draw several squares on the board. Invite volunteers to shade half of each square. Encourage them to find as many different ways of doing this as possible, including:

![Shaded squares](image)

Draw several shapes that include that include various types of half squares and ask students to find the area of each shape. Then ask students to draw designs for each other, and to find and to compare the shaded and unshaded areas of their designs.

Draw a rectangle on a grid. Ask volunteers to shade half of each square in the rectangle using any design they like. **ASK:** What is the shaded area of the rectangle? Do you have to count all the half squares or there is a shortcut? If you know the total number of squares, what part of the total is shaded?

Draw a map of a lake on a grid (or use an overhead projector). Ask your students to find the area of the lake. **ASK:** Which squares contribute a whole 1 km² to the area? Which squares contribute about half a square kilometre? Which squares should not be counted at all because they contribute only a small fraction of a kilometre to the area?

**Bonus**

Have students draw their names using whole squares and different half squares, and calculate the area.

**Extensions**

The next series of extensions fulfils the demands of the Atlantic Curriculum (Expectation E13).

1. If you know the length and height of a parallelogram, how can you find its area?

![Parallelogram](image)
ANSWER: Cut off a triangle at one end and attach it to the other end to form a rectangle.

Area = 4 \times 6 = 24 \text{ cm}^2

2. Use the same method as in the previous question to find the area of an isosceles trapezoid.

3. How can you cut off and rearrange two triangles to find the area of an irregular trapezoid?

4. Jennifer wants to find the area of a triangle:

She cuts through the midpoints of two sides of the triangle and rearranges the pieces:

Then she uses the method of Extension 1 to find the area. What is the area of the triangle?

5. Alex wants to cut a triangle with sides 4 cm, 5 cm and 6 cm once and to rearrange the pieces (slide, rotate or flip them) so that he obtains a quadrilateral or a different triangle. He cuts the triangle as shown below. Does this cut work? Explain why not. Draw at least four different cuts that would allow Alex to rearrange the pieces and to obtain a quadrilateral or a triangle. Copy the triangle onto tracing paper and check your solution by cutting and rearranging the pieces. How many different quadrilaterals or triangles can you produce?
ME5-28
More Area and Perimeter

Draw an irregular shape on a grid and tell students that you want to estimate the area of the shape. Enlarge some of the grid squares that are divided by the shape in unusual or irregular ways, such as:

Discuss with students how they can estimate the perimeter of the lines in these grid squares. In the two leftmost examples above, students can estimate the length of the line as 1 because it is close to being a straight line parallel to one side of the square. In the two examples in the middle, the line is about 1.5 units long. (ASK: Why? What is the length of the diagonal if the square is 1 cm by 1 cm?) If you straightened out the lines in the squares at right, they look like the might be about half the length of one side, so you could estimate their length as half. After discussing these and other estimates, ask students to estimate the perimeter of the shape you have drawn. Repeat with more irregular shapes.

Draw 3 squares on the grid: 1 × 1, 2 × 2, and 3 × 3. Ask your students to find the area and the perimeter for all 3 squares. Invite a volunteer to summarize the properties of the squares in a T-table:

<table>
<thead>
<tr>
<th>Side Length</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>9</td>
</tr>
</tbody>
</table>

Invite students to look for patterns in the numbers. ASK: Do the numbers in the second column vary directly with the numbers in the first column? How? (Perimeter is 4 times side length.) Do the numbers in the third column vary directly with the numbers in the first column? (No) How do you get the area from the length? (Area is side length squared.) What will be the perimeter and the area for the square with side length 4? 6? 8? Extend the T-table to include these quantities.

Present this problem: Paul says that if he doubles the side of a square, both the area and the perimeter double as well. Is that correct? Which part of Paul’s statement is correct? How would you change the incorrect part to make it correct? (The perimeter doubles, the area is multiplied by 4.)
Complete a similar T-table for a set of rectangles: $2 \times 3$, $4 \times 6$, $8 \times 12$. **ASK:** Bill multiplied both the length and the width of a rectangle by 3. How many times larger is the perimeter of the new rectangle? How many times larger is the area of the new rectangle? Let your students draw rectangles of various proportions to check their predictions.

Review the corrected version of Paul’s rule (above): When the sides of a square or rectangle are doubled, the perimeter doubles and the area is multiplied by 4. **ASK:** Is Paul’s rule a general rule or is it good only for squares and rectangles? Invite students to draw a simple shape (like a letter of the alphabet) on grid paper and to enlarge all the dimensions by 2. Ask your students to find the area and the perimeter of both shapes and to check if the rule holds.

**Assessment**

Rocco draws two shapes on grid paper. One of his shapes is 4 times longer and 4 times wider than the other. He found that the perimeter of the larger shape is 6 times larger than the perimeter of the smaller shape, and that the area is 16 times larger. Is he correct? What did he calculate correctly: area, perimeter, or both?
ME5-29
Comparing Area and Perimeter

Draw several rectangles on a grid: $4 \times 6, 5 \times 5, 6 \times 3, 7 \times 2, 3 \times 8$. Label them and ask volunteers to find the area and the perimeter of each one. Ask students to list the rectangles from least to greatest by area. Then ask them to list the rectangles from least to greatest by perimeter.

**ASK:** Are your lists the same? Does the rectangle with the greatest area also have the greatest perimeter? Does the rectangle with the smallest perimeter also have the smallest area? Are there rectangles with the same area? Do they have the same perimeter? Are there rectangles that have the same perimeter? Do they also have the same area?

What do you do to the length and width to calculate area? What do you do to the length and width to calculate perimeter?

**Assessment**
1. Draw 2 rectangles that have the same area—20 cm$^2$—but different perimeters. Calculate their perimeters.
2. Bob drew 2 shapes with the same perimeter but different areas. Is this possible? The sides of Bob’s shapes are whole centimetres. One shape is a square with area 9 cm$^2$. Can you draw this square? The other shape is a rectangle. Can you draw the rectangle?

**Extensions**
1. Mr. Green wants to make a rectangular flower bed with perimeter 24 m. Which dimensions of the flower bed will provide the greatest area?

2. Mr. Brown wants to make a rectangular flower bed with area 36 m$^2$. Which dimensions will give him the least perimeter?
ME5-30
Area and Perimeter

**GOALS**
Students will estimate and find the area and perimeter of rectangles. Students will find the length of a rectangle given the area or perimeter and the width.

**PRIOR KNOWLEDGE REQUIRED**
Area of rectangle
Perimeter

**VOCABULARY**
area
rectangle
width
perimeter
length

Give students several rectangles with lengths and widths that are whole centimetres and say you want to estimate both the area and the perimeter of each one. **ASK:** How can I do that? What can I use to help me? Invite students to share their suggestions and try them out. If necessary, remind students that the area of their thumbnail is approximately 1 cm². How can they use their thumbnail to estimate area and perimeter?

Have students check their estimates by measuring the length and width of the various rectangles and calculating the area and perimeter. Compare the estimates to the actual measurements.

Then tell students you want to find all the rectangles with perimeter 14 cm. The only other requirement is that the lengths and widths are whole centimetres (i.e. 3 or 5, not 2.25 or 4 \( \frac{3}{4} \)). **SAY:** I'm going to start with a rectangle that has width 1 cm. That's the smallest possible measurement I can have. Draw a rectangle and mark the width. **ASK:** If the width of the rectangle is 1 cm, what is the length? Invite the student who answers to explain how he or she came up with the answer. If students need assistance use these **PROMPTS:** The perimeter is the distance all around the shape—if you took a rope that was 14 cm long, it would go all around the sides of our rectangle. If we know the width is 1 cm, how much of the rope have we used? (2 cm, for 2 sides) How much of the rope is left? (12 cm) For how many sides? (2) So how long is each remaining side? (6 cm) Another way to solve the problem is to use the fact that 1 length and 1 width of the rectangle add up to half the perimeter, or 7. If 1 + length = 7, the length must be 6.

Now draw a rectangle with width 2 cm and calculate the length. Repeat for a rectangle with width 3 cm. **ASK:** What is the length of the last rectangle? Do we need to continue making rectangles? Why not? (The last rectangle was 3 \( \times \) 4. A rectangle with width 4 would have length 3—it would be the same rectangle, just rotated or turned on its side!) Do we need to make rectangles with width 5 or 6? What about a rectangle with width 7? How many rectangles with perimeter 14 do we have in total? Ask students to find the area of the rectangles.

**Assessment**
Draw all possible rectangles with perimeter 16 cm. Which one has the greatest area? The least area?

Students could try **QUESTIONS 3 and 4** on the worksheet using a geoboard.
Extensions

1. Describe a situation in which you would have to measure area or perimeter, for instance, to cover a bulletin board or make a border for a picture. Make up a problem based on the situation.
2. A rectangle has area 20 cm$^2$ and length 5 cm. What is its perimeter?
3. The shape has perimeter 24 cm. What is its area?

ME5-31
Problems and Puzzles and
ME5-32
More Area and Perimeter

These are review worksheets that can be supplemented by Activities.

ACTIVITY 1

Students could investigate QUESTION 2 on worksheet ME5-31 using a geoboard.

ACTIVITY 2

Ask students to make as many non-congruent shapes as they can by placing 5 squares edge to edge. (These shapes are called pentaminoes: there are 12 different pentaminoes altogether.) Which pentaminoes have the greatest perimeter? Which have the greatest number of lines of symmetry?

Extension

Find the perimeter and area of the shape.
Use concrete props to review the concepts of length and area, and to introduce the concept of volume. Start with a piece of string. It has only 1 dimension—length. **ASK:** What units do we measure length in? (cm, m, km) Point to the top of a desk or table and explain that it has length and width—that’s 2 dimensions. Anything two-dimensional has area. The area of the desk or table is its surface, the space bound by, or “inside,” the length and width. **ASK:** What units do we measure area in? (cm², m², km²) Now explain that a three-dimensional object, like a cupboard or a box, has length, width, and height. The space taken up by the box or cupboard or any three-dimensional object is called volume, and we can measure it, too. One of the units we use for volume is cubic centimetres, or cm³.

Draw a cube on the board and mark the length of all the sides as 1 cm. Tell students that this is a centimetre cube. It has length, height, and width 1 cm, and its volume is exactly 1 cm³. Tell students that they will be using centimetre cubes to calculate the volume of various shapes.

Ask students what they think larger volumes are measured in. (m³, km³) Explain that these units are very large. For example, to fill an aquarium that has a volume of 1 m³, you would need about 100 pails of water!

Give students some centimeter cubes and ask them to build figures that have volume 4 cm³. Then ask them to build figures with volume 6 cm³. Assist any students who aren’t sure how many cubes to use for each figure, or who use the wrong number. Finally, invite students to build different figures with volume up to 8 cm³. They can work in pairs—one student makes a figure and the other calculates its volume.

Explain that for complicated figures, we can use a top view to calculate the volume. Make or draw the following 3-D figure:

![3D Figure]

Tell students to pretend that they are looking down on the figure from above. How many blocks would they see? (3) Draw 3 squares—the base, or bottom layer, of the figure—and invite volunteers to help you write the number of blocks stacked in each position:

```
3 2 1
```
Explain that this is the mat plan of the figure. You can make or draw a few more figures (like the one below) and invite volunteers to help you draw the mat plans. Start by drawing the shape of the figure's bottom layer, then count the cubes in each position.

Then do the opposite: start with the mat plan and build the figure. Show students this mat plan:

Ask one volunteer to build the bottom layer of the figure. Ask more volunteers to come and add cubes to the stacks that need more than 1 cube.

Then **ASK:** How can we calculate the volume of the figure? Do we need the figure or can we do it from the mat plan? (We can do it from the mat plan. Each cube has volume 1 cm\(^3\), so the total number of cubes gives you the total volume of the figure.)

Ask students to work in pairs. One partner builds a figure with volume no more than 12 cubes and height no more than 4 cubes and the other partner draws the top view for the figure. Partners should swap roles, so that each has a chance to do both the drawing and the building. After everyone has had a chance to do both at least once, have pairs do the reverse: one partner draws the top view for a figure and the other builds the figure accordingly.

**Assessment**

1. Build the figure according to the mat plan:

2. Fill in the mat plan of this figure.

3. Find the volume of both figures above.
Extensions

1. Draw the mat plan for this figure. How many cubes are “hidden” in the picture?

2. Give students the front, top, and side views of a structure and ask them to build it using interlocking cubes. As they work, encourage them to look at their structure from all sides, and to compare what they see to the pictures. Ask them to determine the volume of the completed structure.

   a) 
   
   Front 
   Top 
   Side 
   Possible Answer

   b) 
   
   Front 
   Top 
   Side 
   Possible Answer

   NOTE: You might want to give students the front, top, and side views of a few simpler shapes before they tackle the ones above.

Bonus

Give students the front, top, and side view of a simple figure and ask them to calculate the volume without building the figure.
3. a) A structure has this front and side view and a volume of 6 cm³. Build the structure.

b) Suppose the structure in a) had volume 5 cm³. Can you build 2 structures consistent with the top and side views?

4. Build 3 structures with this front and side view.

**ME5-34**

**Volume of Rectangular Prisms**

Draw a rectangle on a grid (or subdivide a rectangle into equal squares). Ask students to write the addition and multiplication statements needed to calculate the area of this rectangle.

Show students a large rectangular box. Explain that you want to know the volume of the box. (You can say that you want to send something to someone, and the shipping company charges by volume. You need to know the volume to figure how much it will cost to ship this box.) You can fill the box with centimeter cubes, but that is not very practical. Tell students that today they will learn another method for calculating volume.

First, tell students that mathematicians have a fancy name for a rectangular box. They call it a “rectangular prism.” Write the term on the board.

Build a 2 × 4 rectangle using centimetre cubes and **ASK:** How many cubes are in this rectangular prism? What is the volume of this prism? Ask students to explain how they calculated the volume—did they count the cubes one by one or did they count them another way? Because each cube has volume 1 cm³, the total number of cubes gives you the total volume. You can use addition or multiplication to count the total number of cubes:

\[
2 + 2 + 2 + 2 = 8 \text{ cm}^3 \quad (2 \text{ centimetre cubes in each of 4 columns})
\]

\[
2 \times 4 = 15 \text{ cm}^3 \quad (2 \text{ rows of 4 centimetre cubes} \text{ OR length } \times \text{ width})
\]
Add 1 layer to the prism so that it is 2 cubes high. **ASK:** What is the volume of the new prism? Prompt students to use the fact that the prism has 2 horizontal layers. Remind them that they already know the volume of 1 layer. Ask volunteers to write addition and multiplication statements for the volume of the new prism using layers.

\[
8 + 8 = 16 \text{ cm}^3 \\
2 \times 8 = 16 \text{ cm}^3
\]

Add a third layer to the prism and repeat. Invite students to look at this last prism and to calculate the volume by adding vertical layers, or “walls,” instead of horizontal layers.

**ASK:** How many cubes are in the wall at the end of the prism? \((3 \times 2 = 6)\) What is the volume of the wall? \((6)\) How many walls are in the prism? \((4)\) Invite volunteers to write the addition and the multiplication statements for the volume of the prism using the volume of the “wall.” Does this method produce a different result than the previous method? \((\text{no, it’s the same answer})\)

Explain that the third dimension in 3-D figures is called height. Identify the length, width, and height in the prism above. Then use the terms length, width, and height to label the multiplication statement that gives the volume:

\[
3 \times 2 \times 4 = 24 \text{ cm}^3
\]

Draw several prisms on the board, mark the height, width, and length (you can use different units for different prisms), and ask students to find the volume. **SAMPLE PROBLEMS:**

- \(10 \text{ cm} \times 6 \text{ cm} \times 2 \text{ cm}\)
- \(2 \text{ m} \times 3 \text{ m} \times 5 \text{ m}\)
- \(3 \text{ km} \times 4 \text{ km} \times 7 \text{ km}\)

Remind students to include the right units in their answers.

**ASK:** What does the top view for a rectangular prism look like? Is there any difference in heights above each square? \((\text{No, the height of a rectangular prism is the same everywhere.})\) Draw several top views on the board and ask students to identify which are rectangular prisms and which are not. Then ask them to write the length, width, and height for each prism, and to calculate the volume.

**SAMPLE PROBLEMS:**

\[
\begin{array}{ccc}
5 & 5 & 6 \\
5 & 5 & 5
\end{array} \quad \begin{array}{ccc}
7 & 7 & 7 \\
7 & 7 & 7
\end{array} \quad \begin{array}{ccc}
5 & 5 & 5 \\
5 & 5 & 5 \\
5 & 5 & 5 \\
5 & 5 & 5
\end{array} \quad \begin{array}{ccc}
5 & 5 & 5 \\
5 & 5 & 5
\end{array}
\]

**Bonus**

Find the volumes of the figures that aren’t rectangular prisms.
Assessment
Find the volume of the prisms:

a) \[ \begin{array}{|c|c|c|}
\hline
6 & 6 & 6 \\
\hline
6 & 6 & 6 \\
\hline
\end{array} \]

b) \( 4 \text{ m} \times 5 \text{ m} \times 6 \text{ m} \)

c) \[
\begin{array}{|c|c|c|}
\hline
& & \\
\hline
& & \\
\hline
\end{array}
\]

Extensions

1. Find the volume of the shape:
\[
\begin{array}{|c|c|c|}
\hline
5 \text{ cm} & 3 \text{ cm} & 3 \text{ cm} \\
\hline
3 \text{ cm} \\
\hline
\end{array}
\]

2. Find the volume of the prisms:

- \( 1 \text{ m} \times 1 \text{ km} \times 1 \text{ m} \)
- \( 5 \text{ cm} \times 3 \text{ dm} \times 2 \text{ m} \)
- \( 1 \text{ mm} \times 1 \text{ m} \times 1 \text{ km} \)
ME5-35
Mass

NOTE: Mass is a measure of how much substance, or matter, is in a thing. Mass is measured in grams and kilograms. A more commonly used word for mass is weight: elevators list the maximum weight they can carry, package list the weight of their contents, and scales measure your weight. The word weight however, has another very different meaning. To a scientist, weight is a measure of the force of gravity on an object. An object’s mass is the same everywhere—on Earth, on the Moon, in space—but it’s weight changes according to the force of gravity. When we use the term weight in this and subsequent lessons, we use it as a synonym for mass.

Remind students that mass (which we often call weight) is measured in grams (g) and kilograms (kg). Give several examples of things that weigh about 1 gram or about 1 kilogram:

1 g: a paper clip, a dime, a chocolate chip
1 kg: 1L bottle of water, a bag of 200 nickels, a squirrel.

List several objects on the board and ask students to say which unit of measurement is most appropriate for each one—grams or kilograms:

- A whale
- A table
- A napkin
- A cup of tea
- A workbook
- A minivan

Ask students to match these masses to the objects above:

2 000 kg 50 000 kg 10 g 150 g 400 g 10kg

Have students order these objects from heaviest to lightest.

Ask students to think of 3 other objects that they would weigh in grams and 3 objects that would demand kilograms.

Draw a set of scales on the board. Draw various weights on one scale (that is on one side) and ask volunteers to add weights to the other so that the scales are in balance. Then draw weights on both scales, but leave out the numbers on one side and have students fill them in. Include instances where students have to add and divide to find the right weights. SAMPLE PROBLEMS:
Work through the following set of problems as a class. Invite and encourage as many different students as possible to participate by contributing answers, explanations, and suggestions. Break some of the more complicated problems into smaller steps. Use prompts and questions to help students identify what they know and how they can use what they know to solve the problem.

You could keep a list of facts on the board and add to it as you go. For example, problem a) gives the weight of a rabbit (3 kg) and asks you to calculate the weight of a lazy cat (6 kg). Both of these facts could go on the list: Rabbit = 3 kg, Lazy Cat = 2 × Rabbit = 6 kg.

a) A rabbit weighs 3 kg. A lazy cat weighs twice as much as the rabbit. How much does the lazy cat weigh?

b) An angry dog weighs as much as 3 lazy cats. How many rabbits does this dog weigh?

c) What is the dog’s weight in kilograms?

d) An adult raccoon weighs as much as an angry dog and a rabbit. How many rabbits does the raccoon weigh? What is the raccoon’s weight in kilograms?

e) A beaver weighs 3 500 grams less than an adult raccoon. How much does the beaver weigh?

f) A female bear weighs 90 kg. How many rabbits does it weigh? How many lazy cats does it weigh?

How many dogs?

g) How many lazy cats do you need to balance 2 angry dogs and 2 rabbits on a scale?

h) A newborn Siberian tiger weighs about 1 kg. It gains 700 g a week. How much weight does it gain in 4 weeks? How much does the tiger weigh after 4 weeks?

i) A 20-week old Siberian tiger cub is on one scale and an angry dog is on the other. What do you need to do to balance the scales? (Ask a volunteer to draw a model for this problem first.)

ACTIVITY 1

Let students feel the weight of objects that are close to 1 g (EXAMPLE: paperclip) or 1 kg (EXAMPLE: a 1 L bottle of water). They can use these referents to estimate the weight of the objects in the classroom, such as books, erasers, binders, games and calculators. (You could ask students to order the objects from lightest to heaviest.) Students should use scales to weight the objects and check their estimates.
ACTIVITY 2
Weigh an empty container, then weigh the container again with some water in it. Subtract 2 masses to find the mass of the water. Repeat this with a different container but the same amount of water. Point out to students that the mass of a substance doesn’t change even if its shape does.

ACTIVITY 3
Weigh a measuring cup. Pour 10 mL of water in the cup. Calculate the mass of the water by subtracting the weight of the cup from the weight of the water with the cup. How much does 10 mL of water weigh? (How much does 1 mL of water weigh?)

Extensions
1. Jane wants to estimate the mass of one grain of rice. She weighs 100 grains of rice and divides the total by 100. Try to weigh 1 grain of rice. Explain why Jane uses this method above. Use Jane’s method to estimate the mass of a bean or a lentil.

2. When Duncan stands on a scale the arrow points to 45 kg. When he stands on the scale and holds his cat, the arrow points to 50 kg. How could he use these two measurements to find the weight of his cat?
GOALS

Students will determine the capacity of various containers. Students will convert millilitres to litres and vice versa.

PRIOR KNOWLEDGE REQUIRED

Litres (L)
Millilitres (mL)

VOCABULARY

capacity
litre
millilitre

Explain that the capacity of a container is how much it can hold. Write the term on the board. Explain that capacity is measured in litres (L) and millilitres (mL). ASK: Where have you seen the prefix “milli” before and what did it mean? (millimetre; one thousandth) How many millilitres are in 1 litre? In 2 litres? In 7 litres? What do you do to change litres to millilitres? (Multiply by 1 000.) Write on the board:

1 metre = 1 000 millimetres
1 m = 1 000 mm
1 litre = 1 000 millilitres
1 L = 1 000 mL

Put out several containers (EXAMPLES: milk and juice boxes, medicine bottles, measuring cups, cans of paint, cans of pop) with capacities clearly marked on them. Invite students to help you separate the containers into 2 groups: those that can hold 1 or more litres and those that can hold less than 1 litre. Then ask students to help you order the containers by capacity, from least to greatest. The containers can also act as “capacity benchmarks” that you can keep in a class measurement box.

Write the following on the board and ask students whether they would measure the capacity of each container in millilitres or litres:

• a glass of juice
• a bowl of soup
• a pail of water
• a pot of soup
• an aquarium
• a backyard pool

Ask students to think of three more quantities that are measured in litres and three that are measured in millilitres.

Show students a 1 L carton of milk or juice and a small (200 mL) glass. ASK: How can we determine the capacity of the glass? You may ask a volunteer to check how many glasses can be filled from the container. What is the capacity of the glass? Now bring out a larger glass (250 mL). How many glasses of this size can be filled from the carton? Ask a volunteer to check. (NOTE: You will need a large bowl or pot in which to empty out the glasses as they are filled.)

Assessment

a) An aquarium holds two 8 L pails of water. What is its capacity? Write the capacity in litres and millilitres.

b) A jar holds 500 mL of water. How many jars do you need to fill the aquarium?
Measure the capacity of several glasses or containers in your classroom. Estimate the capacity of the containers before you measure their capacity. Students should select and justify appropriate units to measure the capacity of a container. **NOTE:** Students will need a measuring cup and several containers for this activity.

**ACTIVITY 2**

Make two containers: 500 mL and 300 mL (you can use two empty 1 L cartons and cut them at the height of 10 cm and 6 cm respectively). Ask your students to use only these two containers, a tap, and a sink to measure 200 mL of water.

**CHALLENGING:** Measure 400 mL of water using only the same equipment.

**ACTIVITY 3**

Bring in some items from a grocery store. Ask students to look at the labels and sort the items into 2 groups: items where the amount of food is given as mass and items where the amount is given as capacity.

**ACTIVITY 4**

Collect 5 containers (cups, cans, bottles, pails) of different sizes on which capacities were covered or removed.

a) Estimate the capacity of each container.

b) Measure and record the capacity of each container.

c) Order the measurements from greatest to least.

d) Compare your measurements with your estimates.

**Extensions**

1. **JELLYBEAN JARS:** Choose two straight-sided jars of similar size but different dimensions, for example, a tall, thin olive jar and a short, squat salsa jar. Fill both jars with jelly beans. Show students both jars and **ASK:** Which jar has a larger capacity? Why do you think that? How could we check our predictions? Discuss the students’ ideas for determining the answer, then choose one or more and try it! Here are some of the approaches you could investigate:

   a) Open the jars, dump out the contents, and count the jelly beans.

   b) Open the jars, dump out the contents of one jar, and pour the jelly beans from the other into it. Do the jelly beans fill the empty jar? Is there any empty space left over? Are there any jelly beans left in the first jar?

   c) Have a student count the number of jelly beans visible through the bottom of the jar, and record the number. Have another student count the number of layers of jelly beans from the bottom to the top. Have students multiply the two numbers to estimate the total number of beans in each jar.
Fill a large glass container with enough water to submerge either jar. Ask your students to predict what will happen if you place the jar into the bowl. Why does the water level go up? Close both jars tightly. Submerge one jar in the water and mark the new water level with a piece of tape. Remove the first jar and point out what has happened to the water level. Place the second jar in the water and ask students to determine whether the water level is higher or lower than with the first jar. What does this tell us about the size of the jars?

2. **PROJECT:** The origins of SI and its usage throughout the world: When SI was introduced, which countries use it and which do not? Which countries are converting into SI? What are the advantages of SI?

**POSSIBLE SOURCE:**
http://lamar.colostate.edu/~hillger/#metric

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**ME5-37**

**Volume and Capacity**

**GOALS**

Students will use capacity to find volume and vice versa.

**PRIOR KNOWLEDGE REQUIRED**

Volume
Capacity
Volume of rectangular prisms
Converting between litres and millilitres

**VOCABULARY**

volume
capacity
mL
cm³

Hold up a see-through measuring cup with some water in it. Drop a centicube into the cup. Ask your students if they can see how much liquid is displaced by the cube. If they have trouble seeing how much water is displaced by 1 cube, which is likely to happen unless your cup is very small, ask them how they would solve this problem. (One answer: They could drop 10 cubes in and divide the displacement by 10.) Invite volunteers to drop more centicubes into the cup and to measure the displacement. What is the capacity of 1 cm³ cube? (1 mL)

Present a small rectangular box and ask your students how they could measure its capacity. We know the capacity of 1 cm³. What is the capacity of 10 cm³? Of 20 cm³? Invite volunteers to measure the sides of the box and calculate its volume. What is the capacity of the box?

**ASK:** A cube has a capacity of 1 L. What are the dimensions of the cube? How many mL are in 1 L? How do you find the volume of the cube? (You multiply the side by itself 3 times.) Which number is multiplied by itself 3 times to get 1 000? So how long is the side of the cube? (10 cm). Do you know a specific term for this length? Write on the board: Capacity of 1 dm³ = 1 L.

Draw a box on the board and write its dimensions: 30 cm × 40 cm × 50 cm. **ASK:** What is the capacity of the box? Let your students find the capacity in mL first, then ask them to convert it to L. Ask your students if they can solve the problem another way. (They can convert the dimensions to decimetres and get the result in litres: 3 × 4 × 5 = 60 L.)
Assessment
A rectangular box has base 7 cm × 7 cm. It contains .5 L of milk. About how high is the box in centimetres? What is its exact height in millimetres?

Let your students find the volume of small objects like toys, coins, or apples by submerging them in water and measuring the displacement of the water, then converting the capacity of the water displaced to a volume.

Extensions
1. Discuss the story of Archimedes and the Golden Wreath.
   POSSIBLE SOURCE:
   http://math.nyu.edu/~crorres/Archimedes/Crown/CrownIntro.html

2. Daniela wants to find the volume of an apple. She puts the apple into a glass box with 600 mL of water. The box has a square base of 10 × 10 cm. The water reaches a height of 9.8 cm. What is the volume of the apple?
NOTE: Make sure your students understand that you need more of a smaller unit to fill the same amount of space as a larger unit. That’s why, in changing from a larger to a smaller unit, you multiply by a multiple of 10. The multiple depends on which unit you are in and which unit you are changing into. For instance, there are 10 mm in each centimetre, so to change .7 cm to millimetres you multiply by 10 (which, as demonstrated in section NS5-93, shifts the decimal one place right). When you change from a smaller unit to a larger unit you need fewer of the larger unit, so you divide.

Review the material in Lesson ME5-15 with the students. Review the dollar and cent notations, and conversion between units of time (e.g., 1 hour = 60 minutes). Give students a lot of problems requiring conversion between various units of measurement. With every problem, ASK: Is the new unit larger or smaller than the old unit? Do we need more units or less? Are we going to multiply or divide? How many smaller units fit into the larger unit? By how much are we going to multiply or divide?
ME5 Part 2: BLM List

Dot Paper ___________________________________________ 2
Grid Paper (1 cm) ___________________________________ 3
Pentamino Pieces _____________________________________ 4
Grid Paper (1 cm)
Pentamino Pieces

[Diagrams of various pentamino pieces]
GOALS
Students will find the mean of number sets.

PRIOR KNOWLEDGE REQUIRED
Addition
Subtraction
Multiplication
Division

VOCABULARY
mean

SAY: Suppose 5 people picked apples and wanted to share their apples so that everyone had the same number. They might pass apples to each other until everyone had the same number or they might put all of the apples in a bin and hand them out one at a time. Either way, mathematicians would say that the people took the mean, or average, of the number of apples picked.

Suppose that Andrew picked 6 apples, Brian picked 2 apples, Cindy picked 4 apples, Dara picked 1 apple and Erin picked 2 apples. In total, the 5 friends picked 15 apples. Lay out or draw 15 blocks (to represent the 15 apples) in 5 groups (to represent the 5 people):

Ask a volunteer to move blocks from one group to another until all the groups are equal in number. How many blocks are now in each group? (3) The mean of the set of numbers 6, 2, 4, 1, 2, is 3.

Demonstrate another way of finding the mean using the same 15 blocks. Draw a circle for each of the 5 friends, put all the blocks in a pile, and ask a volunteer to distribute the blocks evenly into the 5 circles. Find the mean of a few more sets using either method above. (EXAMPLES: 8, 5, 2; 7, 2, 5, 2; 1, 3, 9, 3)

ASK: How can we find the mean of a set of numbers without using pictures or blocks? How many groups did we divide the blocks into? How did we find that number from the set of numbers? (The number of groups is the number of data values.) How many blocks do we have altogether? How can we find that total from the data values? (Add them together.) How can we find the number of blocks in each group if we know the total number of blocks and the number of groups? (Divide.) Demonstrate with some of the data sets used in the lesson. For EXAMPLE:

\[
\text{Mean} = \frac{\text{Total}}{\text{Number of Groups}} = \frac{6 + 2 + 4 + 1 + 2}{5} = \frac{15}{5} = 3
\]

Write several sets of numbers on the board and have students practise finding the mean without using pictures.

EXAMPLES: 4, 7, 2, 1, 1; 10, 12, 3, 7
Bonus
0, 8, 15, 16, 16, 19, 24

Invite volunteers to share their answers. Occasionally use the term data values when referring to the numbers so that students don’t think of them only as blocks.

Extensions

1. Find the mean. Write your answer as a mixed fraction.
   
   a) 3, 4, 6, 8       b) 4, 7, 8       c) 0, 2, 3, 6, 7
   
   \[ \text{sum of data values} = 3 + 4 + 6 + 8 = 21 \]
   \[ \text{mean} = \frac{21}{4} = 5 \frac{1}{4} \]
   
   number of data values

2. a) The data set 1, 4, 10 has mean \(15 \div 3 = 5\). Add the following data values to this set and decide if the mean increased, decreased, or stayed the same.
   
   i) New data value: 3
   
   New mean: \(18 \div 4 = 4 \frac{1}{2}\)
   
   The mean decreased.

   ii) New data value: 7
   
   New mean: ________
   
   The mean ________

   iii) New data value: 5
   
   New mean: ________
   
   The mean ________

   b) Find the mean of each set, then add a data value so that the mean stays the same.
   
   HINT: If your first guess increases the mean, try a lower value; if your first guess decreases the mean, try a higher value.

   i) 2, 6
   
   Mean: ________
   
   Added value: ________

   ii) 2, 6, 7
   
   Mean: ________
   
   Added value: ________

   iii) 3, 4, 5, 12
   
   Mean: ________
   
   Added value: ________

   c) Look at your answers to part b). How can you know what data value to add if you want to keep the mean the same?

   d) Find the mean.

   i) 1, 4, 10
   
   ii) 1, 4, 5, 10
   
   iii) 1, 4, 5, 5, 10
   
   iv) 1, 4, 5, 5, 5, 10
Finding the Mean

GOALS
Students will understand that the differences between the mean and the values below the mean balance out the differences between the mean and the values above the mean.

PRIOR KNOWLEDGE REQUIRED
Addition
Subtraction
Multiplication
Division
The mean as the sum of the data values divided by the number of data values.

VOCABULARY
sum
data values
mean

Ask students to find the mean of the data set 2, 7, 6 in two different ways:

Using division:

\[
\text{mean} = \frac{\text{sum of data values}}{\text{number of data values}}
\]

\[
= \frac{15}{3}
\]

Using a picture:

\[
\begin{array}{ccc}
2 & 7 & 6 \\
\end{array}
\]

Add a horizontal line in the picture to show where the mean is. ASK: When you moved some blocks to other piles, which piles did you take the blocks from—the piles with more blocks than the mean or fewer blocks than the mean? Which piles did you move them to—the piles with more blocks than the mean or the piles with fewer blocks than the mean? ASK: Why does that make sense? Explain to the students that if all the piles are going to have the same number of blocks, they need to add blocks to the piles that have less than the mean and take blocks away from the piles that have more than the mean. ASK: Is the number of blocks you took away from some piles equal to the number of blocks you added to other piles? How do you know? Did you change the total number of blocks by moving some of them?

Have students calculate the mean for the following set using division: 3, 0, 4, 6, 2. Then draw a picture for the set and add a horizontal line to show the mean:

\[
\begin{array}{cccccc}
3 & 0 & 4 & 6 & 2 \\
\end{array}
\]

ASK: How many blocks are above the horizontal line? How many spaces are below the line? If you moved the blocks above the mean to the spaces below the mean, how many would you have in each pile? (3, the mean)
Then draw a picture in which the horizontal line representing the mean is in the wrong place.

**EXAMPLE:**

![Diagram of a bar graph with a line representing mean]

**ASK:** Is the line in the right place—does it show the mean? How do you know? Right now, there are more spaces below the line than blocks above the line. How can we move the line so that there are fewer spaces below the line and more blocks above the line—should we move the line up or down? (down) How far? (one “row”) Have a volunteer do this and then ask the class: Is the line in the right place now? (yes) How do you know? (There are 5 blocks above the line and 5 spaces below the line, so moving the 5 blocks to the 5 spaces will make all the piles have the same number of blocks.)

Draw more pictures on the board. Have volunteers guess where the mean is by drawing a line, and then have students check the volunteer’s guess independently by counting blocks above the line and spaces below the line. Are these numbers the same? Adjust the line accordingly when necessary.

**Extensions**

1. The data set is 3, 4, 7, 8, 8.
   a) Find the mean.
   b) Is the data value 7 above or below the mean?
   c) Will removing 7 from the data set increase or decrease the mean? Find the mean of the data set 3, 4, 8, 8 to check your prediction.

2. For each data set below:
   a) Find the mean.
   b) Remove the circled data value and predict whether the mean will increase, decrease or stay the same.
   c) Check your answer by calculating the new mean.
      i) 0, 0, 1, 8, 8, 8, 9, 10
      ii) 4, 4, 4, 8
      iii) 1, 1, 2, 3, 8, 9, 20, 28

3. Sonia’s math test scores (out of 10) were: 2, 5, 5, 6, 7, 8, 9, 9, 10.
   a) What is the mean of her test scores? (68 ÷ 10 = 6.8)
   b) Sonia’s teacher decides not to count her lowest score. Predict how removing this data value will change her mean and explain your answer. (This change will increase her average because we are removing a data value that is lower than the average.)
   c) What is the new mean of Sonia’s test scores? Write your answer as a mixed fraction in lowest terms. (66 ÷ 9 = 7 3/9 = 7 1/3)
d) Sonia’s dad said he would buy her tickets to see her favourite rock group if she got an average of at least 7 on her math tests. Will Sonia get her tickets? (Yes.)

4. (Grade 5 Atlantic Curriculum)

a) Ten people work in an office. They get paid different salaries depending on what job they do.

Sales person: $25 000 per year
Secretary: $20 000 per year
Clerk: $17 500 per year

If there are 5 salespeople, 3 secretaries and 2 clerks, find the mean salary.

Predict whether the mean will increase or decrease if:

i) a secretary retires
ii) a clerk retires
iii) 2 salespeople are hired

Then check your predictions.

b) Eleven people work in an office. They get paid different salaries for different jobs.

Salesperson: $18 per hour
Secretary: $17 per hour
Bookkeeper: $16 per hour
Clerk: $12 per hour

If there are 5 salespeople, 2 secretaries, 1 bookkeeper and 3 clerks, find the mean salary.

Predict whether the mean will increase or decrease if:

i) a secretary retires
ii) a salesperson retires
iii) a bookkeeper is hired
iv) 2 new clerks are hired
v) a clerk retires
vi) a secretary is hired
vii) **BONUS:** 2 secretaries and a clerk are hired.

Then check your prediction.
Mean (Advanced)

Show students the data set: 80, 86, 82, 86, 86. Tell students that you want to find the mean without actually adding the data values. Tell them that you guess the mean to be 83 because all the data values are between 80 and 86.

**ASK:** Do you think my guess is reasonable? Is my guess too high or too low? How can I check without actually calculating the mean?

Take a few suggestions. If necessary, guide students to the answer with these **PROMPTS:** If 83 is the mean, how many spaces below or how many blocks above it would each data value have if we were to pile blocks? (Reading the 5 data values in the order given above: 3 spaces below, 3 blocks above, 1 space below, 3 blocks above, and 3 blocks above, for a total of $3 + 1 = 4$ spaces below and $3 + 3 + 3 = 9$ blocks above.) Are there more spaces below or blocks above the predicted mean? (There are more blocks above the predicted mean.) Is the mean too high or too low? How should we move the mean—up or down? (We need more blocks below the line, so we should move the line up.) Have students find the spaces below and the blocks above the new mean, 84, for each data value. (4 below, 2 above, 2 below, 2 above, 2 above, for a total of $4 + 2 = 6$ below and $2 + 2 + 2 = 6$ above) Since the number below is equal to the number above, the mean is indeed 84. Ask students to add the 5 data values and divide by 5 to check their answer. Did they get the same answer both ways? Ask students think about which is easier: adding 4 + 2 and 2 + 2 + 2 or adding the 5 data values and dividing by 5.

Find the mean of another set using a similar method. First make an educated guess, then write the difference between each data value and the mean above or below each number (depending on whether the data value is greater than or less than the mean).

**EXAMPLE:**

Set: 173, 180, 172, 176, 174  
Mean (guess): 175  

| 5  | 1  | Total above = $5 + 1 = 6$  
| 173 | 180 | 172 | 176 | 174 |  
| 2  | 3  | 1  | Total below = $2 + 3 + 1 = 6$  

The total differences above and below the mean are equal, so the mean is indeed 175.

Have your students practise finding the mean of various data sets by guessing and checking rather than calculating. They should write addition statements to verify their guesses.

**EXAMPLES:**

a) 56, 68, 69, 59, 67, 65 (Mean = 64 because $8 + 5 = 4 + 5 + 3 + 1$)

**GOALS**
Students will find the mean of given and self-created number sets. They will create sets of data values with the given mean.

**PRIOR KNOWLEDGE REQUIRED**

- Addition
- Subtraction
- Multiplication
- Division
- Data sets
- Mean

**VOCABULARY**

- mean
- addition
- statement
b) 45, 46, 47, 45, 45, 52, 46, 50 (Mean = 47 because $2 + 1 + 2 + 2 + 1 = 5 + 3$)

c) 128, 131, 127, 129, 136, 131, 128 (Mean = 130 because $2 + 3 + 1 + 2 = 1 + 6 + 1$)

Students can double-check their answers by adding the data values (with a calculator) and then dividing the sum by the number of data values.

Now show students how they can use addition statements to find sets with a given mean. Write the addition statement $1 + 1 = 2$ and tell students that you would like to build a data set with mean 4 using this addition statement. Now show on the board:

Spaces below the mean  Blocks above the mean

1 + 1 = 2

Draw a line to indicate where the mean is, then put in the spaces below the mean and the blocks above the mean. Then fill in the rest of the blocks below the line:

**ASK:** What data set did I create? (3, 3, 6) How did I know the first data value was 3? How did I know the last data value was 6? Ask students to add the data values independently and divide by the number of data values. Is the mean 4?

Have a volunteer draw a picture to find the set with the addition statement $2 = 1 + 1$ and mean 5:

Spaces below the mean  Blocks above the mean

2 = 1 + 1

The data set is 3, 6, 6. Students should check that this data set does indeed have mean 5.

Then ask the class to find other data sets with mean 5 using the following number sentences, where the numbers to the left of the equal sign represent “spaces below” and the numbers to the right represent “blocks above.”

a) $2 + 3 = 1 + 4$  
b) $1 + 4 = 2 + 3$  
c) $1 + 1 + 2 = 4$  
d) $4 = 1 + 1 + 2$

**ASK:** How many data values are in each data set? (4) Why did that happen? (Because there are 4 numbers in each addition statement and each number corresponds to a data value.) How many data values would there be in the data set for this number statement: $1 + 1 + 1 + 1 + 1 + 2 = 3 + 2 + 2$? (9)

**ASK:** Could you use the addition statement $4 + 7 = 3 + 3 + 3 + 2$ to build a data set with mean 5? (No—you can’t have 7 spaces below the 5 unless you use negative numbers.) Can you use $3 + 3 + 3 + 2 = 4 + 7$ to build a data set with mean 5? (Yes, the data values can be greater than the mean by any amount.)

Challenge students to build their own data sets with mean 5 by coming up with their own addition statements.

**Bonus**

Specify the number of data values to make it harder.
Use the following sample test grades to illustrate how the mean helps us to analyze data:

<table>
<thead>
<tr>
<th></th>
<th>Math</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alix</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>Sivan</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Marco</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>Bryan</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>Helen</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>Parvati</td>
<td>10</td>
<td>8</td>
</tr>
</tbody>
</table>

Have students find the mean grade for each test. **ASK:** On which test did students do better overall? Sivan got the same mark on both tests, but her teacher said she did better on English than Math. Why? (She is at the mean for English but below the mean for Math.)

**Extensions**

1. Teach students another way to find a set of 3 numbers with a mean of 5. For example, they might start with the data set 5, 5, 5, and then add and subtract the same number from the data values. Every time you add 1 to a data value, you must subtract 1 from a different data value. One sequence of data sets with mean 5 obtained in this way is:
   
   5, 5, 5  →  4, 5, 6  →  3, 6, 6  →  2, 7, 6  →  2, 8, 5  →  3, 8, 4

2. The mean of a set is 10 and the data values are 2, 19, 7, 4, 15 and ____. What is the missing number? Invite students to solve the problem two different ways.

   **Solution A:**  
   
   Data Set: 2, 19, 7, 4, 15  
   Mean = 10
   
   9  
   2 19 7 4 15
   8 3 6  
   Total value above = 9 + 5 = 14
   Total value below = 8 + 3 + 6 = 17

   Since 9 + 5 = 14, there are 14 blocks above 10. Since 8 + 3 + 6 = 17, there are 17 spaces below 10. We need 3 more spaces above 10, so the final data value must be 13.
Solution B:
Mean = sum of data values ÷ number of data values
= (sum of data values) ÷ 6 = 10

So the sum of data values must be 60. The sum of the given data values is 2 + 19 + 7 + 4 + 15 = 47, so we need 13 to make 60.

3. Create 3 different sets of data for which the mean is 6.

4. Create a set of data with the same mean as that for 36, 48, 52 and 67.

5. Find the mean of these test scores: 49, 49, 49, 50, 51, 52. Is it possible for the mean score on a test to be greater than 50 if more than half of the students have marks less than 50? Explain.

6. Use the number sentence 4 + 3 = 2 + 5 to find data sets with:
   a) mean 5  b) mean 6  c) mean 7  d) mean 8

   What do you notice about your data sets? How do they compare to each other? (You can add 1 to each data value in part a) to obtain the data set for part b), and so on.) Remind students of the discovery they already made in QUESTION 3 of PDM5-13—when you add 1 to each data value, you increase the mean by 1.
PDM5-16
Stem and Leaf Plots

Introduce the words “stem” and “leaf” as they apply in mathematics: the leaf of a number is its rightmost digit and the stem is the number formed by all the remaining digits.

Put an example on the board: stem $\overline{327}$ leaf

Ask volunteers to underline the leaf and circle the stem for various 2-, 3-, and 4-digit numbers, such as 25, 30, 230, 481, 643, 3210, 5403, 5430.

Tell students that a 1-digit number has no stem (or alternately stem 0) because there are no digits except the rightmost one. They can underline the leaf but the 0 isn’t written, so there is nothing to circle. Have students identify the stems and leaves in more numbers, including 1-digit numbers.

Have students identify the pairs with the same stem:

- a) 45 46 63
- b) 79 80 81
- c) 88 89 98
- d) 435 475 431
- e) 86 862 860
- f) 701 70 707
- g) 6423 642 649

**Bonus**

- h) 23 253 235 2 239 2530 2529

Ask your students to circle the numbers with stem 4, underline the numbers with stem 42, and cross out the numbers with neither stem:

$$42 \ 428 \ 4280 \ 4 \ 43 \ 438 \ 420 \ 46 \ 40 \ 4201$$

**Bonus**

Use larger numbers and/or longer lists of numbers.

**ASK:** When the numbers are in order from smallest to largest, are the stems in order, too? Do numbers with the same stem have the same number of digits? Do they have the same number of tens? Point out that the stem is just the number of tens and the leaf is the number of ones. That’s why 1-digit numbers have stem 0—they have 0 tens.

Have students put the following numbers in order:

$$5 \ 19 \ 23 \ 90 \ 107 \ 86 \ 21 \ 45 \ 98 \ 102 \ 43$$

Then demonstrate circling the stems and writing them in the order in which they appear:

$$0 \ 1 \ 2 \ 9 \ 10 \ 8 \ 4$$

(When a stem repeats, such as the 2 in 23 and 21, you don’t have to write it again.) Now have students put the stems in order. **ASK:** Which was easier, writing the numbers in order or writing the stems in order? (writing the stems in order) Why? (There are fewer numbers and the numbers are smaller.)
Have students find the stems of the numbers, order the stems from smallest to largest, and then write the numbers from smallest to largest:

a) 45 46 63  
   b) 567 60 583  
   c) 43 47 92 65 73 70

Draw a T-chart and show students how to make a stem and leaf plot of the above data in 3 steps:

**STEP 1:** Write the stems in order in the left column.

**STEP 2:** Write each leaf in the second column in the same row as its stem. Add the leaves in the order in which they appear. For numbers that have the same stem, put the second leaf next to the first. Think aloud as you do the first few numbers, then have students help you do the rest. **PROMPTS:** Which number is next? What’s the leaf of that number? Where should I write it?

**STEP 3:** Put the leaves in each row in order, from smallest to largest.

<table>
<thead>
<tr>
<th>STEP 1</th>
<th>STEP 2</th>
<th>STEP 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>stem</td>
<td>leaf</td>
<td>stem</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Then have students read the numbers from the finished plot. **ASK:** What are the numbers with stem 0? (just 5) What are the numbers with stem 1? (just 19) With stem 2? (21 and 23) Are these 2 numbers in order? Were they in order in the original list of data values? What did we do to make sure they would be in order in the stem and leaf plot? (We put the leaves in order. Because they have the same stem [the same number of tens], the number with the larger leaf [more ones] is larger.)

Continue reading and writing all the numbers from the plot. Compare the complete set of numbers (now in order) to the original set (unordered). Tell them that the numbers are now in order because we first put them in order according to the number of tens (stems) and then, within groups with the same number of tens, we ordered the ones. Ordering the numbers was easier because we did it in 2 smaller steps.

Have students make stem and leaf plots to order several data sets (include up to 8 numbers in each set). Start with problems where steps 1 and 2 are completed for them, and then move to examples where only step 1 is completed. Finally, have students do all 3 steps themselves. **BONUS:** Use up to 20 data values and/or larger numbers.

Explain to your students that the range is the difference between the largest and the smallest data values. Ask students to tell you the smallest and largest numbers from each of the stem and leaf plots they have drawn, and the range. **ASK:** How does making a stem and leaf plot help you find the range more quickly?
Extensions

1. Tell students that a stem and leaf plot doesn’t have to include only numbers. Here is a stem and leaf plot of friends’ birthdays—the stem is the month and the leaf is the date:

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>March</td>
<td>28 11 14 21</td>
</tr>
<tr>
<td>April</td>
<td>4 4 29 30</td>
</tr>
<tr>
<td>May</td>
<td>8</td>
</tr>
<tr>
<td>June</td>
<td>13 14 22 27</td>
</tr>
</tbody>
</table>

Have students draw a bar graph from this data, with bars for each month. The height of each bar corresponds to the number of birthdays in that month.

Ask the following questions and have students tell you which graph gives you the answer, the stem and leaf plot, the bar graph or both:

a) How many birthdays are in the first half of a month?
b) How many birthdays are in April?
c) Which month has the most birthdays? The fewest?
d) Which month has a birthday the closest to the beginning of the month? The end of the month?

ASK: If you were given only the bar graph of the birthdays, could you have built the stem and leaf plot? Why not? What is the advantage of the stem and leaf plot?

2. John counts the total number of pages in each of his favourite novels:

148 520 589 550 224 562 494 469

Have students draw a stem and leaf plot. What do they notice? (Each stem has only 1 leaf). Tell students that sometimes it is more useful to use the number of hundreds instead of the number of tens as the stem. Re-do the stem and leaf plot with the hundreds as the stem, and the remaining numbers as the leaf. Ask students to explain how using hundreds instead of tens as the stem tells them more about John’s favourite novels (They can see at a glance that most of John’s favourites are 400+ pages long!).
Tell students that they have learned a lot about collecting and classifying data. However, to understand what the data they have collected means, they have to analyze it. Today they will begin learning how to analyze data.

Sometimes knowing the highest and lowest values in a set, or group, of data can be helpful. **SAY:** Suppose you know that the temperature next week is predicted to be 15, 14, 12, 17, 11, 10 and 18 degrees. (Write the temperatures on the board.) **ASK:** What is the highest temperature? What is the lowest temperature? Do you need to know all the temperatures to decide if you need snow pants next week, or is knowing the lowest temperature enough? Do you need all the temperatures to decide if you need a pair of shorts, or is knowing the highest enough?

Write several unordered sets of numbers on the board and invite volunteers to circle the largest number and underline the smallest one. Explain that the **range** of a set is the difference between the highest and the lowest numbers. Calculate the range of the temperatures in the above example with students. (Range = highest – lowest = 18 – 10 = 8) Then ask volunteers to find the ranges for the other sets on the board.

**SAY:** Suppose you are packing for a trip and you know that the range of the temperature at your destination is usually about 5 degrees. This means that the temperature doesn’t change very much; it’s nearly the same every day. **ASK:** How can this help you decide what kinds of clothes to pack? (You might need either snow pants or shorts, but not both!) What if the range at your destination is 20 degrees? What does that tell you? What will you pack? (The temperature changes a lot. You will need clothes for very different temperatures.)

In addition to identifying highest and lowest values, and calculating ranges, students need to be able to order numbers from smallest to largest. Ask volunteers to order the numbers in the sets on the board from smallest to largest.

Tell your students that they will sometimes need to find the number in the middle of a set of data. This number is called the **median**. Find the median of the temperatures in the first example, by first ordering the temperatures and then circling the one in the middle:

10 11 12 **14** 15 17 18

Explain how you determined that 14 is the median. (It is in the middle of the set; there are 3 numbers before it and 3 after it.)

Now write this set of 6 temperatures:

10 11 12 14 15 21
SAY: When the number of terms in a set is even, there are two numbers in the middle and the median is the number halfway between them. The two numbers in the middle of this set are 12 and 14 (circle them) and the number halfway between 12 and 14 is 13. So the median is 13.

ASK: What number is halfway between 2 and 4? Between 5 and 7? 6 and 10? 12 and 16? 13 and 19? (If any students have difficulty finding the number in the middle, ask them to write all the numbers between the two in question and to count in from the sides: 13, 14, 15, 16, 17, 18, 19.)

Ask students to find the median of the sets they ordered previously. Then give them some unordered sets and ask them to find the median. (This would be a good time to do Activity 1, below.) Include sets in which numbers repeat. (SAMPLE SET OF DATA: 5 3 5 5 9 7 11)

Finally, calculate the range above and below the median for the temperatures in the original example:

- Range above median = highest temperature – median = 18 – 14 = 4
- Range below median = median – lowest temperature = 14 – 10 = 4

Look back at the range of the whole set of temperatures (8). Point out that the range of the whole set is the sum of the range above the median and the range below the median.

Explain to students that the range above and below the median is a measure of how much the data values are spread out. Use the first example to illustrate how this information can be useful. Let’s say you knew that the range of temperatures below the median was small. That means the lower temperatures don’t vary much—they’re nearly the same and close to the median. It also means that on nearly half of the days, the temperature will be at or only a little below the median. (Since, by definition, at least half of the data values are at or below the median.)

Give your students several sets (ordered, then unordered) and ask them to find the median and the range above and below the median. Include sets in which numbers repeat and in which the range above or below the median is 0.

SAMPLE SETS OF DATA:

- 2, 3, 3, 4, 6, 7, 8
- 2, 2, 2, 2, 6
- 12, 15, 15, 15

Write a stem and leaf plot where one number occurs twice. EXAMPLE:

<table>
<thead>
<tr>
<th>stem</th>
<th>leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>9</td>
<td>1 8 9</td>
</tr>
<tr>
<td>10</td>
<td>0 2 9 9</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
</tr>
</tbody>
</table>

Have students identify the number that repeats. Then write a stem and leaf plot in which 1 number appears 2 times and another number appears 3 times. Have students identify the numbers that repeat. ASK: Which number appears most often? Explain that this number—the value that occurs most often—is called the mode. Ask if anyone knows the French phrase “a la mode.” Tell students that it means in style or popular. Things that are popular occur often. Similarly, the mode is the most popular number in a set. ASK: Does every set have to have a mode? Remind students of earlier sets in which every value occurred only once—those had no mode.

Ask students to identify the mode in several stem and leaf plots, some of which have more than one repeated data value but only one most common data value. Then introduce data sets with more than one mode. Give them the stem and leaf plot at first and then have them build the stem and leaf plot first to find the mode or modes.
Present the two following sets of students' heights rounded to the nearest centimetre:

Set 1: 148, 152, 153, 155, 155, 156, 156, 163

Explain that one of the sets shows the heights of a grade 5 basketball team, and the other shows the heights of a grade 7 chess team. **ASK:** Which set is which?

Brainstorm with your students what they expect about the heights of basketball players in grade 5 and chess players in grade 7. Are grade 7 students taller on average than grade 5 students? If necessary, explain that basketball players tend to be taller than other people. Are chess players taller or shorter than other people or average? **ASK:** For any set, which is larger, the range or the range above the median? The range or the range above the mean? If you choose 3 data values from the whole set and 3 from the set of values above the mean, which three values might be spread out more? (The values from the whole set. You can choose values from both ends of the whole set, and not from the middle and up only.) If the basketball players are taller than average, do you expect their heights to be more, or less, spread out than the heights of the chess players?

Ask your students to draw bar graphs for both sets using the axes shown below:

![Bar graph axes](image)

**ASK:** How are the graphs different? (In Set 1 the data is grouped around the middle, whereas in Set 2 the data is grouped toward the edges.) Can you decide which set belongs to which team by looking at the graphs? (The heights of the basketball players form Set 1. They are taller than other children of the same age, and most of them are about the same height, so the data is grouped around the centre. The chess players are older, so there is more difference in the heights of the boys and the girls, so their heights make Set 2.) Explain that this property of the graphs that they noticed would be called the shape of the sets.

Ask your students to find the mean, median and range of each set. What do they notice? (The mean and the median are 155 and the range is 15 for both sets.) Could the mean, the median and the range alone help them answer the question which set is which?

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**ACTIVITY 1**

Invite 9 volunteers to be a set of data. Ask them to order themselves from shortest to tallest. Who is the median? Now have one of the volunteers sit down, and ask the class how they can find and represent the median. (Students may suggest making a mark on the board to represent the height halfway between the two middle students, or they may suggest choosing another student in the class whose height is approximately halfway between the two middle students to be the median.)
Students may use Excel and its built-in statistical functions to investigate the result of changes in values on sets of data. For example, let the students create a table with the following values:

<table>
<thead>
<tr>
<th>75</th>
<th>77</th>
<th>69</th>
<th>75</th>
<th>90</th>
<th>75</th>
<th>73</th>
<th>65</th>
<th>68</th>
<th>84</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>73</td>
<td>71</td>
<td>75</td>
<td>70</td>
<td>95</td>
<td>97</td>
<td>65</td>
<td>72</td>
<td>86</td>
</tr>
</tbody>
</table>

The students can find the mean (average), median and mode (all found among statistical functions) of the set. They can change or add data values to see how the changing a value or adding a new data value affects the mean, the median and the mode.
Extensions

1. Find the median and the range below and above the median for each set:
   a) 2, 3, 5, 7, 9, 10
   b) 12, 16, 19, 23, 26, 26, 26

2. Write 3 different sets that have median 9 and range 5.

3. Write 3 different sets that have median 10 with range above the median 0 and range below the median 5.

4. Write a set of data in which:
   a) the largest number is 100.
   b) the smallest number is 100.
   c) the range is 100.
   d) the median is 100.
   e) the mean is 100.
   f) one of the modes is 100.

   Can you satisfy more than one requirement in the same set?

5. Ron counted the number of floors of the buildings in his block: 5, 3, 3, 1, 13
   a) Find the mean, median and modes of the set of data.
   b) The five story building is replaced by a skyscraper of 50 floors. Find the mean, median and the mode of the new data set.
   c) Ron says: The increase in any data value increases the mean. The increase in any data value also increases the median. Are both his statements correct?
   d) The number 50 is much greater than the others. (It is called an outlier). Which value changed the most when you added the outlier, the mean, or the median?
   e) Look at 4 sets of data: 3, 3, 1, 13; 5, 3, 1, 13; 5, 3, 3, 13; 5, 3, 3, 1

   What is the difference between the initial set and these 4 sets? Find the mean for each set. Which of the following statements are always correct?
   • The mean increases if a piece of data below the mean is removed.
   • The mean increases if the piece of data below the median is removed.
   • The mean decreases if a piece of data below the mean is removed.
   • The mean decreases if a piece of data above the mean is removed.
   • The mean increases if a piece of data above the mean is removed.
   • The mean decreases if a piece of data above the median is removed.

6. Can you find a set so that the range below the median is 0, and the mean and the median are the same? Explain your thinking with blocks.

7. Have students make a chart comparing the mean and the median for data sets with only 2 values:

<table>
<thead>
<tr>
<th>Data Set with 2 Values</th>
<th>Mean</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>3, 7</td>
<td>10 ÷ 2 = 5</td>
<td>3 4 5 6 7</td>
</tr>
<tr>
<td>4, 10</td>
<td>7 ÷ 2 = 3 5</td>
<td>4 5 6 7 8 9 10</td>
</tr>
</tbody>
</table>

   Other data sets they should add to the chart:
   a) 0, 12
   b) 5, 7
   c) 9, 21
   d) 11, 15
What do students notice? (the mean and the median are the same) Have students predict whether the same will be true for data sets with 3 or 4 data values and then check their predictions.

8. In a game as in the Activity 2 above, there are 9 envelopes. The minimal amount is $200, the range is $800, the mean and the median are both $600. There is no mode. You pick an envelope with $600. Are you trading or not?

Discuss with your students how the shape of the set might affect the choice using the sets:

Set 1: 200, 300, 400, 500, 600, 700, 800, 900, 1000
Set 2: 200, 597, 598, 599, 600, 601, 602, 603, 1000.

With the first set, the game is very risky – the data is spread evenly, and the gain (or the loss) is large with any trade. In the second case the data bunches up around the median and the mean, and in most cases the gain (or the loss) is small. Presenting the data in bar graphs might be helpful.

PDM5-18
Choices in Data Representation

Give your students a set of data values and invite volunteers to create a bar graph, a line graph and a stem-and-leaf plot for the values. Sample data set:

Katie found some data on the sightings of whales in the Gulf of Maine in the summer of 2007:

<table>
<thead>
<tr>
<th>Date</th>
<th>Jun 1</th>
<th>Jun 8</th>
<th>Jun 16</th>
<th>Jul 3</th>
<th>Jul 4</th>
<th>Jul 13</th>
<th>Jul 21</th>
<th>Aug 1</th>
<th>Aug 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Whales Seen</td>
<td>27</td>
<td>34</td>
<td>17</td>
<td>15</td>
<td>21</td>
<td>17</td>
<td>52</td>
<td>31</td>
<td>27</td>
</tr>
</tbody>
</table>

Discuss with your students what the scale for their bar graph should be and what range they should use. On the horizontal axis of their line graph, should the distance between the successive pairs of dates be the same for all pairs or should some dates have more (or less) space between them than other dates?

On which graph is it easiest to see the range of the values? On which graph is it easiest to see the median? What does the median mean for this set of data? (On half of the dates more whales were seen, and on half of the dates fewer whales were seen.) What information is lost in the stem and leaf plot?

Ask your students to find the mean. What does it mean for this set of data? (The average number of whales seen on a trip.) Is it possible to see this
number of whales? (No, it is not a whole number.) Which graph is most convenient to use to find the mean? On which graph is it easiest to see how far the various values are above and below the mean?

Which trends can be seen in this set of data? On which graph can you see the trends most clearly? If you wanted to estimate how many whales might be seen on a trip on June 23, which graph would you use?

**Extensions**

1. Determine the values of the other bars on the graphs:

   ![Graphs](image)

2. Students may use Excel to plot the graphs of larger sets of data. Let them compare various graphs as they did during the lesson.

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**A Note About Terms in Probability**

**Outcomes and Events**

A simple action such as rolling a die, flipping a coin, or spinning a spinner has various possible results. These results are called the *outcomes* of the action. If you flip a coin, the outcomes are: “You flip a head” and “You flip a tail.” When you describe a specific outcome or set of outcomes, such as rolling a 6, rolling an even number, tossing a head, or spinning red, you identify an *event*.

In probability theory, the terms *outcome* and *event* have very precise meanings. But in elementary texts, outcome is occasionally misused.

This spinner has 3 coloured regions:

![Spinner](image)

There are 3 possible outcomes of spinning the spinner:

1. The pointer lands in the blue region.
2. The pointer lands in the red region.
3. The pointer lands in the green region.
Some textbooks will identify the outcomes as:

1. You spin blue.
2. You spin red.
3. You spin green.

What’s the difference? Identifying outcomes with only colours and not regions can cause confusion when 2 or more regions of a spinner are the same colour. Here is a spinner with 4 coloured regions:

![Spinner Diagram]

There are 4 outcomes of spinning the spinner:

1. The pointer lands in the blue region at top right.
2. The pointer lands in the blue region at bottom right.
3. The pointer lands in the blue region at bottom left.
4. The pointer lands in the red region.

This is clearly not the same as saying that the outcomes are:

1. You spin red.
2. You spin blue.

“You spin blue” and “You spin red” are events, not outcomes. To assess the relative likelihood of spinning red or blue, students must recognize that the pointer may land in 4 distinct regions of the spinner. In 3 of the 4 outcomes, the spinner lands in a blue region. Hence, the event “You spin blue” is more likely than the event “You spin red.”

The outcomes for QUESTION 1 f) on Worksheet PDM5-19 are “The spinner lands in region 1,” “The spinner lands in region 2,” “The spinner lands in region 3,” and “The spinner lands in region 4.” Your student may write something more concise, such as “You spin a 1,” “You spin a 2,” “You spin a 3,” “You spin a 4.” Accept these answers, provided students understand that when different regions of a spinner have the same number or colour, each region must be counted as a distinct outcome.

To avoid confusion, we only use the term “outcome” on the worksheets when the regions of the spinner are uniquely coloured or labelled. When the same colour or label appears more than once on the spinner, we use phrases like “ways of spinning red” instead of “outcomes”.

**Probability and Outcomes**

In probability theory, an “event” is a particular “subset” of a set of outcomes. For instance, on the spinner above, the event “You spin blue” is the subset of outcomes in which the pointer lands in a blue region. On a die, the event “You roll an even number” is the subset of outcomes: {“You roll a 2,” “You roll a 4,” “You roll a 6”}. Your student needn’t learn the technical meaning of the term “event” until they are older. (For now, tell them an event is any particular result of an action or occurrence they wish to assess the likelihood of.) But they should know that the “outcomes” of an occurrence are all the ways the occurrence could happen.

The probability or likelihood of an event may be expressed as a fraction: given a set of outcomes (where each outcome is equally likely), the denominator of the fraction is the total number of outcomes and the numerator is the number of ways the event could happen.
Probability = \frac{\# \text{ of ways the event can happen}}{\# \text{ of outcomes}}

If you flip a coin, the probability of flipping a head is \( \frac{1}{2} \), since there are two outcomes of flipping the coin, but only one way to flip a head.

If you roll a die, the probability of rolling a five is \( \frac{1}{6} \), since there are six outcomes but only one way to roll a five.

On the spinner below, the probability of spinning red is \( \frac{3}{8} \), since there are eight regions (all of the same size) in which the pointer might land, but only three of the regions are red.

![Spinner](image)

**Expectation**

The probability of spinning blue on the spinner below is \( \frac{1}{3} \).

![Spinner](image)

If you were to spin the spinner 12 times you would expect to spin blue \( \frac{1}{3} \) of the time, or \( 12 \div 3 = 4 \) times.
Tell students that today they will start learning how to predict the future!

Hold up a die and ask students to predict what will happen when you roll it. Can it land on a vertex? On an edge? No, the die will land on one of its sides. Ask students to predict which number you will roll. Then roll the die (more than once, if necessary) to show that the prediction about landing on a side works, but the number they picked does not necessarily come. Explain that the possible results of rolling the die are called outcomes, and to predict the future students must learn to identify which outcomes of various actions are more likely to happen and which are not. But first, they must learn to identify outcomes correctly.

Hold up a coin and **ASK:** What are the possible outcomes of tossing a coin? How many outcomes are there? Show a spinner and a set of marbles. What are the possible outcomes of spinning the spinner or picking a marble with your eyes closed? Ask students to identify the possible outcomes of a soccer game. How many outcomes are there? (3 outcomes: team A wins, team B wins, a draw)

**ASK:** You have to make a spinner with 5 possible outcomes. How would you do this? Invite volunteers to draw possible spinners. If the spinner in the picture at right does not arise, draw it and **ASK:** How many outcomes are there for this spinner?

Are all the outcomes bound to come equally, or is there an outcome that they think will happen more often than the other ones? Draw the second spinner and **ASK:** How many outcomes does the second spinner have? (4) Will the pointer ever be in the grey region? (no, never)

**Extensions**

1. Bill has a pentagonal pyramid with numbers on the sides. He puts a non-base side on the table and rolls the pyramid. How many outcomes are there? Make a pentagonal pyramid, number the sides, and check your answer. Are there sides the pyramid never lands on if you roll it in the manner described? (Yes—the base. It will not roll if it stands on a base.) Repeat with a hexagonal prism and an octagonal prism: predict the number of outcomes then make a model to check your prediction.

2. Ed and George are playing “Rock, paper, scissors.” Describe all the possible outcomes of the game. (**NOTE:** “Ed wins” is not an outcome, it is an event. “Ed has paper, George has rock” is an outcome.)
Have students make some predictions. Ask them to tell you if the following events are likely or unlikely:

- The sun will rise tomorrow.
- The teacher will give the answers to the test before giving the test.
- An alien will walk into the class in the next minute.
- They will have lunch in half an hour.
- It will rain tomorrow.
- It will snow in June.

Invite students to name some events and have other students tell if the events are likely or unlikely. You could ask students to compare the likelihood of events. For example, it is unlikely to snow in June, but it is more unlikely that an alien will walk into the class!

Explain to your students that mathematicians call an event **likely** if it is expected to happen more than half the time and **unlikely** if it is expected to happen less than half the time.

Ask your students which word people would use to describe an event like meeting a live dinosaur in the street. Can that happen at all? Write the terms **likely**, **unlikely** and **impossible** on the board. Ask your students which words describe an event that will definitely happen, like rolling a number less than 7 on a die. Add the word **certain** to the list.

**NOTE:** Although students might use the word impossible to describe the likelihood of meeting a dinosaur, this event is not necessarily impossible (scientists might find a way to clone dinosaurs). The only events that are strictly impossible are events that are contradictory—like rolling a number greater than 6 on a die.

Ask students to give examples of various events and explain whether they are likely, unlikely, certain, or impossible. Encourage students to think of events using marbles, dice, money, and other objects, as well events from daily life, such as meeting a tiger or an astronaut on the way to school.

Hold up a coin. **ASK:** What are the possible outcomes of flipping this coin? How many outcomes are there? What is more likely—to flip a head or a tail? Explain that the chances are the same—you have **even chances** of flipping a head or a tail. Add the term to the list and explain that the chances of an event are even when the event happens in exactly half of the outcomes.

Flipping a tail is 1 out of 2 possible outcomes; 1 is half of 2. **ASK:** How many outcomes are there when you roll a die? (6) How many outcomes are even numbers? (3) Since half of the outcomes are even numbers, you have even chances of rolling an even number (and an even chance of rolling an odd number).

**SAY:** We have 8 marbles in a box. I take out a marble. How many outcomes are possible? (8, regardless of the colour of the marbles) What is half of
8? So if I want even chances to take out a green marble, exactly 4 marbles should be green. Does it matter what colour the other marbles are? (No, provided they’re not green.)

Draw the spinner below on the board and ask students to describe the possibility of spinning each of the colors. Is it equally likely that they will spin green or that they will spin blue? Is it equally likely that they will spin yellow or blue? Why? (HINT: Look at the angle at the centre)

Ask students to draw a spinner to match this description:

1. It is likely you will spin on the yellow.
2. It is unlikely to get green.
3. It is equally likely to get green and red.
4. It is impossible to spin blue.

Now write this list of properties:

1. It is impossible to get green.
2. It is certain to get blue.
3. It is likely to get red.
4. It is unlikely to get white.
5. It is equally likely to get white and red.
6. It is equally likely to get red and purple.
7. It is equally likely to get white and yellow.

ASK: Do you think we can make a spinner that matches all of these? Do any of the properties contradict each other? Look at 3 and 6: can a spinner match 3 and 6? (No. If the spinner is likely to give red, more than half of the spinner should be red. Then only less than half of it can be purple. But if it is equally likely to give red and purple, the red and the purple should have the same area. But more than half is never equal to less than half!) Have students pick 3 of the properties on the list and create a spinner that will match them. If the all-blue spinner does not arise, show that example yourself and ask students to identify the properties that describe it (1, 2, 5, 6 and 7).

Which properties does this spinner match? (Same as the all-blue spinner: the pointer never reaches any other region)

Show students a collection of marbles or coloured counters:

G Y G B G
R B G G Y

ASK: Which colour are you most likely to pick? Which colour is less likely to be picked: yellow or red? So which colour is least likely to be picked? Which colours are equally likely?
Draw a line on the board:

```
impossible                  certain
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Ask your students to mark on the line the probability of picking...

- a green marble
- a blue marble
- a yellow marble
- a red marble
- a pink marble
- a marble of any colour

**Assessment**

1. Make a spinner that matches this description:
   - It is most likely to spin green.
   - It is unlikely to spin red.
   - It is most unlikely to spin purple.
   - It is equally likely to spin purple and blue.
   - It is impossible to spin yellow.

2. Explain why it is impossible to make a collection of marbles to match this description:
   - It is impossible to pick red. It is certain to pick blue. It is very unlikely to pick green.

3. Change one word in the description of the collection of marbles above to make it possible. Draw the collection that matches the new description.

**A Game for Two**

Give students several marbles or counters of various colours. One player chooses a set of 12 counters or marbles, unseen to the partner. The player describes the set to the partner, using probability terms (certain, likely, unlikely, and so on). The second player has to reconstruct the set from the first player’s description.
Divide students into 2 groups. Each group will conduct a different experiment and share the results with the second group afterwards.

**Group 1**

Students will each need a die. Ask them to list all the outcomes of rolling a die and to count the outcomes that suit each event listed below. Then ask them to describe each event as likely, unlikely, or having even chances:

1. Roll a number greater than 2.  
2. Roll a number greater than 5.  
3. Roll an odd number.

Each student rolls the die 12 times and tallies the results. Do the results match the predictions? Have students combine their results. (This group tally chart—how many times the group rolled 1, 2, and so on—may be useful during the next lessons.) How many rolls did the whole group make? How many times does the group expect to get an even number? A number greater than 5? A number greater than 2? Students should explain their answers. Do the group’s results match the predictions better than the individual results?

Students can create a table like this to summarize predictions and results as they conduct the experiment: **TOTAL OUTCOMES: 1, 2, 3, 4, 5, 6**  
**NUMBER OF POSSIBLE OUTCOMES: 6**

<table>
<thead>
<tr>
<th>Event</th>
<th>Suitable Outcomes</th>
<th>Individual Results (12 Rolls)</th>
<th>Group Results ( ___ Rolls)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll a number &gt; 2</td>
<td>3 4 5 6</td>
<td>__ out of 12</td>
<td></td>
</tr>
<tr>
<td>Roll a number &gt; 5</td>
<td>6</td>
<td>__ out of 12</td>
<td></td>
</tr>
<tr>
<td>Roll an odd number</td>
<td>1 3 5</td>
<td>__ out of 12</td>
<td></td>
</tr>
</tbody>
</table>

**Group 2**

Students will each need 3 red marbles, 2 yellow marbles, 1 green marble, and a non-transparent bag or box (e.g., a lunchbox).

Ask students to list all the outcomes of drawing a marble from the box and to count all the outcomes that suit each event from the list below. Ask students to describe each event as likely, unlikely, or having even chances:

1. Draw a red marble.  
2. Draw a green marble.  
3. Draw a marble that is not yellow.

Each student draws a marble 12 times (returning the marble after each draw and shaking the bag) and records the results in a tally chart. Do the individual results suit the prediction? Have students combine their results. Do the group’s combined results suit the predictions? Students can create a table similar to that used by Group 1 to summarize their results.

Ask the students to write a summary of the experiments. Prompts for Group 1:

- I performed an experiment with a die. I rolled the die ___ times.  
- I expected that ___ of ___ rolls will give a number greater than 2. I got a number > 2 in ___ rolls.  
- I thought that it is ___________ to get a number > 2, because ___________  
- I learned that __________________________________________________________________________

Discuss the similarities and differences between the experiments of the two groups.
Extensions

1. If you roll a die, are your chances of rolling a number greater than 2: unlikely, even or likely? Explain your answer.

2. Write the numbers from 1 to 10 on ten cards, one number on each card. Ask students to select 6 cards at random. Let them check which of the following statements hold. Repeat the experiment 10 times. Are these events certain, impossible, likely or unlikely? Have the students explain their answers.
   - The sum of the numbers will be greater than 60.
   - All the numbers will be even.
   - Two numbers will be neighbours.
   - The sum of the numbers will be greater than 20.
   - No numbers will be neighbours.
   - The sum of the numbers will be less than 12.
   - Three cards will be odd.

3. Invent or describe a game where a certain player’s chance of winning is very close to certain. What are the chances of the other player(s) to win?

4. Doug has a total of $95 in his wallet ($50, $20, $20, and $5). Which bill is he most likely to draw? Which bills are equally likely to be drawn?

5. Match the spinner with the correct statement:

   - A
     - W
     - R
     - B
   - B
     - W
     - R
     - G
   - C
     - B
     - B
   - D
     - R
     - B
     - W

   ____ Spinning blue is three times as likely as spinning white
   ____ Spinning blue and white are equally likely
   ____ Spinning any colour is equally likely
   ____ Spinning one colour is twice as likely as spinning any other colour
PDM5-21
Probability

Show your students two pencils of different length. Ask them how they could determine which pencil is longer. Then show two objects where direct comparison is impossible, such as a ruler and the circumference of a cup. Students might suggest measuring the length with a measuring tape. Then ask your students how they could compare the weight of two objects, say a book and a cup. What could they do to compare the temperature in two different places? Point out that in all cases they tried to attach numbers to each object and to compare the numbers. They used different tools for that purpose—measurement tape, scale or thermometer. Each tool provided a number, a measurement. What would they do to compare the likelihood of two events, such as the likelihood of rolling 8 on a pair of dice and the likelihood that their favourite hockey team wins 5 to 3 in the next game? Is there a tool to measure likelihood? No. Explain that probability is the branch of mathematics that studies likelihood of events and expresses the likelihood of various events in numbers. The measure of likelihood of an event is called probability.

Draw the following spinner on the board:

```
B
R
B
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**ASK:** How many different regions does the spinner have? How many different ways can you spin red? (Only one.) How many different ways can you spin blue? (Two ways) Since the regions of the spinner are all the same size, it is equally likely that the spinner will land in any of the regions. Since two regions are coloured blue and only one is coloured red, students should see that it is twice as likely that they will spin blue as it is that they will spin red.

Explain that in mathematics we describe probability as a fraction:

\[
\text{Probability} = \frac{\# \text{ of ways the event can happen}}{\# \text{ of possibilities}}
\]

For our spinner, there are three possible outcomes. (Even though there are only two colours there are three regions). So the probability of spinning red is \(\frac{1}{3}\) and the probability of spinning blue is \(\frac{2}{3}\). **ASK:** Which fraction of the spinner is coloured red? Which fraction of the spinner is coloured blue?

Give your students several exercises such as:

```
R  W  R  Y  R  W
```

How many ways can you draw a yellow marble?

How many ways can you draw a marble of any colour?
What is the probability of drawing a yellow marble?

How many ways can you draw a white marble? What is the probability of drawing a white marble?

Show your students the following spinner:

![Spinner with colors: R, B, B, B, W]

**ASK:** Which colour are you more likely to spin: red or white? Why? How many regions does this spinner have? How many are coloured red? How many are coloured white?

What do we get if we write a fraction where the numerator is just the number of white regions and the denominator is the number of all regions? \( \frac{1}{5} \) Repeat with the red colour. Did we get the same fractions? (yes) However, we want these fractions to measure the likelihood of spinning red and spinning white. If one of them is more likely to happen than the other, the fractions should be different. This means that we made a mistake in our calculations. Ask your students if they can guess what was wrong.

Explain that the number of possibilities in the definition of probability refers to equal possibilities. The red part of the spinner is smaller than the white part. Can we divide the spinner into equal parts so that each part is coloured in one colour? (The spinner now is divided into quarters, but one of the quarters is coloured in two colours.) Invite a volunteer to cut the spinner into equal parts.

**ASK:** How many regions does the spinner have now? (8) How many regions are coloured red? What is the probability of spinning red? \( \frac{1}{8} \) White? \( \frac{2}{8} \) Blue? \( \frac{5}{8} \)

Review the concept of reducing fractions with the students and ask them to write the probability of spinning white on the spinner above as a reduced fraction.

Let your students solve several more complicated questions like:

Write a fraction that gives the probability of rolling a multiple of 3 on a die.

To solve this question, ask your students to write out all the possible outcomes of rolling a die. Then ask them to circle the numbers that are multiples of 3. How many are there? Ask students to write the probability as a fraction and to reduce the fraction to lowest terms.

More questions to practice:

What is the probability of a letter in the alphabet to be a vowel?

What is the probability to roll a number greater than 4 on a die?

Give your students two dice. Ask them to roll the dice and to write a fraction where the numerator is given by the least number rolled, and the denominator is given by the larger number rolled. The students then have to draw a spinner or a collection of marbles so that the probability of spinning blue or drawing a blue marble equals the fraction.
Extension

Match the net for a tetrahedron (△) to the correct statement:

A  B  C  D
2 3 4
4 3 2
3 4 2

1. The probability of rolling a 1 is \( \frac{1}{4} \)
2. The probability of rolling an even number is \( \frac{3}{4} \)
3. The probability of rolling an odd number is \( \frac{3}{4} \)
4. The probability of rolling a 3 is \( \frac{1}{2} \)
SAY: I would like to play a game with you. The rules of the game are simple. I will spin a spinner. If I get red, I win; if I get blue, the class wins. Ask students if they agree to play by these rules. Now show them the spinner. Do they still want to play? Why not?

Write the term **fair game** on the board. Ask students to explain what they think this term might mean. Encourage students to use math vocabulary in their explanations. Point out that in a fair game, both players have equal chances, or are equally likely, to win. Does this mean there can never be a draw? No. Use this game to illustrate: Flip a nickel and a penny. If both give heads, you win. If both give tails, the class wins. If there is one head and one tail, it is a draw—no one wins. Explain the rules to the class, then ask a volunteer to list all the four outcomes. **ASK:** Is the game fair? (Yes) Why? (Each player wins in one case only, so they have even chances to win.) Would the game stay fair if we added a third player who wins when one of the coins gives a head and the other a tail? (No - the third player will be more likely to win than the other two - he will have two winning outcomes.) Would the game be fair if the third player wins with head on the nickel and tail on the penny? (Yes)

Give the rules for another game: There are 6 marbles in a box. If you draw a red marble, you win; if it is blue, the class wins. Otherwise, the game is a draw. There are 2 red marbles in the box. **ASK:** To make the game fair, how many blue marbles should be in the box? What about the rest of the marbles—what colour can they be? Can you think of another combination of marbles that will make the game fair?

Vary the game: If you draw a red marble you win, but if you draw any other colour, the class wins. If 2 of the 6 marbles are red, who has more chances to win—you or the class? What if 5 marbles are red? What should be in the box to make the game fair?

Let your students describe the probability of throwing a dart (assume the dart always hits the board) and landing on each of the colours:
You may also ask some more complicated questions, such as: What is the probability of getting red or yellow? What is the probability of getting any colour other than blue? What is the probability of getting a colour that is on the flag of Canada? On the flag of your province? Which two colours have the same probability of occurring? Find a combination of colours such that the probability of the dart landing on one of the colours is \( \frac{5}{8} \). Ask your students to make up their own problems using this chart.

**Assessment**

1. Two players are spinning this spinner. Invent two different rules of play to ensure that the game is fair.

2. What is the probability to draw a red marble from the box with 3 red marbles, 2 white marbles and 3 green marbles?

**Extensions**

1. Emily has a bag of 8 marbles. 5 are blue. Peter has a bag of 7 marbles. 3 are blue.
   a) What is the probability of drawing a blue marble from Emily’s bag?
   b) What is the probability of drawing a blue marble from Peter’s bag?
   c) Emily and Peter pour all of their marbles into one bag. What is the probability of drawing a blue marble from the bag?

2. Mark draws all possible rectangles with a perimeter of 12 cm (with whole number sides). If he picks one of the rectangles at random, what is the probability that it will have length of 3 cm?

3. Carl and Clara played a game based on luck. Carl won 15 times and Clara won 12 times. Does this mean the game is not fair?

4. Carmel wants to express probability by drawing rectangles. For example, he draws a rectangle to represent the probability of tossing a head on a coin. The probability of tossing a head is \( \frac{1}{2} \), so he shades \( \frac{1}{2} \) of the rectangle.

   Use Carmel’s method to show the probability of:
   a) Drawing a red marble from a box with 4 red marbles and 8 green marbles
   b) Picking a consonant from the word “rectangle”
   c) Spinning blue on the spinner below.
Show students this spinner and ask them if the chances of spinning red and spinning blue are equal. What about the chances of spinning green and spinning blue? Why? Write on the board:

Number of possible outcomes: 3 (1 green, 1 red, 1 blue)
Chances of spinning blue: 1 out of 3

SAY: I am going to spin the spinner 12 times. How many times do you expect me to spin blue? Why? Write the calculation on the board:

\[ \frac{1}{3} \text{ of } 12 = 4 \quad \text{OR:} \quad 12 \div 3 = 4 \text{ times} \]

Remind students that actual outcomes usually differ from expected outcomes. You might not get blue 4 times every time you make 12 spins, but 4 times is the most likely number of times you will spin blue.

Review with students how to find a fraction of a set and a fraction of a number as well as visual representation of fractions. Practise by solving these and other questions:

\[ \frac{1}{2} \text{ of } 14 \quad \frac{1}{3} \text{ of } 15 \quad \frac{1}{5} \text{ of } 25 \]
\[ \frac{2}{3} \text{ of } 20 \quad \frac{2}{3} \text{ of } 9 \quad \text{Quarter of } 16 \]

Three eighths of 24
\[ \frac{1}{2} \text{ of } | | | | | | | | | | | | \]
\[ \frac{1}{3} \text{ of } | | | | | | | | | | | | | | \]

Which part of the set “R G R G G R Y Y G Y” is G?

ASK: Which part of this spinner is blue? What are the chances of spinning blue? (3 out of 4) In mathematics we say “The probability of spinning blue on this spinner is \( \frac{3}{4} \).” Write that on the board. Explain that you can determine the number of times you would expect to spin blue out of 20 spins as follows:

STEP 1: \( \frac{1}{2} \) of 20 is \( 20 \div 4 = 5 \).
This is how many times you expect the spinner to land in each region.

STEP 2: Three regions are blue. You expect to spin blue \( \frac{3}{4} \) of all times.
If you expect the spinner to land in each region 5 times, and 3 of the regions are blue, then the spinner will land in a blue region \( 3 \times 5 \) times:

\[ \frac{3}{4} \text{ of } 20 \text{ is } 3 \times 5 = 15. \]
You can show this with a picture:
\[
\frac{1}{4} \text{ of } 20 = 20 \div 4 = 5
\]

\[\begin{array}{cccc}
\text{I} & \text{I} & \text{I} & \text{I} \\
\text{I} & \text{I} & \text{I} & \text{I} \\
\text{I} & \text{I} & \text{I} & \text{I} \\
\text{I} & \text{I} & \text{I} & \text{I} \\
\end{array}\]

Therefore, \(\frac{3}{4} \text{ of } 20 = 3 \times 5 = 15\)

Have students practice calculating expected outcomes with these and similar questions:

- If you flip a coin 16 times, how many times do you expect to get a tail?
- Hong wants to know how many times he is likely to spin green if he spins this spinner 24 times. He knows that \(\frac{1}{3} \text{ of } 24 = 8\) (\(24 \div 3 = 8\)). How can he use this information to find how many times he is likely to spin green?
- If you roll a die 18 times, how many times do you expect to get a 4? To get a 1?
- How many times would you expect to spin blue if you spin this spinner 50 times? How many times would you expect to spin green?

**CHALLENGING:** If you roll a die 30 times, how many times do you expect to roll an even number? How many times do you expect to roll either 4 or 6?

You should review long division with your students before you assign the worksheets on expectation.

**Assessment**

Rea spins this spinner 30 times. How many times is she likely to get blue?

**Bonus**

1. You flip two coins, a nickel and a dime, 12 times. List the possible outcomes. How many times do you expect to get one head and one tail?
2. Jack and Jill play “Rock, paper, scissors” 18 times. List all possible outcomes of the game. (HINT: “Jack has rock and Jill has paper” is different from “Jack has paper, Jill has rock”! Why?) How many times do you expect to see a draw? What is the probability of a draw?
Extensions

1. Design an experiment with three possible outcomes in which one of the outcomes has a probability near \( \frac{1}{2} \).

2. Choose a novel. Open it to any page and note whether or not the first letter is a “t”. Check 10 pages in this manner. Describe the probability that “t” is the first letter on a page of the book.

3. Give your students two dice of different colors, say red and blue. The red die will give the numerator of a fraction and the blue die will give the denominator. Roll the dice 40 times and record the fractions. How many times did you get a fraction that was reduced to lowest terms? What is the experimental probability of getting a reduced fraction? Pool the results of the whole class. Are they all the same? What is the experimental probability of getting a reduced fraction from the results of the whole class? Which result is more likely to match the theoretical probability?

List all the possible combinations of the two dice as fractions (there are 6 different results of the red die for every result of the blue die, so the total number of combinations is 36). Count the reduced fractions. What is the theoretical probability of getting a reduced fraction? Compare with the experimental results and write a report about what you’ve learnt.

ACTIVITY

Have students flip a coin ten times and keep a tally of the number of heads. Point out that (unless a miracle occurred in your class) that not every student flipped heads exactly half the time. Ask the students to identify the result that was furthest from the expected number of 5 heads. Then combine all the results of the entire class (i.e. add up the total number of heads from all the tallies). The overall proportion of heads should be closer to half of the total number of tosses. Explain to your students that the more trials you conduct (i.e. the more times you repeat an experiment) the more closely the actual result will match the expected result for the event.
PDM5-24 is a review worksheet, which can be used for practice. Here is an additional activity:

Pair up students and give each pair a container with 1 red counter and 5 blue counters inside. Player 1 wins if they draw a red counter and Player 2 wins if they draw a blue counter. Ask students to explain whether the game is fair or not. Have students play the game 20 times (replacing the counter each time) and keep a tally of who wins each time. Ask students if the results were what they expected.

Then ask students to keep track of who wins and loses in 20 repetitions of the following game:

Players roll a die. Player 1 wins if a 1 is rolled. Player 2 wins otherwise.

Ask students the following questions: Is the second game fair? Are the results what you expected?

How are the games in questions 1 and 2 similar? Is the first game less fair than the second?

(When looked at from the point of view of probability, the two games are identical. In both games the second player has 5 out of 6 chances to win: the probability of Player 2 winning is $\frac{5}{6}$.)

Students could predict the number of reds they would expect to spin for a given number of trials (i.e. for 15 trials, 30 trials, etc.) They could keep a tally of their results and compare them to the expected number of reds.

**Extension**

a) Look at the second game in the activity. How many times do you expect Player 1 to win, if the game is played 24 times?

b) Compare the second game in the activity with the following game:

Players take turns rolling a die. If a player rolls a 1, he gets 1 point. If he rolls any other number, his partner gets 1 point. The die is rolled 24 times. Player with more points wins.

Is this game fair? Explain.

c) How many points is each player expected to get in the game in part b)?
To illustrate the idea of a coordinate system you can start with the following card trick:

1. First, deal out nine cards—face up—in the arrangement shown below:

   \[
   \begin{array}{ccc}
   \text{Row 3} & \text{Row 2} & \text{Row 1} \\
   \text{Column 1} & \text{Column 2} & \text{Column 3}
   \end{array}
   \]

2. Next ask a student to select a card in the array and then tell you what column it’s in (but not the name of the card).

3. Gather up the cards, with the three cards in the column your student selected on the top of the deck. Show clearly how you do that.

4. Deal the cards face up in another \(3 \times 3\) array making sure the top three cards of the deck end up in the top row of the array.

5. Ask your student to tell you what column their card is in now. The top card in that column is their card, which you can now identify!

6. Repeat the trick several times and ask your students to try to figure out how it works. You might give them hints by telling them to watch how you place the cards, or even by repeating the trick with a \(2 \times 2\) array.

When your students understand how the trick works, you can ask the following questions:

- Would there be any point to the trick if the subject told the person performing the trick both the row and the column number of the card they had selected? Clearly there would be no trick if the performer knew...
both numbers. Two pieces of information are enough to unambiguously identify a position in an array or graph. This is why graphs are such an efficient means of representation: two numbers can identify any location in two-dimensional space (in other words, on a flat sheet of paper). This discovery, made over 300 years ago by the French mathematician René Descartes, was one of the simplest and most revolutionary steps in the history of mathematics and science: his idea of representing position using numbers underlies virtually all modern mathematics, science, and technology.

You might ask your students how many numbers would be required to represent the position of an object relative to an origin in three-dimensional space. (The answer is three. Think of the origin as being situated on a plane or flat piece of paper that has a grid or graph on it. You need two numbers to tell you how to travel from the origin along the grid lines on the plane to situate yourself directly above or below the object, and one more number to tell you how far you have to travel up or down from the plane to reach the object.)

• Ask your students if the trick would work with a larger array. Have them try the trick with a 4 × 4 array. They should see that as long as the array is square (with an equal number of rows and columns), the trick works for any number of cards. Ask your students to explain why this is so and why the trick doesn’t work if the array isn’t square (for instance, try it with 2 columns and 6 rows).

• Ask your students if the original trick (i.e. with a square array) would work if the subject told the performer which row the card was in rather than which column. Have your students show you how the new trick would be performed. The fact that the trick works equally well in both cases illustrates a very deep principle of invariance in mathematics. In a square array, there is no real difference between the rows and columns. In fact, if you rotate the array by a quarter turn, the rows become columns and vice versa. More generally, once you fix an origin in space, it doesn’t matter how you set up your grid (the lines representing the rows and columns). In all cases you need only two numbers to identify a position.

Now draw an array of three columns and rows on the board and number the columns and rows:

```
  3   ●   ●   ●       C
  2   ●   ●   ●       R   O   W
  1   ●   ●   ●       L   U   M   N
  1   2   3
```

Point out a row and a column, and stress that we order rows from bottom to top, and columns from right to left. Ask several volunteers to locate the third column, second row, etc. Then ask your students to do the worksheets. The QUESTION 7 on the worksheets can be played as a game—one of the players gives the column and the row, the other has to mark the point according to the numbers. They might play “Hangman” hanging each other for incorrect answers.

**Assessment**

1. Join the dots in the given column and row:

   a) Column 3, Row 2
      
   b) Column 1, Row 3
      
   c) Column 3, Row 3
      
      ```
      ●   ●   ●
      ●   ●   ●
      ●   ●   ●
      ```
2. Circle the dot where the two lines meet:
   a) Column 2, Row 3  
   b) Column 3, Row 1  
   c) Column 1, Row 1

3. Identify the proper column and row for the circled dot:
   a)   
   b)   
   c)   
   d)   

   Column _____  
   Row _____  

   Bonus
Which letters of the alphabet can be written on the grid and described in terms of rows and columns only? (See QUESTION 3 of the worksheet.) Which numbers can be written this way?

Ball Game
The students are the points in a coordinate system, ask each student to say which column and row they are in. Then throw a ball according to directions—start by giving a student a ball, and a column and row number. The student holding the ball has to throw it to the student sitting at the given point.

Extension
The card trick can be modified for non-square arrays if one allows one extra rearrangement. Deal out an array of 3 columns, 9 rows. Have a student select a card and tell you what column it’s in. Re-deal the cards so that all of the nine cards from the chosen column land in the top three rows of the new array. Ask the student to tell you what column their card is in now, and re-deal the top three cards in that column into the top row of a new array. Once the student tells you what column their card is in, you can identify the top card in that column as the one they selected.

This version of the trick illustrates a powerful general principle in science and mathematics: when you are looking for a solution to a problem, it is often possible to eliminate a great many possibilities by asking a well-formulated question. In the card trick one is able to single out one of 27 possibilities by asking only three questions. Repeat the trick, asking your students how many possibilities were eliminated by the first question (18), by the second question (6), and by the third (2).
Review the previous lesson. Draw an array of dots on the board, circle a point and ask your students to write the coordinates of the point: Column___, Row___. Ask your students: Imagine you have to write the coordinates of 100 points. Would you like to write the words “column” and “row” 100 times? What could you do to shorten the notation? Students might suggest making a T-table or even writing a pair of numbers, because the column is always the first number.

ASK: How do you know, which is first, column or row? What if you have to ask a partner to find a dot given a pair of numbers, without telling them which number belongs to the column and which number is the row number? Give your students a pair of numbers, such as 2, 3 and do not tell them which one is the column number and which one is the row number. How many points can they find that could go with these two numbers? Suppose your partner does not speak English and you cannot show him, that you use column number first. How would they overcome this difficulty?

Explain to your students that mathematicians found the way out – they made an international convention: The place of the dot will be given by two numbers, put in parenthesis, and the column number is always on the left, and the row number is always on the right: (column, row). Give your students several pairs of numbers and ask them to identify the points on the grid. Explain that the pair of numbers is called “coordinates of the point”. Include this term into the spelling test.

Draw an array of dots and label the columns by letters and the rows by numbers. Ask your students to identify some points, such as (A, 4), (B, 2), (C, 3). Mark the points (B, 3) and (C, 2) and ask your students to identify them. Repeat the exercise with a grid instead of an array. As a variation, you might mark both rows and columns with letters.

Students need lots of practice.

Review the names of special quadrilaterals and triangles before assigning these questions:

1. Graph the vertices A (1,2), B (2,4), C (4,4), D (5,2). Draw lines to join the vertices. What kind of polygon did you draw? How many lines of symmetry does the shape have? How many pairs of parallel sides does it have?

2. On grid paper, draw a coordinate grid. Graph the vertices of the triangle A (1,2), B (1,5), C (4,2). Draw lines to join the vertices. What kind of triangle did you make?

Ask your students if they have seen any of these methods of marking points with letters or numbers in real life. You might show them a map with a grid on it.
Assessment
1. Mark the positions: (3, 2), (4, 1).
2. Write the coordinates of the marked positions: _____, _____.

![Graph with coordinates](image)

Bonus
Draw the points below on a grid, then join the points in the order you’ve drawn them. Join the first and the last points. What shape did you make? Find the area of the shape.

a) (A, 4), (A, 5), (B, 5), (C, 4), (D, 4), (E, 3), (D, 3), (C, 2), (C, 1), (B, 2), (B, 3).

b) (0, 1), (2, 3), (2, 6), (3, 7), (4, 6), (4, 3), (6, 1), (6, 0), (5, 0), (4, 1), (3, 0), (2, 1), (1, 0), (0, 0).

**ACTIVITY**

Students will need a pair of dice of different colours. The player rolls the dice and records the results as a pair of coordinates: (the number on the red die, the number on the blue die). He plots a point that has this pair of coordinates on grid paper. He rolls the dice for the second time and obtains the second point in the same way. The player joins the points with a line. After that he has to draw a rectangle so that the line he drew is a diagonal of the rectangle.

There could be several rectangles drawn this way. If the line is neither vertical, nor horizontal, the simplest solution is to make the sides of the rectangle horizontal and vertical. In this case, ask your students if they see a pattern in the coordinates of the vertices.

ADVANCED: Students will need a pair of dice of different colours and a spinner below.

![Spinner](image)

Player rolls the dice twice and plots the points the same way he did in the previous activity. He also spins a spinner. He has to draw a quadrilateral of the type the spinner shows, so that the line he drew is the diagonal of the quadrilateral.
For this lesson, a magnetic board with a grid on it (or an overhead projector with a grid drawn on a transparent slide) would be helpful. Let your students practice sliding dots in the form of a small circular magnet right and left, then up and down. Students should be able to identify how far a dot slid in a particular direction and also be able to slide a dot a given distance. If any students have difficulty in distinguishing between right and left, write the letters L and R on the left and right sides of the board.

After students can slide a dot in given direction, show them how to slide a dot in a combination of directions.

You might draw a hockey rink on a magnetic grid and invite volunteers to move a small circular magnet as if they were passing a puck. Sample questions and tasks:

Pass the puck three units right; five units left; seven units down; two units up.
Pass the puck two units left and five units up.
Position several small figures of players on grid intersections in various points of the rink and ask your students such questions as:
Player 3 passes the puck 5 units right and two units up. Who receives the pass?
Player 5 wants to pass the puck to Player 7. How many units left and how many units down should the puck go?

**Bonus**
Player 4 sent the puck 3 units up. How many units left should Player 7 move to get the pass?

**Assessment**
Slide the dot:

a) 3 units right; 3 units up

b) 6 units left; 3 units down

c) 7 units left; 2 units up
**ACTIVITY 1**

**Ball Game**
The students are points on the grid, and you give directions such as: “The ball slides three units to the right”; the student with the ball has to throw it to the right place in the grid.

**ACTIVITY 2**

In the school yard, draw a grid on the ground. Ask your students to move a certain number of units in various combinations of directions by hopping from point to point in the grid.

**ACTIVITY 3**

**Memory Game**
Students will need a grid and several (1 to 4) small objects (play money of different values or beads of different colours could be used). The objects are placed on the intersections of the grid. Player 1 slides one of the objects while Player 2’s back is turned, and Player 2 then has to guess which object was moved and describe the slide. This game will become much easier when coordinates are placed on the grid and the students are familiar with the coordinate system. When students learn coordinate systems (in section G5-23), they will be able to memorize the coordinates of the objects. They can then compare the coordinates after the objects were moved with the coordinates before the objects were moved to determine exactly which object moved and how.
Tell your students the following story. You might use two actual figures to demonstrate the movements in the story.

Suppose you have a pair of two-dimensional figures and you wish to place one of the figures on top of the other. But the figures are very heavy and very hot sheets of metal. You need to program a robot to move the sheets: to write the program you have to divide the process into very simple steps. It is always possible to move a figure into any position in space by using some combination of the following three movements:

1. You may slide the figure in a straight line (without allowing the object to turn at all):

   ![Slide Diagram]

2. You may turn the figure around some fixed point (usually on the figure):

   ![Turn Diagram]

3. And you may flip the figure over:

   ![Flip Diagram]
Two figures are congruent if the figures can be made to coincide by some sequence of flips, slides and turns. For instance, the figures in the picture below can be brought into alignment by rotating the right hand figure counter clockwise a quarter turn around the indicated point, then sliding it to the left.

It is not always possible to align two figure using only slides and turns. To align the figures below you must, at some point, flip one of the figures:

One way to flip a figure is to reflect the figure through a line that passes through an edge or a vertex of the figure. Tell your students that today you are going to teach them about slides.

Show students the following picture and ask them how far the rectangle slid to the right. Ask for several answers and record them on the board. You may even call a vote.

Students might say the shape moved anywhere between one and seven units right. Take a rectangular block and perform the actual slide, counting the units with the students. The correct answer is 4.

Show another picture:
This figure has a dot on its corner. How much did it slide? This time it is easier to describe the slide—just use the benchmark dot on the corner. Check with the block.

Show a third picture.

Is this a slide? The answer is NO, this is a slide together with a rotation. You cannot slide this block from one position to the other, without turning it.

**ACTIVITY**

Give your students a set of pattern blocks or Pentamino pieces and ask them to trace a shape on dot paper so that at least one of the corners of the shape touches a dot. Ask students to slide the shape a given combination of directions. After the slide, trace the pattern block again.
### GOALS
Students will slide shapes on a grid, and describe the slide.

### PRIOR KNOWLEDGE REQUIRED
- Slide a dot on a grid
- Distinguish between right and left

### VOCABULARY
- slide, translation
- translation arrow

**Slides (Advanced)**

Draw a shape on a grid on the board and perform a slide, say three units right and two units up. Draw a translation arrow as shown on the worksheet. Ask your students if they can describe the slide you’ve made. If they have trouble, suggest that they look at how the vertex of the figure moved (as shown by the transition arrow). To help students describe the slide, you might tell them that the grid lines represent streets and they have to explain to a truck driver how to get from the location at the tail of the arrow to the location at the tip of the arrow. The arrow shows the direction as the crow flies, but the truck has to follow the streets.

Make sure your students know that a slide is also called a “translation”. Students should also understand that a shape and its image under a translation are congruent.

### Extensions

1. Slide the figures however you want, and then describe the slide:

   ![Diagram of a slide transformation](image)

2. Describe a move made by a chess knight as a slide. Describe some typical moves of other pieces such as a pawn or a rook (castle).
Assign a letter to each row of desks in your class and a number to each column. Ask your students to give the coordinates of their desks. Then play “postman”—a student writes a short message to another student and writes the student’s “address” in coordinates. A volunteer postman then delivers the letter. The postman has to describe how the letter moved (two to the front and one to the left, for example).

Place a slide with a map of Saskatchewan on the overhead projector (see the BLM). Ask volunteers to find the cities on the map and to answer the questions:

- What are the coordinates of Saskatoon?
- What are the coordinates of Regina?
- What are the coordinates of Uranium City?
- What are the coordinates of Prince Albert?
- What can you find in the square A4? D5? D1?

**Battleship Game**

This game may be either played in pairs or a teacher can play against the whole class, when the class is guessing the teacher’s ships. You might also give some tips—when a player hits something, where can the other squares of the ship be? When the ship is sunk, where there are no ships?

**Sample Placement:**

(Continued on next page.)
Let your students draw their own map, possibly based on a book they are reading. Ask them to make their own questions of the same kind as on the worksheet and ask their partners to answer them.

**G5-24 Reflections**

**GOALS**

Students will perform reflections of points and shapes through a line.

**PRIOR KNOWLEDGE REQUIRED**

Symmetry

**VOCABULARY**

reflection  mirror line  symmetry  symmetry line

Give your students an assortment of Pentamino pieces. Ask them to trace each piece on grid paper, draw a mirror line through a side of the piece and then draw the reflection of the piece in the mirror line. Students could check if they have drawn the image correctly by flipping the grouping of Pentamino pieces over the mirror line and seeing if it matches the image. Let your students know that a “flip” is also called a “reflection.” Students should notice that each vertex on the original shape is the same distance from the mirror line as the corresponding vertex of the image. Let your students practice reflecting shapes with partners: Each student draws a shape of no more than 10 squares, and chooses the mirror line. The partner has to reflect the shape over the given mirror line.

**Advanced game:** One student draws two shapes of no more than 10 squares so that the shapes are symmetric in a line but one square is misplaced. The partner has to correct the mistake.

Draw four points as shown and explain that two of these points are reflections of the other two. Challenge students to draw the mirror line. How do they know that the line they have drawn is the mirror line? Which point is reflection of which?
Write several words, such as MOODY CAT IN A WOODEN BOX, on a transparency sheet and project it onto the board in an incorrect way (flipped horizontally or vertically). Ask your students if they can read the text. Which words are still readable? Which letters look normal? Which transformation should be performed to make the text look completely normal? (A reflection.) Ask your students to draw the mirror line. Show them that a reflection in a mirror and a flip of the transparency sheet both achieve the goal.

**Bonus**
1. Students could try to copy and reflect a shape in a slant line, for example:

![Example shapes](image)

2. Sort all the capital letters of the alphabet into a Venn diagram:
   - 1. Letters that look the same after a reflection in a horizontal line.
   - 2. Letters that look the same after a reflection in a vertical line.

**Extensions**
1. Which letters of the alphabet look different after a horizontal or vertical reflection, but look the same after two reflections? **Example:** If you reflect the letter E or L through a vertical line, the image faces backwards. If you reflect the image through a second vertical line, you produce the original letter. What happens if you reflect a letter first through a horizontal line, then through a vertical line?

2. Draw an equilateral triangle. If you reflect it through one of the sides and look at two shapes together, what shape will you get? Write down your prediction and check it. Is the result different for the other sides? Repeat with an isosceles triangle and a triangle with a right angle. Are the results different for different sides? Check all sides.

The following five extensions answer the demands of The Atlantic Curriculum for Grade 5.

3. Cyril experiments with a mirror and a straight line. He draws a straight line and puts the mirror across it. He looks at the angle between the line he drew and the mirror and at the angle between the reflection of the line and the mirror. He thinks that these angles are the same. Is he correct?

Cyril turns the mirror and looks at the angle between the line he drew and its reflection in the mirror. The angle between the line and the mirror is 20°. How large is the angle between the line and its reflection? Cyril wants to put a mirror so that it is at a right angle with the line. What is the degree measure of a right angle? How large is the angle between the line and its reflection be when the mirror is at a right angle to the line? What does Cyril see in the mirror?
4. Boris experiments with a Mira and an angle. He draws an angle and places the Mira so that it touches the vertex on his angle and divides the angle in two. He rotates the Mira around the vertex until the angle on one side of the Mira and its reflection are the same. Using the mirror as a ruler he draws a line through the angle (starting at the vertex of the angle). He says that the line cuts the angle into two equal parts. Is he correct? The line that divides an angle into two equal parts is called a bisector.

5. Which of the points on the line L is closest to the point A? Estimate, then measure the distances between the points to check your prediction. Connect the points on the line with the point A. Measure the angles between the lines you drew and the line L. What is the angle at the point that is nearest to A? Angela measures the acute angles. She says: The further the point from A, the less the angle between the line I drew from the point to A and the line L. Is she correct? Can you draw a point on L that is nearer to A than D? The distance from a point A to a line is the shortest among the distances from A to the points on the line. What is the distance from A to L?

CHALLENGING: How can you use Cyril’s method to find a point on a line that is nearest to a given point? ANSWER: Put a mirror across the line so that it touches the given point (which we will call point A). Turn the mirror (around the point A) until you see that the reflection of the line continues the line itself. At that point you know that the mirror is perpendicular (i.e., at a right angle) to the line. Draw a line through A using the mirror as a ruler. The point where your line meets the given line is the point nearest to A.

6. Gleb wants to find the midpoint of a line segment AB using symmetry. He knows that a point and its image in a mirror are the same distance from the mirror. He puts a Mira across the segment AB and looks at the point A’ (the mirror image of the point A). He also sees the point B through the Mira. He makes sure that the mirror is perpendicular to the line and he moves the mirror between the points A and B. What does he see in the Mira when it is in the middle of the line segment? Why does this happen exactly at the midpoint of the segment?

ANSWER: When the Mira is at the midpoint of the segment, Gleb sees that the points A and B coincide. This happens because the distance between the mirror and the point A (which is the same as the distance between the mirror and A’) is now the same as the distance between the Mira and the point B.

7. A line that is both perpendicular to the given segment and passes through its middle is called a perpendicular bisector of a segment. How can Gleb use Mira to draw a perpendicular bisector of a segment? (HINT: Gleb and Cyril are friends.)
G5-25
Rotations

Review the meaning of the terms “clockwise” and “counter clockwise” using a large clock or by drawing arrows on the board. If you have a large clock, ask volunteers to rotate the minute hand—clockwise and counter clockwise—a full turn, half turn, and a quarter of turn. You might also ask your students to be the clocks: each student stands with a hand forward and turns clockwise (CW) or counter clockwise (CCW) according to your commands.

Draw several clocks on the board as shown below and ask your students to tell you how far and in which direction each hand moved from start to finish:

Then draw examples with only one arrow and ask students to turn the arrow:

a) $\frac{1}{4}$ turn CCW  b) $\frac{1}{2}$ turn CCW  c) $\frac{3}{4}$ turn CW  d) $\frac{3}{4}$ turn CCW

Assessment
1. Describe the rotation of an arrow:

2. Show the position of the arrow after each turn:

a) $\frac{1}{4}$ turn CW  b) $\frac{1}{2}$ turn CW  c) $\frac{3}{4}$ turn CCW  d) $\frac{3}{4}$ turn CW
G5-26
Rotations (Advanced)

GOALS
Students will describe and perform rotations of shapes that are multiples of a quarter turn.

PRIOR KNOWLEDGE REQUIRED
Fractions: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$
Clockwise
Counter clockwise

VOCABULARY
clockwise
counter clockwise
rotation

Review the previous lesson by drawing several arrows or clock hands. To help your students visualize the effect of a rotation of a shape, have them make a small flag (as in QUESTION 1 on the worksheet) by taping a triangular piece of paper to a straw. Ask students to rotate the flag and trace its image after the rotation. Students could also cut out shapes similar to the other ones on the worksheet and trace the images of these shapes after a rotation. Then ask your students to trace or draw a figure, decide on rotation and draw the new shape without the prop. Students could also practice rotating pattern blocks or Pentamino shapes (around vertices of the shapes) on a grid.

Assessment
1. Draw the shape after each turn:
   a) $\frac{1}{4}$ turn CW
   b) $\frac{1}{2}$ turn CW
   c) $\frac{3}{4}$ turn CCW
   d) $\frac{3}{4}$ turn CW

Extensions
1. Using pattern blocks or cardboard polygons, trace the figure on a sheet of paper. Then choose a vertex and rotate the shape around the vertex $\frac{1}{4}$ turn. Trace the figure again. Would you get the same result if you had reflected the figure? Shapes that are particularly interesting in this case are right-angled trapezoids.

2. Trace the flower onto tracing paper and cut it out. Try to reflect it vertically, then rotate it $\frac{1}{4}$ turn CW. Draw the result. Now rotate and reflect the flower vertically. Is the result the same? Try various combinations of $\frac{1}{4}$ turn rotations and reflections. Which combinations give the same results? Why?

3. What happens to the line of symmetry of a figure after a rotation of $\frac{1}{4}$ turn? $\frac{1}{2}$ turn? If the shape has a vertical line of symmetry, will it have a vertical line of symmetry after either a $\frac{1}{4}$ or $\frac{1}{2}$ turn?
The following three extensions satisfy the demands of The Atlantic Curriculum (Expectation E11).

4. Hold up a large paper rectangle. Pin it to a piece of bristol board with a single pin (not in the middle of the rectangle) and trace the rectangle. Explain to your students that you want to turn the rectangle so that it returns to its original position. Explain that the place where the pin is stuck is called the centre of rotation. Ask a volunteer to trace the position of the rectangle after a \( \frac{1}{4} \) clockwise turn. Does the image look exactly the same as the original? No—the image is oriented differently. Make another \( \frac{1}{4} \) clockwise turn. Trace the image again and compare it with the original again. This time the rectangle is displaced. Repeat this exercise several times. Students should see that the rectangle will not return to its original position after a rotation that is less than 360° unless the centre of rotation is the centre of the rectangle.

Stick the pin in the centre of the rectangle. Ask: How much should I rotate the rectangle so that the image is exactly the same as the original? I rotate the rectangle and stop every time the position of the shape coincides with the original position. How many times do I stop until I make the full 360° turn? (2 times: once for a 180° turn and once for a 360° turn)

Explain to your students that a shape that returns to the original position before it completes the full turn is said to have rotational symmetry. Explain also that the number of times we stop during the complete rotation around the centre of the shape is called the order of the rotational symmetry. So a rectangle has rotational symmetry of order 2: we stop once after a half-turn and a second time after the full turn. Ask your students to predict the order of rotational symmetry for a square. Repeat the rotational exercise above (with the pin in the middle of the square) and check their prediction. The square has rotational symmetry of order 4, because we stop 4 times during the full turn. As a challenge, let your students find the order of rotational symmetry for an equilateral triangle (3) and a regular pentagon (5) and regular hexagon (6). Can they see a pattern?

5. What is the order of rotational symmetry of the flower in Extension 2? (Answer: One. This shape has no centre, and rotation around any point brings the flower to its original position only after a whole 360° turn.)

6. What is the order of rotational symmetry for other special quadrilaterals? (Answer: Parallelogram and rhombus have rotational symmetry of order 2. The trapezoid, kite and any other quadrilateral do not have rotational symmetry, so the order is 1).
G5-27
Rotations and Reflections

GOALS
Students will perform rotations, reflections and their combinations using shapes on grids.

PRIOR KNOWLEDGE REQUIRED
Perform a slide, a rotation or a reflection of a shape

VOCABULARY
rotation
reflection
slide
centre of rotation
mirror line

Write on the board:

- Half-turn 270° \(\frac{3}{4}\) turn
- Quarter of turn 180° \(\frac{1}{4}\) turn
- Three quarters of turn 90° \(\frac{1}{2}\) turn

Invite volunteers to join the degrees to the verbal descriptions of turns.

Explain to your students that today their task will be harder than the one you assigned in the last lesson—they will have to rotate and reflect shapes using grid paper. Draw a simple shape on a grid on the board and mark one of the corners with a dot. Highlight one of the sides passing through the dot. **ASK:** where will the side be located after a 180° rotation? Invite a volunteer to draw the rotated side. Highlight another side and ask another volunteer to draw its image after the rotation. Ask your students to finish the rotation of the shape.

Ask your students to describe the shape before and after the rotation.

**EXAMPLE:** The shape is three squares long and two squares wide. It is longer horizontally than vertically. It has an indentation of 1 square at the top-right corner.) Repeat the exercise with several more complicated shapes, including triangles (and with quarter turns).

Let your students practice combinations of rotations and reflections (only two transformations at a time). After some practice, draw a shape like the one above and ask your students the following question: Pam performed a reflection of a shape in a horizontal line through the top side and then turned the shape 90° clockwise around the marked corner. Padma first turned the shape 90° clockwise around the same corner and then reflected it in a horizontal line through the top side. Did they get the same result? You may call a vote, and after that check the results. **ASK:** Which transformation should Padma perform after the \(\frac{1}{4}\) turn to get the same result as Pam? Why? (She should use a reflection through the right side, because it is the image of the top side after rotation.)
Assessment
Rotate the shape \( \frac{1}{4} \) turn clockwise around the bottom corner and reflect it through the left side.

BONUS: Which single transformation is the same as a reflection in a vertical line and then in a horizontal line?

A Game for Pairs
Ask your students to draw a non-symmetric shape on a piece of grid paper. The original shape is visible to both players at all times. Player 1 rotates the shape \( \frac{1}{2} \) or \( \frac{1}{4} \) turn in the direction of his choice or reflects it through a vertical or a horizontal line (the player can only use a single transformation!), so that his partner does not see the result. Then he describes the position and orientation of the shape to his partner. Player 2 has to draw the shape and to identify the transformation performed by his partner.

Extensions
1. Draw a coordinate grid on grid paper. Draw a triangle with vertices A (1,5), B(4,5), C(4,7). Draw a vertical mirror line through the points (5,0) and (5,7). Reflect the triangle in the mirror line. Then rotate the triangle \( \frac{1}{4} \) turn counter clockwise around the image of vertex C. Write the coordinates of the vertices of the new triangle.

2. To rotate a shape \( \frac{1}{4} \) turn clockwise around a point that is not a vertex:

   **STEP 1:** Highlight the side that contains the point (the point is called the rotation centre).

   **STEP 2:** Rotate the side around the point.

   **STEP 3:** Draw an arrow from the rotation centre to one of the vertices of the shape.
STEP 4: Rotate the arrow to get the image of the vertex.

Repeat steps 3 and 4 if necessary for the other vertices.

STEP 5: Draw the image of the shape using the images of the vertices.

Rotate the figures 90° clockwise and counter clockwise around the shown points:
G5-28
Slides, Rotations and Reflections

**GOALS**
Students will perform and identify combinations of two transformations.

**PRIOR KNOWLEDGE REQUIRED**
Perform and identify a slide, a rotation or a reflection of a shape

**VOCABULARY**
- rotation
- reflection
- slide
- centre of rotation
- mirror line
- translation arrow

Draw a triangle in a square as shown and several results of transformations, such as:

![Triangle transformations](image)

Ask your students to identify the transformation used to get from the left triangle to the right triangle. If the transformation used was a reflection, the students should also identify the line of reflection, and if it was a rotation—its centre. Show a slide as well and remind your students how you identify the slide using a marked corner. Students could cut out copies of the triangle and use it to test their guesses.

Repeat the exercise above with two transformations made one after the other. These questions often have more than one answer, so encourage your students to find as many answers as they can.

**Extension**
Draw a coordinate grid on grid paper. Draw a trapezoid with vertices A (1,5), B(4,5), C(3,6), D(2,6). Rotate the trapezoid 90° counter clockwise around point D. Then slide the image 3 units right and 1 down. Write the coordinates of the vertices of the new trapezoid.

**A Game for Pairs**
Ask your students to draw a non-symmetric shape on a piece of grid paper. The original shape is visible to both players at all times. Player 1 performs two transformations of her choice, and shows the result to her partner. Player 2 has to identify the transformations performed by her partner. She may present several answers, and each answer that gives the same result grants her a point. She has to guess the exact transformations that Player 1 used as well. For example, suppose Player 1 used the transformations “Rotate clockwise \(\frac{1}{4}\) turn, then reflect through the right side.” If Player 2 says: “Reflect through the top side, rotate \(\frac{1}{4}\) turn clockwise,” she gets 1 point but has to continue guessing.
G5-29
Slides, Rotations and Reflections (Advanced)

**GOALS**
Students will perform and identify combinations of two transformations.

**PRIOR KNOWLEDGE REQUIRED**
Perform and identify a slide, a rotation or a reflection of a shape

**VOCABULARY**
- rotation
- reflection
- centre of rotation
- mirror line
- slide
- translation arrow

Draw the shapes shown below on the board or on an overhead. Ask your students to identify the single transformation that takes each shape into the others.

Draw the shapes shown below on the board or on an overhead.

Draw a translation arrow between the shapes C and D. Ask your students to identify the slide. Mark another vertex on shape C and ask your students to identify the vertex of D which is the image of the marked vertex. Ask them to draw the translation arrow between the new vertices. Students should notice that the second translation arrow is the same length and points in the same direction as the first.

Ask your students to find a pair of shapes that can be transformed into one another by a reflection, (A and B, for instance) and ask a volunteer to draw the mirror line. Draw a vertical mirror line through the left-hand side of A and ASK: What should I do to the image of A (after the reflection) to move it onto shape B?

Return to the shapes A and B. ASK: Can I pass from A to B using only reflections through the sides? How many reflections will I need? Can you tell which way the letter L (shape A) will point after each reflection without actually making the reflections? I reflected shape A seven times through vertical lines. Which way does it point now? I slid shape A in some direction. Which way does it point now? I turned it 90° clockwise. Which way does it point now? A harder question: I slid it, turned it 180° and reflected it through a vertical line. Which way does it point now?
Assessment

1. Describe a series of transformations that could be used to get shape A onto shape C. Give at least two answers.

2. Which transformations were used to move shape B onto shape D? How many answers can you find?

G5-30
Building Pyramids

GOALS
Students will build a skeleton of a pyramid and describe the properties of pyramids.

PRIOR KNOWLEDGE REQUIRED
Geometrical shapes:
- triangle
- square
- rectangle
- pentagon
- hexagon

VOCABULARY
- edge
- vertex
- triangular
- hexagonal
- pyramid
- pentagonal
- skeleton
- base

Start with a riddle: “You have 6 toothpicks. Make 4 triangles with them. The toothpicks must touch each other only at the ends.” Let your students try to solve the riddle using toothpicks and modelling clay to hold the toothpicks together at the vertices of the triangles. The answer, of course, is the triangular pyramid. You might give your students the hint that the solution is three-dimensional.

Sketch a rectangular pyramid on the board and shade the base. Ask volunteers to mark the edges and the vertices (counting them, and making a tally chart). Write the words “base”, “edges”, “vertex” and “vertices” on the board.

Give your students modelling clay and toothpicks. Show them how to make a pyramid—first a base, then add an edge to each vertex of the base and join the edges at a point. The students should make triangular, square and pentagonal pyramids. Then let them fill in the chart and answer the questions on the worksheet.

After finishing the worksheet, they may check their prediction for the hexagonal pyramid by making one.

Tell your students that the shapes they have built are called “skeletons” of pyramids. You might write on the board the “equation”: “SKELETON = Edges + Vertices”. As animal skeletons are covered with flesh and skin, the skeleton of a pyramid can be covered with paper or glass or other substances and will have faces. Show a pyramid (with faces) and write the word “faces” on the board as well.

Assessment
Create and fill in the fifth row of the chart on the worksheet—for the heptagonal (7-sided) based pyramid.
Extensions

1. How many faces, edges, and vertices would a pyramid with a ten-sided base have?

2. Give a rule for calculating the number of edges in a pyramid that has a base with n sides. (Use n in your answer.)

   SOLUTION: A pyramid with n edges in the base also has n vertices in the base. But attached to each vertex in the base there is one non-base edge.

   Hence there are n non-base edges and n base edges. Therefore there are 2 × n edges altogether in a pyramid with n base edges. (So, for example, a pyramid with 5 base edges would have 2 × 5 = 10 edges altogether.)

3. Ask your students to bring to class pyramids or pictures of pyramids (Egypt, Mexico, Japan, entrance to Louvre, Paris, France and any others) that they can find at home. You can use the pyramids they brought in the lessons G5-33 and G5-34.

4. HOMEWORK PROJECT: Ask your students to find a picture of a pyramidal structure and give a presentation about it—what was the structure used for, when and where was it built, why does it have the pyramidal form.
Building Prisms

GOALS
Students will build a skeleton of a prism and describe the properties of prisms.

PRIOR KNOWLEDGE REQUIRED
Geometrical shapes: triangle, square, rectangle, pentagon, hexagon

VOCABULARY
| face    | edge |
| vertex  | vertices |
| prism   | cube |
| triangular | pentagonal |
| hexagonal | base |

Sketch a prism on the board, shade the bases. Ask volunteers to mark the edges and the vertices (counting them and making a tally chart). Write the words “base”, “edges”, “vertex” and “vertices” on the board.

Give your students modelling clay and toothpicks. Show them how to make a prism—first make two copies of the base, and then join each vertex on one base to a vertex on the other base with an edge. Students should make triangular and pentagonal prisms and a cube. Let them fill in the chart and answer the questions on the worksheet.

After finishing the worksheet, students may check their prediction for the hexagonal prism by making one.

Ask your students what they have built. (Skeletons of prisms.) What are the “bones”? (The edges.) What do the skeletons need to become prisms? (Write the word “faces” on the board.)

Assessment
Create and fill in a fifth row of the chart—for the heptagonal (7-sided) based prism.

Extensions
The following two extensions were adapted from The Atlantic Curriculum.

1. Give each student two boxes that are both rectangular prisms (you may ask them in advance to bring various boxes from home). The length, width and height of the boxes should be different. Ask your students to tell you the proper mathematical name for the boxes (rectangular prisms) and ask them if they can find any faces that are congruent. Your students might find the congruent faces by tracing the faces on paper. How many congruent faces does each prism have? Ask your students to label the faces, so that the congruent faces are marked with the same letter. Then ask them to measure the edges and to write down the dimensions for each face. What happens if they join a pair of prisms along congruent faces? Ask your students to predict the dimensions of the new prisms and to check their predictions. How many faces of the prism double in size? (4) How many dimensions of the prism double? (1)

2. Hold up a triangular or pentagonal prism. Join it to the board by one of the faces and ask your students: Suppose the board is a mirror. Let’s look at the solid that is composed of both the prism and the reflection. Describe the solid and name it if you can. Use only prisms with equilateral bases for this activity. (More complicated bases could be used for very advanced students.) Let your students practice with a mirror. You can use a large mirror at the front of the class or smaller mirrors and smaller shapes that students could work with in groups. Are the results different if
you attach the side face or a base of the prism to the mirror? (Yes—the base gives a prism of double height, and the side face creates a prism with an irregular base.

For instance, if you use a triangular prism with an equilateral base, the resulting prism will have a rhombus as the base, and a pentagonal prism will produce an octagonal prism with an irregular base: \( \square \square \). What happens if you repeat the process with a **pyramid** with an equilateral base? Will you still get a pyramid? (Generally, no.) Is it a prism? (No as well.)

3. Ask students to build the following shapes using interlocking cubes:

   a) a rectangular prism with a square base and side faces that are not square.
   b) a rectangular prism (with a square base) that is 3 times as high as the length of its base.

4. Take two copies of the Triangular Prism with a Scalene Base (see “Right Prisms” BLM). Join one of the non-base faces of one prism to a congruent face of the other prism. What 3-D figure do you get? What is the shape of its base? Turn one of the prisms upside down and repeat the exercise. Is the result different? Students can make predictions of the results they get with other non-base faces of the same prism and then check their predictions.

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**G5-32 Vertices, Edges and Faces**

Remind your students that there are lots of 3-D shapes in the world around us that are either pyramids or prisms. As an example you might show them a photo of the pyramids in Egypt.

![Pyramids in Egypt](image)

Hold up a 3-D shape and draw a picture of the shape on the board. Write the words “3-D shape”, ask volunteers to show the edges, the faces and the vertices, both on the shape itself and on the drawing, and write the terms “edge”, “face” and “vertex” on the board. Remind your students that the plural of “vertex” is “vertices”.

Your students will need the skeletons of the cubes they made during the last two lessons. Give each student 2 squares made of paper and 4 squares made of transparent material. Ask them to add:

- The non-transparent squares as the bottom face and the back face
- The transparent ones—as the top, the front and the side faces
Add the faces step by step, one face at a time, emphasizing the positions and names of the faces. It is a good idea to show the students how to add faces on a larger model.

Ask students to identify the edges of the cube that they see only through the transparent paper. If the transparent faces were made of paper, would they see these edges? No. The edges that would be invisible if all the faces were non-transparent are called the “hidden edges”. On a two-dimensional drawing of a cube these hidden edges are marked with dotted lines.

**ACTIVITY**

Ask students to hold their cubes in various positions (on the table, on the floor looked at from above, slightly above them and so on.) Ask students to describe what the faces look like when seen from different angles (they look like square, parallelogram, etc). The outline of the shape itself can look like a square, a rectangle, a hexagon, a trapezoid and a rhombus.

**Extension**

The following exercise was adapted from the Atlantic Curriculum.

Suggest that your students make models of various prisms and pyramids with play dough or modelling clay. Start with a model of a cube. Ask your students to cut off a single vertex of the model with piano wire or a plastic cutter.

![Cube with vertex cut off]

Explain that the new face that appeared is called the **cross-section**. What shape does the cross section have? Ask your students: How many faces meet at a vertex of a cube? How many edges? (3) How many sides does the cross-section have? How many vertices? Why? (Each face of the cube produces an edge of the cross-section. Each edge of the cube produces a vertex of the cross-section.)

Ask your students to predict the shape of the cross-sections (resulting from cutting off vertices) for other shapes, first prisms, and then pyramids. For pyramids, first ask your students if they expect that all vertices will produce the same cross section, or there might be exceptions? (The point of the pyramid produces the shape with the same number of sides as the base, whereas the other vertices produce triangles.)
Prisms and Pyramids

GOALS
Students will identify and count faces, edges and vertices of 3-D shapes and their drawings.

PRIOR KNOWLEDGE REQUIRED
Faces, edges, vertices

VOCABULARY
3-D shape
edge
vertices
prism

face
vertex
pyramid
cube

Divide your students into groups. Give each group several 3-D shapes, so that each group has some rectangular and triangular pyramids, rectangular and triangular prisms and a cube. Ask your students to count the faces of the shapes. If some students are having trouble keeping track of the number of faces, they might mark each face with a chalk dot or a small sticker. Ask your students to count the edges and vertices on the 3-D shapes as well (they also might shade edges with chalk and mark vertices with stickers.) Ask them to write the results of their count in the table on the worksheet.

Draw a pentagonal pyramid and a triangular prism on the board and let volunteers count the edges, faces and vertices of these figures. Ask them to mark the edges and circle the vertices as they count.

Extensions

1. a) Pick 3 shapes and trace each of their faces (e.g. if you picked a square-based pyramid, trace all 5 of the faces on the pyramid—even if some of them are congruent). Compare the number of faces in your tracings with those in your chart from QUESTION 1: do you have the correct number of faces? Be sure to organize your work neatly so you can tell which faces go with which shapes.

   b) Underneath the faces for each shape, answer the following questions:

   (i) How many different-shaped faces does this 3-D shape have? What are they?

   (ii) Circle the face (or faces) that form the base of the 3-D shape. Copy and complete this sentence:

   “The base of a ________________ is a ______________.”

2. Ask your students to try and add the number of faces and vertices of a cube and subtract the number of edges. The result is 2. What happens if you do that to another solid? (It will be 2 as well. This fact is known as Euler’s formula and was discovered by the great Swiss mathematician Leonard Euler in the 18th century.)

ACTIVITY 1

Show your students an example of a cone and a cylinder. Explain that a cone has one curved surface and one flat surface, while a cylinder has two flat surfaces and one curved surface. Ask students to find as many examples of pyramids, prisms, cones, and cylinders in the classroom as they can.
3. Let your students construct shapes from Polydrons—from regular 3-D shapes like prisms and pyramids to animals and castles. Count the faces, edges and vertices. Check if the Euler’s formula holds.

4. This exercise was adapted from the Atlantic Curriculum.

Suggest that your students make models of various prisms and pyramids with play dough or model clay. Start with a model of a cube. SAY: I want to make a cross-section of a cube, but this time I will cut off a single edge of the model. Ask your students to predict the shape of the cross-section that is parallel to an edge. Let them check the prediction by actual cutting their models with piano wire or plastic cutters.

![Diagram of a cube with a cross-section](image)

Ask your students to explain why the cross section has four sides. CHALLENGING: Why is it rectangular? (In the diagram, the edge that was cut off was perpendicular to face E. If the cross-section is parallel to the edge that was cut off then edges a and b will also be perpendicular to face E, and also to any line that lies in face E, such as line c. Hence lines a and b are perpendicular to line c (and by the same argument to line d).

5. Ask your students to predict what shape the cross-section of a cube will have if it is made by a plane parallel to one of the faces of the cube. Let your students check their predictions. Invite them to make cross-sections using various other planes, such as planes through various diagonals of the cube. Do these planes produce different shapes? What different quadrilaterals can they produce? (Rectangles, squares, trapezoids, parallelograms and rhombuses.)
G5-34
Prism and Pyramid Bases

GOALS
Students will distinguish between prisms and pyramids, and identify their bases.

PRIOR KNOWLEDGE REQUIRED
Pyramid
Prism
Geometric shapes: triangle, rectangle, quadrilateral, pentagon, hexagon

VOCABULARY
triangle rectangle
quadrilateral pentagon
hexagon prism
pyramid base
point

Hold up a pentagonal pyramid. Ask a volunteer to count the faces. What shapes are they? (A pentagon and five triangles.) Ask a volunteer to draw a pentagon and a triangle on the board. Ask another volunteer to write the number of faces that have each shape inside the shape itself.

Tell your students that in the pentagonal pyramid the face that is not the same shape as the others is the base. Hence in this pyramid a pentagon is the base.

In a pyramid there is always one base, unless the pyramid has all triangular faces (in which case it must be a triangular pyramid). The shape of the base of a pyramid gives the pyramid its name. Hence if a pyramid has a base with 5 sides, it is called a “pentagonal pyramid”. A prism has two bases. The non-base sides of the prism are always rectangles.

Give your students a set of 3-D shapes (you can use the shapes from your collection or ask students to construct shapes using the nets from the “Nets for 3-D Shapes” BLM). Ask your students to place each shape on a piece of paper and trace the faces. Ask them to write the number of faces of the traced shape inside the shape the way you did on the board. Students should also write the name of the figure beside the base. If your students have trouble spelling the names of the figures, write them on the board. Later include them into a spelling test.

Assessment
1. Use three shapes for each student, each set should include a pyramid and a prism, and one shape with all faces congruent, like a cube or a triangular pyramid with equilateral faces. A good set: a pentagonal pyramid, a triangular prism and a cube.
   a) Place each shape—base downward—on a piece of paper and trace the base. (That way you can verify that each student knows how to find the base.)
   b) Write the name of the figure beside the base and indicate whether the figure has one or two bases.
   c) If all faces of the figure are congruent, indicate this.
2. Look at the shapes in QUESTION 3 of the worksheet. Which shapes are pyramids? Which shapes are prisms? Can there be a shape that is both prism and pyramid? Why not?
GOALS

Students will compare pyramids and prisms systematically. They will also describe the similarities and differences between polyhedra, cylinders and cones.

PRIOR KNOWLEDGE REQUIRED

Prism and pyramid
Counting faces, edges, vertices of polyhedron
Bases of prism and pyramid

VOCABULARY

cylinder  cone
prism    pyramid
vertex   vertices
edge    face
base  net

Give your students a collection of 3-D shapes or have them make a set of shapes using the “Nets for 3-D Shapes” BLM. Ask your students to order the shapes (pyramids and prisms separately) so that the number of edges in the base increases. After that they should fill in the chart:

<table>
<thead>
<tr>
<th>Picture of Base</th>
<th>Number of...</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>edges</td>
<td>vertices</td>
</tr>
</tbody>
</table>

Ask the students if they can see any patterns in the number of edges, vertices and faces of pyramids and prisms.

Look for the pattern rules in the chart not only vertically, but horizontally:

**FOR PYRAMIDS:** What do you have to do to the number of vertices in the base to get the number of vertices in the whole pyramid? (Add 1.) What do you have to do to the number of edges in the base to get the number of edges in the whole pyramid? (Multiply by 2.)

**FOR PRISMS:** What do you have to do to the number of vertices in the base to get the number of vertices in the whole prism? (Multiply by 2.) What do you have to do to the number of edges in the base to get the number of edges in the whole prism? (Multiply by 3.) What do you have to do to the number of edges in the base to get the number of faces in the whole prism? (Add 2.)

Ask your students to add a row to each chart and to fill it in so you can see if they can extend the patterns in the chart.

**ASK:** If you have a prism with 100-gon in the base, how many vertices does this prism have?

How many faces does a pyramid with 200-sided base have? And how many vertices?

**Bonus**

Fill in a row of your chart (no need to draw the base) for 1000-gon pyramid and prism.
Ask your students to draw the following chart in their notebooks and to fill it in, using the shapes they used for the previous exercise:

<table>
<thead>
<tr>
<th>Property</th>
<th>Rectangular Pyramid</th>
<th>Triangular Pyramid</th>
<th>Same?</th>
<th>Different?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of faces</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shape of base</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of bases</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of faces that are not bases</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shape of faces that are not bases</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of edges</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of vertices</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Show your students how you can make a cone from a piece of paper. Ask them where they have seen this shape (cone of ice-cream, clown hat, etc). Does a cone remind some other geometric shape that they have studied in the lesson? (a pyramid) Let volunteers fill in the following chart on the board:

<table>
<thead>
<tr>
<th>Property</th>
<th>Pyramid</th>
<th>Cone</th>
<th>Same?</th>
<th>Different?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of faces</td>
<td>many</td>
<td>two</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Shape of base</td>
<td>polygon</td>
<td>circle</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Number of bases</td>
<td>1</td>
<td>1</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of faces that are not bases</td>
<td>many</td>
<td>1, curved</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Number of edges</td>
<td>many</td>
<td>1, curved</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Number of vertices</td>
<td>many</td>
<td>1</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Has a vertex opposite to the base</td>
<td>yes</td>
<td>yes</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td># of vertices = # of vertices in the base + 1</td>
<td>yes</td>
<td>yes</td>
<td></td>
<td>✓</td>
</tr>
</tbody>
</table>
Ask your students to imagine a pyramid with 1 000-sided base. Does it look like a cone? Write a comparison paragraph on the board:

- **SIMILARITIES:** Both shapes have one base and a vertex at the opposite end. A cone is like a pyramid with a circular base. The more sides the base of a pyramid has, the nearer it is to a circle and so the nearer the pyramid is to a cone.

- **DIFFERENCES:** Pyramid has many faces, one polygonal (base), and the other triangular. It has many edges that are straight lines. It has many vertices, not just the one at the point. A cone has only one flat face, that is a circle, and one curved “face”. It has only one curved “edge”.

Show your students how to make a cylinder from a piece of paper. Ask volunteers to fill in the comparison chart:

<table>
<thead>
<tr>
<th>Property</th>
<th>Prism</th>
<th>Cylinder</th>
<th>Same?</th>
<th>Different?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of faces</td>
<td>many</td>
<td>3</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Shape of base</td>
<td>polygon</td>
<td>circle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of vertices in the base</td>
<td>many</td>
<td>0</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Number of bases</td>
<td>2</td>
<td>2</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of faces that are not bases</td>
<td>many</td>
<td>1, curved</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Number of edges</td>
<td>many</td>
<td>2, curved</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Number of vertices</td>
<td>many</td>
<td>0</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td># of vertices = 2 x # of vertices in the base</td>
<td>yes</td>
<td>yes</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td># of faces = # of edges in the base + 2</td>
<td>yes</td>
<td>yes</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

Ask your students to imagine a prism with 100-sided base. Does it look similar to a cylinder?

Write a comparison paragraph on the board:

**SIMILARITIES:** The cylinder and the prism both have two bases. A cylinder is like a prism with a circular base. The more sides the base of a prism has, the nearer the base is to a circle. The nearer the base is to a circle, the nearer the prism is to a cylinder.

**DIFFERENCES:** A prism has many faces, two polygonal (bases), and the other rectangular. It has many edges that are straight lines. It has many vertices. A cylinder has only two flat faces, that are circles, and one curved “face”. It has only two curved “edges” and no vertices at all.
After doing the exercise of comparing cones and cylinders to pyramids and prisms your students should have a better idea how to answer the question on the worksheet that asks them to compare two 3-D shapes.

Students may try the first activity after that.

**Assessment**

1. Make a property chart for the rectangular and triangular prisms.
2. Write a paragraph comparing rectangular and triangular prisms using the chart.
3. Who am I?
   - a) I have only rectangular faces.
   - b) I have 8 faces, 6 of them are rectangles.
   - c) I have 6 edges. No pyramid or prism has fewer vertices than I do!
   - d) I am a prism with 9 edges.
   - e) I have one circular base.
4. Which of these are right prisms?

---

**ACTIVITY 1**

**A Game in Pairs**

One player gives a description of a shape; the other has to name the shape. **ADVANCED:** the player gives only two numbers of the three possibilities: number of edges, vertices and faces. **EXAMPLE:** 12 edges and 6 faces: a rectangular prism. 12 edges and 7 faces: a hexagonal pyramid. Your students might need a table of the number of faces, edges and vertices they made building prisms and pyramids.

---

**ACTIVITY 2**

This activity satisfies one of the demands of the Ontario and Atlantic Curricula.

Place two skeletons of triangular prisms on a table (see lesson G5-31). Push the sides of one of the prisms so that the entire prism is tilted to one side. Ask students to compare the number of faces, edges and vertices of the two prisms. Students should see that some of the faces of the new prisms are parallelograms rather than rectangles. Explain to your students that the original figure is called a **right prism** because the angles between the base and the vertical edges are right angles. Students can also create prisms that are not right prisms using the nets provided in the “Skew Prisms” BLM. For assessment, provide your students with a collection of prisms, some of them right prisms and some of them not, and ask your students to sort the shapes into right prisms and not right prisms.
Extensions

1. Word Search Puzzle (3-D Shapes): Please refer to the BLM.

2. Sketch a skeleton of a prism or pyramid. For EXAMPLE:

![Example Diagrams]

Ask students to answer true or false to the following questions:

- My bases are all triangles.
- I have more vertices than edges.
- All of my faces are congruent

Ask more questions of this sort.

3. Select a prism and say how many pairs of parallel edges each face has.

4. Repeat the exercise from Extension 5 of the lesson G5-33 with a prism that is not a right prism. What is the shape of the cross-section parallel to an edge of the prism that joins the bases? Do different edges produce different cross-sections?
**G5-36**

**Nets and Faces**

**GOALS**

Students will create nets for pyramids, prisms and cubes.

**PRIOR KNOWLEDGE REQUIRED**

Geometrical shapes: triangle, square, rectangle, pentagon, hexagon
Faces, edges, vertices of a 3-D shape

**VOCABULARY**

<table>
<thead>
<tr>
<th>net</th>
<th>vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>face</td>
<td>pyramid</td>
</tr>
<tr>
<td>edge</td>
<td>prism</td>
</tr>
<tr>
<td>vertex</td>
<td>cube</td>
</tr>
<tr>
<td>triangular</td>
<td>pentagonal</td>
</tr>
</tbody>
</table>

Draw a copy of the charts your students made in the previous lesson. Ask students if they see any relationships between the number of vertices, edges or faces in a prism or pyramid and the shape of the base. Ask your students what patterns they see, and ask them to give geometrical reasons for these patterns. For **EXAMPLE**, the number of vertices in a pyramid is one more than the number of vertices in the base, and the number of edges in a pyramid is twice the number of edges in the base. You might prompt your students to think about the construction of a pyramid to see if they can explain these relationships.

Pyramids are constructed by connecting all vertices in the base to an additional vertex at the top of the pyramid. Hence the number of vertices in a pyramid is one more than the number of vertices in the base. Each pyramid has two types of edges—the base edges and the non-base edges. The number on non-base edges is equal to the number of base vertices (since each base vertex is connected to the vertex at the top of the pyramid by a non-base edge). But the number of base vertices is equal to the number of base edges. Hence the total number of edges in the pyramid is twice the number of edges in the base.

The number of vertices in a prism is twice the number of vertices in the base, because all the vertices belong either to the top base or to the bottom base. Each prism has three types of edges—the edges in the top base, the edges in the bottom base and the edges that connect the vertices in the top with the vertices in the bottom. The top and bottom bases are congruent, so they have the same number of edges. The connecting edges join each vertex in the bottom with a single vertex in the top. Hence the number of connecting edges is equal to the number of edges in each base (in either the top or bottom base), so the total number of edges in the prism is three times the number of edges in the base. The number of faces in the prism is two more than the number of edges in the base, because the number of side faces of the prism is the same as the number of edges in the base, (then you add two for the top and bottom base).

Draw several shapes on the board and ask your students which 3-D shape they make.

**SAMPLES:**

![Shapes](image-url)
Hold up a rectangular or a pentagonal pyramid. Ask your students to tell you which two types of faces it has. How many bases does it have? And what is the shape of the side faces? How many side faces does it have? If you need to make a net for this pyramid, the easiest way would be to start with a base (draw it on the board and write “base” on it) and to add a side face along each edge of the base (draw one side face and ask volunteers to draw the rest). Ask students to cut out the nets of the pyramids from the BLM “Nets for 3-D Shapes” in this manual—they can use the shapes later to help find the answers to QUESTION 1 on the worksheet.

Ask your students, what happens if you cut off one of the side faces and try to re-glue it at some other place. They might actually cut off one of the triangles and try to fit the triangle to some other edge. You might draw an example on the board. Try the following positions: will the shape fold into a pyramid if the face is glued in this position?

- (no)
- (yes)
- (no)

At this point, you may let your students do the first activity. When students have finished the activity, draw the picture below on the board and ask if this will work as the pentagonal pyramid net:

- (yes)

Invite volunteers to come up to the board and draw pictures that will not make a net for a pyramid (not necessarily pentagonal). For each drawing, ask your students to explain why this picture is not a net of a pyramid. Invite volunteers to change the drawing so that it will make a net.

Hold up a triangular or a pentagonal prism. Ask your students, which two types of faces it has. How many bases does it have? What is the shape of the side faces? How many side faces does it have? If you wanted to make a net for this prism, the easiest way would be to start with the band of rectangles for side faces and to add the bases. Illustrate this on the board.
Repeat the exercise of cutting off a face of the prism and attaching it in some other place. Do that separately for a base and a side face. Ask students to cut out the nets of the prisms from the BLM “Nets for 3-D Shapes”. Let them cut off the faces and try to rearrange them at other places.

Draw several examples of “nets” of triangular prism on the board and ask volunteers to explain why these drawings cannot serve as nets for prisms:

![Nets for Triangular Prism](image)

- The bases are not the same
- The middle face is too short
- The bottom base is flipped
- Side face is missing

Invite volunteers to draw more pictures that will not work as prism nets, and ask the class to guess why these drawings cannot be prism nets. For a more challenging task, ask volunteers to draw pictures that might or might not work as nets, and let the class guess if these are nets of prisms.

**ACTIVITY 1**

Give students square or pentagonal pyramids and ask them to trace the faces on a piece of paper, so that they create a net. Ask them to cut out the nets they have drawn. Let them cut off faces of the net and re-attach the faces at different places. Will the new net fold into the same pyramid? Which edges are places where you would want to re-glue the faces and which are not? Repeat this exercise with a prism. This activity is important—students will explore various ways to create nets for the same solid, rather than memorizing a single net shape.

**ACTIVITY 2**

Allow your students to play the following game: one partner (or group) draws a net, the other has to guess what shape the net can be folded into. (Use the nets they have created in the previous activity.)

**ACTIVITY 3**

Give your students Pentamino pieces made of paper or bristol board. Which ones can be folded into a square box without a lid? Let them first try to predict the result, then check it. This activity is a good preparation for Extension 1.
ACTIVITY 4

Build-a-Net Game:
Divide your class into groups of 3 to 4 students. Give each group a set of polygons from the BLM “Build-a-Net Game”. Each student receives 6 shapes, the rest are left in a pile on the side. If a student has a set of polygons that form the net of a pyramid or a prism, she is allowed to turn the set in for a point and take an equal number of new cards from the side pile. Students take turns placing polygons in the central pile. Each time a student places a polygon in the central pile, they pick up a new shape from the side pile. As soon as a student places a polygon in the central pile that can be combined with other polygons in the pile to make a net, that player is allowed to remove the net and scores a point. The game ends when there are no cards left in the side pile. The winner is the player who has created the greatest number of nets.

NOTE: Review the nets of prisms and pyramids that can be built from the shapes in the “Build-a-Net Game” BLM before starting the game. Ask your students: Which shapes can only be used for one or two nets? (pentagon and hexagon) Which shapes can be used for more than two nets?

ACTIVITY 5

After students have constructed the pyramids and prisms from the BLM “Nets for 3-D Shapes”, ask them to sketch the nets from memory. You might also ask them to sketch what they think the net for a hexagonal pyramid would look like.

ACTIVITY 6

Predict whether the net shown will make a pyramid. Copy the shape onto grid paper. Cut the shape out and try to construct a pyramid. Was your prediction correct?

ACTIVITY 7

Sketch the net for a prism by rolling a 3-D prism on paper and tracing its faces. For reference, use the steps shown (at right) for a pentagonal prism:

- Step 1: Trace one of the non-base faces.
- Step 2: Roll the shape onto each of its bases and trace the bases.
- Step 3: Roll the shape onto each of its remaining rectangular faces and trace each face.
ACTIVITY 8

Find a prism or pyramid at home or at school and sketch its faces.

The next two activities fulfil the demands of the Ontario Curriculum.

ACTIVITY 9

Does this make a net of a 3-D shape? Cut out and check your prediction.

(no)

ACTIVITY 10

What shapes do these nets make? Cut them out and check.

Extensions

1. Copy the following nets onto centimetre grid paper (use 4 grid squares for each face).

   Predict which nets will make cubes. Cut out each net and fold it to check your predictions.

   After that draw as many different nets for a cube as you can.
2. Give your students many toothpicks of various lengths and some modelling clay. Ask them
to make several triangular pyramids with different bases. For example, students might make
acute-angled, right-angled and obtuse-angled triangle bases. Then ask your students to choose
two different triangular pyramids and trace their faces to obtain nets for them. Ask your students
to compare the nets. Students might also measure the angles of the faces with protractors, and
the sides with rulers. Ask your students to make a triangular pyramid with...

   a) …more than one right angle.
   b) …at least two obtuse angles.

   ADVANCED:

   c) …two right angles and one obtuse angle.
   d) …two right angles and two obtuse angles.

3. Copy the shapes into your notebook. How many hidden faces does each shape have? How
many hidden edges does each shape have? Draw in the missing edges with dotted lines.

4. How many faces of the little cubes are on...

   a) the outside of the figure? (including the hidden faces)
   b) the interior of the figure?

5. On a cube draw a line from vertex A to B, then a line from B to C. What is the measure
of ABC?

   HINT: Imagine drawing a line from vertex A to C on the hidden face. What kind of triangle
is \( \triangle ABC \)?

   ANSWER: \( \triangle ABC \) is an equilateral triangle. So \( \angle ABC = 60^\circ \)

6. Each edge of the cube was made with 5 cm of wire.

   How much wire was needed to make the cube? (Don’t forget the hidden edges.)
GOALS

Students will sort 3-D shapes according to their properties.

PRIOR KNOWLEDGE REQUIRED

Properties of pyramids and prisms
Venn diagrams

VOCABULARY

- cylinder
- cone
- prism
- pyramid
- base
- vertex
- vertices
- edge
- face

Give each student (or team of students) a deck of shape cards and a deck of property cards. These cards are in the “3-D Shape Sorting Game” BLM. (If you have enough 3-D shapes have students use 3-D shapes instead of the cards.) Let them play the following games:

3-D Shape Sorting Game: Each student flips over a property card and then sorts the shapes onto two piles according to whether a shape on a card has the property or not.

Students get a point for each card that is on the correct pile. (If you prefer, you could choose a single property for the class and have everyone sort the shapes using that property.)

Once students have mastered this sorting game they can play the next game.

3-D Venn Diagram Game: Give each student a copy of the Venn diagram sheet in the BLM section (or have students create their own Venn diagram on a sheet of construction paper or bristol board). Ask students to choose two property cards and place one beside each circle of the Venn diagram. Students should then sort their shape cards using the Venn diagrams. Give 1 point for each shape that is placed in the correct region of the Venn diagram.

Assessment

Use the shapes below to complete the following Venn diagram:

1. One or more rectangular faces
2. One or more triangular faces

Extension

Draw a Venn diagram to sort the shapes on the worksheet according to the properties:

1. Pyramid
2. One or more triangular faces.

What do you notice about your Venn Diagram? Explain why part of one of the circles is empty.
G5-38
Tessellations

GOALS
Students will identify polygons that tessellate. They will create tessellation patterns using a variety of shapes.

PRIOR KNOWLEDGE REQUIRED
Degrees
Sum of angles in a triangle is 180°
A straight angle has measure 180°
Polygons

VOCABULARY
polygon vertex quadrilateral
vertices hexagon octagon
tessellate degree

Explain to your students that a tessellation is a pattern made up of one or more shapes that completely covers a surface, without any gaps or overlaps. One example of a tessellation is a floor tiling with polygons (or other shapes). If you can tile the floor with one shape, this shape is said to tessellate. Ask your students: Do you know of any shape that tessellates? Your students will almost certainly say "square", but they might also name other shapes, such as rectangles (ask them to draw a picture: there is more than one way to tessellate with rectangles), diamonds, other parallelograms, triangles or hexagons. Show a picture of a beehive. Is it a tessellation? If there are any interesting tessellation patterns around the schools, mention them as well. Use the first two activities.

Present a problem:

Joshua says that he can tile the floor of the bathroom using only regular octagons. Is this correct?

Josef says that he can tile the floor using only regular pentagons. Is this correct? Do they encounter the same problem?

Let your students use paper pentagons, hexagons and octagons to check whether these figures tessellate. Ask volunteers to sketch tessellations with triangles, squares and hexagons on the board.

Ask your students: what is the degree measure of a straight angle? Draw a straight angle and ask what the degree measure around the vertex is. There are two straight angles, so there are 360° around the point. Draw a picture of three line segments shaped like the letter Y and ask your students what the sum of the angles around the vertex should be. If the three angles in the Y are the same, what is the measure of each angle?

Draw a table on the board with the following headings (first five columns) and the shapes below (add the rightmost column later, as part of the discussion outlined below). Use as many volunteers as possible to fill in the table:
### Regular Polygon Table

<table>
<thead>
<tr>
<th>Regular polygon</th>
<th>Number of vertices</th>
<th>Number of triangles</th>
<th>Sum of angles</th>
<th>Degree measure of one angle</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td>1</td>
<td>180°</td>
<td>60°</td>
<td>6 \times 60° = 360°, 6 shapes tessellate</td>
</tr>
<tr>
<td>Square</td>
<td>4</td>
<td>2</td>
<td>360°</td>
<td>90°</td>
<td>4 \times 90° = 360°, 4 shapes tessellate</td>
</tr>
<tr>
<td>Pentagon</td>
<td>5</td>
<td>3</td>
<td>540°</td>
<td>108°</td>
<td>3 \times 108° = 324°, 4 \times 108° = 432°, does not tessellate</td>
</tr>
<tr>
<td>Hexagon</td>
<td>6</td>
<td>4</td>
<td>720°</td>
<td>120°</td>
<td>3 \times 120° = 360°, 3 shapes tessellate</td>
</tr>
<tr>
<td>Regular octagon</td>
<td>8</td>
<td>6</td>
<td>1080°</td>
<td>135°</td>
<td>2 \times 135° = 270°, 3 \times 135° = 405°, does not tessellate</td>
</tr>
</tbody>
</table>

Ask your students to identify the rules for the sequences they see in the columns of the table. You might wish to ask them to write a formula for the sequences in the second, third and fourth columns. Ask them to use the formulas to add a line for the regular octagon to the table.

Ask your students which polygons tessellate and how many polygons meet at a vertex in each tessellation. **ASK:** Could you predict that 6 equilateral triangles are needed to fill the space around a vertex from the data in the table? Which column would you use? Can you write a multiplication statement that explains why 6 equilateral triangles around a vertex leave no gaps and do not overlap? Add the sixth column and fill in the information about triangles. Repeat with squares and hexagons.

**ASK:** What happens when you try to arrange a set of pentagons around a vertex? Ask students what will happen if they try to place three or four pentagons around a vertex. Students should see that the sum of the interior angles of three pentagons arranged around a vertex is 324°. Since this is less than 360° the three pentagons will not completely fill the space around the vertex. Four pentagons will not tessellate the space around the vertex either, since the sum of the four interior angles is 432°, which is greater than 360°, so one of the pentagons will overlap the others.

Ask your students: Have you ever seen floor patterns made with shapes that are not regular polygons? Or patterns made with several kinds of tiles?

Draw a grid and the following pattern on the board (or use an overhead projector) and ask your students to extend the design:
Is this the only way to tessellate with this shape? (No, you can also use a reflection, a rotation or a combination of two or three transformations.)

Suggest your students to draw various letters of alphabet on grid paper and to check whether they tessellate.

**Bonus**

Suggest that your students create a design using regular octagons and squares with equal sides on grid paper. **HINT:** Two regular octagons fill 270° around a vertex. How many degrees do you need to make the full 360°? What shape has this angle?

---

**ACTIVITY 1**

John says that he can tile the floor of the bathroom with quadrilaterals. Any quadrilaterals!

Fold a sheet of paper three times, so that there are eight layers; draw a quadrilateral on it and cut it out, cutting through all the 8 layers. Try to arrange the 8 quadrilaterals so that they do not have gaps and do not overlap to check if John is right. Check various quadrilaterals, not only the special ones!

---

**ACTIVITY 2**

Repeat the activity above with triangles. Note that two congruent triangles create a quadrilateral.

---

**ACTIVITY 3**

Divide your students into groups and give each group one of the sets of shapes listed below. Ask them to create as many different tessellations as possible using one or more shapes from their set. Let the students share their designs when they are finished. You might ask them to construct the shapes using a ruler and a protractor as well.

- Set 1: A square and a regular octagon. Side length 5 cm for both.
- Set 2: A rhombus with angles 60° and 120° and an equilateral triangle. Side length 5 cm for both.

*(Continued on next page.)*
(Continued from previous page.)

- Set 3: An isosceles trapezoid with sides and smaller base of 5 cm and base angles of 135° and a square with side of 5 cm.
- Set 4: An isosceles trapezoid with sides and smaller base of 5 cm and base angles of 60° and an equilateral triangle with side of 5 cm.
- Set 5: A regular hexagon and an equilateral triangle. Side length 5 cm for both.
- Set 6: A rectangle with sides of 5 and 10 cm, and a square with side length of 5 cm.

**Extension**

**PROJECT:** Research the designs by M. C. Escher, choose a drawing and check:
- Which polygons were used to create a tessellation?
- Which transformations were used to create the tessellation?

**POSSIBLE SOURCE:**
http://www.mcescher.com/
GOALS
Students will perform simple transformations on patterns. They will create patterns that look the same after various transformations.

PRIOR KNOWLEDGE REQUIRED
Clockwise, counter clockwise
\(\frac{1}{2}, \frac{1}{4}, \frac{3}{4}\) turn
Reflection
Mirror line
Lines of symmetry

VOCABULARY
pattern
clockwise
counter clockwise
\(\frac{1}{2}, \frac{1}{4}, \frac{3}{4}\) turn

Review the meaning of the terms clockwise and counter clockwise. Draw several arrows at various positions and ask your students to draw the final position after \(\frac{1}{2}\) turn clockwise, \(\frac{1}{4}\) turn counter clockwise, \(\frac{3}{4}\) turn clockwise, \(\frac{3}{4}\) turn counter clockwise.

Draw a \(2 \times 2\) grid on the board and make a simple pattern on the grid, such as:

```
    ●●
```

Ask your students to draw this pattern on grid paper and to cut it out. Ask them to turn the design \(\frac{1}{2}\) turn clockwise around a given vertex and to draw the result in a separate grid. Repeat with several other turns.

Show your students a method to draw the result without the prop. Start with a simple \(2 \times 2\) grid like the one shown, mark the top-left corner with a dot and shade the top side. **ASK:** Where will the dot be after \(\frac{1}{4}\) clockwise turn? Invite a volunteer to draw the dot on a separate grid, then ask where the shaded line will go and invite another volunteer to draw it. After that ask a volunteer to shade the squares after the turn. Draw several simple designs on \(2 \times 2\) and \(3 \times 3\) grids and ask your students to draw the results of various turns using this method.

An alternative method to form mental pictures of rotations of figures:

**STEP 1:** Shade some of the squares on a 3 by 3 grid.

```
    ●●●
   ●●●
 ●●●
```

**STEP 2:** Imagine rotating the grid by a certain amount around any of the vertices; say, \(\frac{1}{4}\) turn counter clockwise. Mark the side of the grid that would be on top after the rotation. (Students should notice that the same side will be on top no matter which of the vertices they use as the centre of rotation.)

```
    ●●●
   ●● ●
 ●●●
```

If students have trouble with this step, have them practice it with different directions and sizes of rotations.

**STEP 3:** Show what the figure would look like if the side marked with an \(\times\) was on top.

```
    ●●●●
   ●●●●
 ●●●●
```

```
    ●●●●
   ●●●●
 ●●●●
```

```
    ●●●●
   ●●●●
 ●●●●
```

```
    ●●●●
   ●●●●
 ●●●●
```
(You can turn your head when you look at the original shape to see what it would look like if the side marked with an × was on top.)

Here is a set of SAMPLE QUESTIONS you could assign to help with this step. In each question, show what the grid would look like if the side marked with an × was on top.

a)  

b)  

c)  

d)  

e)  

f)  

For variation you could ask what the shape would look like if the marked side was rotated to various positions, for instance:

Draw a 3 × 3 design, like the one in the picture below and ask your students to reflect it in each of the sides.

Which results look the same? Repeat with several designs, including ones with horizontal, vertical or both lines of symmetry. Include also a design that does not have a line of symmetry.

Ask your students to draw lines of symmetry in each of the designs they used that have lines of symmetry. You can suggest that your students make a table with headings:

<table>
<thead>
<tr>
<th>Design</th>
<th>Has a horizontal line of symmetry</th>
<th>Has a vertical line of symmetry</th>
<th>Does not change after reflection in a horizontal line</th>
<th>Does not change after reflection in a vertical line</th>
</tr>
</thead>
</table>

Ask your students if they see any relations in the columns. Then ask them to draw a design that would look the same after a reflection in both a horizontal line and a vertical line.

For a challenge, ask them to draw a design that looks different in a vertical or a horizontal reflection, but such that the results of both reflections are the same. You might suggest that students should look at shapes that have lines of symmetry that are not horizontal or vertical. (ANSWER: A design that does not have a vertical or a horizontal line of symmetry, but has both diagonals as lines of symmetry.)
Ask your students to make a design that will look the same after a $\frac{1}{2}$ turn. Invite volunteers to draw their designs on the board. Ask your students to draw lines of symmetry in their designs.

Then ask the following question: Suppose a design does not change after a $\frac{1}{2}$ turn. Does it mean it has a line of symmetry? If students cannot find an example of a shape that looks the same after a half turn, but does not also have a line of symmetry, draw the following EXAMPLE:

![example diagram]

Challenge students to find a pair of squares such that the first square will be rotated onto the second and the second onto the first. Students can create more complicated solutions by finding other pairs of squares that are the images of one another under rotation.

Ask your students to make some $5 \times 5$ designs that stay the same after a $\frac{1}{4}$ turn, where not every square is shaded. (Note that it does not matter whether the turn is clockwise or counter clockwise. If the pattern stays the same after a $\frac{1}{4}$ turn in one direction, it will stay the same after a $\frac{1}{4}$ turn in the other direction.)

Students might also shade a corner square and shade successive images of the square after a $\frac{1}{4}$ turn. After they have done this three times they will have shaded all the corner squares and will have a pattern that does not change.

If students have trouble finding a solution you might suggest the following method:

Start by shading a square and ask where it will be located in the $5 \times 5$ grid after a $\frac{1}{4}$ turn. If we want our design to stay the same, then the square that is the image of the original square should be shaded in the pattern as well. However, this square will not stay in its place—it will move. Where to? Ask a volunteer to shade the place where the second square is located after the rotation. Repeat shading squares until you get the original square shaded.

**Assessment**

1. Circle the designs that do not change after $\frac{1}{2}$ turn. Put an $\times$ through the designs that do not change after $\frac{1}{4}$ turn.

![designs for assessment]

2. Choose a design that was not circled or crossed and reflect it through a horizontal line. Then turn the result $\frac{1}{4}$ turn clockwise.

3. Create your own $3 \times 3$ design that stays the same after a reflection in a vertical line.

4. Create your own $4 \times 4$ design that stays the same after $\frac{1}{2}$ turn.

**Extension**

How many lines of symmetry can be in a square pattern that does not change after $\frac{1}{2}$ turn? (0, 2 or 4). Try to create a $3 \times 3$ pattern with 1 line of symmetry that does not change after $\frac{1}{2}$ turn. Shade a square on one side of the line. Shade its image after the rotation. Shade the images of both after reflection through the line of symmetry. How many lines of symmetry does the resulting shape have? Repeat with $5 \times 5$ grid, start by shading the square with coordinates (4, 5).
G5-40
Patterns with Transformations (Advanced)

GOALS
Students will create patterns with transformations.

PRIOR KNOWLEDGE REQUIRED
Clockwise, counter clockwise
\(\frac{1}{2}, \frac{1}{4}, \frac{3}{4}\) turn
Reflection
Mirror line

VOCABULARY
pattern
clockwise
counter clockwise
\(\frac{1}{2}, \frac{1}{4}, \frac{3}{4}\) turn

Hold up a square grid (with a pattern of shaded squares) like one of those you used last lesson. Pin the square to a piece of bristol board with a single pin (not in the middle) and trace it. Explain to your students that you want to turn the square \(\frac{1}{4}\) turn so that it returns to its original position. Remind your students that the place where the pin is stuck is called the centre of rotation. Ask a volunteer to trace the position of the square after the rotation. Repeat this exercise several times. Students should see that the square will not return to its original position after a rotation unless the centre of rotation is the centre of the square.

Now place the pin in the corner of the square and ask students to imagine the image after a \(\frac{1}{4}\) turn. Shade one of the sides of the square passing through the centre of rotation and invite a volunteer to shade its image after rotation. What is the angle between the shaded lines? (90°) Then invite volunteers to draw the image of the rotated square. Repeat with more complicated designs, with triangles and various colours or even striped patterns.

Ask your students to draw a \(3 \times 3\) square on grid paper with an asymmetric design. Ask them to create a larger design by rotating the square repeatedly counter clockwise around the top-right corner and drawing the result.

Draw a simple asymmetric shape, like the one below:

Ask your students to create a pattern by: sliding the shape repeatedly 2 units right, reflecting the shape repeatedly in a line through the right side or rotating it half-turn (how many degrees is that?) around the marked point.

As a bonus question, ask your students to create a pattern using two transformations, say, first a reflection, then a \(\frac{1}{2}\) turn, then repeat.

ACTIVITY 1
Have students answer questions 1, 2 and 4 by making designs on grid paper, cutting them out and turning them.
Extensions

1. Mark the centres of rotation to get:

- Shape B from Shape A (Note that the centre will be in the middle of the bottom edge of A, not on a vertex)
- Shape C from Shape A
- Shape D from Shape B
- Shape D from Shape C

2. Which transformation was used to get Shape D from Shape A?

3. Vinijaa moved shape A to Shape C without using a rotation. Describe what she did. If she used reflection—draw the mirror line, and if she used a slide—show the translation arrow and describe the slide.

4. Draw reflection of shape A through the right (slanted) side.
Isoparametric Drawings

Project a sheet of isometric dot paper onto the board using the overhead projector. Show students how to draw a cube using the dots. Start from the top face, then draw the vertical edges (no hidden ones!), and then draw the visible bottom edges.

```
  Step 1   Step 2   Step 3
```

Explain that to create an isoparametric drawing, it helps to start from the top. Look at the topmost layer and draw the top face or faces first. Then draw the vertical edges that are part of the topmost layer as you did with the single cube.

Hold up a shape made with three cubes:

Invite a volunteer to draw the top layer (a single cube). What does the next layer look like? It consists of two cubes. Take two cubes locked together and compare this shape to the original shape: Which edges of the new shape are hidden (by the top cube) in the original shape? Which visible edges of the original shape are already drawn (because they are the bottom edges of the cube)? Ask a volunteer to draw the remaining visible edges of the second layer.

You may wish to do the worksheets as a class—so that volunteers draw pictures from the worksheet on the board. Students may find it easier to copy a shape onto isoparametric dot paper if they start by shading the top layer of the shape.

Give your students some interlocking cubes and ask them to build the figures from QUESTION 2 on the worksheet.
Building and Drawing Figures

**GOALS**
Students will build shapes from interlocking cubes and draw top views of shapes drawn on isometric dot paper.

**PRIOR KNOWLEDGE REQUIRED**
Understand a drawing on isometric dot paper

**VOCABULARY**
isometric dot paper
top view
interlocking cubes

Explain that sometimes it is hard to read the drawing on the isometric dot paper, because some cubes are hidden and some edges overlap. Project the drawing given here on the board as an example:

Explain that a very convenient way to understand the drawing is to try to construct the “mat plan” of the shape. A mat plan represents the bottom level of the shape. In this case the mat plan would look like this:

Shade one square on the mat plan, as shown. Invite a volunteer to shade the column that stands above this square in the isoparametric diagram. How many cubes are in it? (3) Ask the volunteer to write “3” in the shaded square. Ask more volunteers to finish the mat plan. Remove the drawing and ask another volunteer to construct the shape using the mat plan. Give your students more examples like those in QUESTION 2 of the worksheet.

**Assessment**
Draw the mat plan and build the shape from interlocking cubes:
Extension

Draw the figures from the worksheets on regular dot paper. For example:

![Figure example]

**G5-43**

**Geometry in the World**

The worksheet G5-43: *Geometry in the World* is an extension and review worksheet. It can complement the presentation of cross-curricular projects. Here are some project ideas:

**Symmetry:**

1. Flags and Coats of Arms of Canadian provinces/cities. Which ones have lines of symmetry? Which ones have more than one line of symmetry? (None!)
2. Flags of the world. Make a list of world countries with flags that have 2 lines of symmetry.
3. Coats of Arms of Soccer/Baseball/Hockey clubs. Which ones have lines of symmetry? Which ones have more than one line of symmetry?
4. Cultural diversity: Alphabets. Use a non-Latin alphabet to find letters that have lines of symmetry. Are there symbols that have more than one line of symmetry?
5. Make several designs of snowflakes. How many lines of symmetry do your snowflakes have?

**Geometry in everyday life:**

1. Bee hives and hexagons—Research why bees build hexagonal shapes in the hive: Why not rectangular or triangular shapes?
2. Bridges—Which geometric shapes are used in bridges design? Try to build a bridge with rectangles. Put a heavy weight on it and watch the bridge collapse. Triangles are rigid—you cannot change the shape of the triangle without altering the side lengths. If you add diagonals to the rectangles, the bridge will stand, but then it is built with triangles.)
3. Floor patterns: Make your own floor pattern. Choose a tessellation made by regular polygons using slides only (no rotations!) Cut the
tessellating polygon along a line of your choice, slide and re-glue the pieces, as in Activity 4 of G5-38. Create a picture based on the shape that you’ve got and create a floor pattern with that picture. **EXAMPLE:**

**STEP 1:** Take a tessellation with rhombuses:

STEP 2: Cut off and slide several times:

STEP 3: Add drawing to a shape:

STEP 4: Arrange copies of your shape in a floor pattern!

---

**G5-44**

**Problems and Puzzles**

The worksheet **G5-44: Problems and Puzzles** is a review worksheet and may be used for extra practice.
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3-D Shape Sorting Game

- Pyramid
- Cone
- Rectangular Prism
- Cube
- Triangular Prism
- Octahedron
3-D Shape Sorting Game (continued)

- Hexagonal Prism
- Triangular Pyramid
- Cylinder
- Octahedron
- Conical Frustum
- Octagonal Pyramid
3-D Shape Sorting Game (continued)
### 3-D Shape Sorting Game (continued)

<table>
<thead>
<tr>
<th>More than four faces</th>
<th>Square-shaped base</th>
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<tr>
<td>Triangular-shaped base</td>
<td>Fewer than six faces</td>
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<tr>
<td>Two or more square-shaped faces</td>
<td>Four or more triangular-shaped faces</td>
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<tr>
<td>Ten or more edges</td>
<td>Six or fewer vertices</td>
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<td>Four or more vertices</td>
<td>Exactly twelve edges</td>
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<td>Pyramids</td>
<td>Prisms</td>
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### 3-D Shape Sorting Game (continued)

<table>
<thead>
<tr>
<th>All faces congruent</th>
<th>No rectangular faces</th>
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<tbody>
<tr>
<td>Circular base</td>
<td>Exactly one pair of parallel faces</td>
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<tr>
<td>At least 3 pairs of parallel faces</td>
<td>No parallel edges</td>
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</tbody>
</table>
Build-a-Net Game
Build-a-Net Game (continued)
Grid Paper (1 cm)
Isometric Dot Paper
Nets for 3-D Shapes

Square Pyramid

Triangular Pyramid
Nets for 3-D Shapes (continued)
Nets for 3-D Shapes (continued)

- Pentagonal Pyramid
- Pentagonal Prism
Pattern Blocks

Triangles

Squares

Rhombuses

Trapezoids

Hexagons
Pentamino Pieces
Right Prisms

Right Prism with a Parallelogram Base
Right Prisms (continued)

Triangular Prism with a Scalene Base
Skew Prisms

Parallelepiped with Three Different Pairs of Parallelogram Faces
Skew Prisms (continued)

Skew Square Prism
Skew Prisms (continued)
Venn Diagram
Word Search Puzzle (3-D Shapes)

WORDS TO SEARCH:
- base
- edge
- face
- hexagonal
- net
- pentagonal
- prism
- pyramid
- rectangular
- skeleton
- triangular
- vertex
- vertices

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