Turning parallelograms into rectangles of the same area. Give each student a parallelogram drawn on grid paper. Ask students to cut a piece off the parallelogram and to reattach it so as to turn the parallelogram into a rectangle. Students may need to experiment to find the answer; provide extra copies of parallelograms as needed. If students need a hint, you can suggest cutting off a triangle as shown in the margin. Give everyone a chance to find the answer, and, if necessary, invite one or more students to explain it to the class. ASK: Are the sides of the rectangles the same length as the sides of the parallelogram? How many sides preserve their length after being rearranged? (2)

Introduce base and height. Explain that the sides of the parallelogram that have the same length as the sides of the rectangle students created are called the bases of the parallelogram. A base is something that an object can stand on. If we think of a parallelogram as standing on its base (hold one of the parallelograms used earlier vertically, as if it was standing on a desk on its base), we can ask how high the parallelogram is. Draw a height on the parallelogram you are using, and explain that the height of a parallelogram is always measured at a right angle to the base. Point out that “base” is also short for “the length of the base” and “height” refers to both the distance between the bases and the perpendicular it is measured along. The parallelogram students were using is drawn on a grid, and the base runs along the grid lines, so it is really easy to draw the height of the parallelogram. Give students another copy of the parallelogram and ask them to find the base and the height. Then have students find and record
the base and the height of the parallelograms below.

a)  

b)  

c)  

Ask students to copy the parallelograms above onto grid paper and draw where they would cut them so as to convert them to rectangles. Ask students to draw each piece they cut off in the position they would reattach it, creating pictures like the one in the margin.

**The area of a rectangle can be expressed as width \times height.** Remind students that the area of a rectangle has a formula, “length \times width.” Point out that it is sometimes hard to say which side is the “length” and which side is the “width.” Usually, the longer side is the length and the shorter side is the width. However, in real life we often think of width as distance across. For a rectangle such as the one at left, the distance across is 5, so is the width 4—the length of the shorter side—or is it 5—the distance across? Luckily, we can write the formula for the area of a rectangle as “width \times height” as well as “length \times width.” When we use the first version, there is no confusion about which side shows what. The width in this formula is 5, and the height is 4. Point out that in both cases you multiply the lengths of the adjacent sides (sides that share a vertex) to get the area of a rectangle.

**The formula for the area of a parallelogram.** ASK: Did the area of the parallelogram change when you cut and rearranged the pieces? (no) Remind students that the base of the parallelogram is the side that does not change when the parallelogram is converted into a rectangle. Have students write the height for each rectangle and compare it to the height and the base of the parallelogram. What is the height of each rectangle they produced equal to? (the height of the original parallelogram) SAY: To find the area of a rectangle, we multiply width by height. What should we multiply to find the area of a parallelogram? (base and height) Ask students to try to write a formula for the area of a parallelogram using the base and height. (Area = base \times height) Remind students that sometimes we use a shortcut for formulas. For example, we could use \(A\) for area, \(b\) for base, and \(h\) for height. In this case we would write the formula as \(A = b \times h\).

**Finding the area of parallelograms using a formula.** Remind students to write the units when multiplying base and height. This habit becomes very useful when students work with different units, as it helps to avoid multiplying lengths in incompatible units (such as meters and centimeters).

Demonstrate finding the area of a parallelogram using the formula. Example:

\[
\text{Area of parallelogram} = \text{base} \times \text{height} \\
\text{Base} = 5 \text{ cm} \\
\text{Height} = 4 \text{ cm} \\
\text{Area of parallelogram} = 5 \text{ cm} \times 4 \text{ cm} = 20 \text{ cm}^2
\]
Exercises: Find the area of each parallelogram.

a) base 3 cm, height 4 cm  
b) base 3 m, height 4.5 m

c) base 3.5 km, height 3 km  
d) base 3.4 cm, height 4.3 cm

e) base 4 ft, height 2.5 ft  
f) base 5.5 in, height 4.5 in

Answers: a) 12 cm², b) 13.5 m², c) 10.5 km², d) 14.62 cm², e) 10 ft², f) 24.75 in²

The area of a parallelogram is not defined by side lengths. Draw two identical rectangles using different colors. Mark the lengths of the sides and have students find the area of one of the rectangles. Ask: What is the area of the second rectangle? How do you know? (the rectangles have the same sides, so they have the same area)

Have on hand a large, flexible rectangle made from cardboard strips and paper fasteners. Ask: Do you think two parallelograms with the same side lengths have the same area? How could we find out? Take various answers, then hold up the rectangle and begin to deform it into a shape that has area close to zero.

Point out that you aren’t changing the length of the sides, you’re just moving them and creating different parallelograms. Ask: Have we answered the question? How? (Students should see, qualitatively if not quantitatively, that parallelograms with the same side lengths can have different areas.) Emphasize that the length of the sides does not define the area of a parallelogram the way it does the area of a rectangle.

Any side of a parallelogram could be a base. Draw a parallelogram on the board so that it does not have any vertical or horizontal sides. Explain that since this parallelogram is not on a grid, it does not matter which pair of opposite sides you chose as bases. Actually, you can choose any pair of opposite sides to be the bases. Pick one pair of sides and highlight it, saying that you want to use these sides as bases. How could you now find the height of the parallelogram?

Explain that if your parallelogram was made of paper, you could turn it so that your chosen base was horizontal and then measure the height vertically. However, you can’t turn the drawing on the board. What could you do instead? Take suggestions, then remind students that the height is perpendicular to the base, so you need to draw a line that is perpendicular to the base.

Constructing a perpendicular to a given line. Remind students how to use a set square to construct a perpendicular.
The distance between parallel lines. Ask students to look at the parallelograms on grid paper that they used at the beginning of the lesson. Ask them to draw the height in these parallelograms in more than one place. 

ASK: Is the height the same no matter where you draw it in a parallelogram? (yes) When you work on a grid, the height does not depend on where you measure it. Do you think the same will happen when you are working on blank paper?

Ask students to draw a pair of slant parallel lines by drawing lines along both sides of a ruler. Have students draw perpendiculars to one of the lines (so that they intersect with the other line) in three different places. Ask students to check the angle between the new lines and the other line, parallel to the first. Are these right angles? (yes) Explain that when a line intersects one of the parallel lines at a right angle, it will also intersect the other parallel line at a right angle. Then ask students to measure the distance between the lines along the three perpendiculars. Do students get the same answer? If not, have students check the angles between the lines (they should be right angles) and the measurements. Students can also exchange their pictures with a partner and check each other’s work. Point out to students that they have discovered an important fact—the distance between parallel lines (measured along a perpendicular) does not depend on where the perpendicular is drawn.

The height of a parallelogram can be measured anywhere. Return to the parallelogram already on the board (the one you used to show that any side can be a base). ASK: Does the height of a parallelogram depend on where it is measured? (no) Why or why not? (because the opposite sides are parallel lines and the height is the distance between the parallel lines, so it is the same everywhere)

Parallelograms with height falling outside the parallelogram. Have students copy the parallelograms below on grid paper. Explain that you want to find the area of these parallelograms. Is it possible to convert them to rectangles using the method from the beginning of the lesson? Why not? (the triangle that you cut off does not contain all of the slanted side of the parallelogram)

Finding the area of parallelograms in different ways. Ask students to draw a parallelogram and to find its area in two ways, using different sides as bases. Should the two methods give the same answer? Why? (Yes, because the area is the same no matter how you measure it.) Are the two answers the same? If not, what could cause the answers to be different? (imprecise measurements, a mistake in drawing the heights)
NOTE: In this section, students learned that the area of a parallelogram can be written using variables:

\[ A = b \times h \]

where \( b \) is the base and \( h \) is the height. They also learned the formula using words:

\[ \text{Area} = \text{base} \times \text{height} \]

In the following sections, students will learn formulas for the area of various shapes. We recommend you give students practice writing and applying these formulas using both words and variables.

Extensions

1. Cutting and rearranging parallelograms with height outside the parallelogram to find the area. ASK: Will cutting and rearranging a parallelogram change its area? (no) Use the two parallelograms in the examples from the end of the lesson for this Extension. Students can’t cut and rearrange these parallelograms the same way they did those at the beginning of the lesson, but invite them to try cutting and rearranging them a different way. The goal is to create a new parallelogram with at least one pair of parallel sides that align with the grid lines. Students should start by cutting each parallelogram out and tracing it (to preserve the original shape). They may want to cut out several copies (or you may want to provide multiple copies), so that they can test various solutions. They should find the heights and bases of the old and new parallelograms to calculate and compare their areas.

Sample solutions

2. On BLM Area of Parallelograms (Advanced) (p. G-76) students will find the area of parallelograms using non-horizontal sides as bases.

(MP3) 3. A parallelogram has height 80 cm and base 1.5 m. Tom says: The area is \( 80 \text{ cm} \times 1.5 \text{ m} = 120 \text{ cm}^2 \). Is he correct? Explain.

Sample answer: Tom is incorrect because he multiplied two numbers that represent different units. He should have converted one of the dimensions to have the same units as the other. For example, he could have converted the base into cm: \( 1.5 \text{ m} = 150 \text{ cm} \). The area is \( 80 \text{ cm} \times 150 \text{ cm} = 12,000 \text{ cm}^2 \).